MIMO-Empowered Secondary Networks for Efficient Spectrum Sharing

30

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W. Zhang (ed.), *Handbook of Cognitive Radio*,
https://doi.org/10.1007/978-981-10-1394-2_29

Abstract

Cognitive radio (CR) and multiple-input multiple-output (MIMO) are two independent physical layer technologies that have made significant impact on wireless networks. In particular, CR operates on the channel level to exploit efficiency across spectrum dimension, while MIMO operates within the same channel to exploit efficiency across spatial dimension. In this chapter, we explore MIMO-empowered CR technique to enhance spectrum access in wireless networks. Specially, we study how to apply MIMO-empowered CR for both interweave and underlay paradigms in multi-hop network environment. With MIMO interference cancelation (IC) capability, we first show how multiple secondary links achieve simultaneous transmission on the same channel under the interweave paradigm. Next, we show how secondary networks achieve simultaneously transmission with the primary network on same channel to achieve transparent coexistence under the underlay paradigm. Through rigorous mathematical modeling, problem formulation, and extensive simulation results, we find that MIMO-empowered CR can offer significant improvement in terms of spectrum access and throughput performance under both interweave and underlay paradigms.

Keywords

Cognitive radio · MIMO · Interference cancelation · Spectrum sharing · Coexistence · Interweave · Underlay · Multi-hop network

Introduction

Since its inception, cognitive radio (CR) has quickly been accepted as the enabling radiotechnology for next-generation wireless communications [8, 22]. A CR promises unprecedented flexibility in radio functionalities via programmability at the lowest layer, which was once done in hardware. Due to its spectrum sensing, learning, and adaptation capabilities, CR has the potential to address the heart of the problem associated with spectrum scarcity (via dynamic spectrum access (DSA)) and interoperability (via channel switching). Already, CR or its predecessor, software-defined radio (SDR), has been implemented for cellular communications [20], the military [10], and public safety communications [13]. It is envisioned that CR will be employed as a general radio platform upon which numerous wireless applications can be implemented.

In parallel to the development of CR, MIMO [2, 19] has already been widely implemented in commercial wireless products to increase capacity. The goal of

MIMO and how it operates are largely independent of and orthogonal to CR. Instead of exploiting idle channels for wireless communications, MIMO attempts to increase capacity within the same channel via space-time processing [6]. In particular, by employing multiple antennas at both the transmit and receive nodes, wireless channel capacity can scale almost linearly with the number of antennas (via spatial multiplexing) [4, 18]. Further, with zero-forcing beamforming (ZFBF) [3, 21], a node may use its degrees of freedom (DoFs) to cancel interference from other nodes or its own interference to other nodes.

In this chapter, we explore MIMO in CR-based secondary networks in both interweave (i.e., interference avoidance or DSA) and underlay paradigms [7] to enhance spectrum efficiency and spatial reuse. In interweave paradigm, to avoid interference to primary network, the secondary networks can only operate on spectrum holes. However, with MIMO IC capability, secondary nodes are allowed to be active simultaneously on the same band in the secondary network. If we assume that each node in a cognitive radio network (CRN) is equipped with A_{MIMO} antennas, then one would expect at least A_{MIMO}-fold capacity increase when compared to a CRN with only a single antenna at each node, due to spatial multiplexing gain from MIMO. Now observing that CR and MIMO handle interference differently (with CR on the channel level and MIMO within a channel), we ask the following fundamental question: *Will joint optimization of CR (via channel assignment) and MIMO (via DoF allocation) offer more than A_{MIMO}-fold in capacity?*

In underlay paradigm, we explore the potential of *simultaneous activation* of a secondary network with the primary network, as long as the interference produced by the secondary nodes can be properly "controlled" (e.g., canceled) by secondary nodes. In other words, secondary nodes are allowed to access the spectrum as long as they can cancel their interference to the primary nodes in such a way that primary nodes do not feel the presence of secondary nodes. In other words, activities by the secondary nodes are made transparent (or "invisible") to the primary nodes. We call this *transparent coexistence*. Although the idea of the transparent coexistence has been explored in the information theory (IT) community, results from the IT and communication (COMM) communities have mainly limited to very simple network settings, e.g., several nodes or link pairs, all for *single-hop* communications [1, 5, 11, 23, 24]. The more interesting problem of how transparent coexistence can be achieved in a *multi-hop* secondary network remains open. As shown in [9, 14], the problem complexity associated with multi-hop CR networks is much higher than single-hop CR networks.

The remainder of this chapter is organized as follows. In sections "MIMO-Based Secondary Network in Interweave Paradigm" and "MIMO-Based Secondary Network in Transparent-Coexistence Paradigm", we explore MIMO-empowered CR for a multi-hop secondary network under the interweave and underlay paradigms, respectively. Through case studies, we demonstrate that MIMO-empowered secondary networks can significantly improve spectrum efficiency and spatial reuse under both interweave and underlay paradigms. Section "Summary and Future Directions" summarizes this chapter.

MIMO-Based Secondary Network in Interweave Paradigm

In this section, we study MIMO-based secondary multi-hop network in interweave paradigm. Our discussion consists of two levels: The first is on channel level, i.e., how does a secondary network exploit available spectrum and handle interference via the use of different channels. The second is within a channel, i.e., how does MIMO mitigate co-channel interference via ZFBF (i.e., using DoF). A thorough understanding of these interference avoidance/cancelation techniques across/within channels is critical to mathematical modeling and ultimately fully exploit the potential of MIMO and CR. Based on this background, we then develop a rigorous mathematical model and study a throughput maximization problem to exploit the potential benefit of MIMO-based secondary network.

Co-channel Interference Cancelation with MIMO DoFs

The total number of antennas at a node is called *degrees of freedom* (or DoFs) [12] at the node. A node can use some or all of its DoFs for either spatial multiplexing (SM) (to achieve multiple concurrent data streams over a link) or co-channel IC (to enable multiple links on the same band), as long as the number of DoFs being used does not exceed the number of antennas at the node.

The allocation of DoFs at a node for SM and IC can be based on an ordering of all nodes in the network [16]. For a given ordered node list, the DoFs at a node can be allocated as follows.

- *DoF Allocation at A Transmit Node.* First, the transmit node needs to allocate DoFs for SM. The number of DoFs to be allocated equals to the number of data streams to be transmitted. Then, for IC, this transmit node must ensure that its transmission does not interfere with those receive nodes that are before this node in the ordered list. To cancel its interference to these receive nodes, this node needs to allocate a number of DoFs that are equal to the received data streams by those nodes. This transmit node does not need to allocate any of its DoFs to null potential interference to those receive nodes that are after itself in the ordered node list.
- *DoF Allocation at A Receive Node.* First, the receive node needs to allocate DoFs for data reception (SM). The number of DoFs to be allocated equals to the number of data streams to be received. Then, for IC, this node must ensure that its reception is not interfered by those transmit nodes that are ordered before this node in the list. To cancel the interference from these transmit nodes, this node needs to allocate a number of DoFs that are equal to the transmitted data streams by those nodes. This receive node does not need to allocate any of its DoFs to null potential interference from those transmit nodes that are after itself in the ordered node list.

An example is given in Fig. 1, where there are four nodes, each equipped with 4 antennas. All nodes operate on the same channel, and there are two mutually

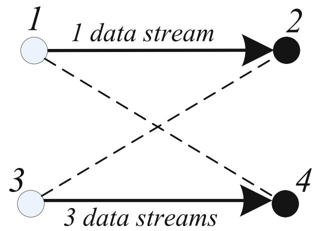

Fig. 1 Simultaneous activation of two secondary links with IC

interfering links in the network: $1 \rightarrow 2$ and $3 \rightarrow 4$. Suppose the ordered node list for DoF allocation is 1, 2, 3, and 4. Further, node 1 is transmitting 1 data stream to node 2 and node 3 is transmitting 3 DoF to node 4 . Now we show how the DoFs at each node are allocated for interference cancelation and spatial multiplexing:

- Starting with node 1, it is the first node in the list and it is a transmit node. Then it allocates 1 DoF for its transmission with 1 data stream. It does not need to allocate any DoF to cancel potential interference to other receive nodes that are after itself in the ordered node list, i.e., node 4.
- The next node in the list is node 2. As a receive node, it allocates 1 DoF for receiving 1 data stream from node 1. It does not need to consider allocating any DoF to cancel interference from other transmit nodes that are after itself in the ordered node list, i.e., node 3.
- The next node in the list is node 3. As a transmit node, it needs to ensure that its transmission does not interfere with any receive node before itself in the list, i.e., node 2. Thus, node 3 uses 1 DoF (equals to the number of received data streams by node 2) to cancel its interference to node 2. Now it has 3 remaining DoFs, which can all be used to transmit data streams (up to 3) to node 4.
- The last node in the list is node 4. As a receive node, node 4 needs to allocate 3 of its DoFs for receiving 3 data streams from node 3. Node 4 also needs to use its remaining 1 DoF (equals to the number of transmitted data streams by node 1) to cancel interference from node 1. This completes the DoF allocation at each node.

Mathematical Modeling

We consider a secondary multi-hop network consisting of a set of $\mathcal{N}$ nodes. At each node $i \in \mathcal{N}$, there is a set of $\mathcal{B}_i$ available frequency bands that can be used for communications. As discussed, $\mathcal{B}_i$ may represent the set of bands that are unused by the primary users and may be different at each node due to geographical difference. Denote the set of commonly available bands between nodes i and j as $\mathcal{B}_{ij} = \mathcal{B}_i \bigcap \mathcal{B}_j$. Also, denote A_i as the number of antennas at node i. Suppose there are multiple sessions in this network. Denote $\mathcal{Q}$ the set of sessions in the network. For a session $q \in \mathcal{Q}$, denote $s(q)$ the source node, $d(q)$ the destination node, and $f(q)$ the flow rate (in b/s). Table 1 lists all notation used in the interweave paradigm.

Table 1 Notation in interweave paradigm

Symbol	Definition
A_i	The number of antennas at node $i \in \mathscr{N}$
A_{MIMO}	The number of antennas at each node
$\mathscr{B}_i$	The set of available bands at node $i \in \mathscr{N}$
$\mathscr{B}_{ij}$	The set of common available bands at nodes $i, j \in \mathscr{N}$
c	The capacity when 1 DoF is used for data transmission on a band over a link
$d(q)$	Destination node of session q
$f(q)$	The rate of session q
g_i^b	A binary indicator. g_i^b is 1 if node i is transmitting or 0 otherwise
h_i^b	A binary indicator. h_i^b is 1 if node i is receiving or 0 otherwise
$\mathscr{I}_i^b$	The set of nodes in the interference range of node i on band b
$\mathscr{L}_{i,b}^{\text{Out}}$	The set of outgoing links on band b at node i
$\mathscr{L}_{i,b}^{\text{In}}$	The set of incoming links on band b at node i
$\mathscr{L}_{\text{Active}}$	The set of links used for routing
$\mathscr{N}$	The set of all nodes in the network
$\mathscr{Q}$	The set of active sessions in the network
$\text{Rx}(l)$	Receiving node of link l
$s(q)$	Source node of session q
$\text{Tx}(l)$	Transmitting node of link l
z_l^b	The number of data streams over link l on band b
θ_{ji}^b	Binary indicator showing the relationship between nodes i and j in the ordered list on band b
λ_{ji}^b	The number of DoFs on band b used by transmitting node i to cancel its interference to node j
μ_{ji}^b	The number of DoFs on band b used by receiving node i to cancel the interference from node j

Half-Duplex Constraint. To model the half-duplex nature of each node in a band, we use two binary variables g_i^b and h_i^b to indicate node i's transmission/reception status on band b, i.e.,

$$g_i^b = \begin{cases} 1 & \text{if node } i \text{ is transmitting on band } b, \\ 0 & \text{otherwise.} \end{cases}$$

$$h_i^b = \begin{cases} 1 & \text{if node } i \text{ is receiving on band } b, \\ 0 & \text{otherwise.} \end{cases}$$

where $i \in \mathscr{N}, b \in \mathscr{B}_i$. Then the half-duplex constraint (i.e., a node cannot transmit and receive at the same time in the same band) can be represented as follows:

$$g_i^b + h_i^b \le 1, \qquad (i \in \mathscr{N}, b \in \mathscr{B}_i). \tag{1}$$

Node Ordering for IC. As discussed in section "Co-channel Interference Cancelation with MIMO DoFs", the DoF allocation (for SM and IC) at each node is determined sequentially based on an ordered node list. This ordering determines DoF allocation behavior in the final solution and should be part of the optimization problem. We point out that such a node ordering approach for DoF allocation is the most efficient approach among all existing DoF models that can guarantee feasibility. As pointed out in [16], an optimal ordering of secondary nodes can be found by inserting a formulation of the ordering relationship into the specific optimization problem.

Denote π^b as an ordered list of the nodes in the secondary network on channel $b \in \mathscr{B}$, and denote π_i^b as the position of node $i \in \mathscr{S}$ in π^b. Therefore, $1 \leq \pi_i^b \leq S$, where $S = |\mathscr{S}|$. For example, if $\pi_i^b = 3$, then it means that node i is in the third position in the list π^b.

To model the relative ordering between any two secondary nodes i and j in π^b, we define a binary indication variable $\theta_{j,i}^b$ and define it as follows:

$$\theta_{j,i}^b = \begin{cases} 1 \text{ if node } j \text{ is before node } i \text{ in } \pi^b \text{ on channel } b; \\ 0 \text{ otherwise.} \end{cases}$$

It was shown in [16] that the following relationships hold:

$$\pi_i^b - S \cdot \theta_{j,i}^b + 1 \leq \pi_j^b \leq \pi_i^b - S \cdot \theta_{j,i}^b + S - 1 \ , \quad (i, j \in \mathscr{S}, b \in \mathscr{B}). \tag{2}$$

Transmitter DoF Constraint. Now we consider DoF allocation at a node, which includes DoFs allocated for SM and DoFs allocated for IC.

For transmission and reception, the number of required DoFs is $\sum_{l \in \mathscr{L}_{i,b}^{\text{Out}}} z_l^b$ for a transmit node i and $\sum_{m \in \mathscr{L}_{j,b}^{\text{In}}} z_m^b$ for a receive node j, respectively. As we discussed in section "Co-channel Interference Cancelation with MIMO DoFs", in any given band, the total number of data streams for transmission or reception at a node is limited by its number of antennas. Denote l as a directional link in the network and z_l^b as the number of data streams over link l in band b. Then we have the following two constraints:

$$g_i^b \leq \sum_{l \in \mathscr{L}_{i,b}^{\text{Out}}} z_l^b \leq g_i^b A_i \qquad (i \in \mathscr{N}, b \in \mathscr{B}_i) \ , \tag{3}$$

$$h_i^b \leq \sum_{l \in \mathscr{L}_{i,b}^{\text{In}}} z_l^b \leq h_i^b A_i \qquad (i \in \mathscr{N}, b \in \mathscr{B}_i) \ , \tag{4}$$

where $\mathscr{L}_{i,b}^{\text{Out}}$ and $\mathscr{L}_{i,b}^{\text{In}}$ represent the sets of outgoing and incoming links at node i in band b, respectively.

For IC, as discussed in section "Co-channel Interference Cancelation with MIMO DoFs", a transmit node needs to allocate its DoFs to cancel its interference to all receive nodes before itself in the ordered node list. Denote $\mathcal{I}_i^b$ as the set of nodes to which node i can interfere within band b. Then the number of DoFs that node i allocates for IC can be computed as $\sum_{j \in \mathcal{I}_i^b} (\theta_{ji}^b \cdot \sum_{m \in \mathcal{L}_{j,b}^{\text{In}}}^{\text{Tx}(m) \neq i} z_m^b)$, where the inner summation $\sum_{m \in \mathcal{L}_{j,b}^{\text{In}}}^{\text{Tx}(m) \neq i} z_m^b$ gives the number of data streams for a given receive node j, while the outer summation is taken only over those receive nodes that are before node i in the ordered node list. Now considering both the DoFs at a node allocated for SM and IC, we have the following constraint:

$$\sum_{l \in \mathcal{L}_{i,b}^{\text{Out}}} z_l^b + \sum_{j \in \mathcal{I}_i^b} \left(\theta_{ji}^b \cdot \sum_{m \in \mathcal{L}_{j,b}^{\text{In}}}^{\text{Tx}(m) \neq i} z_m^b \right) \leq A_i g_i^b + (1 - g_i^b) M, \tag{5}$$

where $i \in \mathcal{N}, b \in \mathcal{B}_i$, and M is a sufficiently large number to ensure the constraint holds when node i is not a transmitnode (e.g., we can set $M = \sum_{j \in \mathcal{I}_i^b} A_j$).

Receiver DoF Constraint. Similarly, if node i is a receive node, we have the following constraint for its DoF allocation:

$$\sum_{l \in \mathcal{L}_{i,b}^{\text{In}}} z_l^b + \sum_{j \in \mathcal{I}_i^b} \left(\theta_{ji}^b \cdot \sum_{m \in \mathcal{L}_{j,b}^{\text{Out}}}^{\text{Rx}(m) \neq i} z_m^b \right) \leq A_i h_i^b + (1 - h_i^b) M, \tag{6}$$

where $i \in \mathcal{N}, b \in \mathcal{B}_i$.

For a given route for each session, we can identify the set of links on this route. Denote $\mathcal{L}_{\text{Active}}$ as the set of links that are used by all these routes in the network. Then we have the following capacity constraint on link $l \in \mathcal{L}_{\text{Active}}$.

$$\sum_{q \in \mathcal{Q}}^{l \text{ is traversed by } q} f(q) \leq c \sum_{b \in \mathcal{B}_{\text{Tx}(l),\text{Rx}(l)}} z_l^b \quad (l \in \mathcal{L}_{\text{Active}}), \tag{7}$$

where $f(q)$ is the flow rate of session $q \in \mathcal{Q}$ and c is the capacity when 1 DoF is used for data transmission on a band over link l.

Problem Formulation

Based on the above mathematical model, various problems can be formulated. In this chapter, we study a throughput optimization problem with the objective of maximizing the minimum session rate among all secondary sessions. The optimization problem can be written as follows:

OPT max $f_{\min}$
s.t $f_{\min} \le f(q) \quad (q \in \mathscr{Q})$;
Half-duplex constraints: (1);
Node ordering constraints: (2);
Transmitter DoF constraints: (3), (5);
Receiver DoF constraints: (4), (6);
Link capacity constraints: (7).

In this formulation, $f_{\min}$, $f(q)$, g_i^b, h_i^b, z_l^b, and θ_{ji}^b are optimization variables and A_i, M, and c are given constants. Due to the nonlinear product term $\sum_{j\in\mathscr{I}_i^b}(\theta_{ji}^b \cdot \sum_{m\in\mathscr{L}_{j,b}^{\text{In}}}^{\text{Tx}(m)\neq i} z_m^b)$ in (5), $\sum_{j\in\mathscr{I}_i^b}(\theta_{ji}^b \cdot \sum_{m\in\mathscr{L}_{j,b}^{\text{Out}}}^{\text{Rx}(m)\neq i} z_m^b)$ in (6), and integer variables, the problem is in the form of mixed-integer nonlinear programming (MINLP).

Mathematical Reformulation

Note that the constraints in (5) and (6) have nonlinear terms (product of variables), which bring in extra complexity in problem formulation. We now show how these nonlinear terms can be removed via linearization. For the nonlinear term in (5), we define a new variable λ_{ji}^b as follows:

$$\lambda_{ji}^b = \theta_{ji}^b \cdot \sum_{m\in\mathscr{L}_{j,b}^{\text{In}}}^{\text{Tx}(m)\neq i} z_m^b \qquad (i \in \mathscr{N}, b \in \mathscr{B}_i, j \in \mathscr{I}_i^b), \tag{8}$$

which is the number of DoFs that transmit node i uses to cancel the interference to receive node j. With λ_{ji}^b, (5) can be rewritten as:

$$\sum_{l\in\mathscr{L}_{i,b}^{\text{Out}}} z_l^b + \sum_{j\in\mathscr{I}_i^b} \lambda_{ji}^b \le A_i g_i^b + (1 - g_i^b)M, \tag{9}$$

where $i \in \mathscr{N}, b \in \mathscr{B}_i$. Now, we need to add some constraints for λ_{ji}^b. This can be done by examining the definition of λ_{ji}^b in (8). For binary variable θ_{ji}^b, we have the following relaxed constraints: $\theta_{ji}^b \ge 0$, $1 - \theta_{ji}^b \ge 0$. For $\sum_{m\in\mathscr{L}_{j,b}^{\text{In}}}^{\text{Tx}(m)\neq i} z_m^b$, we have $\sum_{m\in\mathscr{L}_{j,b}^{\text{In}}}^{\text{Tx}(m)\neq i} z_m^b \ge 0$, $A_j - \sum_{m\in\mathscr{L}_{j,b}^{\text{In}}}^{\text{Tx}(m)\neq i} z_m^b \ge 0$. Multiplying each constraint involving θ_{ji}^b by one of the two constraints involving $\sum_{m\in\mathscr{L}_{j,b}^{\text{In}}}^{\text{Tx}(m)\neq i} z_m^b$, and replacing the product term $\theta_{ji}^b \cdot \sum_{m\in\mathscr{L}_{j,b}^{\text{In}}}^{\text{Tx}(m)\neq i} z_m^b$ with the new variable λ_{ji}^b, we obtain the following four constraints:

$$\lambda_{ji}^b \ge 0 \tag{10}$$

$$\lambda_{ji}^b \le \sum_{m\in\mathscr{L}_{j,b}^{\text{In}}}^{\text{Tx}(m)\neq i} z_m^b \tag{11}$$

$$\lambda_{ji}^b \le A_j \cdot \theta_{ji}^b \tag{12}$$

$$\lambda_{ji}^b \ge A_j \cdot \theta_{ji}^b - A_j + \sum_{m \in \mathscr{L}_{j,b}^{\mathrm{In}}}^{\mathrm{Tx}(m) \ne i} z_m^b \tag{13}$$

where $i \in \mathscr{N}, b \in \mathscr{B}_i, j \in \mathscr{I}_i^b$. Note that due to the relaxation of integer variable θ_{ji}^b, $\sum_{m \in \mathscr{L}_{j,b}^{\mathrm{In}}}^{\mathrm{Tx}(m) \ne i} z_m^b$, and product operations, the above four constraints for λ_{ji}^b might be looser than (8). However, for the special case when θ_{ji}^b is a binary variable, it can be easily verified that (8) is equivalent to the four constraints in (10), (11), (12), and (13). Therefore, it is sufficient to have linear constraints (9), (10), (11), (12), and (13) to replace (5).

Similarly, to remove the nonlinear term in (6), we define μ_{ji}^b as the number of DoFs that receive node i uses to cancel the interference from transmit node j. Following the same token, (6) can be replaced by the following linear constraints:

$$\sum_{l \in \mathscr{L}_{i,b}^{\mathrm{In}}} z_l^b + \sum_{j \in \mathscr{I}_i^b} \mu_{ji}^b \le A_i h_i^b + (1 - h_i^b) M \tag{14}$$

$$\mu_{ji}^b \ge 0 \tag{15}$$

$$\mu_{ji}^b \le \sum_{m \in \mathscr{L}_{j,b}^{\mathrm{Out}}}^{\mathrm{Rx}(m) \ne i} z_m^b \tag{16}$$

$$\mu_{ji}^b \le A_j \cdot \theta_{ji}^b \tag{17}$$

$$\mu_{ji}^b \ge A_j \cdot \theta_{ji}^b - A_j + \sum_{m \in \mathscr{L}_{j,b}^{\mathrm{Out}}}^{\mathrm{Rx}(m) \ne i} z_m^b \tag{18}$$

where $i \in \mathscr{N}, b \in \mathscr{B}_i, j \in \mathscr{I}_i^b$.

With the above linearization, we have a revised optimization problem formulation (denoted as OPT-R).

OPT-R max $f_{\min}$

s.t $f_{\min} \le f(q) \quad (q \in \mathscr{Q})$;

Half-duplex constraints: (1);

Node ordering constraints: (2);

Transmitter DoF constraints: (3), (10), (11), (12), and (13);

Receiver DoF constraints: (4), (14), (15), (16), (17), and (18);

Link capacity constraints: (7).

In this formulation, $f_{\min}$, $f(q)$, g_i^b, h_i^b, z_l^b, θ_{ji}^b, λ_{ji}^b, and μ_{ji}^b are optimization variables and A_i, M, and c are constants. The problem is now in the form of mixed-integer linear programming (MILP), which is NP-hard in general. The computation complexity of MILP is exponential, but fortunately, the branch-and-cut based solution procedure used by a commercial solver such as CPLEX is very efficient. Therefore, we will use CPLEX to solve our MILP problems, which turns out to be very successful for all practical purposes.

Anticipated Results

Before we present numerical results, we offer the following discussion on the possible solution to our problem. Consider a CRN with only a single transmit/receive antenna at each node (i.e., $A_i = 1, i \in \mathcal{N}$). Denote f_{CRN} as the optimal objective value for this CRN with our problem formulation. Now consider a CRN^{MIMO} with the same topology as the above CRN but now with A_{MIMO} transmit/receive antennas at each node. This CRN^{MIMO} is a special case of our CRN^{MIMO} network with all $A_i = A_{\text{MIMO}}, i \in \mathcal{N}$. Denote $f_{\text{CRN}^{\text{MIMO}}}$ as the optimal objective value for this CRN^{MIMO} under our problem formulation. Comparing $f_{\text{CRN}^{\text{MIMO}}}$ with f_{CRN}, we have the following observation:

$$f_{\text{CRN}^{\text{MIMO}}} \geq A_{\text{MIMO}} \times f_{\text{CRN}} \tag{19}$$

The equality part in (19) can be easily explained by exploiting SM, i.e., constructing the same solution in CRN^{MIMO} as that in the CRN but with A_{MIMO} data streams on each link.

However, we are more interested in the possible *inequality* part in (19), i.e., with joint channel level (via CR) and co-channel level (via MIMO DoF) optimization within a CRN^{MIMO}, we should anticipate more than A_{MIMO}-fold increase in the optimal solution. The greater the gap is in this inequality, the more potential in the joint CR and MIMO that can be exploited. We shall look into this potential gain via numerical results on various networks in the next section.

A Case Study

In this section, we present some numerical results for various network configurations. We consider randomly generated secondary networks with $|\mathcal{N}| = 30$ nodes in an 100×100 area. For ease of scalability and generality, we normalize all units for distance, bandwidth, and rate with appropriate dimensions. In this case study, we assume there are four sessions in the network with the source node and destination node for each session which are randomly selected. There are $|\mathcal{B}| = 15$ frequency bands available in the network. The set of available bands at each node is being

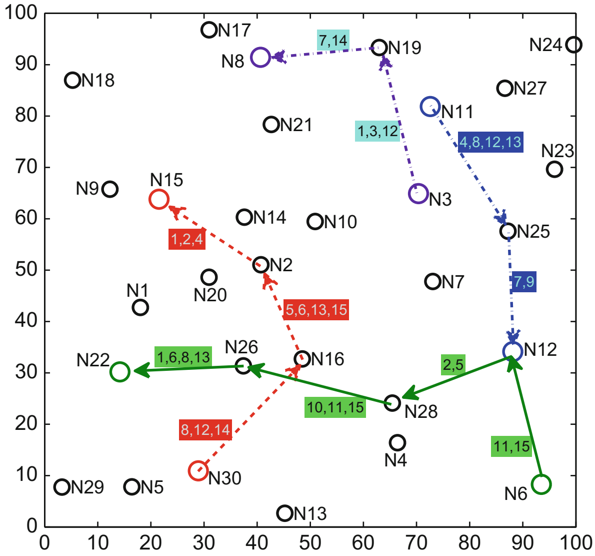

Fig. 2 Assigned bands on each link for the 30-node secondary network

randomly selected from the 15-band pool. The capacity achieved by one band and 1 DoF is normalized to 1. We assume that the transmission range is 30 and the interference range is 60.

Results Before we present results to validate for all 30-node network instance, we select one network instance and explain the details of its optimal solution. This will offer us thorough understanding on what is behind MIMO-based CRN.

The particular network configuration that we will examine is shown in Fig. 2. The location and available bands for each node are listed in Table 2. Table 3 specifies the source and destination nodes for each session. For MIMO, we assume each node in the network is equipped with 4 antennas. We assume minimum-hop routing is used in the network.

Using CPLEX, we can obtain optimal solution to the OPT-R problem. The optimal objective value for this secondary network is 6, which means each session can send at least 6 data streams from its source to its destination.

In addition to the optimal objective value, we show channel level and co-channel level solution to achieve this objective. Figure 2 shows the optimal band assignment to each link for each session. The bands assigned on each link are shown in shaded boxes. This result is also shown in Table 4 (first 3 columns). Also shown in column 4 of Table 4 is the capacity on each band under the optimal solution. In column 5, we show the capacity (in terms of sum of capacity on each band) over each link. Note that this capacity is at least 6, thus guaranteeing each session can transport at least 6 data streams.

Table 2 Each node's location and available frequency bands for the 30-node CRN^{MIMO}

Node	Location	Available bands	Node	Location	Available bands
N1	(18.0, 42.7)	1,2,4,5,6,8,9,10,11,12,13,14	N16	(48.5, 32.7)	2,4,5,6,8,10,11,12,13, 14,15
N2	(40.7, 51.0)	1,2,4,5,6,7,8,9,10,11,13,14,15	N17	(31.0, 96.8)	4,6,7,12,15
N3	(70.4, 64.9)	1,2,3,4,5,7,8,9,12,13,14,15	N18	(5.3, 87.0)	6,7,15
N4	(66.4, 16.4)	2,7,10	N19	(63.0, 93.3)	1,3,4,7,12,14
N5	(16.4, 7.8)	5,6,9,10,12,13,14	N20	(30.9, 48.6)	1,2,4,6,8,9,10,11,12, 13,14
N6	(93.5, 8.3)	11,15	N21	(42.7, 78.4)	1,3,4,7,12,14
N7	(73.1, 47.8)	1,2,3,4,5,7,8,9,13,15	N22	(14.2, 30.2)	1,2,5,6,8,9,10,11,12, 13,14
N8	(40.6, 91.4)	4,6,7,12,14	N23	(99.0, 69.6)	1,2,3,4,7,9,12,13,15
N9	(12.3, 65.8)	1,2,7,14	N24	(99.6, 93.9)	3,9,12
N10	(50.9, 59.5)	1,2,3,4,5,6,7,8,11,14,15	N25	(87.2, 57.6)	1,2,4,5,7,8,9,12,13,15
N11	(72.6, 81.9)	1,2,3,4,5,7,8,9,12,13,14,15	N26	(37.4, 31.4)	1,2,4,5,6,8,9,10,11, 12,13,14,15
N12	(88.1, 34.1)	2,5,7,9,11,15	N27	(86.6, 85.4)	1,2,3,4,7,9,12,13
N13	(45.2, 2.7)	10,12,13,14	N28	(65.5, 24.1)	2,5,7,10,11,15
N14	(37.6, 60.3)	1,2,4,5,6,7,8,10,11,13,14,15	N29	(3.3, 7.8)	5,6,9,13
N15	(21.5, 63.8)	1,2,4,7,8,10,14	N30	(28.9, 10.9)	5,6,8,9,10,11,12,13,14

Table 3 Source and destination nodes of each session in the 30-node CRN^{MIMO}

Session q	Source node $s(q)$	Destination node $d(q)$
1	N30	N15
2	N6	N22
3	N11	N12
4	N3	N8

We now examine co-channel DoF allocation in the optimal solution. Recall that DoF allocation is performed within the same band. Given that we have a total of 15 bands in the network, we shall have DoF allocation within each of the 15 bands. Let's first show DoF allocation in one particular band, say band 1. Note that band 1 is used by links N2 → N15, N3 → N19, N26 → N22 in Fig. 2. The DoF allocation on these 6 nodes are given in Fig. 3 and Table 5. As shown in Fig. 3, there are 2 data streams in each of these 3 links on band 1. The dashed lines in Fig. 3 show the interference relationship among the nodes, i.e., node N2 interferes with N19 and N22, node N3 interferes with N15, and node N26 interferes with N15. These transmission links and interference relationships are also listed in Table 5 (row 1), where "N2 → N15 (N19, N22)" denotes N2 transmits to N15 and interferes with N19, N22, etc. Also shown in the first column of Table 5 is the optimal order for the 6 nodes for DoF allocation in the optimal solution, i.e., N2, N3, N15, N19, N26, and N22. Based on this order, the DoFs at each node are allocated as follows (also see Fig. 3):

Table 4 Details of band assignment, capacity on each band, and capacity on each link in the 30-node CRN^{MIMO}

Session q	Link	Assigned band	Capacity on band	Link capacity
1	N30 → N16	8	1	6
		12	1	
		14	4	
	N16 → N2	5	1	6
		6	3	
		13	1	
		15	1	
	N2 → N15	1	2	6
		2	1	
		4	3	
2	N6 → N12	11	3	6
		15	3	
	N12 → N28	2	3	6
		5	3	
	N28 → N26	10	4	6
		11	1	
		15	1	
	N26 → N22	1	2	7
		6	1	
		8	1	
		13	3	
3	N11 → N25	4	1	6
		8	2	
		12	2	
		13	1	
	N25 → N12	7	2	6
		9	4	
4	N3 → N19	1	2	7
		3	4	
		12	1	
	N19 → N8	7	2	6
		14	4	

- Starting with node N2, it is the first node in the ordered node list and it is a transmit node. Then it allocates 2 DoFs to transmit 2 data streams to node N15. It does not need to allocate any DoF to cancel potential interference to other receive nodes after itself in the node list, i.e., nodes N19 and N22.
- The next node in the list is N3. As a transmit node, it allocates 2 DoFs to transmit 2 data streams to node N19. It does not need to allocate any DoF to cancel potential interference to receive node N15, which is after itself in the ordered node list.

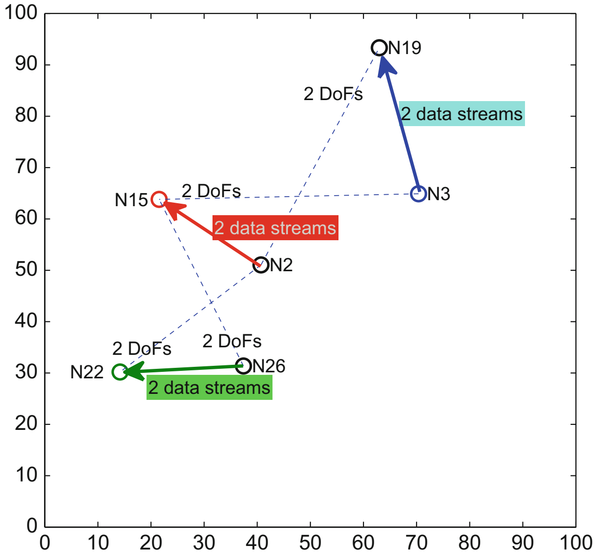

Fig. 3 The DoF allocation in the optimal solution in band 1 for the 30-node CRN$^{\text{MIMO}}$ example

Table 5 The DoF allocation in band 1 in the 30-node CRN$^{\text{MIMO}}$ example

Transmission and interference	N2 → N15 (N19, N22), N3 → N19 (N15), N26 → N22 (N15)	
Ordered node list	Interference cancelation (# of DoFs, to/from, node)	Spatial multiplexing (# of DoFs, transmit/receive, node)
N2		(2, Transmit, N15)
N3		(2, Transmit, N19)
N15	(2, From, N3)	(2, Receive, N2)
N19	(2, From, N2)	(2, Receive, N3)
N26	(2, To, N15)	(2, Transmit, N22)
N22	(2, To, N2)	(2, Receive, N26)

- The next node in the list is N15. As a receive node, it needs to allocate 2 DoFs to receive 2 data streams from node N2. In addition, it must ensure that its reception is not interfered with by any transmit node before itself in the list, i.e., N3. Thus it allocates the remaining 2 DoFs to cancel the interference from node N3.
- The next node in the list is N19. As a receive node, it allocates 2 DoFs for receiving 2 data streams from node N3. In addition, it allocates the remaining 2 DoFs to cancel interference from transmit node N2 which is before itself in the list.
- The next node in the list is N26. As a transmit node, it needs to ensure that its transmission does not interfere with any receive node before itself in the list, i.e., N15. For this purpose, it allocates 2 DoFs to cancel its interference to node N15. Then it allocates the remaining 2 DoFs to transmit 2 data streams to node N22.

- The last node in the list is N22. As a receive node, it allocates 2 DoFs to receiving 2 data streams from node N22. In addition, it must ensure that its reception is not interfered with by any transmit node before itself in the list, i.e., N2. Thus, it allocates the remaining 2 DoFs to cancel the interference from node N2.

This completes the DoF allocation for each node in the list in band 1. The DoF allocation for the 6 nodes is also listed in Table 5, where we employ the following two abbreviated notations:

- We use the tuple (# of DoFs, From/To, Node) to denote the IC relationship between the nodes. For example, (2, From, N3) denotes current node (in the first column of the same row) allocates 2 DoFs to cancel the interference from N3, whereas (2, To, N15) denotes current node allocates 2 DoFs to cancel its interference to N15.
- We use the tuple (# of DoFs, Transmit/Receive, Node) to denote data transmission relationship between the nodes. For example, (2, Transmit, N15) denotes the current node (in the first column of the same row) allocates 2 DoFs to transmit data streams to N15, whereas (2, Receive, N2) denotes the current node uses 2 DoFs to receive data streams from N2.

Discussions in bands 2 to 15 are similar and are omitted to conserve space.

$f_{\mathrm{CRN}^{\mathrm{MIMO}}}\, vs. A_{\mathrm{MIMO}} \times f_{\mathrm{CRN}}$

The results above show an optimal solution for a 30-node secondary network with $A_{\mathrm{MIMO}} = 4$ antennas at each node. We now validate the result in (19) under different number of antennas at each node. Figure 4 shows the optimal objective values under different number of antennas for the same 30-node network discussed in the last section. Also shown in this figure is a dashed line with a slope of f_{CRN}.

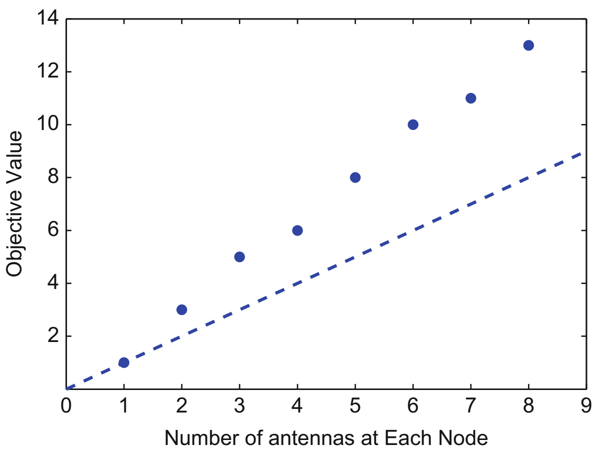

Fig. 4 Objective value under different antennas for the 30-node $\mathrm{CRN}^{\mathrm{MIMO}}$

Note that the equality in (19) only coincides for the first point, i.e., single antenna at each node. When the number of antennas at each node is greater than 1, we have an inequality, i.e., $f_{\mathrm{CRN}^{\mathrm{MIMO}}} > A_{\mathrm{MIMO}} \times f_{\mathrm{CRN}}$. That is, with joint channel level (via CR) and co-channel level (via MIMO DoF) optimization within a $\mathrm{CRN}^{\mathrm{MIMO}}$ network, we have more than A_{MIMO}-fold increase in the optimal solution.

MIMO-Based Secondary Network in Transparent-Coexistence Paradigm

In section "MIMO-Based Secondary Network in Interweave Paradigm", we have studied how multi-hop secondary CR network achieve simultaneous transmission on the same channel among secondary nodes with MIMO-empowered CR under the interweave paradigm. In this section, we study how a multi-hop secondary CR network can coexist with a primary network transparently. A MIMO node's ability to use a subset of its DoFs to cancel interference while using the remaining subset of DoFs for data transmission allows the possibility of simultaneous activation of the secondary nodes with the primary nodes. For a set of channels owned by the primary networks, the primary nodes may use them in whatever manner to suit their needs. On the other hand, the secondary nodes are only allowed to use these channels if they can cancel their interference to the primary nodes. Further, to ensure successful transmission among the secondary nodes, the secondary nodes also need to perform IC to/from the primary nodes as well as potential interference among the secondary nodes. Simply put, all IC burden should rest solely on the secondary nodes and remain invisible to the primary nodes.

It is important to realize that we strive to put all IC burden on the secondary node side. Specifically, the transmitter of a secondary node needs to cancel its interference to all neighboring primary receive nodes who are interfered with by this secondary transmitter; the receiver of a secondary node needs to cancel interference from all neighboring primary transmit nodes who interfere with this secondary receiver. To achieve transparency to primary nodes, it is important for the secondary nodes to have accurate channel state information (CSI). The problem here is: how can a secondary node obtain the CSI between itself and its neighboring primary nodes while remaining transparent to the primary nodes? We propose the following solution to resolve this problem. For each primary node, it typically sends out a pilot sequence (training sequence) to its neighboring primary nodes such that those primary nodes can estimate the CSI for communication. This is the practice for current cellular networks, and we assume such a mechanism is available for a primary network. Then, the secondary nodes can *overhear* the pilot sequence signal from the primary node while staying transparent. Suppose the pilot sequence from the primary nodes is publicly available (as in cellular networks) and is thus known to the secondary nodes. Then the secondary nodes can use this information and the actual received pilot sequence signal from the primary node for channel estimation. Based on the reciprocity property of a wireless channel [17], the estimated CSI can also be used as CSIT (channel state information at transmitter side). Therefore, a secondary node can obtain complete CSI between itself and a primary node.

Problem Scope

Although the new coexistence paradigm has been explored at the physical layer, its application to a multi-hop network environment is far from trivial. Consider a primary multi-hop ad hoc network $\mathscr{P}$ shown in Fig. 5, which is colocated with a secondary multi-hop network $\mathscr{S}$ in the same geographical region. Suppose that there is a set of channels $\mathscr{B}$ available to the primary network. The primary nodes can use this set of frequency channels freely as if they were the only nodes in the network. The primary nodes do not need to be MIMO-capable. The secondary nodes, however, are allowed to use a channel in $\mathscr{B}$ only if their interference to the primary nodes is canceled properly, with complete transparency to the primary nodes. As discussed, the secondary nodes are assumed to be equipped with MIMO. In this context, we have a number of challenges for the secondary network.

- **Channel Selection** In a secondary network, an intermediate relay node is both a transmitter and a receiver. Due to half-duplex, a node cannot transmit and receive on the same channel at the same time. Therefore, scheduling (either in time slot or channel) is needed. In this chapter, we assume scheduling is performed in the form of channel assignment. Therefore, a secondary relay node needs to select different channels for transmission and reception. Note that scheduling transmission and reception of a secondary node will lead to a different interference relationship among the primary and secondary nodes in the network. This brings in considerable complexity to the mathematical modeling of interference relationship.
- **IC to/from Primary Network** A secondary transmitter needs to cancel its interference to its neighboring primary receivers, while a secondary receiver

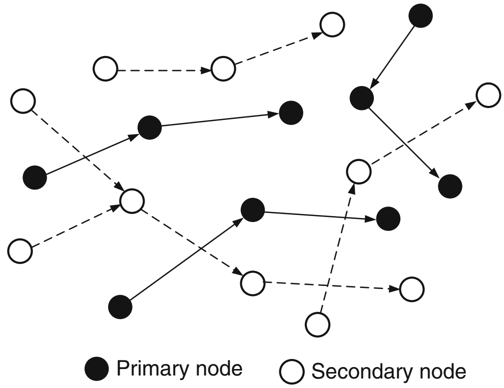

Fig. 5 A multi-hop secondary network colocated with a multi-hop primary network

needs to cancel the interference from its neighboring primary transmitters. Such challenge magnifies when the secondary network is a multi-hop network.

- **IC Within Secondary Network** In addition to interference between the primary and secondary nodes, interference from a secondary node may also interfere another secondary node within their own network. Such interference must also be canceled properly (either by a secondary transmitter or the secondary receiver that is being interfered with) to ensure successful data communications inside the secondary network. Resource allocation to account for such IC is clearly not a trivial problem.

It is important to realize that the above three challenges are not independent, but rather deeply intertwined with each other. In particular, channel selection at a secondary node is directly tied to the interference relationship between primary and secondary nodes as well as interference among the secondary nodes within each channel. Further, the combined channel resource and total DoFs at each node determine a complete resource space in the network: an optimal DoF allocation and channel selection at each secondary node for both IC to/from the primary nodes and within the secondary nodes are critical to achieve the desired network performance objective. A modeling and formulation of transparent-coexistence paradigm would call for a joint consideration of all these components.

Mathematical Modeling

In this section, we develop a mathematical model for the transparent-coexistence paradigm where a multi-hop secondary network shares the same spectrum as a primary network (see Fig. 5).

Referring to Fig. 5, we consider a secondary multi-hop network consisting of a set of nodes $\mathscr{S}$, which is colocated with a primary multi-hop network consisting of a set of nodes $\mathscr{P}$. Suppose that there is a set of channels $\mathscr{B}$ available to the primary network. For the primary network, there is no special node requirement, and we assume that each primary node is a traditional single-antenna node. A primary node may transmit and receive on the same channel but in different time slots or transmit and receive on different channels. We consider the latter in this chapter. Consider a set of multi-hop sessions $\tilde{\mathscr{F}}$ among the primary nodes. For a given routing for each session, denote $\tilde{\mathscr{L}}$ the set of active links in the primary network (shown in solid arrow lines in Fig. 5). Denote $\tilde{z}^b(\tilde{l})$ as the number of data streams over primary link $\tilde{l} \in \tilde{\mathscr{L}}$ on channel b. Then due to single antenna on each primary node, $\tilde{z}^b(\tilde{l}) = 1$ if link $\tilde{l}$ is active on channel b and 0 otherwise.

For the secondary network, we assume MIMO's capability at each node. Denote A_i as the number of antennas on a secondary node $i \in \mathscr{S}$. Suppose there is a set of multi-hop sessions $\mathscr{F}$ in $\mathscr{S}$. For a given routing for each session, denote $\mathscr{L}$ as the set of secondary links (shown in dashed arrow line in Fig. 5). Denote $r(f)$ as the rate of session $f \in \mathscr{F}$. A general goal of throughput maximization is to maximize a function of $r(f)$, $f \in \mathscr{F}$. Table 6 lists all notations used in the underlay paradigm.

Table 6 Notation in underlay paradigm

Primary network	
$\mathscr{P}$	The set of nodes in the primary network
T	The number of time slots in a frame
$\mathscr{B}$	The sets of channels owned by the primary network
B	The number of channels in set $\mathscr{B}$, $B = \|\mathscr{B}\|$
$\tilde{\mathscr{F}}$	The set of sessions in the primary network
$\tilde{\mathscr{I}}_i$	The set of primary nodes within the interference range of secondary node i
$\tilde{\mathscr{L}}_i^{\text{In}}$	The set of incoming links (from other primary nodes) at node $i \in \mathscr{P}$
$\tilde{\mathscr{L}}_i^{\text{Out}}$	The set of outgoing links (to other primary nodes) at node $i \in \mathscr{P}$
$\tilde{\mathscr{L}}$	The set of links in the primary network
$\tilde{z}_{(\tilde{l})}^b$	The number of data streams over primary link $\tilde{l}$ on channel b
Secondary network	
$\mathscr{S}$	The set of nodes in the secondary network
S	The number of secondary nodes in the network, $S = \|\mathscr{S}\|$
A_i	The number of antennas at secondary node $i \in \mathscr{S}$
c	The minimum data rate carried by a data stream
$\mathscr{F}$	The set of sessions in the secondary network
$\mathscr{I}_i$	The set of node in $\mathscr{S}$ that are within the interference range of secondary node i
$\mathscr{L}_i^{\text{In}}$	The set of incoming links (from other secondary nodes) at node $i \in \mathscr{S}$
$\mathscr{L}_i^{\text{Out}}$	The set of outgoing links (to other secondary nodes) at node $i \in \mathscr{S}$
$\mathscr{L}$	The set of secondary links
$r(f)$	The data rate of the session $f \in \mathscr{F}$
$r_{\min}$	The minimum data rate among all secondary sessions
$\text{Rx}(l)$	The receiver of link $l \in \mathscr{L}$
$\text{Tx}(l)$	The transmitter of link $l \in \mathscr{L}$
x_i^b	$= 1$ if node $i \in \mathscr{S}$ is a transmitter on channel b and is 0 otherwise
y_i^b	$= 1$ if node $i \in \mathscr{S}$ is a receiver on channel b and is 0 otherwise
$z_{(l)}^b$	The number of data streams over link $l \in \mathscr{L}$ on channel b
$\lambda_{j,i}^b$	The number of DoFs used by transmit node $i \in \mathscr{S}$ to cancel its interference to receive node $j \in \mathscr{S}$ on channel b
$\mu_{j,i}^b$	The number of DoFs used by receive node $i \in \mathscr{S}$ to cancel the interference from transmit node $j \in \mathscr{S}$ on channel b
$\theta_{j,i}^b$	Binary indicator showing the relationship between nodes i and j in ordered list on channel b, $i, j \in \mathscr{S}$
π^b	An ordering for IC among the secondary nodes on channel b
π_i^b	The position of node $i \in \mathscr{S}$ in π^b

Channel Selection. To model channel use behavior at a secondary node for transmission or reception, we denote x_i^b and y_i^b ($i \in \mathscr{S}$ and $b \in \mathscr{B}$) as whether node i selects channel b for transmission or reception, respectively. We have

$$x_i^b = \begin{cases} 1 & \text{if node } i \text{ uses channel } b \text{ for transmission;} \\ 0 & \text{otherwise.} \end{cases}$$

$$y_i^b = \begin{cases} 1 \text{ if node } i \text{ uses channel } b \text{ for reception;} \\ 0 \text{ otherwise.} \end{cases}$$

To consider half-duplex (a node cannot transmit and receive on the same channel at the same time), we have the following constraint on x_i^b and y_i^b:

$$x_i^b + y_i^b \leq 1 \qquad (i \in \mathscr{S}, b \in \mathscr{B}). \tag{20}$$

DoF Allocation at a Secondary Transmitter. Recall that the secondary network is solely responsible for IC to/from the primary network as well as IC within itself. At a secondary transmitter, it needs to expend DoFs for SM, IC to primary receivers, and IC to other secondary receivers:

- For SM, denote $z^b(l)$ and $\mathscr{L}_{i,\text{Out}}^b$ as the number of data streams over link $l \in \mathscr{L}$ and the set of outgoing links from secondary node i on channel b. Then the number of DoFs at secondary node $i \in \mathscr{S}$ for SM on channel b is $\sum_{l \in \mathscr{L}_{i,\text{Out}}^b} z^b(l)$.
- For IC to primary receivers, recall $\tilde{z}^b(\tilde{l})$ is the number of data streams over primary link $\tilde{l}$ on channel b. For a primary node $p \in \mathscr{P}$, denote $\tilde{\mathscr{L}}_{p,\text{In}}^b$ as the set of incoming primary links on channel b. Denote $\tilde{\mathscr{I}}_i$ as the set of neighboring primary nodes that are located within the interference range of secondary transmitter i. Then at node i, the number of DoFs required for IC to primary receivers is $\left(\sum_{p \in \tilde{\mathscr{I}}_i} \sum_{\tilde{l} \in \tilde{\mathscr{L}}_{p,\text{In}}^b} \tilde{z}^b(\tilde{l})\right)$ on channel b.
- For IC to secondary receivers, as discussed earlier, a secondary transmitter i only needs to cancel its interference to those nodes that are before itself in the ordered list. For a secondary node $j \in \mathscr{S}$, denote $\mathscr{L}_{j,\text{In}}^b$ as the set of incoming secondary links. Denote $\mathscr{I}_i$ as the set of neighboring secondary nodes that are located within the interference range of secondary transmitter i. Then at node i, the number of DoFs required for IC to secondary receivers is $\sum_{j \in \mathscr{I}_i} \left(\theta_{j,i}^b \cdot \sum_{k \in \mathscr{L}_{j,\text{In}}^b}^{\text{Tx}(k) \neq i} z^b(k)\right)$ in channel b, and $\text{Tx}(k)$ represents the transmitter of link k.

Putting all three DoF consumptions together at a secondary transmitter i, we have the following constraints:

$$x_i^b \leq \sum_{l \in \mathscr{L}_{i,\text{Out}}^b} z^b(l) + \left[\left(\sum_{p \in \tilde{\mathscr{I}}_i} \sum_{\tilde{l} \in \tilde{\mathscr{L}}_{p,\text{In}}^b} \tilde{z}^b(\tilde{l})\right) + \sum_{j \in \mathscr{I}_i} \left(\theta_{j,i}^b \cdot \sum_{k \in \mathscr{L}_{j,\text{In}}^b}^{\text{Tx}(k) \neq i} z^b(k)\right)\right] \cdot x_i^b \leq x_i^b A_i, \tag{21}$$

which means that if node i is transmitting, its DoF consumptions cannot exceed the total number of DoFs at node i; if node i is not transmitting, there is no

DoF consumption for transmissions, and $\sum_{l \in \mathscr{L}^b_{i,\text{Out}}} z^b(l) = 0$. By introducing a large constant M, which is an upper bound of $\left[\left(\sum_{p \in \tilde{\mathscr{I}}_i} \sum_{\tilde{l} \in \tilde{\mathscr{L}}^b_{p,\text{In}}} \tilde{z}^b(\tilde{l})\right) + \sum_{j \in \mathscr{I}_i} \left(\theta^b_{j,i} \cdot \sum^{\text{Tx}(k) \neq i}_{k \in \mathscr{L}^b_{j,\text{In}}} z^b(k)\right)\right]$ (e.g., $M = \sum_{j \in \mathscr{I}_i} A_j + \sum_{p \in \tilde{I}_i} \sum_{\tilde{l} \in \tilde{\mathscr{L}}^b_{p,\text{In}}} \tilde{z}^b(\tilde{l})$), we can use the following two sets of constraints to replace (21):

$$x^b_i \leq \sum_{l \in \mathscr{L}^b_{i,\text{Out}}} z^b(l) + \left(\sum_{p \in \tilde{\mathscr{I}}_i} \sum_{\tilde{l} \in \tilde{\mathscr{L}}^b_{p,\text{In}}} \tilde{z}^b(\tilde{l})\right) + \sum_{j \in \mathscr{I}_i} \left(\theta^b_{j,i} \cdot \sum^{\text{Tx}(k) \neq i}_{k \in \mathscr{L}^b_{j,\text{In}}} z^b(k)\right) \leq A_i x^b_i + (1 - x^b_i) M, \tag{22}$$

$$\sum_{l \in \mathscr{L}^b_{i,\text{Out}}} z^b(l) \leq x^b_i A_i. \tag{23}$$

We can see that when node i is transmitting (i.e., $x^b_i = 1$), (22) becomes (21) and (23) holds trivially; if node i is not transmitting (i.e., $x^b_i = 0$), (23) and (21) are equivalent, which is $\sum_{l \in \mathscr{L}^b_{i,\text{Out}}} z^b(l) = 0$, and (22) holds trivially.

Since (22) has a nonlinear term $\left(\theta^b_{j,i} \cdot \sum^{\text{Tx}(k) \neq i}_{k \in \mathscr{L}^b_{j,\text{In}}} z^b(k)\right)$, we can use *reformulation-linearization technique* (RLT) [15] to reformulate this nonlinear term as several linear terms. We define a new variable $\lambda^b_{j,i}$ as follows:

$$\lambda^b_{j,i} = \theta^b_{j,i} \cdot \sum^{\text{Tx}(k) \neq i}_{k \in \mathscr{L}^b_{j,\text{In}}} z^b(k), \qquad (i \in \mathscr{S}, j \in \mathscr{I}_i, b \in \mathscr{B}).$$

For binary variable $\theta^b_{j,i}$, we have the following related constraints: $\theta^b_{j,i} \geq 0, (1 - \theta^b_{j,i}) \geq 0$. For $\sum^{\text{Tx}(k) \neq i}_{k \in \mathscr{L}^b_{j,\text{In}}} z^b(k)$, we have $\sum^{\text{Tx}(k) \neq i}_{k \in \mathscr{L}^b_{j,\text{In}}} z^b(k) \geq 0$ and $A_j - \sum^{\text{Tx}(k) \neq i}_{k \in \mathscr{L}^b_{j,\text{In}}} z^b(k) \geq 0$. We can multiply each constraint involving $\theta^b_{j,i}$ by one of the two constraints involving $\sum^{\text{Tx}(k) \neq i}_{k \in \mathscr{L}^b_{j,\text{In}}} z^b(k)$, replacing the product term $\left(\theta^b_{j,i} \cdot \sum^{\text{Tx}(k) \neq i}_{k \in \mathscr{L}^b_{j,\text{In}}} z^b(k)\right)$ with a new variable $\lambda^b_{j,i}$. Then (22) can be replaced by the following linear constraints:

$$x^b_i \leq \sum_{l \in \mathscr{L}^b_{i,\text{Out}}} z^b(l) + \left(\sum_{p \in \tilde{\mathscr{I}}_i} \sum_{\tilde{l} \in \tilde{\mathscr{L}}^b_{p,\text{In}}} \tilde{z}^b(\tilde{l})\right) + \sum_{j \in \mathscr{I}_i} \lambda^b_{j,i} \leq A_i x^b_i + (1 - x^b_i) M \quad (i \in \mathscr{S}, b \in \mathscr{B}), \tag{24}$$

$$\lambda_{j,i}^b \geq 0 \qquad (i \in \mathscr{S}, j \in \mathscr{I}_i, b \in \mathscr{B}), \tag{25}$$

$$\lambda_{j,i}^b \leq \sum_{k \in \mathscr{L}_{j,\mathrm{In}}^b}^{\mathrm{Tx}(k) \neq i} z^b(k) \qquad (i \in \mathscr{S}, j \in \mathscr{I}_i, b \in \mathscr{B}), \tag{26}$$

$$\lambda_{j,i}^b \leq A_j \cdot \theta_{j,i}^b \qquad (i \in \mathscr{S}, j \in \mathscr{I}_i, b \in \mathscr{B}), \tag{27}$$

$$\lambda_{j,i}^b \geq A_j \cdot \theta_{j,i}^b - A_j + \sum_{k \in \mathscr{L}_{j,\mathrm{In}}^b}^{\mathrm{Tx}(k) \neq i} z^b(k) \qquad (i \in \mathscr{S}, j \in \mathscr{I}_i, b \in \mathscr{B}). \tag{28}$$

DoF Allocation at a Secondary Receiver. At a secondary receiver, it needs to expend DoFs for SM, for IC from primary transmitters, and for IC from other secondary transmitters. For a primary node $p \in \mathscr{P}$, denote $\tilde{\mathscr{L}}_{p,\mathrm{Out}}^b$ as the set of outgoing primary links. Following the same token as our discussion for a secondary transmitter, we can put all DoF consumption at a secondary receiver as follows:

$$\begin{aligned} y_i^b &\leq \sum_{k \in \mathscr{L}_{i,\mathrm{In}}^b} z^b(k) + \left(\sum_{p \in \tilde{\mathscr{I}}_i} \sum_{\tilde{l} \in \tilde{\mathscr{L}}_{p,\mathrm{Out}}^b} \tilde{z}^b(\tilde{l}) \right) + \\ &\sum_{j \in \mathscr{I}_i} \left(\theta_{j,i}^b \cdot \sum_{l \in \mathscr{L}_{j,\mathrm{Out}}^b}^{\mathrm{Rx}(l) \neq i} z^b(l) \right) \leq A_i y_i^b + (1 - y_i^b) N, \end{aligned} \tag{29}$$

$$\sum_{k \in \mathscr{L}_{i,\mathrm{In}}^b} z^b(k) \leq y_i^b A_i, \tag{30}$$

where $\sum_{k \in \mathscr{L}_{i,\mathrm{In}}^b} z^b(k)$ represents the number of DoFs used for SM, $\left(\sum_{p \in \tilde{\mathscr{I}}_i} \sum_{\tilde{l} \in \tilde{\mathscr{L}}_{p,\mathrm{Out}}^b} \tilde{z}^b(\tilde{l}) \right)$ represents the number of DoFs used for suppressing interference from primary transmitters, and $\sum_{j \in \mathscr{I}_i} \left(\theta_{j,i}^b \cdot \sum_{l \in \mathscr{L}_{j,\mathrm{Out}}^b}^{\mathrm{Rx}(l) \neq i} z^b(l) \right)$ represents the number of DoFs consumed for canceling interference from other secondary transmitters, and N represents a large constant, and $\mathrm{Rx}(l)$ represents the receiver of link l.

Again, we can use RLT to linearize the nonlinear term $\left(\theta_{j,i}^b \cdot \sum_{l \in \mathscr{L}_{j,\mathrm{Out}}^b}^{\mathrm{Rx}(l) \neq i} z^b(l) \right)$ in (29). Denote $\mu_{j,i}^b$ as $\left(\theta_{j,i}^b \cdot \sum_{l \in \mathscr{L}_{j,\mathrm{Out}}^b}^{\mathrm{Rx}(l) \neq i} z^b(l) \right)$. Then (29) can be replaced by the following linear constraints:

$$\begin{aligned} y_i^b \leq \sum_{k \in \mathscr{L}_{i,\mathrm{In}}^b} z^b(k) + \left(\sum_{p \in \tilde{\mathscr{I}}_i} \sum_{\tilde{l} \in \mathscr{L}_{p,\mathrm{Out}}^b} \tilde{z}^b(\tilde{l}) \right) + \sum_{j \in \mathscr{I}_i} \mu_{j,i}^b \\ \leq A_i y_i^b + (1 - y_i^b) N \qquad (i \in \mathscr{S}, b \in \mathscr{B}), \end{aligned} \tag{31}$$

$$\mu_{j,i}^b \geq 0 \qquad (i \in \mathscr{S}, j \in \mathscr{I}_i, b \in \mathscr{B}), \tag{32}$$

$$\mu_{j,i}^b \leq \sum_{l \in \mathscr{L}_{j,\text{Out}}^b}^{\text{Rx}(l) \neq i} z^b(l) \qquad (i \in \mathscr{S}, j \in \mathscr{I}_i, b \in \mathscr{B}), \tag{33}$$

$$\mu_{j,i}^b \leq A_j \cdot \theta_{j,i}^b \qquad (i \in \mathscr{S}, j \in \mathscr{I}_i, b \in \mathscr{B}), \tag{34}$$

$$\mu_{j,i}^b \geq A_j \cdot \theta_{j,i}^b - A_j + \sum_{l \in \mathscr{L}_{j,\text{Out}}^b}^{\text{Rx}(l) \neq i} z^b(l) \qquad (i \in \mathscr{S}, j \in \mathscr{I}_i, b \in \mathscr{B}). \tag{35}$$

Link Capacity Constraint. For link $l \in \mathscr{L}$, we have the following link capacity constraint:

$$\sum_{f \in \mathscr{F}}^{f \text{ traversing } l} r(f) \leq c \cdot \sum_{b \in \mathscr{B}} z^b(l) \qquad (l \in \mathscr{L}), \tag{36}$$

where c is the data rate carried by a data stream.

Formulation

Based on the above mathematical model, various problems can be formulated. In this chapter, we study a throughput optimization problem with the objective of maximizing the minimum session rate among all secondary sessions. The optimization problem can be written as follows:

OPT
max $r_{\min}$
s.t $r_{\min} \leq r(f) \qquad (f \in \mathscr{F})$;
Half-duplex constraints: (20);
Node ordering constraints: (2);
Transmitter DoF constraints: (23), (22), (24), (25), (26), (27), and (28);
Receiver DoF constraints: (30), (31), (32), (33), (34), and (35);
Link capacity constraints: (36).

In this formulation, $r_{\min}, r(f), x_i^b, y_i^b, z^b(l), \pi_i^b, \lambda_{j,i}^b, \mu_{j,i}^b$ and $\theta_{j,i}^b$ are optimization variables, and $A_i, M, N, \tilde{z}^b(\tilde{l})$ and c are given constants. This optimization problem is in the form of a mixed-integer linear program (MILP), which is NP-hard in general. The computation complexity of MILP is exponential but can be solved efficiently by CPLEX solver.

A Case Study

The goal of this section is twofold. First, we want to use numerical results to demonstrate how a secondary network can operate simultaneously with the primary network while being transparent to the primary network. Second, we will show the tremendous benefits in terms of throughput gain under the transparent-coexistent paradigm.

Consider a 20-node primary network and a 30-node secondary network randomly deployed in the same 100×100 area (see Fig. 6). For the ease of scalability and generality, we normalize all units for distance, bandwidth, and throughput with appropriate dimensions. As discussed in section "Mathematical Modeling", the primary nodes are traditional single-antenna device, while the secondary nodes are equipped with MIMO. We assume there are 4 antennas for transmission or reception on each secondary node. Further, we assume all nodes' transmission range and interference range are 30 and 50, respectively, on all channels. There are $|\mathscr{B}| = 10$ channels available in the network. For simplicity, we assume the achievable rate of 1 DoF on a channel is 1 unit. In this case study, we assume there are three active sessions in the primary network and four active sessions in the secondary network and that minimum-hop routing is used for each primary and secondary session. Further, the channel allocation on each hop for a primary session is known a priori (see Fig. 6).

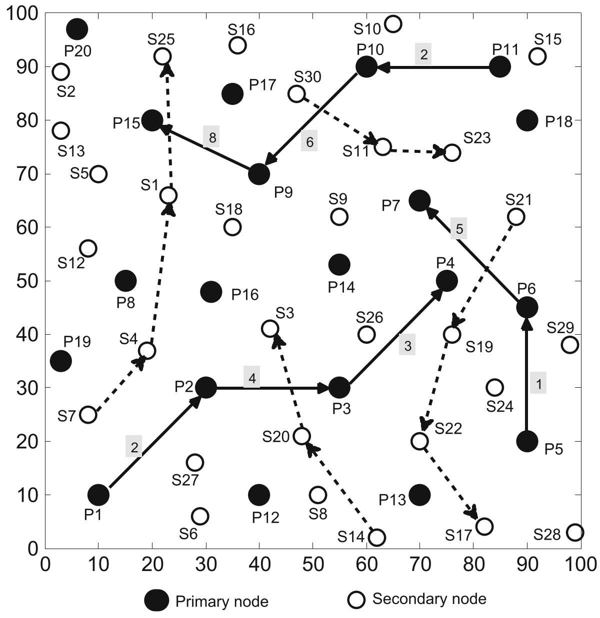

Fig. 6 Active sessons in the primary and secondary networks

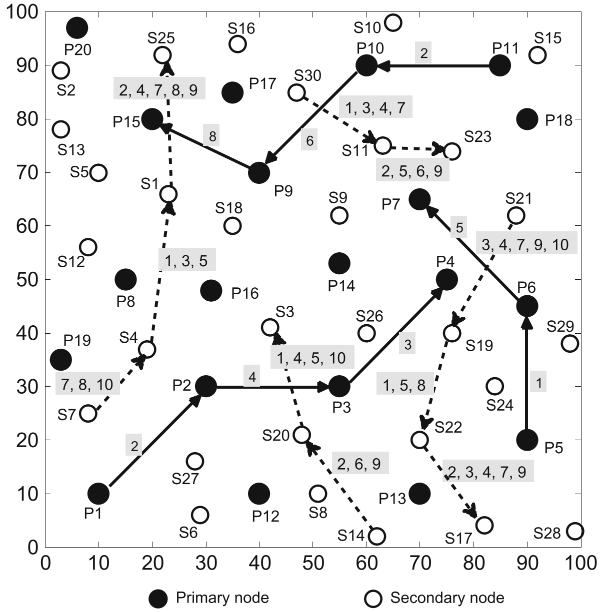

Fig. 7 Channel allocation on each link for the secondary sessions. Channel allocation on each link for the primary sessions is given a priori

For this network setting for the primary and secondary networks, the obtained objective value is 7. The channel allocation on each link for each secondary session is shown in Fig. 7. The details of DoFs used for SM on each channel at each link are shown in Table 7. The achievable rate (i.e., total number of DoFs used for SM) on a link is also shown in this table.

To see how links in the primary and secondary networks can be active in the same channel at the same time, consider channel 2 in Fig. 7. For channel 2, it is active on $P_1 \to P_2$ and $P_{11} \to P_{10}$ in the primary network and $S_{14} \to S_{20}$, $S_{22} \to S_{17}$, $S_1 \to S_{25}$ and $S_{11} \to S_{23}$ in the secondary network. The interference relationships among these 6 links are shown in Fig. 8, where the dotted arrow lines show the interference relationships among them. The 2 primary links $P_1 \to P_2$ and $P_{11} \to P_{10}$ do not interfere with each other as the receiver of each link is outside the interference range of the other link's transmitter. But each of these two primary links is within the interference range of its neighboring secondary links. Now consider link $P_1 \to P_2$:

- To cancel interference from secondary nodes (S_1, S_{14}, and S_{22}) to primary node P_2, secondary transmitters S_1, S_{14}, and S_{22} use 1 DoF to cancel their interference to primary receiver P_2. Consequently, the transmissions on $S_1 \to S_{25}$, $S_{14} \to S_{20}$, and $S_{22} \to S_{17}$ will be transparent to primary node P_2.

Table 7 Channel allocation on each link, DoF allocation on each channel for SM, and achievable data streams on each link for the secondary sessions

Session	Link	Channel allocation	DoF for SM	Achievable data streams
1	$S_7 \longrightarrow S_4$	7	3	7
		8	2	
		10	2	
	$S_4 \longrightarrow S_1$	1	2	7
		3	2	
		5	3	
	$S_1 \longrightarrow S_{25}$	2	1	7
		4	1	
		7	1	
		8	1	
		9	3	
2	$S_{21} \longrightarrow S_{19}$	3	2	7
		4	1	
		7	1	
		9	1	
		10	2	
	$S_{19} \longrightarrow S_{22}$	1	1	7
		5	2	
		8	4	
	$S_{22} \longrightarrow S_{17}$	2	1	7
		3	1	
		4	1	
		7	3	
		9	1	
3	$S_{14} \longrightarrow S_{20}$	2	2	7
		6	4	
		9	1	
	$S_{20} \longrightarrow S_3$	1	2	7
		4	2	
		5	1	
		10	2	
4	$S_{30} \longrightarrow S_{11}$	1	2	7
		3	1	
		4	1	
		7	3	
	$S_{11} \longrightarrow S_{23}$	2	2	7
		5	1	
		6	3	
		9	1	

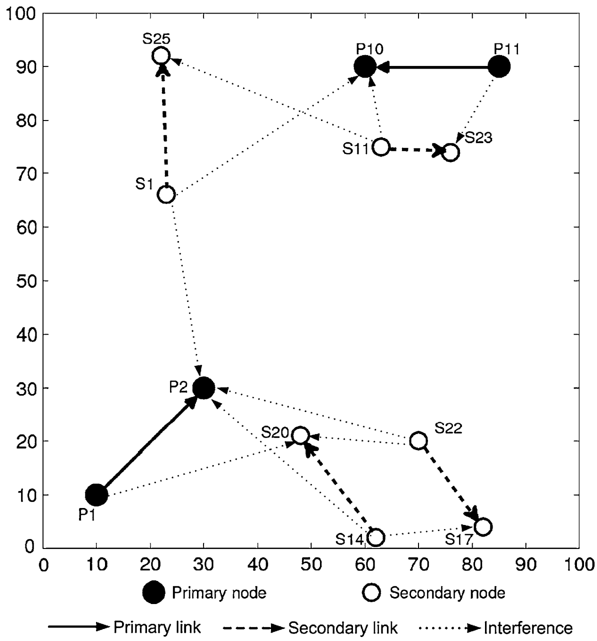

Fig. 8 Illustration of interference relationships among the primary and secondary links on channel 2 in the case study

- To cancel interference from primary node P_1 to secondary receive node S_{20}, S_{20} uses 1 DoF to cancel this interference.
- Among the secondary links, $S_{14} \rightarrow S_{20}$ and $S_{22} \rightarrow S_{17}$ interfere with each other since the receiver of each link falls within the interference range of the transmitter of the other link. To cancel its interference to S_{17}, transmitter S_{14} uses 1 DoF to cancel this interference. On the other hand, to cancel the interference from S_{22}, receiver S_{20} uses 1 DoF to cancel this interference. After IC, nodes S_{14} and S_{20} can use the remaining 2 DoFs for SM (on both transmitter and receiver sides), while nodes S_{22} and S_{17} can only use 1 DoF for SM to meet IC constraints (24) and (31).

The discussion for primary link $P_{11} \rightarrow P_{10}$ is similar and is omitted to conserve space. In addition to channel 2, other channels that exhibit transparent coexistence between the primary and secondary links include channels 1, 3, 4, 5, 6, and 8.

Comparison to Interference-Avoidance Paradigm To see the benefits of transparent-coexistence paradigm, we compare to the interference-avoidance paradigm. Under the interference-avoidance paradigm, a secondary node is not allowed to transmit (receive) on a channel when a nearby primary receiver (transmitter) is using the same channel. Therefore, the set of available channels

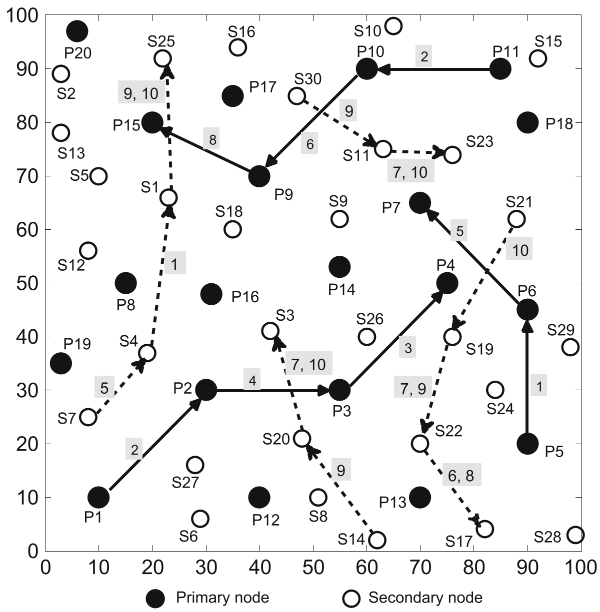

Fig. 9 Channel allocation on each link under the interference-avoidance paradigm

that can be used for secondary nodes is smaller. The problem formulation for this paradigm is simpler (although somewhat similar) than OPT and was presented in section "Mathematical Modeling". In particular, we can remove the second term $(\sum_{p\in\tilde{\mathscr{I}}_i}\sum_{\tilde{l}\in\mathscr{L}^b_{p,\mathrm{In}}}\tilde{z}^b(\tilde{l})$ and $\sum_{p\in\tilde{\mathscr{I}}_i}\sum_{\tilde{l}\in\mathscr{L}^b_{p,\mathrm{Out}}}\tilde{z}^b(\tilde{l})$) in constraints (24) and (31) in OPT that are used for secondary nodes to cancel interference to/from the primary nodes.

Following the same setting as in the case study above, we solve the above optimization problem under the interference-avoidance paradigm. The obtained objective value is 3 (compared to 7 in transparent-coexistence paradigm). The channel allocation on each link for each secondary session is shown in Fig. 9. Comparing Figs. 7 with 9, we find that the set of channels used on each secondary link under interference-avoidance paradigm is smaller than that under transparent-coexistence paradigm.

Summary and Future Directions

In this chapter, we explore MIMO-empowered CR in multi-hop networks for efficient spectrum sharing. Specially, we study joint optimization of CR and MIMO under both interweave and underlay paradigms in multi-hop ad hoc network environment. In the interweave paradigm, by exploiting CR's behavior at channel level and MIMO's capability within a channel, we showed that we can have much bigger design space to mitigate interference in the network. In the underlay paradigm, by employing the MIMO IC capability, we can achieve simultaneous transmission of primary and secondary networks in the same channel. Under both paradigms, we offer the systematic mathematical modeling, problem formulation, and performance evaluation. The extensive simulation results show that the MIMO-empowered secondary networks offer significant improvement in spectrum efficiency and throughput performance.

Much work remains to be done to transition the ideas in this paper into reality. In particular, the focus of this paper has been on exploring performance gain under idealized network setting (by ignoring many details that may arise from practical operations). We briefly discuss some of the practical issues that must be addressed to apply the MIMO-empowered CR in interweave and underlay paradigms in the real world. First, to perform IC, we assume each secondary node could obtain the accurate CSI based on channel reciprocity. But in reality, the communication channel not only consists of the physical channel but also the antennas, RF mixers, filters, A/D converters, etc., which are not necessarily identical on all the nodes. Therefore, complex calibration among the nodes is needed to achieve channel reciprocity. Such calibration is not a simple task even for a pair of transmitter and receiver and certainly is more complicated among a network of nodes. Second, zero-forcing based IC may not be perfect even if we have perfect CSI. A consequence of non-perfect IC is interference leakage, which is undesirable for both primary and secondary receivers. How to mitigate such interference leakage to a minimal acceptable level should be a key consideration when deploying MIMO-empowered CR technique into secondary network for real applications. Third, the IC and DoF allocation algorithm that we designed for the secondary network is a centralized one. Such a centralized solution serves our purpose of introducing new concepts. It bears similar pros and cons of other centralized algorithm for a wireless network. If a centralized solution is adopted in practice, those issues must be carefully addressed. On the other hand, if a distributed solution is desired, then a different set of issues need to be addressed. These issues include partial network knowledge, limited information sharing, communication overheard, and ensuring IC feasibility at each secondary node, among others. Regardless centralized or distributed solution, flow dynamics (new session initiation, existing session termination) will add additional complexity on information update and algorithm execution. Clearly, there is a large landscape for further research on these practical operation issues. We expect to see more follow-up research in this area in the near future.

Acknowledgements This work was supported in part by the US National Science Foundation (NSF) under Grants 1642873, 1617634, 1443889, 1343222, 1102013, and 1443434 and the Office of Naval Research Grant N00014-15-1-2926. Part of W. Lou's work was completed while she was serving as a Program Director at the NSF. Any opinion, findings, and conclusions or recommendations expressed in this chapter are those of the authors and do not reflect the views of the NSF. The authors thank Virginia Tech Advanced Research Computing for giving them access to the BlueRidge computer cluster.

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Further Reading

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