Chapter 8
On Throughput Maximization Problem for UWB-Based Sensor Networks via Reformulation–Linearization Technique

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Abstract Nonlinear optimization problems (if not convex) are NP-hard in general. One effective approach to develop efficient solutions for these problems is to apply the branch-and-bound (BB) framework. A key step in BB is to obtain a tight linear relaxation for each nonlinear term. In this chapter, we show how to apply a powerful technique, called Reformulation–Linearization Technique (RLT), for this purpose. We consider a throughput maximization problem for an ultra-wideband (UWB)-based sensor network. Given a set of source sensor nodes in the network with each node generating a certain data rate, we want to determine whether or not it is possible to relay all these rates successfully to the base station. We formulate an optimization problem, with joint consideration of physical layer power control, link layer scheduling, and network layer routing. We show how to solve this nonlinear optimization problem by applying RLT and BB. We also use numerical results to demonstrate the efficacy of the proposed solution.

8.1 Introduction

In the first decade of the twenty-first century, there was a flourish of research and development efforts on UWB [17] for military and commercial applications. These applications include tactical handheld and network LPI/D radios, non-LOS LPI/D groundwave communications, precision geolocation systems, high-speed wireless

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LANs, collision avoidance sensors, and intelligent tags, among others. There are some significant benefits of UWB for wireless communications, such as extremely simple design (and thus cost) of radio, large processing gain in the presence of interference, extremely low power spectral density for covert operations, and fine time resolution for accurate position sensing [14, 17].

In this chapter, we consider a UWB-based sensor network for surveillance and monitoring applications. For these network applications, upon an event detection, all sensing data must be relayed to a central data collection point, which we call a base-station. The multi-hop nature of a sensor network introduces some unique challenges. Specifically, due to interference from neighboring links, a change of power level on one link will produce a change in achievable rate in all neighboring links. As a result, the capacity-based routing problem at the network layer is deeply coupled with link layer and physical layer problems such as scheduling and power control. An optimal solution to a network level problem thus must be pursued via a cross-layer approach for such networks.

In this chapter, we study the data collection problem associated with a UWB-based sensor network. For such a network, although the bit rate for each UWB-based sensor node could be high, the total rate that can be collected by the single base-station is limited due to the network resource bottleneck near the base-station as well as interference among the incoming data traffic. Therefore, a fundamental question is the following: *Given a set of source sensor nodes in the network with each node generating a certain data rate, is it possible to relay all these rates successfully to the base-station?*

A naive approach to this problem is to calculate the maximum bit rate that the base station can receive and then perform a simple comparison between this limit with the sum of bit rates produced by the set of source sensor nodes. Indeed, if this limit is exceeded, it is impossible to relay all these rates successfully to the base station. But even if the sum of bit rates generated by source sensor nodes is less than this limit, it may still be infeasible to relay all these rates successfully to the base station. Due to interference and the fact that a node cannot transmit and receive at the same time and in the same band, the actual sum of bit rates that can be relayed to the base station can be substantially smaller than the raw bit rate limit that a base station can receive. Further, such limit is highly dependent upon the network topology, locations of source sensor nodes, bit rates produced by source sensor nodes, and other network parameters. As a result, testing for this feasibility is not trivial and it is important to devise a solution procedure to address this problem.

In this chapter, we study this feasibility problem through a cross-layer optimization approach, with joint consideration of physical layer power control, link layer scheduling, and network layer routing. The link layer scheduling problem deals with how to allocate resources for access among the nodes. Motivated by the work in [10], we consider how to allocate frequency sub-bands, although this approach can also be applied to a time-slot based system. For a total available UWB spectrum of W, we divide it into M sub-bands. For a given M, the scheduling problem considers how to allocate bandwidth to each sub-band and in which sub-bands a node should transmit or receive data. Note that a node cannot transmit and receive within the same sub-band. The physical layer power control problem considers how much power a node should use to transmit data in a particular sub-band. Finally, the routing problem at the network layer considers which set of paths a flow should take from the source sensor node toward the base-station. For optimality, we allow a flow from a source node to be split into sub-flows that take different paths to the base-station.

We formulate this feasibility problem as an optimization problem, which turns out to be a mixed-integer non-polynomial program. To reduce the problem complexity, we modify the integrality and the non-polynomial components in the constraints by exploiting a reformulation technique and the linear property between the rate and SNR, which is unique to UWB. The resulting new optimization problem is then cast into a form of a *non-linear program* (NLP). Since an NLP is NP-hard in general, our specific NLP is likely to be NP-hard, although its formal proof is not given in this chapter. The contribution of this chapter is the development of an approximation solution procedure to this feasibility problem based on a branch-and-bound framework and the powerful Reformulation–Linearization Technique (RLT) [19].

The remainder of the chapter is organized as follows. In Sect. 8.2, we give details of the network model for our problem and discuss its inherent cross-layer nature. Section 8.3 presents a mathematical formulation of the cross-layer optimization problem and a solution procedure based on the branch-and-bound and RLT procedures. In Sect. 8.4, we present numerical results to demonstrate the efficacy of our proposed solution procedure and give insights on the impact of the different optimization components. Section 8.5 reviews related work and Sect. 8.6 concludes this chapter.

8.2 Network Model

We consider a UWB-based sensor network. Although the size of the network (in terms of the number of sensor nodes N) is potentially large, we expect the number of simultaneous source sensor nodes that produce sensing data to be limited. That is, we assume the number of simultaneous events that need to be reported in different part of the network is not large. Nevertheless, the number of nodes involved in relaying (routing) may still be significant due to the limited transmission range of a UWB-based sensor node and the coverage of the network.

Within such a sensor network, there is a base-station (or sink node) to which all collected data from source sensor nodes must be sent. For simplicity, we denote the base-station as node 0 in the network.

Under this network setting, we are interested in answering the following questions:

- Suppose we have a small group of nodes $\mathcal{S}$ that have detected certain events and each of these nodes is generating data. Can we determine if the bit rates from these source sensor nodes can be successfully sent to the base station (under the capacity limit)?

- If the answer "yes," how should we relay data from each source sensor node to the base station?

Before we further explore this problem, we give the following definition for the feasibility of a rate vector $\mathbf{r}$, where each element, r_i, of the vector corresponds to the rate of a continuous data flow produced by node $i \in \mathscr{S}$.

Definition 8.1. For a given rate vector $\mathbf{r}$ having $r_i > 0$ for $i \in \mathscr{S}$, we say that this rate vector is *feasible* if and only if there exists a solution such that all r_i, $i \in \mathscr{S}$, can be relayed to the base-station.

To determine whether or not a given rate vector $\mathbf{r}$ is feasible, there are several issues at different layers that must be considered. At the network layer, we need to find a multi-hop route (likely multi-paths) from a source node to the sink node. At the link and physical layers, we need to find a scheduling policy and power control for each node such that certain constraints are met satisfactorily. Clearly, this is a cross-layer problem that couples routing, scheduling, and power control. In the rest of this section, we will take a closer look at each problem. Table 8.1 lists notation used in this chapter.

8.2.1 Scheduling

At the link layer, our scheduling problem deals with how to allocate link media for access among the nodes. Motivated by Negi and Rajeswaran's work in [10], we consider how to allocate frequency sub-bands, although this approach can also be applied to time-slot based systems. For the total available UWB spectrum of $W = 7.5$ GHz (from 3.1 GHz to 10.6 GHz), we divide it into M sub-bands. Since the minimum bandwidth of a UWB sub-band is 500 MHz, we have $1 \le M \le 15$. For a given number of total sub-bands M, the scheduling problem considers how to allocate the total spectrum of W into M sub-bands and in which sub-bands a node should transmit or receive data. More formally, for a sub-band m with normalized bandwidth $\lambda^{(m)}$, we have

$$\sum_{m=1}^{M} \lambda^{(m)} = 1$$

and

$$\lambda_{\min} \le \lambda^{(m)} \le \lambda_{\max} \text{ for } 1 \le m \le M,$$

where $\lambda_{\min} = 1/15$ and $\lambda_{\max} = 1 - (M-1) \cdot \lambda_{\min}$.

Table 8.1 Notation

Symbol	Definition
b_{ij}^m	Achievable rate from node i to node j in sub-band m under transmission power p_{ij}^m
$\mathbf{b}$	The vector of b_{ij}^m, $1 \le i \le N, j \in \mathscr{N}_i, 1 \le m \le M$
b_{ij}	Total achievable rate from node i to node j in all sub-bands
f_{ij}	Flow rate from node i to node j
g_{ij}	Propagation gain from node i to node j
g_{jj}	Self-interference parameter at node j
g_{nom}	Propagation gain at a nominal distance
$\mathscr{I}_i$	The set of nodes that can produce interference on node i
K	The feasible scaling factor used in optimization problem formulation
$\mathscr{L}$	The problem list in the branch-and-bound procedure
LB_z	The lower bound of problem z in the branch-and-bound procedure
LB	The global lower bound among all problems in the branch-and-bound procedure
M	Total number of sub-bands for scheduling
N	Total number of sensor nodes in the network
$\mathscr{N}_i$	The set of one-hop neighboring nodes of node i
$p_{\max}$	$= W\pi_{\max}/g_{\text{nom}}$, the power limit
p_{ij}^m	Transmission power used by node i in sub-band m for transmitting data to node j
$\mathbf{p}$	The vector of p_{ij}^m, $1 \le i \le N, j \in \mathscr{N}_i, 1 \le m \le M$
q_j^m	Total power (signal and noise) received by node j in sub-band m
$\mathbf{q}$	The vector for q_j^m, $1 \le j \le N, 1 \le m \le M$
r_i	Bit rate generated at source sensor node $i \in \mathscr{S}$
$\mathscr{S}$	The set of source sensor nodes in the network
UB_z	The upper bound of problem z in the branch-and-bound procedure
UB	The global upper bound among all problems in the branch-and-bound procedure
W	$= 7.5$ GHz, the entire spectrum for UWB networks
$\lambda^{(m)}$	Normalized length of sub-band m, $\sum_{m=1}^{M} \lambda^{(m)} = 1$.
Λ	The vector of $\lambda^{(m)}$, $1 \le m \le M$
$\lambda_{\min}$	The minimum value of $\lambda^{(m)}$
$\lambda_{\max}$	The maximum value of $\lambda^{(m)}$
η	Power spectral density of ambient Gaussian noise
$\pi_{\max}$	Limit of power spectral density at a node

8.2.2 Power Control

The power control problem considers how much power a node should use in a particular sub-band to transmit data. Denote p_{ij}^m as the transmission power that node i uses in sub-band m for transmitting data to node j. Since a node cannot transmit and receive data within the same sub-band, we have the following constraint: if $p_{ik}^m > 0$ for any node k, then p_{ji}^m must be 0 for each node j.

The power density limit for each node i must satisfy

$$\frac{g_{\text{nom}} \cdot \sum_{j \in \mathscr{N}_i} p_{ij}^m}{W \cdot \lambda^{(m)}} \le \pi_{\max},$$

where $\pi_{\max}$ is the maximum allowed power spatial density, g_{nom} is the gain at some fixed nominal distance $d_{\text{nom}} \geq 1$, and $\mathcal{N}_i$ is the set of one-hop neighboring nodes of node i (under the maximum allowed transmission power). A popular model for gain is

$$g_{ij} = \min\{d_{ij}^{-n}, 1\}, \tag{8.1}$$

where d_{ij} is the distance between nodes i and j and n is the path loss index. Note that the nominal gain should also follow the same propagation gain model (8.1). Thus, we have $g_{ij} = (\frac{d_{\text{nom}}}{d_{ij}})^n g_{\text{nom}}$, when $d_{ij} \geq 1$. Denote

$$p_{\max} = \frac{W \cdot \pi_{\max}}{g_{\text{nom}}}. \tag{8.2}$$

Then the total power that a node i can use at sub-band m must satisfy the following power limit,

$$\sum_{j \in \mathcal{N}_i} p_{ij}^m \leq p_{\max} \lambda^{(m)}. \tag{8.3}$$

Denote $\mathcal{I}_i$ as the set of nodes that can make interference at node i when they use the maximum allowed transmission power. The achievable rate from node i to node j within sub-band m is then

$$b_{ij}^m = W\lambda^{(m)} \cdot \log_2 \left(1 + \frac{g_{ij} \cdot p_{ij}^m}{\eta W \lambda^{(m)} + \sum_{k \in \mathcal{I}_j, l \in \mathcal{N}_k}^{(k,l) \neq (i,j)} g_{kj} p_{kl}^m}\right), \tag{8.4}$$

where η is the ambient Gaussian noise density. Denote b_{ij} as the total achievable rate from node i to node j among all M sub-bands. We have

$$b_{ij} = \sum_{m=1}^{M} b_{ij}^m. \tag{8.5}$$

8.2.3 Routing

The routing problem at the network layer considers the set of paths that a flow takes from the source node toward the base station. For optimality, we allow a flow from a source node to be split into sub-flows and take different paths to the base station. Denote the flow rate from node i to node j as f_{ij}. We have

$$f_{ij} \leq b_{ij},$$
$$\sum_{j \in \mathcal{N}_i} f_{ij} - \sum_{j \in \mathcal{N}_i} f_{ji} = r_i.$$

The constraint $f_{ij} \leq b_{ij}$ says that a flow's bit rate is upper bounded by the achievable rate on this link and the second constraint is for flow balance at node i.

8.3 Feasibility and Solution Procedure

We can develop an upper bound on the maximum rate (denoted as C_0) that the base-station can receive [21]. For a given source rate vector $\mathbf{r}$, where $r_i > 0$ denotes that node i is a source sensor node that produces sensing data at rate r_i, if $\sum_{i=1}^N r_i > C_0$, then the rate vector must be infeasible. But $\sum_{i=1}^N r_i \leq C_0$ does not guarantee the feasibility of the rate vector $\mathbf{r}$ and further determination is needed. Moreover, if we indeed find that a given rate vector $\mathbf{r}$ is feasible, we also would like to obtain a complete solution that implements $\mathbf{r}$ over the network, i.e., a solution showing the power control, scheduling, and routing for each node.

8.3.1 Rate Feasibility Problem Formulation

Our approach to this feasibility determination problem is to solve an optimization (maximization) problem, which aims to find the optimal power control, scheduling, and routing such that K, called *feasible scaling factor*, is maximized while $K \cdot \mathbf{r}$ is feasible. If the optimal solution yields $K \geq 1$, then the rate vector $\mathbf{r}$ is feasible; otherwise (i.e., $K < 1$), the rate vector $\mathbf{r}$ is infeasible.

Since a node is not allowed to transmit and receive within the same sub-band, we have that if $p_{jl}^m > 0$ for any $l \in \mathcal{N}_j$ then p_{ij}^m should be 0 for all $i \in \mathcal{N}_j$. Mathematically, this property can be formulated as follows. Denote x_j^m $(1 \leq j \leq N$ and $1 \leq m \leq M)$ as a binary variable with the following definition: if sub-band m is used for receiving data at node j then $x_j^m = 1$; otherwise, $x_j^m = 0$. Since $\sum_{i \in \mathcal{N}_j} p_{ij}^m \leq |\mathcal{N}_j| p_{\max} \lambda^{(m)}$ and $\sum_{l \in \mathcal{N}_j} p_{jl}^m \leq p_{\max} \lambda^{(m)}$, we have the following constraints, which capture both the constraint that a node j cannot transmit and receive within the same sub-band m and the constraint on the power level.

$$\sum_{i \in \mathcal{N}_j} p_{ij}^m \leq |\mathcal{N}_j| \cdot p_{\max} \cdot \lambda^{(m)} \cdot x_j^m,$$
$$\sum_{l \in \mathcal{N}_j} p_{jl}^m \leq p_{\max} \cdot \lambda^{(m)} \cdot (1 - x_j^m).$$

The rate feasibility problem (RFP) can now be formulated as follows:

8.3.1.1 Rate Feasibility Problem

Maximize $\quad K$

subject to $\quad \sum_{m=1}^{M} \lambda^{(m)} = 1$

$$\sum_{j\in\mathscr{N}_i} p_{ij}^m - p_{\max}\lambda^{(m)} \le 0 \qquad (1 \le i \le N, 1 \le m \le M)$$

$$b_{ij}^m = W\lambda^{(m)}\log_2\left(1 + \frac{g_{ij}p_{ij}^m}{\eta W\lambda^{(m)} + \sum_{k\in\mathscr{I}_j, l\in\mathscr{N}_k}^{(k,l)\neq(i,j)} g_{kj}p_{kl}^m}\right)$$

$$(1 \le i \le N, j \in \mathscr{N}_i, 1 \le m \le M)$$

$$\sum_{i\in\mathscr{N}_j} p_{ij}^m \le |\mathscr{N}_j| p_{\max}\lambda^{(m)}x_j^m \qquad (1 \le j \le N, 1 \le m \le M) \tag{8.6}$$

$$\sum_{l\in\mathscr{N}_j} p_{jl}^m \le p_{\max}\lambda^{(m)}(1 - x_j^m) \qquad (1 \le j \le N, 1 \le m \le M) \tag{8.7}$$

$$\sum_{m=1}^{M} b_{ij}^m - f_{ij} \ge 0 \qquad (1 \le i \le N, j \in \mathscr{N}_i)$$

$$\sum_{j\in\mathscr{N}_i} f_{ij} - \sum_{j\in\mathscr{N}_i} f_{ji} - r_i K = 0 \qquad (1 \le i \le N)$$

$$\lambda_{\min} \le \lambda^{(m)} \le \lambda_{\max} \qquad (1 \le m \le M)$$

$$x_j^m = 0 \text{ or } 1 \qquad (1 \le j \le N, 1 \le m \le M)$$

$$K, p_{ij}^m, b_{ij}^m, f_{ij} \ge 0 \ (1 \le i \le N, j \in \mathscr{N}_i, 1 \le m \le M).$$

The formulation for problem RFP is a *mixed-integer non-polynomial program*, which is NP-hard in general [5]. We conjecture that the RFP problem is also NP-hard, although its formal proof is not given in this chapter. Our approach to this problem is as follows. As a first step, we show how to remove the integer (binary) variables and the non-polynomial terms in the RFP problem formulation and reformulate the RFP problem as a non-linear program (NLP). Since an NLP problem remains NP-hard in general, in Sect. 8.3.3, we devise a solution by exploring a branch-and-bound framework and the so-called *Reformulation–Linearization Technique* (RLT) [19].

8.3.2 Reformulation of Integer and Non-Polynomial Constraints

The purpose of integer (binary) variables x_j^m is to capture the fact that a node cannot transmit and receive within the same sub-band, i.e., if a node j transmits data to any node l in a sub-band m, then the data rate that can be received by node j within this sub-band must be 0. Instead of using integer (binary) variables, we use the following approach to achieve the same purpose. We introduce a notion called *self-interference parameter* g_{jj}, with the following property:

$$g_{jj}\cdot p_{jl}^m \gg \eta W\lambda^{(m)}.$$

We incorporate this into the bit rate calculation in (8.4), i.e.,

$$b_{ij}^m = W\lambda^{(m)}\cdot\log_2\left(1 + \frac{g_{ij}p_{ij}^m}{\eta W\lambda^{(m)} + \sum_{k\in\mathscr{I}_j, l\in\mathscr{N}_k}^{(k,l)\neq(i,j)} g_{kj}p_{kl}^m + \sum_{l\in\mathscr{N}_j} g_{jj}p_{jl}^m}\right). \tag{8.8}$$

Thus, when $p_{jl}^m > 0$, i.e., node j is transmitting to some node l, then in (8.8), we have $b_{ij}^m \approx 0$ even if $p_{ij}^m > 0$. In other words, when node j is transmitting to any node l, the achievable rate from node i to node j is *effectively* shut down to 0.

With this new notion of g_{jj}, we can capture the same tramission/receiving behavior of a node without the need of using integer (binary) variables x_j^m as in the RFP formulation. As a result, we can remove constraints (8.6) and (8.7).

To write (8.8) in a more compact form, we re-define $\mathscr{I}_j$ to include node j as long as j is not the base-station node (i.e., node 0). Thus, (8.8) is now in the same form as (8.4). Denote

$$q_j^m = \sum_{k\in\mathscr{I}_j, l\in\mathscr{N}_k} g_{kj}p_{kl}^m. \tag{8.9}$$

Then we have

$$b_{ij}^m = W\lambda^{(m)}\log_2\left(1 + \frac{g_{ij}p_{ij}^m}{\eta W\lambda^{(m)} + \sum_{k\in\mathscr{I}_j, l\in\mathscr{N}_k}^{(k,l)\neq(i,j)} g_{kj}p_{kl}^m}\right)$$

$$= W\lambda^{(m)}\log_2\left(1 + \frac{g_{ij}p_{ij}^m}{\eta W\lambda^{(m)} + q_j^m - g_{ij}p_{ij}^m}\right).$$

To remove the non-polynomial terms, we apply the low SNR property that is unique to UWB [16] and the linearity approximation of the log function, i.e., $\ln(1+x) \approx x$ for $x > 0$ and $x \ll 1$. We have

$$b_{ij}^m \approx \frac{W\lambda^{(m)}}{\ln 2}\cdot\frac{g_{ij}p_{ij}^m}{\eta W\lambda^{(m)} + q_j^m - g_{ij}p_{ij}^m},$$

which is equivalent to

$$\eta W\lambda^{(m)}b_{ij}^m + q_j^m b_{ij}^m - g_{ij}p_{ij}^m b_{ij}^m - \frac{W}{\ln 2}g_{ij}\lambda^{(m)}p_{ij}^m = 0.$$

Finally, without loss of generality, we let $\lambda^{(m)}$ conform the following property.

$$\lambda^{(1)} \le \lambda^{(2)} \le \cdots \le \lambda^{(M)}.$$

Although this additional constraint does not affect the optimal result, it will help speed up the computational time in our algorithm.

With the above re-formulations, we can now re-write the RFP problem as follows:

8.3.2.1 RFP-2

$$
\begin{aligned}
&\text{Maximize} && K \\
&\text{subject to} && \sum_{m=1}^{M} \lambda^{(m)} = 1 \\
& && \lambda^{(m)} - \lambda^{(m-1)} \geq 0 \qquad (2 \leq m \leq M) \\
& && \sum_{j \in \mathcal{N}_i} p_{ij}^m - p_{\max}\lambda^{(m)} \leq 0 \qquad (1 \leq i \leq N, 1 \leq m \leq M) \\
& && \sum_{k \in \mathcal{I}_j, l \in \mathcal{N}_k} g_{kj} p_{kl}^m - q_j^m = 0 \qquad (0 \leq j \leq N, 1 \leq m \leq M) \\
& && \eta W \lambda^{(m)} b_{ij}^m + q_j^m b_{ij}^m - g_{ij} p_{ij}^m b_{ij}^m - \frac{W}{\ln 2} g_{ij} \lambda^{(m)} p_{ij}^m = 0 \\
& && \qquad (1 \leq i \leq N, j \in \mathcal{N}_i, 1 \leq m \leq M) \qquad (8.10) \\
& && \sum_{m=1}^{M} b_{ij}^m - f_{ij} \geq 0 \qquad (1 \leq i \leq N, j \in \mathcal{N}_i) \\
& && \sum_{j \in \mathcal{N}_i} f_{ij} - \sum_{j \in \mathcal{N}_i} f_{ji} - r_i K = 0 \qquad (1 \leq i \leq N) \\
& && K, p_{ij}^m, b_{ij}^m, q_j^m, f_{ij} \geq 0 \; (1 \leq i \leq N, j \in \mathcal{N}_i, 1 \leq m \leq M) \\
& && \lambda^{(1)} \geq \lambda_{\min}, \; \lambda^{(M)} \leq \lambda_{\max}.
\end{aligned}
$$

Although problem RFP-2 is simpler than the original RFP problem, it is still a *non-linear program* (NLP), which remains NP-hard in general [5]. For certain NLP problems, it is possible to find a solution via its Lagrange dual problem. Specifically, if the objective function and constraint functions in the primal problem satisfy suitable *convexity* requirements, then the primal and dual problem have the same optimal objective value [2]. Unfortunately, such a duality-based approach, although attractive, is not applicable to our problem. This is because RFP-2 is a *non-convex* optimization problem (see (8.10)). There is likely a *duality gap* between the objective values of the optimal primal and dual solutions. As a result, a solution approach for the Lagrange dual problem cannot be used to solve our problem. Although there are efforts on solving non-convex optimization problem via a duality (see, e.g., [18] by Rubinov and Yang, where Lagrange-type dual problems are formulated with zero duality gap), we find that the complexity of such an approach is prohibitively high (much higher than the branch-and-bound solution approach proposed in this chapter).

In the next section, we develop a solution procedure based on the branch-and-bound framework [11] and the so-called *Reformulation–Linearization Technique* (RLT) [19, 20] to solve this NLP optimization problem.

8.3.3 A Solution Procedure

8.3.3.1 Branch-and-Bound

Using the branch-and-bound framework, we aim to provide a $(1-\varepsilon)$-optimal solution, where ε is a small pre-defined constant reflecting our tolerance for approximation in the final solution. Initially, we determine suitable intervals for each variable that appears in nonlinear terms. By using a *relaxation technique*, we then obtain an upper bound UB on the objective function value. Although the solution to such a relaxation usually yields infeasibility to the original NLP, we can apply a *local search algorithm* starting from this solution to find a feasible solution to the original NLP. This feasible solution now provides a lower bound LB on the objective function value.

If the distance between the above two bounds is small enough, i.e., $LB \geq (1-\varepsilon)UB$, then we are done with the $(1-\varepsilon)$-optimal solution obtained by the local search. Otherwise, we will use the branch-and-bound framework to find a $(1-\varepsilon)$-optimal solution. The branch-and-bound framework is based on the divide-and-conquer idea. That is, although the original problem is hard to solve, it may be easier to solve a problem with a smaller solution search space, e.g., if we can further limit $\lambda^{(1)} \leq 0.1$. So, we divide the original problem into sub-problems, each with a smaller solution search space. We solve the original problem by solving all these sub-problems. The branch-and-bound framework can remove certain sub-problems before solving them entirely and thus, can provide a solution much faster than a general divide-and-conquer approach.

During the branch-and-bound framework, we put all these sub-problems into a problem list $\mathcal{L}$. Initially, there is only Problem 1 in $\mathcal{L}$, which is the original problem. For each problem in the list, we can obtain an upper bound and a lower bound with a feasible solution, just as we did initially. Then, the global upper bound for all the problems in the list is $UB = \max_{z \in \mathcal{L}}\{UB_z\}$ and the global lower bound for all the problems in the list is $LB = \max_{z \in \mathcal{L}}\{\text{LB}_z\}$. We choose Problem z having the current worst (maximum) upper bound $UB_z = UB$ and then partition this problem into two new Problems z_1 and z_2 that replace Problem z. This partitioning is done by choosing a variable and partitioning the interval of this variable into two new intervals, e.g., $0 \leq \lambda^{(1)} \leq 0.2$ to $0 \leq \lambda^{(1)} \leq 0.1$ and $0.1 \leq \lambda^{(1)} \leq 0.2$. For each new problem created, we obtain an upper bound and a lower bound with a feasible solution. Then we can update both UB and LB.

Once $LB \geq (1-\varepsilon)UB$, the current feasible solution is $(1-\varepsilon)$-optimal and we are done. This is the termination criterion. Otherwise, for any Problem z', if we have $(1-\varepsilon)UB_{z'} < LB$, where UB'_z is the upper bound obtained for Problem z', then we can remove Problem z' from the problem list $\mathcal{L}$ for future consideration. The method then proceeds to the next iteration.

Note that since we are interested in determining whether or not K is greater than or equal to 1 (to check feasibility), we can terminate the branch-and-bound framework if any of the following two cases holds: (1) if the upper bound of K is smaller than 1, then RFP-2 is infeasible or (2) if we find any feasible solution with $K \geq 1$, then RFP-2 is feasible.

8.3.3.2 Relaxation with RLT Technique

Throughout the branch-and-bound framework (both initially and during each iteration), we need a relaxation technique to obtain an upper bound of the objective function. For this purpose, we apply a method based on *Reformulation–Linearization Technique* (RLT) [19, 20], which can provide a linear relaxation for a polynomial NLP problem. Specifically, in (8.10), RLT introduces new variables to replace the inherent polynomial terms and adds linear constraints for these new variables. These new RLT constraints are derived from the intervals of the original variables.

In particular, for nonlinear term $\lambda^{(m)}b_{ij}^m$ in (8.10), we introduce a new variable y_{ij}^m to replace $\lambda^{(m)}b_{ij}^m$. Since $\lambda^{(m)}$ and b_{ij}^m are each bounded by $(\lambda^{(m)})_L \leq \lambda^{(m)} \leq (\lambda^{(m)})_U$ and $(b_{ij}^m)_L \leq b_{ij}^m \leq (b_{ij}^m)_U$, respectively, we have $[\lambda^{(m)} - (\lambda^{(m)})_L] \cdot [b_{ij}^m - (b_{ij}^m)_L] \geq 0$, $[\lambda^{(m)} - (\lambda^{(m)})_L] \cdot [(b_{ij}^m)_U - b_{ij}^m] \geq 0$, $[(\lambda^{(m)})_U - \lambda^{(m)}] \cdot [b_{ij}^m - (b_{ij}^m)_L] \geq 0$, and $[(\lambda^{(m)})_U - \lambda^{(m)}] \cdot [(b_{ij}^m)_U - b_{ij}^m] \geq 0$. From the above relationships and substituting $y_{ij}^m = \lambda^{(m)}b_{ij}^m$, we have the following RLT constraints for y_{ij}^m.

$$
\begin{aligned}
&(\lambda^{(m)})_L \cdot b_{ij}^m + (b_{ij}^m)_L \cdot \lambda^{(m)} - y_{ij}^m \leq (\lambda^{(m)})_L \cdot (b_{ij}^m)_L \\
&(\lambda^{(m)})_U \cdot b_{ij}^m + (b_{ij}^m)_L \cdot \lambda^{(m)} - y_{ij}^m \geq (\lambda^{(m)})_U \cdot (b_{ij}^m)_L \\
&(\lambda^{(m)})_L \cdot b_{ij}^m + (b_{ij}^m)_U \cdot \lambda^{(m)} - y_{ij}^m \geq (\lambda^{(m)})_L \cdot (b_{ij}^m)_U \\
&(\lambda^{(m)})_U \cdot b_{ij}^m + (b_{ij}^m)_U \cdot \lambda^{(m)} - y_{ij}^m \leq (\lambda^{(m)})_U \cdot (b_{ij}^m)_U .
\end{aligned}
$$

We, therefore, replace $\lambda^{(m)}b_{ij}^m$ with y_{ij}^m in (8.10) and add the above RLT constraints for y_{ij}^m into the RFP-2 problem formulation. Similarly, we let $u_{ij}^m = q_j^m b_{ij}^m$, $v_{ij}^m = p_{ij}^m b_{ij}^m$, and $w_{ij}^m = \lambda^{(m)} p_{ij}^m$. From $(p_{ij}^m)_L \leq p_{ij}^m \leq (p_{ij}^m)_U$ and $(q_j^m)_L \leq q_j^m \leq (q_j^m)_U$, we can obtain the RLT constraints for u_{ij}^m, v_{ij}^m, and w_{ij}^m as well.

Denote Λ, $\mathbf{p}$, $\mathbf{b}$, and $\mathbf{q}$ as vectors for $\lambda^{(m)}$, p_{ij}^m, b_{ij}^m, and q_j^m, respectively. After we replace all non-linear terms as above and add the corresponding RLT constraints into the RFP-2 problem formulation, we obtain the following LP.

$$
\begin{aligned}
\text{Maximize} \quad & K \\
\text{subject to} \quad & \textstyle\sum_{m=1}^{M} \lambda^{(m)} = 1 \\
& \lambda^{(m)} - \lambda^{(m-1)} \geq 0 \quad (2 \leq m \leq M) \\
& \textstyle\sum_{j \in \mathcal{N}_i} p_{ij}^m - p_{\max}\lambda^{(m)} \leq 0 \ (1 \leq i \leq N, 1 \leq m \leq M) \\
& \textstyle\sum_{k \in \mathcal{I}_j, l \in \mathcal{N}_k} g_{kj} p_{kl}^m - q_j^m = 0 \quad (0 \leq j \leq N, 1 \leq m \leq M) \\
& \eta W y_{ij}^m + u_{ij}^m - g_{ij} v_{ij}^m - \tfrac{W}{\ln 2} g_{ij} w_{ij}^m = 0 \quad (1 \leq i \leq N, j \in \mathcal{N}_i, 1 \leq m \leq M) \\
& \text{RLT constraints for } y_{ij}^m, u_{ij}^m, v_{ij}^m, \text{ and } w_{ij}^m \ (1 \leq i \leq N, j \in \mathcal{N}_i, 1 \leq m \leq M) \\
& \textstyle\sum_{m=1}^{M} b_{ij}^m - f_{ij} \geq 0 \quad (1 \leq i \leq N, j \in \mathcal{N}_i) \\
& \textstyle\sum_{j \in \mathcal{N}_i} f_{ij} - \sum_{j \in \mathcal{N}_i} f_{ji} - r_i K = 0 \ (1 \leq i \leq N) \\
& K, f_{ij}, y_{ij}^m, u_{ij}^m, v_{ij}^m, w_{ij}^m \geq 0 \quad (1 \leq i \leq N, j \in \mathcal{N}_i, 1 \leq m \leq M) \\
& (\Lambda, \mathbf{p}, \mathbf{b}, \mathbf{q}) \in \Omega,
\end{aligned}
$$

where $\Omega = \{(\Lambda, \mathbf{p}, \mathbf{b}, \mathbf{q}) : (\lambda^{(m)})_L \leq \lambda^{(m)} \leq (\lambda^{(m)})_U, (p_{ij}^m)_L \leq p_{ij}^m \leq (p_{ij}^m)_U, (b_{ij}^m)_L \leq b_{ij}^m \leq (b_{ij}^m)_U, (q_j^m)_L \leq q_j^m \leq (q_j^m)_U\}$.

The details of the proposed branch-and-bound solution procedure with RLT are given in Fig. 8.1. Note that in Step 14 of the Feasibility Check Algorithm, the method chooses a partitioning variable based on the maximum relaxation error. Clearly, $\lambda^{(m)}$ is a key variable in the problem formulation. As a result, the algorithm will run much more efficiently if we give the highest priority to $\lambda^{(m)}$ when it comes to choosing a partitioning variable.[1]

8.3.3.3 Local Search Algorithm

In the branch-and-bound framework, we need to find a solution to the original problem from the solution to the relaxation problem (see Step 9 in Fig. 8.1). In particular, we need to obtain a feasible solution from $\hat{\Lambda}$ and $\hat{\mathbf{p}}$. We now show how to obtain such a feasible solution.

We can let $\Lambda = \hat{\Lambda}$. Note that in RFP-2, we introduced the notion of a self-interference parameter to remove the binary variables in RFP. Then in $\hat{\mathbf{p}}$, it is possible that $p_{il}^m > 0$ and $p_{ji}^m > 0$ for a certain node i within some sub-band m. Therefore, it is necessary to find a new $\mathbf{p}$ from $\hat{\mathbf{p}}$ such that no node is allowed to transmit and receive within the same sub-band. An algorithm that achieves this purpose is shown in Fig. 8.2. The basic idea is to split the total bandwidth used at node i into two groups of equal bandwidth: one group for transmission and the other group for receiving.

After we obtain Λ and $\mathbf{p}$ by the algorithm in Fig. 8.2, constraints on sub-band division, scheduling, and power control are all satisfied. Now we can compute b_{ij} from (8.4) and (8.5). Then, we solve the following simple LP for K.

[1] In our implementation of the algorithm, we give the highest priority to $\lambda^{(m)}$, the second highest priority to p_{ij}^m, and consider q_j^m last when we choose a partitioning variable. This does not hamper the convergence property of the algorithm [19].

Feasibility Check Algorithm

1. Initialize()
{
2. Let the lower bound $LB = -\infty$ and the initial problem list $\mathscr{L}$ include only the original problem, denoted as Problem 1.
3. The initial value sets for $(\Lambda, \mathbf{p}, \mathbf{b}, \mathbf{q})$ are $\lambda_{min} \le \lambda^{(m)} \le \lambda_{max}, 0 \le p_{ij}^m \le p_{max} \cdot \lambda_{max}$, $0 \le b_{ij}^m \le \frac{g_{ij}}{\eta \ln 2} p_{max} \lambda_{max}$, and $0 \le q_j^m \le p_{max} \lambda_{max} \sum_{k \in \mathscr{I}_j} g_{kj}$.
4. Solve the RLT relaxation for Problem 1 and obtain its solution $(\hat{\Lambda}, \hat{\mathbf{p}}, \hat{\mathbf{b}}, \hat{\mathbf{q}})$.
5. The objective value of the solution is an upper bound UB_1 to problem 1.
}
6. MainIteration()
{
7. Select Problem z that has the maximum UB_z among all problems in list $\mathscr{L}$.
8. Update the global upper bound $UB = UB_z$.
9. Find a feasible solution ψ_z from $\hat{\Lambda}$ and $\hat{\mathbf{p}}$ via a local search algorithm and denote its objective value as LB_z.
10. If $(LB_z > LB)$ {
11. Update $\psi^* = \psi$ and $LB = LB_z$.
12. If $(LB \ge (1-\varepsilon)UB)$, we stop with the $(1-\varepsilon)$-optimal solution ψ^*.
13. Otherwise, remove each Problem z' with $(1-\varepsilon)UB_{z'} \le LB$ from list $\mathscr{L}$. }
14. Find the maximum relaxation error among $|\hat{\lambda}^{(m)} \hat{b}_{ij}^m - \hat{y}_{ij}^m|, |\hat{q}_j^m \hat{b}_{ij}^m - \hat{u}_{ij}^m|$, $|\hat{p}_{ij}^m \hat{b}_{ij}^m - \hat{v}_{ij}^m|$, and $|\hat{\lambda}^{(m)} \hat{p}_{ij}^m - \hat{w}_{ij}^m|$, for $1 \le i \le N, j \in \mathscr{N}_i, 1 \le m \le M$.
15. In the case that the maximum relaxation error is $|\hat{\lambda}^{(m)} \hat{b}_{ij}^m - \hat{y}_{ij}^m|$,
16. if $[(\lambda^{(m)})_U - (\lambda^{(m)})_L] \cdot \min\{\hat{\lambda}^{(m)} - (\lambda^{(m)})_L, (\lambda^{(m)})_U - \hat{\lambda}^{(m)}\} \ge [(b_{ij}^m)_U - (b_{ij}^m)_L] \cdot \min\{\hat{b}_{ij}^m - (b_{ij}^m)_L, (b_{ij}^m)_U - \hat{b}_{ij}^m\}$, we partition Ω_z into two new regions Ω_{z1} and Ω_{z2} by dividing $[(\lambda^{(m)})_L, (\lambda^{(m)})_U]$ into $[(\lambda^{(m)})_L, \hat{\lambda}^{(m)}]$ and $[\hat{\lambda}^{(m)}, (\lambda^{(m)})_U]$.
17. Otherwise, we partition Ω_z into two new regions by dividing $[(b_{ij}^m)_L, (b_{ij}^m)_U]$ into $[(b_{ij}^m)_L, \hat{b}_{ij}^m]$ and $[\hat{b}_{ij}^m, (b_{ij}^m)_U]$.
18. Similarly, we can perform a corresponding partition if the maximum relaxation error is $|\hat{q}_j^m \hat{b}_{ij}^m - \hat{u}_{ij}^m|$, $|\hat{p}_{ij}^m \hat{b}_{ij}^m - \hat{v}_{ij}^m|$, or $|\hat{\lambda}^{(m)} \hat{p}_{ij}^m - \hat{w}_{ij}^m|$.
19. Solve the RLT relaxation for Problems z_1 and z_2 and obtain their upper bounds UB_{z1} and UB_{z2}.
20. Remove Problem z from the problem list $\mathscr{L}$.
21. If $(1-\varepsilon)UB_{z1} > LB$, add Problem z_1 into the problem list $\mathscr{L}$.
22. If $(1-\varepsilon)UB_{z2} > LB$, add Problem z_2 into the problem list $\mathscr{L}$.
23. If $\mathscr{L} = \emptyset$, stop with the $(1-\varepsilon)$-optimal solution ψ^*.
24. Otherwise, go to Step 7 for the next iteration.
}

Fig. 8.1 A solution procedure to the RFP-2 problem based on branch-and-bound and RLT

$$
\begin{array}{lll}
\text{Maximize} & K & \\
\text{subject to} & f_{ij} \le b_{ij} & (1 \le i \le N, j \in \mathscr{N}_i) \\
& \sum_{j \in \mathscr{N}_i} f_{ij} - \sum_{j \in \mathscr{N}_i} f_{ji} - r_i K = 0 & (1 \le i \le N) \\
& K, f_{ij} \ge 0 & (1 \le i \le N, j \in \mathscr{N}_i).
\end{array}
$$

If an LP solution provides a $K \ge 1$, then this rate vector $\mathbf{r}$ is feasible.

Power Update Algorithm

1. Choose a node i that meets one of the following requirements. If no such node exists, we are done and the updated power vector is $\mathbf{p}$.
2. First, identify a node such that all nodes that receive data from this node already have their transmission power updated.
3. If no such node exists, choose the node among all nodes that do not have their transmission power updated and is closest to the base-station.
4. If there is no sub-band used for both transmission and receiving at node i under $\hat{\mathbf{p}}$, then do not update its power and go to Step 1.
5. Otherwise, define Γ_{out} as the total bandwidth used by node i for transmitting data and Γ_{in-out} as the total bandwidth used by node i only for receiving data.
6. If $\Gamma_{out} > \Gamma_{in-out}$, node i tries to release some sub-bands used for both transmission and receiving under $\hat{\mathbf{p}}$ and reduce the total used bandwidth to $(\Gamma_{out} + \Gamma_{in-out})/2$ in the following order.
7. First it releases sub-band m with $\sum_{j \in \mathscr{N}_i} (p_{ij}^m)_L = 0$, in non-decreasing order of $\sum_{j \in \mathscr{N}_i} p_{ij}^m$.
8. Second it reduces the transmission power in sub-band m, i.e., from p_{ij}^m to $(p_{ij}^m)_L$, in non-decreasing order of $\sum_{j \in \mathscr{N}_i} (p_{ij}^m)_L$.
9. If node i decides to use a sub-band, then all nodes should not use this sub-band to transmit data to node i.
10. Go to Step 1.

Fig. 8.2 An algorithm to obtain $\mathbf{p}$ from $\hat{\mathbf{p}}$

8.4 Numerical Results

8.4.1 Simulation Setting

In this section, we present numerical results for our solution procedure and compare it to other possible approaches. Given that the total UWB spectrum is $W = 7.5$ GHz and that each sub-band is at least 500 MHz, we have that the maximum number of sub-bands is $M = 15$. The gain model for a link (i, j) is $g_{ij} = \min(d_{ij}^{-2}, 1)$ and the nominal gain is chosen as $g_{\text{nom}} = 0.02$. The power density limit π_{max} is assumed to be 1% of the white noise η [16].

We consider a randomly generated network of 100 nodes (see Fig. 8.3) over a 50×50 area, where the distance is based on normalized length in (8.1). The base-station is located at the origin. The details for this network will be elaborated shortly when we present the results.

We investigate the impact of scheduling and routing. We are interested in comparing a cross-layer approach to a decoupled approach to our problem.

8.4.2 Impact of Scheduling

For the 100-node network shown in Fig. 8.3, there are eight source sensor nodes (marked as stars) in the network. The randomly generated data rate are $r_1 = 5$,

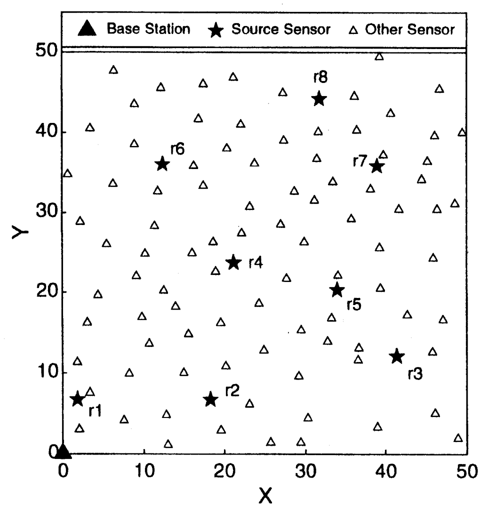

Fig. 8.3 Network topology for a 100-node network

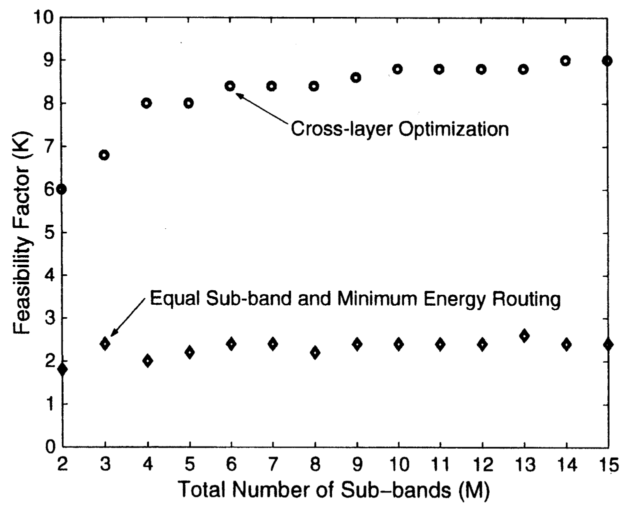

Fig. 8.4 The maximum achievable K as a function of M for the 100-node network

$r_2 = 2$, $r_3 = 2$, $r_4 = 4$, $r_5 = 5$, $r_6 = 3$, $r_7 = 3$, and $r_8 = 1$, with units defined in an appropriate manner. To show performance limits, we consider whether the network can transmit $K \cdot r_i$ from source sensor node i to the base station and investigate the maximum K (feasible scaling factor) under different approaches. Figure 8.4 (upper curve) shows the maximum achievable K for different M under our solution procedure. Clearly, K is a non-decreasing function of M, which states

Table 8.2 Performance of feasible scaling factor K under different spectrum allocations with $M = 5$ for the 100-node network

Spectrum allocation	K	Rate
Optimal: (0.4256, 0.2339, 0.1660 0.1066 0.0679)	8.0	200
Equal: (0.20, 0.20, 0.20, 0.20, 0.20)	4.2	105
Random 1: (0.36, 0.23, 0.20, 0.11, 0.10)	2.8	70
Random 2: (0.27, 0.24, 0.21, 0.17, 0.11)	4.2	105

that the more sub-bands available, the larger traffic volume that the network can support. The physical explanation for this is that the more sub-bands available, the more opportunity for each node to avoid interference from other nodes within the same sub-band, and thus more throughput in the network. Also, note that there is a noticeable increase in K when M is small. But when $M \geq 4$, the increase in K is no longer significant. This suggests that for simplicity, we could just choose a small value (e.g., $M = 5$) for the number of sub-bands instead of the maximum $M = 15$.

To show the importance of joint optimization of physical layer power control, link layer scheduling, and network layer routing, in Fig. 8.4, we also plot K as a function of M for a pre-defined routing strategy, namely, the minimum-energy routing with equal sub-band scheduling. Here, the energy cost is defined as g_{ij}^{-1} for link (i, j). Under this approach, we find a minimum-energy path for each source sensor node and determine which sub-band to use for each link and with how much power. When a node cannot find a feasible solution to transmit data to the next hop, it declares that the given rate vector is infeasible. In Fig. 8.4, we find that the minimum-energy routing with equal sub-band scheduling approach is significantly inferior than the proposed cross-layer optimization approach.

Table 8.2 shows the results for K under different spectrum allocations for $M = 5$. The routes are the same as those obtained under optimal routing from our cross-layer optimal solution (see Fig. 8.5 (a)) and are fixed in this study. The first optimal spectrum allocation is obtained from the cross-layer optimal solution. The second is an equal spectrum allocation and the following two are random spectrum allocations. Clearly, the cross-layer optimal spectrum allocation provides the best performance among all these spectrum allocations. It is important to realize that in addition to the number of sub-bands M, the way how the spectrum is allocated for a given M also has a profound impact on the performance. In Table 8.3, we perform the same study for $M = 10$ and obtain the same conclusion.

8.4.3 *Impact of Routing*

We study the impact of routing on our cross-layer optimization problem under a given optimal schedule (obtained through our solution procedure). Table 8.4 shows the results in this study. In addition to our cross-layer optimal routing, we also

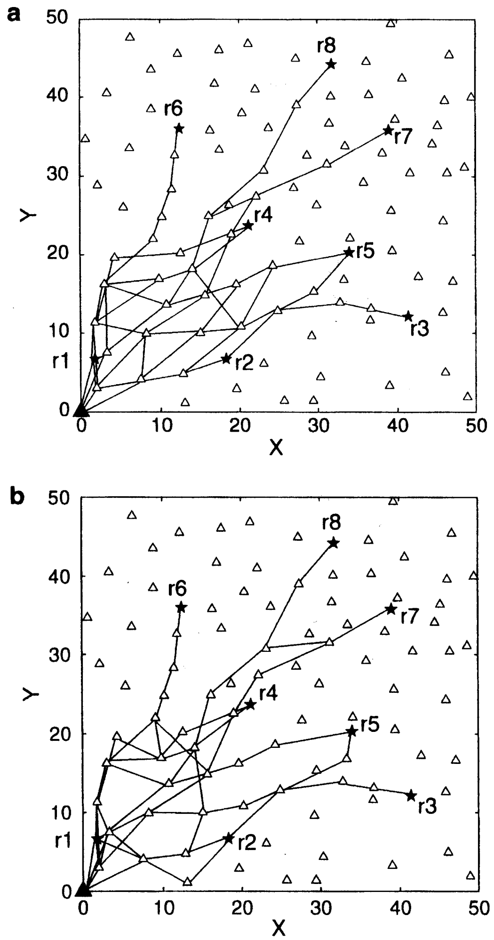

Fig. 8.5 Optimal routing for the 100-node network. (a) $M = 5$, (b) $M = 10$

consider the following two routing approaches, namely, minimum-energy routing and minimum-hop routing. The minimum-hop routing is similar to the minimum-energy routing, except the cost here is measured in the number of hops.

In Table 8.4, the spectrum allocation is chosen as the optimal spectrum allocation from our cross-layer optimal solution (see Tables 8.2 and 8.3) and is fixed. Clearly, the cross-layer optimal routing outperforms both minimum-energy and minimum-hop routing approaches. Both minimum-energy routing and minimum-hop routing are minimum-cost routing (with different link cost). Minimum-cost routing only uses a single-path, i.e., multi-path routing is not allowed, which is not likely to provide an optimal solution. Moreover, it is very likely that multiple source sensors share a "good" path. Thus, the rates for these sensors are bounded by the achievable

Table 8.3 Performance of feasible scaling factor K under different spectrum allocations with $M = 10$ for the 100-node network

Spectrum allocation	K	Rate
Optimal: (0.1551, 0.1365, 0.1283, 0.0962, 0.0952, 0.0916, 0.0901, 0.0702, 0.0689, 0.0679)	8.8	220
Equal: (0.10, 0.10, 0.10, 0.10, 0.10, 0.10, 0.10, 0.10, 0.10, 0.10)	3.6	90
Random 1: (0.14, 0.13, 0.12, 0.11, 0.09, 0.09, 0.09, 0.08, 0.08, 0.07)	4.0	100
Random 2: (0.17, 0.13, 0.11, 0.10, 0.10, 0.09, 0.08, 0.08, 0.07, 0.07)	3.8	95

Table 8.4 Performance of feasible scaling factor K under different routing strategies for the 100-node network

	$M = 5$		$M = 10$	
Routing Strategy	K	Rate	K	Rate
Optimal Routing	8.0	200	8.8	220
Minimum-Energy Routing	2.2	55	2.4	60
Minimum-Hop Routing	1.4	35	2.0	50

rate of this path. Further, minimum-hop routing has its own problem. Minimum-hop routing prefers small number of hops (with a long distance on each hop) toward the destination node. Clearly, a long-distance hop will reduce its corresponding link's achievable rate, due to the distance gain factor.

8.5 Related Work

A good overview paper on UWB is given in [14]. Physical layer issues associated with UWB-based multiple access communications can be found in [3, 7, 8, 12, 22] and references therein. In this section, we focus on related work addressing networking problems with UWB.

In [9], Negi and Rajeswaran first showed that, in contrast to previously published results, the throughput for UWB-based ad hoc networks increases with node density. This important result is mainly due to the large bandwidth and the ability of power and rate adaptation of UWB-based nodes, which alleviate interference. More importantly, this result demonstrates the significance of physical layer properties on network layer metrics such as network capacity. In [1], Baldi et al. considered the admission control problem based on a flexible cost function in UWB-based networks. Under their approach, a communication cost is attached to each path and the cost of a path is the sum of costs associated with the links it comprises. An admissibility test is then made based on the cost of a path. However, there is no explicit consideration of joint cross-layer optimization of power control, scheduling, and routing in this admissibility test. In [4], Cuomo et al. studied a multiple access scheme for UWB. Power control and rate allocation problems were formulated for both elastic bandwidth data traffic and guaranteed-service traffic. The impact of routing, however, was not addressed.

The most closely related research to our work are [10] and [15]. In [10], Negi and Rajeswaran studied how to maximize proportional rate allocation in a single-hop UWB network (each node can communicate to any other node in a single hop). The problem was formulated as a cross-layer optimization problem with similar scheduling and power control constraints as in this chapter. In contrast, our focus in this chapter is on a feasibility test for a rate vector in a sensor network and we consider a multi-hop network environment where routing is also part of the cross-layer optimization problem. As a result, the problem in this chapter is more difficult. In [15], Radunovic and Le Boudec studied how to maximize the total log-utility of flow rates in multi-hop ad hoc networks. The cross-layer optimization space consists of scheduling, power control, and routing. As the optimization problem is NP-hard, the authors then studied a simple ring network as well as a small-sized network with pre-defined scheduling and routing policies. On the other hand, in this chapter, we have developed a novel solution procedure to our cross-layer optimization problem.

8.6 Conclusion

In this chapter, we studied the important problem of routing data traffic in a UWB-based sensor network. We followed a cross-layer optimization approach with joint consideration of physical layer power control, link layer scheduling, and network layer routing. We developed a solution procedure based on the branch-and-bound framework and the RLT technique. Our numerical results demonstrated the efficacy of our proposed solution procedure and substantiated the importance of cross-layer optimization for UWB-based sensor networks.

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