

Increasing User Throughput in Cellular Networks with Interference Alignment

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Abstract—Recent advances in information theory (IT) have shown great promises of interference alignment (IA) for cellular networks. However, due to a number of assumptions, these IT results cannot be directly applied to address practical problems. The goal of this paper is to fill in this gap by studying IA for cellular networks with more practical settings. We propose an IA scheme that includes constraints at each user and each base station (BS) for the uplink communication of a cellular network. We prove the feasibility of the IA scheme by constructing the encoding and decoding vectors for each data stream so that it can be transported free of interference. Based on this IA scheme, we study an uplink user throughput maximization problem and show the throughput improvement of the IA scheme over two other schemes.

I. INTRODUCTION

In recent years, interference alignment (IA) has become a promising technique for interference management in wireless networks. The basic idea of IA is to construct the signals at the transmitters so that at each receiver, the undesired signals (interference) are overlapping while the desired signals remain resolvable. It was shown in [1] that IA allows the aggregate degrees-of-freedom (DoFs) of K -user interference channel to increase linearly with the number of users K (rather than being a constant). Since its inception, the benefits of IA have been recognized and exploited for a variety of interference channels and networks, such as the K -user $M \times N$ interference channel [3], the MIMO Y channel [8], ergodic capacity in fading channel [7], [9], and the multi-hop MIMO network [18].

The potential benefits of IA have also been studied for the cellular networks [4], [10], [14], [15], [16], [17]. The most significant results in this area were developed by Suh and Tse in [14], where they showed that an IA scheme can achieve $K/(\sigma\sqrt{K}+1)^{G-1}$ DoFs for each cell, where G is the number of cells and K is the number of users in a cell. As the number of per-cell users is large enough ($K \rightarrow +\infty$), each cell can achieve one DoF, meaning that each cell (base station) can serve its users as if there were no interference in the network.

The results in [14] are significant from information theoretic perspective. However, the underlying network settings and assumptions are far from what may happen in practice. Specifically, the work in [14] was based on the following assumptions: (i) Each user in the network is restricted to one data stream. This is not likely to hold in a practical cellular

network, particularly when the demand from each user may vary widely due to different applications. (ii) The number of users served by each BS is identical. This assumption makes it convenient to design an IA scheme at a BS, but it does not hold in practice. (iii) The number of available frequency subcarriers in the network is equal to the number of users under a BS plus one. The authors assume this setting so that all of the interference streams are aligned on the same (one) direction at a BS. But in reality, the number of available frequency subcarriers in the network is not dependent on the number of users under a BS. (iv) Each receiver is in the interference range of all transmitters, i.e., symmetric interference pattern. But this is hardly true for a cellular network in practice, where a user (or BS) is only within the interference range of a subset of BSs (or users). In summary, due to the above assumptions, there remains a gap between the theoretical findings in [14] and how IA can be applied to a cellular network in practice.

The goal of this paper is to bridge the gap between the information theoretical results in [14] and how IA can be exploited for more practical settings. We consider a cellular network consisting of a set of BSs and a set of users, each of which has a single antenna. User population is randomly distributed in the area. A user may fall into the service areas of multiple BSs and will choose one BS as its service provider. A user can transmit/receive any number of data streams, which is only limited by the subcarrier resources. The total number of available subcarriers are user-independent and depends on the communication standards (e.g., 1024 in LTE). Our objective is to exploit IA in the frequency domain so as to maximize the uplink user throughput in the cellular networks.

Under the above settings, this paper makes the following contributions:

- We propose an IA scheme for each user (transmitter) and BS (receiver) in the uplink communication. At each user, we propose an approach to determine which subset of its interfering streams should be selected for alignment at a BS. At each BS, we propose a procedure for IA so that the desired data streams remain resolvable. For the proposed IA scheme, we develop a set of IA constraints for each user and BS. We also prove the feasibility of the proposed IA scheme at the PHY layer.
- Based on the proposed IA scheme, we develop a mathematical model for the uplink user throughput maximization problem. This model incorporates BS selection in the

formulation. To remove nonlinear terms in the formulation, we employ *Reformulation-Linearization Technique* (RLT). We show that the final formulation is in a form that is suitable for a commercial solver.

- We study the performance of our proposed IA scheme by solving the uplink user throughput maximization problem for different network instances. For comparison, we compare it to two other schemes: “no-IA” scheme and “crude-IA” scheme. Results from 100 network instances show that our IA scheme achieves an average 98% throughput improvement over the no-IA scheme, and an average 39% improvement over the crude-IA scheme.

The remainder of this paper is organized as follows. Section II offers some essential background on IA in cellular networks. Section III describes a user throughput maximization problem. In Section IV, we propose an IA scheme and prove its feasibility. In Section V, we incorporate BS selection into our IA scheme. In Section VI, we formulate the uplink user throughput maximization problem. In Section VII, we offer numerical results to show the benefits of the IA scheme. Section VIII concludes this paper.

II. IA IN CELLULAR NETWORKS: A PRIMER

In general, IA refers to the construction of transmit signals so that (i) they overlap at the *unintended* receivers, and (ii) they remain resolvable at the *intended* receivers. In the context of cellular networks, we consider an IA scheme in the frequency domain by mapping each transmit stream onto all of the available subcarriers. Suppose that there are K subcarriers available in the network. Then the encoding vector for each outgoing stream has a dimension of $K \times 1$. At each transmitter, one needs to design its encoding vectors to ensure that its outgoing signals overlap at their unintended receivers while remaining resolvable at their intended receivers.

An Example. Consider the uplink of a small cellular network with 2 BSs and 4 users as shown in Fig. 1. A solid arrow line represents a directed link and a dashed arrow line represents a directed interference. Each user and BS have a single antenna. The network is well-synchronized in both time and frequency domain. To show the benefits of IA, let’s start with a simple case by assuming $K = 3$. Note that we take $K = 3$ only for ease of illustration and we will consider the cases with larger value of K (as in a practical system) later in the example. For the case of $K = 3$, we will show that by using IA, a total of 4 data streams can be sent from the users to their BSs, with 1 data stream from each user. In comparison, when IA is not used, at most 3 data streams can be sent (with 1 data stream on each subcarrier), since putting more than one data stream on a subcarrier will result in collision on that subcarrier.

To show how to send 4 data streams by using IA, we first introduce some notation. For vectors \mathbf{a} and \mathbf{b} , denote $\mathbf{a} := \mathbf{b}$ if there exists a nonzero complex number c such that $\mathbf{a} = c\mathbf{b}$. Denote $\{\mathbf{u}_b^1 \cdots \mathbf{u}_b^K\}$ as a set of linearly independent basis vectors with dimension $K \times 1$ and nonzero entries. Denote \mathbf{H}_{ji} as the (frequency-domain) channel matrix between user i to

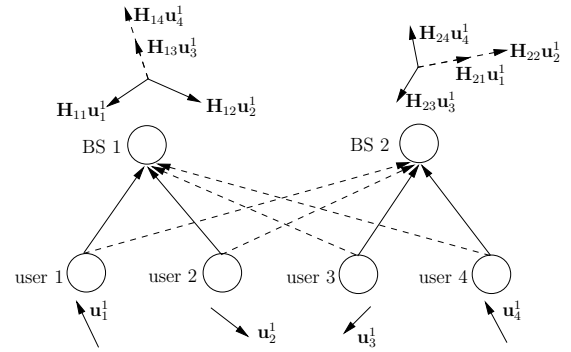


Fig. 1: An example of IA in the frequency domain.

BS j .¹ Denote \mathbf{u}_i^k as the encoding vector for the k -th outgoing stream at user i . We construct the encoding vectors at user 1 and user 2 as follows: $\mathbf{u}_1^1 := \mathbf{u}_b^1$ and $\mathbf{u}_2^1 := \mathbf{H}_{22}^{-1}\mathbf{H}_{21}\mathbf{u}_1^1$. As a result, at BS 2, the interfering stream from user 1 is aligned to the interfering stream from user 2, as shown in Fig. 1. Likewise, we construct the encoding vectors at user 3 and user 4 as follows: $\mathbf{u}_3^1 := \mathbf{u}_b^1$ and $\mathbf{u}_4^1 := \mathbf{H}_{24}^{-1}\mathbf{H}_{23}\mathbf{u}_3^1$. Then at BS 1, the interfering stream from user 3 is aligned to the interfering stream from user 4, as shown in Fig. 1. By using the above encoding vectors at the 4 users, the received data and interfering streams at each BS are on 3 different directions, as shown in Fig. 1. Therefore, 3 subcarriers are sufficient to send 4 data streams from the users to their BSs.

If the total number of subcarriers in the network is 12 (i.e., $K = 12$), 16 data streams can be sent from the users to the BSs (with 4 data streams from each user) by using IA. In comparison, when IA is not used, at most 12 data streams can be sent from the users to their BSs. If the total number of subcarriers in the network is 1024 (i.e., $K = 1024$), 1364 data streams can be sent from the users to the BSs (with 341 from each user) by using IA. In comparison, when IA is not used, at most 1024 data streams can be sent from the users to the BSs.

III. PROBLEM STATEMENT AND CHALLENGES

A. Goals and Problem Statement

The goal of this paper is to exploit the benefits of frequency-domain IA for increasing user throughput in the uplink of a cellular network. Instead of following an information theoretic approach as in [14], we are interested in addressing more practical problems, which we contrast as follows:

- The number of users served by each BS can be arbitrary (not necessary to be equal).
- A user’s transmitter only interferes with those BSs’ receivers within its interference range (rather than all BSs’ receivers).
- The number of data streams at each user can vary, depending on application requirements (rather than being identical).
- The number of total available subcarriers in the network is user-independent (e.g., $K = 256, 1024, \text{ or } 2048$).

¹ \mathbf{H}_{ji} is a $K \times K$ diagonal matrix.

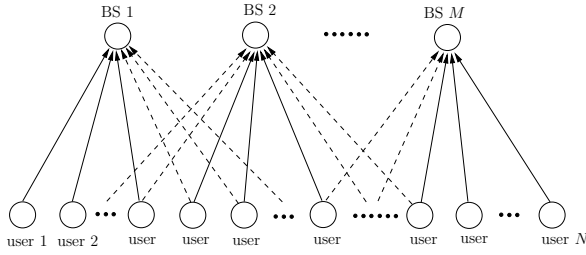


Fig. 2: The uplink transmission in a cellular network.

Note that the consideration of these practical issues is an important step to bridge the gap between results in IT to practical problems for future cellular networks. As such, new mathematical models need to be developed and new optimization problems need to be studied.

We consider the uplink of a cellular network consisting of a set of BSs and a set of users (see, e.g., Fig. 5). A user may fall into the service area of multiple BSs but can choose only one BS as its service provider. Our objective is to exploit IA in the frequency domain so as to maximize the uplink minimum throughput among all the users in a cellular network.

B. Challenges

We identify a number of challenges in the uplink user throughput maximization problem as follows:

- **IA scheme.** How to perform IA at the users is not a trivial problem, as the signal alignment behavior from a user's data stream is different at different BSs. Therefore, at each user (transmitter), one needs to determine which subset of its interfering streams should be selected for alignment at a BS within its interference range. Further, one needs to design an alignment scheme at each BS so that the desired data streams are resolvable at all BSs while the undesirable data streams can be aligned to some predefined directions whenever possible.
- **IA Feasibility.** While designing an IA scheme, one needs to ensure its feasibility at the PHY layer. This is not a trivial problem either. An improperly designed IA scheme may turn out to be infeasible at the PHY layer. To prove feasibility of an IA scheme, one needs to show that there exist an encoding vector (at a user) and a decoding vector (at the BS) for each data stream such that all of the data streams in the network can be transported free of interference.
- **BS Selection.** In a cellular network, a user may fall into the service area of multiple BSs. Since an IA scheme is tightly coupled with a user's choice of a BS, making an optimal choice of a BS (so as to maximize the objective function) is not an easy problem.

IV. AN IA SCHEME AND ITS FEASIBILITY

In this section, we develop an IA scheme for the uplink communication in a cellular network. The IA scheme includes IA constraints at each user and BS, as well as how to construct encoding and decoding vectors for each stream. In Section IV-A, we present such an IA scheme. In Section IV-B, we give a feasibility proof of this IA scheme at the PHY layer.

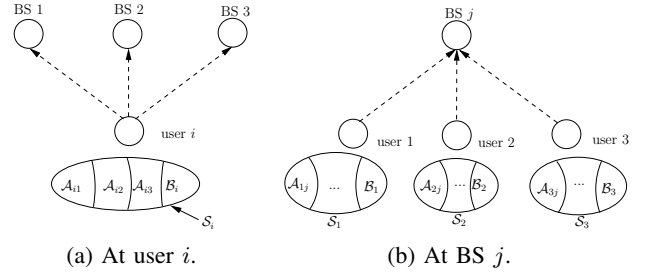


Fig. 3: An example illustrating IA constraints at user i and BS $j \in \mathcal{I}_i^{\text{bs}}$.

A. An IA Scheme

Consider a cellular network shown in Fig. 2. Each BS and user has a single antenna. Denote \mathcal{N} as the set of all users in the network and N as its cardinality (i.e., $N = |\mathcal{N}|$). Denote \mathcal{M} as the set of all BSs in the network and M as its cardinality (i.e., $M = |\mathcal{M}|$). Denote $\mathcal{T}_j^{\text{usr}}$ as the set of users who choose BS j as their service provider. Denote $\mathcal{I}_j^{\text{usr}}$ as the set of users that interfere with BS j , i.e., BS j is within the interference range of these users and BS j is not the service provider of these users. Denote $\mathcal{I}_i^{\text{bs}}$ as the set of BSs that are interfered with by user i , i.e., those BSs are within the interference range of user i but are not chosen by user i as its service provider. We assume that the channel state information (CSI) is globally available for the users and the BSs.

Suppose that user i is interfering with BS j , i.e., $j \in \mathcal{I}_i^{\text{bs}}$. Denote $\mathcal{S}_i = \{s_i^k : 1 \leq k \leq \sigma_i\}$ as the set of streams at user i , where s_i^k is the k -th stream and σ_i is the number of streams in set \mathcal{S}_i . Then each stream in \mathcal{S}_i is an interfering stream for BS j . At BS j , we hope that as many interfering streams from user i can be aligned to some predefined interference directions as possible.

Among the interfering streams in \mathcal{S}_i , denote \mathcal{A}_{ij} as the subset of interfering streams that can be aligned to some predefined interference directions at BS j . Denote α_{ij} as the cardinality of \mathcal{A}_{ij} , i.e., $\alpha_{ij} = |\mathcal{A}_{ij}|$. Among the streams in \mathcal{S}_i , there may be a subset \mathcal{B}_i of streams that are not aligned to any predefined interference direction at all BSs in $\mathcal{I}_i^{\text{bs}}$. Denote β_i as the cardinality of \mathcal{B}_i , i.e., $\beta_i = |\mathcal{B}_i|$. Thus we have

$$\mathcal{B}_i = \mathcal{S}_i \setminus (\cup_{j \in \mathcal{I}_i^{\text{bs}}} \mathcal{A}_{ij}).$$

Now we present an IA scheme, which includes three constraints and encoding/decoding vectors.

User Constraints. At user i (see Fig. 3(a) for example), there are σ_i outgoing streams, each of which is an interfering stream to all BSs in $\mathcal{I}_i^{\text{bs}}$. For each interfering stream $s_i^k \in \mathcal{S}_i$, we will construct a feasible encoding vector for this stream so that it is successfully aligned at most at *one* BS in $\mathcal{I}_i^{\text{bs}}$ (see Section IV-B). That is, our encoding vector is only required to guarantee the alignment of a stream at one BS. Based on this requirement, we define $\mathcal{A}_{ij_1} \cap \mathcal{A}_{ij_2} = \emptyset$ ($j_1, j_2 \in \mathcal{I}_i^{\text{bs}}$, $j_1 \neq j_2$). Therefore, we have following constraints at user i :

$$\beta_i + \sum_{j \in \mathcal{I}_i^{\text{bs}}} \alpha_{ij} = \sigma_i, \quad \text{for } i \in \mathcal{N}. \quad (1)$$

BS Constraints. At BS j (see Fig. 3(b) for example), we need to align the interfering streams in \mathcal{A}_{ij} (for each $i \in \mathcal{I}_j^{\text{usr}}$) to some predefined interference directions. To do this, one must answer two questions: (i) what should be the set of predefined interference directions at BS j ; (ii) how to align the interfering streams in \mathcal{A}_{ij} to the set of predefined interference directions.

There may be many possible solutions to the above two questions. Here, we show one solution for which we can offer a feasibility proof at the PHY layer (see Section IV-B). In our solution, for the first question, we use $\cup_{i \in \mathcal{I}_j^{\text{usr}}} \mathcal{B}_i$ as the set of predefined interference directions at BS j . That is, each interfering stream in \mathcal{A}_{ij} will be aligned to an interfering stream in $\cup_{i \in \mathcal{I}_j^{\text{usr}}} \mathcal{B}_i$. For the second question, we align each interfering streams in \mathcal{A}_{ij} for each $i \in \mathcal{I}_j^{\text{usr}}$ to a unique interference stream in $\cup_{k \in \mathcal{I}_j^{\text{usr}}} \mathcal{B}_k$. That is, each interfering streams in \mathcal{A}_{ij} is aligned uniquely into the interference subspace formed by the union of \mathcal{B}_k over $k \in \mathcal{I}_j^{\text{usr}}$ except its own \mathcal{B}_i . Here, “uniquely” refers that any two interfering streams in \mathcal{A}_{ij} will not be aligned to the same interfering stream in $\cup_{k \in \mathcal{I}_j^{\text{usr}}} \mathcal{B}_k$. Based on our proposed solution to questions (i) and (ii), we have the following constraints at BS j :

$$\alpha_{ij} \leq \sum_{\substack{k \neq i \\ k \in \mathcal{I}_j^{\text{usr}}}} \beta_k, \quad \text{for } i \in \mathcal{I}_j^{\text{usr}}, j \in \mathcal{M}. \quad (2)$$

Dimension Constraints. At BS j , the total number of its desired data streams is $\sum_{i \in \mathcal{I}_j^{\text{usr}}} \sigma_i$, while the number of its unaligned interfering streams is $\sum_{i \in \mathcal{I}_j^{\text{usr}}} (\sigma_i - \alpha_{ij})$. Since the number of directions for desired data streams and unaligned interfering streams cannot exceed the number of available subcarriers, we have the following constraints at BS j :

$$\sum_{i \in \mathcal{I}_j^{\text{usr}}} \sigma_i + \sum_{i \in \mathcal{I}_j^{\text{usr}}} (\sigma_i - \alpha_{ij}) \leq K \quad \text{for } j \in \mathcal{M}. \quad (3)$$

In the rest of section, we show the feasibility of this IA scheme at the PHY layer by constructing an encoding/decoding vector for each data stream so that each data stream can be transported free of interference.

B. Feasibility of the IA Scheme

Denote \mathbf{H}_{ji} as the channel matrix between user i and BS j over K subcarriers. \mathbf{H}_{ji} is a diagonal complex matrix with the k -th diagonal entry representing the channel coefficient of the k -th subcarrier. For each stream s_i^k , denote $\mathbf{u}_i^k \in \mathbb{C}^{K \times 1}$ as its encoding vector at user i and $\mathbf{v}_j^l \in \mathbb{C}^{K \times 1}$ as its decoding vector at its intended BS j . Denote π as an IA scheme that meets the constraints in (1), (2), and (3), with corresponding encoding vector \mathbf{u}_i^k and decoding vector \mathbf{v}_j^l for each stream s_i^k ($i \in \mathcal{N}$, $1 \leq k \leq \sigma_i$).

To decode data stream s_i^k at BS j (zero-forcing), decoding vector \mathbf{v}_j^l should be able to filter out interfering streams from two types of users. The first type is the interfering users who do not choose BS j as their service provider, i.e., for $i' \in \mathcal{I}_j^{\text{usr}}$, $(\mathbf{v}_j^l)^T \mathbf{H}_{ji'} \mathbf{u}_{i'}^{k'} = 0$ holds for $1 \leq k' \leq \sigma_{i'}$. The second type is the users who choose BS j as their service provider, i.e., for

$i' \in \mathcal{I}_j^{\text{usr}}$ and $(i', k') \neq (i, k)$, $(\mathbf{v}_j^l)^T \mathbf{H}_{ji'} \mathbf{u}_{i'}^{k'} = 0$ holds for $1 \leq k' \leq \sigma_{i'}$. More formally, we have the following definition:

Definition 1: An IA scheme π is feasible at the PHY layer if for $i' \in (\mathcal{I}_j^{\text{usr}} \cup \mathcal{I}_j^{\text{usr}})$ and $(i', k') \neq (i, k)$,

$$(\mathbf{v}_j^l)^T \mathbf{H}_{ji} \mathbf{u}_i^k = 1, \quad (4)$$

$$(\mathbf{v}_j^l)^T \mathbf{H}_{ji'} \mathbf{u}_{i'}^{k'} = 0, \quad (5)$$

hold for $1 \leq k' \leq \sigma_{i'}$.

The following theorem is the main result of the feasibility.

Theorem 1: There exists at least one set of encoding and decoding vectors such that IA scheme π is feasible at the PHY layer.

The rest of this section will be devoted to a proof of this theorem. Here is a road map. Our proof is based on construction. First, we construct an encoding vector for each stream. Then we give two lemmas characterizing the dimensions of such encoding vectors. Based on these lemmas, we show that there always exists an decoding vector for each stream such that constraints (4) and (5) in Definition 1 are satisfied.

Encoding Vector Construction. Denote $\mathcal{E}^{S_i} = \{\mathbf{u}_i^k : 1 \leq k \leq \sigma_i\}$ as the set of encoding vectors for the set of streams S_i at user i . Among the encoding vectors in \mathcal{E}^{S_i} , denote $\mathcal{E}^{A_{ij}}$ as the subset of encoding vectors that correspond to the interfering streams in \mathcal{A}_{ij} ; denote \mathcal{E}^{B_i} as the subset of encoding vectors that correspond to the interfering streams in \mathcal{B}_i . Since we define a unique encoding vector for each stream, we have

$$|\mathcal{E}^{A_{ij}}| = \alpha_{ij} \quad \text{for } j \in \mathcal{M}, i \in \mathcal{I}_j^{\text{usr}};$$

$$|\mathcal{E}^{B_i}| = \beta_i \quad \text{for } i \in \mathcal{N};$$

$$\mathcal{E}^{B_i} = \mathcal{E}^{S_i} \setminus (\cup_{j \in \mathcal{I}_i^{\text{bs}}} \mathcal{E}^{A_{ij}}) \quad \text{for } i \in \mathcal{N};$$

$$\mathcal{E}^{A_{ij_1}} \cap \mathcal{E}^{A_{ij_2}} = \emptyset, \quad \text{for } i \in \mathcal{N}, j_1, j_2 \in \mathcal{I}_i^{\text{bs}}, j_1 \neq j_2.$$

We define $\mathcal{E}^A = \cup_{i \in \mathcal{N}, j \in \mathcal{I}_i^{\text{bs}}} \mathcal{E}^{A_{ij}}$ and $\mathcal{E}^B = \cup_{i \in \mathcal{N}} \mathcal{E}^{B_i}$. Then we have $\cup_{i \in \mathcal{N}} \mathcal{E}^{S_i} = \mathcal{E}^A \cup \mathcal{E}^B$. We first construct the encoding vectors in \mathcal{E}^B and then construct the encoding vectors in \mathcal{E}^A .

Denote $\{\mathbf{u}_b^k : 1 \leq k \leq K\}$ as a set of *linear independent* complex vectors with dimension $K \times 1$ and *nonzero* entries. Then, for each $\mathbf{u}_i^k \in \mathcal{E}^B$, we construct it by having

$$\mathbf{u}_i^k := \mathbf{u}_b^k. \quad (6)$$

Now we construct the encoding vectors in \mathcal{E}^A . Recall that in IA scheme π , each interfering stream in \mathcal{A}_{ij} is aligned to an interfering stream in $\cup_{k \in \mathcal{I}_j^{\text{usr}}} \mathcal{B}_k$. Therefore, for each $\mathbf{u}_i^k \in \mathcal{E}^A$, we construct it by having

$$\mathbf{u}_i^k := \mathbf{H}_{ji}^{-1} \mathbf{H}_{ji'} \mathbf{u}_{i'}^{k'}, \quad (7)$$

where $\mathbf{u}_{i'}^{k'}$ is an encoding vector in \mathcal{E}^B (i.e., $\mathbf{u}_{i'}^{k'} := \mathbf{u}_b^{k'}$) and $i' \neq i$.

Encoding Vector Properties. Denote $\dim(\mathcal{E}^{S_i})$ as the dimension of the subspace spanned by the vectors in \mathcal{E}^{S_i} . Then we have the following lemma.

Lemma 1: At each user $i \in \mathcal{N}$, the constructed encoding vectors in \mathcal{E}^{S_i} are linearly independent, i.e., $\dim(\mathcal{E}^{S_i}) =$

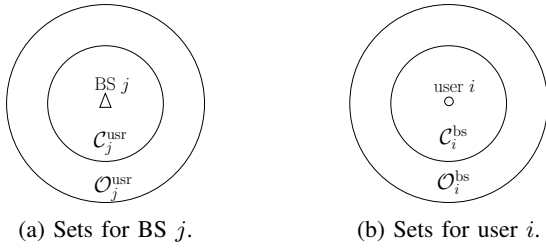


Fig. 4: Sets illustration at BS j and user i .

$|\mathcal{E}^{S_i}|$.

We give a sketch of our proof here. A complete proof of this lemma is given in [19]. First, since the encoding vectors in \mathcal{E}^{B_i} are independent of any channel while the encoding vectors in $\mathcal{E}^{A_{ij}}$ are dependent on the channel, we have $\dim(\mathcal{E}^{S_i}) = \dim(\mathcal{E}^{B_i}) + \dim(\cup_{j \in \mathcal{I}_i^{\text{bs}}} \mathcal{E}^{A_{ij}})$. Second, since the channel matrices in $\{\mathbf{H}_{ji} : j \in \mathcal{I}_i^{\text{bs}}\}$ are random and thus independent of each other, we have $\dim(\cup_{j \in \mathcal{I}_i^{\text{bs}}} \mathcal{E}^{A_{ij}}) = \sum_{j \in \mathcal{I}_i^{\text{bs}}} \dim(\mathcal{E}^{A_{ij}})$. Third, based on the construction procedure, we have $\dim(\mathcal{E}^{B_i}) = |\mathcal{E}^{B_i}|$ and $\dim(\mathcal{E}^{A_{ij}}) = |\mathcal{E}^{A_{ij}}|$. Therefore, we conclude that $\dim(\mathcal{E}^{S_i}) = |\mathcal{E}^{S_i}|$.

At BS j , denote \mathcal{Q}_j^{T} as the set of directions for its desired data streams and \mathcal{Q}_j^{I} as the set of directions for its interfering streams. We have

$$\begin{aligned} \mathcal{Q}_j^{\text{T}} &= \cup_{i \in \mathcal{I}_j^{\text{usr}}} \{\mathbf{H}_{ji} \mathbf{u}_i^k : \mathbf{u}_i^k \in \mathcal{E}^{S_i}\}, \\ \mathcal{Q}_j^{\text{I}} &= \cup_{i \in \mathcal{I}_j^{\text{usr}}} \{\mathbf{H}_{ji} \mathbf{u}_i^k : \mathbf{u}_i^k \in \mathcal{E}^{S_i}\}. \end{aligned}$$

Then, we have the following lemma.

Lemma 2: At each BS $j \in \mathcal{M}$, each of its desired data stream occupies an independent direction, i.e.,

$$\dim(\mathcal{Q}_j^{\text{T}} \cup \mathcal{Q}_j^{\text{I}}) = \sum_{i \in \mathcal{I}_j^{\text{usr}}} \sigma_i + \dim(\mathcal{Q}_j^{\text{I}}), \text{ for } j \in \mathcal{M}. \quad (8)$$

We give a proof sketch here. A complete proof of this lemma is given in [19]. Denote $\mathcal{Q}_j^{\text{I, Eff}} = \cup_{i \in \mathcal{I}_j^{\text{usr}}} \{\mathbf{H}_{ji} \mathbf{u}_i^k : \mathbf{u}_i^k \in \mathcal{E}^{S_i} \setminus \mathcal{E}^{A_{ij}}\}$. Then we have $\text{span}(\mathcal{Q}_j^{\text{I}}) = \text{span}(\mathcal{Q}_j^{\text{I, Eff}})$. Since the channel matrices $\{\mathbf{H}_{ji} : i \in \mathcal{I}_j^{\text{usr}} \cup \mathcal{I}_j^{\text{bs}}\}$ are random and thus independent of each other, we have $\dim(\mathcal{Q}_j^{\text{T}} \cup \mathcal{Q}_j^{\text{I, Eff}}) = \dim(\mathcal{Q}_j^{\text{T}}) + \dim(\mathcal{Q}_j^{\text{I, Eff}})$ and $\dim(\mathcal{Q}_j^{\text{T}}) = \sum_{i \in \mathcal{I}_j^{\text{usr}}} \sigma_i$. Therefore, we conclude $\dim(\mathcal{Q}_j^{\text{T}} \cup \mathcal{Q}_j^{\text{I}}) = \sum_{i \in \mathcal{I}_j^{\text{usr}}} \sigma_i + \dim(\mathcal{Q}_j^{\text{I}})$.

Decoding Vector. For the decoding vectors, we have the following proposition:

Proposition 1: If the encoding vectors satisfy (8), then there exists a decoding vector for each stream such that constraints (4) and (5) are satisfied.

A proof of Proposition 1 is given in Appendix A. This completes the proof of Theorem 1.

V. BS SELECTION AND ITS IMPACT ON IA

As stated earlier, a user may be within the service area of multiple BSs and thus must choose one BS as its service provider. For each user, how to choose a BS as its service provider is a part of our optimization problem.

As shown in Fig. 4(a), denote $\mathcal{C}_j^{\text{usr}}$ as the set of users within the service area of BS j ; denote $\mathcal{O}_j^{\text{usr}}$ as the set of users that

are outside the service area of BS j but can still interfere with BS j . As shown in Fig. 4(b), denote $\mathcal{C}_i^{\text{bs}}$ as the set of BSs that user i can choose as its service provider; denote $\mathcal{O}_i^{\text{bs}}$ as the set of BSs whose service areas do not cover user i but are still inside the interference range of user i .

BS Selection. Denote x_{ij} as a binary variable to indicate whether or not user i chooses BS $j \in \mathcal{C}_i^{\text{bs}}$ as its service provider, i.e., $x_{ij} = 1$ if user i chooses BS j and 0 otherwise. Since user i can choose at most one BS as its service provider, we have

$$\sum_{j \in \mathcal{C}_i^{\text{bs}}} x_{ij} = 1, \quad i \in \mathcal{N}. \quad (9)$$

Denote y_{ij} as the complementary binary variable of x_{ij} . That is, $y_{ij} = 1$ if user i does *not* choose BS $j \in \mathcal{C}_i^{\text{bs}}$ as its service provider and 0 otherwise. Then we have the following constraints:

$$x_{ij} + y_{ij} = 1, \quad j \in \mathcal{C}_i^{\text{bs}}, i \in \mathcal{N}. \quad (10)$$

Impact on IA. We now show that the above BS selection variables can be incorporated into (1), (2), and (3) in our IA scheme.

- To incorporate BS selection variables in (1), we need to first clarify $\mathcal{I}_i^{\text{bs}}$, i.e., the set of BSs that are interfered with by user i . Based on the definitions of $\mathcal{O}_i^{\text{bs}}$, $\mathcal{C}_i^{\text{bs}}$, and y_{ij} , we have

$$\mathcal{I}_i^{\text{bs}} = \mathcal{O}_i^{\text{bs}} \cup \{j : y_{ij} = 1, j \in \mathcal{C}_i^{\text{bs}}\}.$$

Then, (1) can be re-written as:

$$\beta_i + \sum_{j \in \mathcal{O}_i^{\text{bs}}} \alpha_{ij} + \sum_{j \in \mathcal{C}_i^{\text{bs}}} \alpha_{ij} \cdot y_{ij} = \sigma_i, \quad i \in \mathcal{N}. \quad (11)$$

- Likewise, for (2), we need to first clarify $\mathcal{I}_j^{\text{usr}}$, i.e., the set of users that are interfering with BS j . Based on the definitions of $\mathcal{O}_j^{\text{usr}}$, $\mathcal{C}_j^{\text{usr}}$, and y_{ij} , we have

$$\mathcal{I}_j^{\text{usr}} = \mathcal{O}_j^{\text{usr}} \cup \{i : y_{ij} = 1, i \in \mathcal{C}_j^{\text{usr}}\}. \quad (12)$$

Depending on whether user $i \in \mathcal{O}_j^{\text{usr}}$ or $i \in \mathcal{C}_j^{\text{usr}}$, (2) can be re-written as:

$$\alpha_{ij} \leq \sum_{k \in \mathcal{O}_j^{\text{usr}}, k \neq i} \beta_k + \sum_{k \in \mathcal{C}_j^{\text{usr}}} \beta_k \cdot y_{kj}, \quad i \in \mathcal{O}_j^{\text{usr}}, j \in \mathcal{M}, \quad (13)$$

$$\alpha_{ij} \cdot y_{ij} \leq \sum_{k \in \mathcal{O}_j^{\text{usr}}} \beta_k + \sum_{k \in \mathcal{C}_j^{\text{usr}}, k \neq i} \beta_k \cdot y_{kj}, \quad i \in \mathcal{C}_j^{\text{usr}}, j \in \mathcal{M}, \quad (14)$$

- Finally, for (3), we need to first clarify $\mathcal{T}_j^{\text{usr}}$, i.e., the set of users that choose BS j as their service provider. Based on the definitions of $\mathcal{C}_j^{\text{usr}}$ and x_{ij} , we have

$$\mathcal{T}_j^{\text{usr}} = \{i : x_{ij} = 1, i \in \mathcal{C}_j^{\text{usr}}\}.$$

Then, (3) can be re-written as:

$$\sum_{i \in \mathcal{C}_j^{\text{usr}}} \sigma_i \cdot x_{ij} + \sum_{i \in \mathcal{I}_j^{\text{usr}}} (\sigma_i - \alpha_{ij}) \leq K \quad j \in \mathcal{M}.$$

which is equivalent to

$$\sum_{i \in \mathcal{C}_j^{\text{usr}}} \sigma_i \cdot x_{ij} + \sum_{i \in \mathcal{C}_j^{\text{usr}}} (\sigma_i - \alpha_{ij}) \cdot y_{ij} + \sum_{i \in \mathcal{O}_j^{\text{usr}}} (\sigma_i - \alpha_{ij}) \leq K, \quad j \in \mathcal{M}, \quad (15)$$

based on $\mathcal{I}_j^{\text{usr}}$ in (12).

VI. USER THROUGHPUT MAXIMIZATION PROBLEM

In this section, we employ the IA scheme in Section IV-A to study an uplink user throughput maximization problem in a cellular network. For simplicity, we assume that fixed modulation and coding scheme (MCS) is used for each data stream and that each data stream corresponds to one unit data rate. The goal is to maximize the minimum rate among all the users. Denote r_{\min} as the minimum rate among all users. Then we have:

$$\sigma_i \geq r_{\min}, \quad i \in \mathcal{N}. \quad (16)$$

Based on the constraints in Section V, the user throughput maximization problem can be formulated as follows:

$$\begin{aligned} \text{OPT-IA}^{\text{raw}} : \quad & \text{Max } r_{\min} \\ \text{s.t.} \quad & \text{BS selection: (9), (10);} \\ & \text{IA constraints: (11), (13), (14), (15);} \\ & \text{Minimum rate constraints: (16).} \end{aligned}$$

OPT-IA^{raw} is a mixed integer nonlinear programming (MINLP). To eliminate the nonlinear terms in the constraints, we employ the *Reformulation-Linearization Technique (RLT)* in [13].

To eliminate the nonlinear term $\alpha_{ij} \cdot y_{ij}$ in the constraints, we define $\lambda_{ij} = \alpha_{ij} \cdot y_{ij}$. This replacement requires to add the following two constraints:

$$0 \leq \lambda_{ij} \leq \alpha_{ij}, \quad j \in \mathcal{C}_i^{\text{bs}}, i \in \mathcal{N}, \quad (17)$$

$$\alpha_{ij} - (1 - y_{ij}) \cdot K \leq \lambda_{ij} \leq y_{ij} \cdot K, \quad j \in \mathcal{C}_i^{\text{bs}}, i \in \mathcal{N}. \quad (18)$$

Similarly, to eliminate the nonlinear term $\beta_i \cdot y_{ij}$ in the constraints, we define $\mu_{ij} = \beta_i \cdot y_{ij}$. This replacement requires to add the following two constraints:

$$0 \leq \mu_{ij} \leq \beta_i, \quad j \in \mathcal{C}_i^{\text{bs}}, i \in \mathcal{N}, \quad (19)$$

$$\beta_i - (1 - y_{ij}) \cdot K \leq \mu_{ij} \leq y_{ij} \cdot K, \quad j \in \mathcal{C}_i^{\text{bs}}, i \in \mathcal{N}. \quad (20)$$

By replacing $\lambda_{ij} = \alpha_{ij} \cdot y_{ij}$ and $\mu_{ij} = \beta_i \cdot y_{ij}$ in the IA constraints (11), (13), (14), (15), we have the following linear IA constraints:

$$\beta_i + \sum_{j \in \mathcal{O}_i^{\text{bs}}} \alpha_{ij} + \sum_{j \in \mathcal{C}_i^{\text{bs}}} \lambda_{ij} = \sigma_i, \quad i \in \mathcal{N}, \quad (21)$$

$$\alpha_{ij} \leq \sum_{k \in \mathcal{O}_j^{\text{usr}}, k \neq i} \beta_k + \sum_{k \in \mathcal{C}_j^{\text{usr}}} \mu_{kj}, \quad i \in \mathcal{O}_j^{\text{usr}}, j \in \mathcal{M}, \quad (22)$$

$$\lambda_{ij} \leq \sum_{k \in \mathcal{O}_j^{\text{usr}}} \beta_k + \sum_{k \in \mathcal{C}_j^{\text{usr}}, k \neq i} \mu_{kj}, \quad i \in \mathcal{C}_j^{\text{usr}}, j \in \mathcal{M}, \quad (23)$$

$$\sum_{i \in \mathcal{C}_j^{\text{usr}}} (\sigma_i - \lambda_{ij}) + \sum_{i \in \mathcal{O}_j^{\text{usr}}} (\sigma_i - \alpha_{ij}) \leq K, \quad j \in \mathcal{M}. \quad (24)$$

Then, OPT-IA^{raw} can be reformulated as follows:

$$\begin{aligned} \text{OPT-IA:} \quad & \text{Max } r_{\min} \\ \text{s.t.} \quad & \text{BS selection: (9), (10),} \\ & \text{IA constraints: (17), (18), (19), (20),} \\ & \quad \quad \quad (21), (22), (23), (24), \\ & \text{Minimum rate constraints: (16),} \end{aligned}$$

where \mathcal{N} , \mathcal{M} , $\mathcal{C}_i^{\text{bs}}$, $\mathcal{O}_i^{\text{bs}}$, $\mathcal{C}_j^{\text{usr}}$, $\mathcal{O}_j^{\text{usr}}$, and K are known; x_{ij} and y_{ij} are binary variables; r_{\min} , σ_i , α_{ij} , β_i , λ_{ij} , and μ_{ij} are non-negative integer variables.

OPT-IA is a mixed integer linear programming (MILP). Although the theoretical worst-case complexity to a general MILP problem is exponential [2], [11], there exist highly efficient optimal/approximation algorithms (e.g., branch-and-bound with cutting planes [12]) and heuristics (e.g., sequential fixing algorithm [5], [6]). Another approach is to apply an off-the-shelf solver (CPLEX [20]), which can successfully handle a moderate-sized network. We will adopt this approach as it is sufficient to serve our purpose in this paper.

VII. PERFORMANCE EVALUATION

In this section, we use a case study to illustrate how IA scheme works in a cellular network to maximize uplink user throughput. We also compare the user throughput performance of our IA scheme to two other schemes: “no-IA” scheme and “crude-IA” scheme. In the no-IA scheme, a subset of subcarriers is allocated to each user for its data transmission such that at each BS, each data or interfering stream occupies a *unique* subcarrier. That is, there is a complete absence of overlapping of interfering streams on any subcarrier. We denote the user throughput maximization problem under the no-IA scheme as OPT-noIA and its formulation is given in [19]. In the crude-IA scheme, a subset of subcarriers is allocated to each user for its data transmission such that at a BS, each of its desired data streams is on a unique subcarrier while the interfering streams are allowed to overlap. This problem is similar to ours except that each data stream in our IA scheme occupies *all* subcarriers and there is an optimization on the design of directions for intended data streams and interfering data streams. In light of this key difference, we denote the user throughput maximization problem under the crude-IA scheme as OPT-crudeIA and its formulation is given in [19].

A. Simulation Setting

For ease of exposition, we normalize all units for distance, time, bandwidth, and data rate with appropriate dimensions. We consider a cellular network with 9 BSs and 100 users within a 1000×1000 area (see Fig. 5 for example). We divide the whole area into 9 equal-sized grids and deploy the BSs at the center of the grids. The 100 users are randomly distributed in the area with a uniform probability. A user can be in “active” or “inactive” state, with equal probability. When

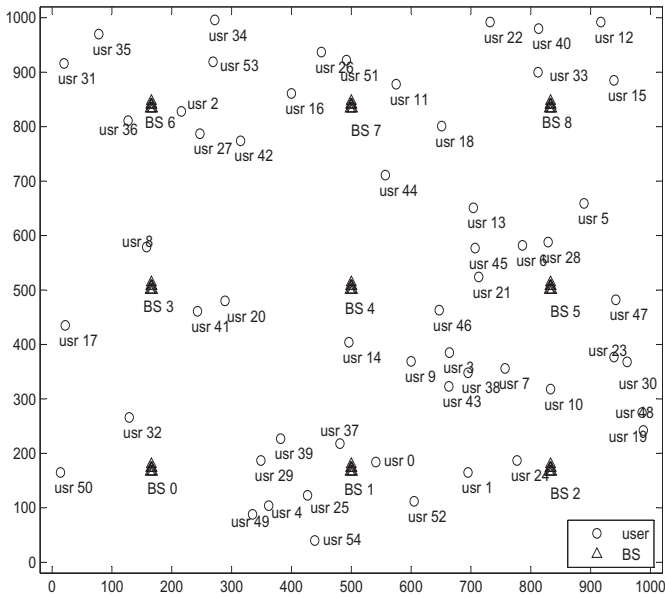


Fig. 5: Network topology in the case study.

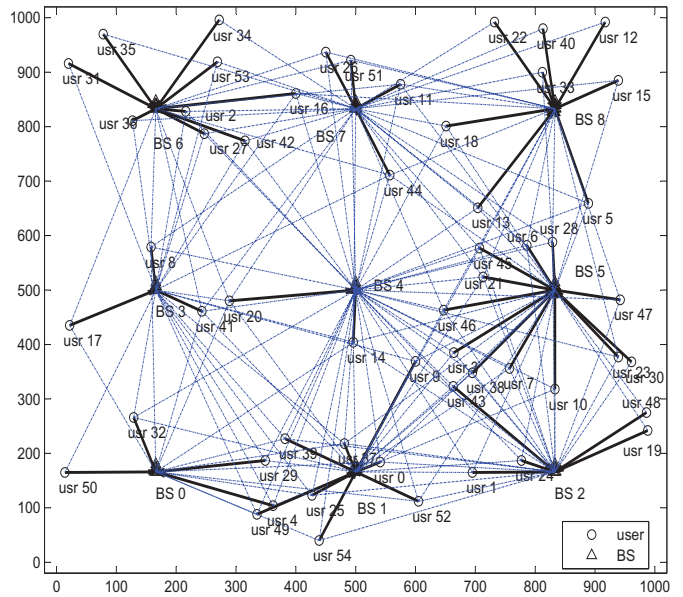


Fig. 6: BS selection and interference of each user.

“active”, a user has a persistent traffic for transmission; when “inactive”, a user is not served by any BS. For any comparison study (or “instance”), the state of a user is the same under all three schemes. We assume that the transmission range and interference range of a user are 240 and 480, respectively. The total number of subcarriers in the network is 256.

B. A Case Study

We first show the results for one network instance with 55 active users in Fig. 5 (the other 45 inactive users are not shown in this figure). By solving the OPT-IA problem for this network instance, we obtain the optimal objective value of 13. We then solve the OPT-noIA problem for this network instance, and we obtain the optimal objective value of 6. This indicates that our IA scheme can increase the user throughput by 117% when compared to the no-IA scheme. We also solve the OPT-crudeIA problem for this network instance, and we obtain the optimal objective value of 9. This indicates that our IA scheme can increase the user throughput by 44% when compared to the crude-IA scheme.

We now give some details in the solution to the OPT-IA problem. Fig. 6 shows the BS selection by each user and interference by the users on each BS. In this figure, a solid arrow line represents an established link from a user to its chosen BS and a dashed line represents an interference. Table I summarizes the IA behavior at each BS. In this table, the first column lists the BSs in the network; the second column lists the number of users that choose this BS as their service provider; the third column lists the number of desired data streams at this BS, where each user has 13 data streams to its BS; the fourth column lists the dimension of the subspace for the interfering streams at this BS, which is 256 minus the number in the third column; the fifth column lists the number of undesired interfering streams (from neighboring interfering users) at this BS; the sixth column lists the interference

overlapping ratio, which is the ratio of the fifth column to the fourth column. In the sixth column, a value greater than 1 indicates the existence of interference overlapping. The large the ratio is, the more IA has been achieved at the corresponding BS.

Now let’s take a look at the row for BS 5 in Table I as an example. As shown in Fig. 5, BS 5 is used as service provider by 12 users. Since each user has 13 outgoing data streams, the number of desired data streams at BS 5 is 156. Thus, the dimension of the subspace for the interfering streams is upper bounded by 100 ($= 256 - 156$). As shown in Fig. 5, BS 5 is being interfered by 17 users and thus has 221 ($= 17 \times 13$) interfering streams. Therefore, the interference overlapping ratio at BS 5 is $221/100 = 2.21$ (as shown in the table).

C. Complete Simulation Results

We have also done comparison study over 100 network instances, where each network instance represents a unique active/inactive behavior among the 100 users. The results for 100 network instances are given in [19]. The results show that our IA scheme has an average 98% throughput improvement over the no-IA scheme, and an average 39% throughput improvement over the crude-IA scheme.

VIII. CONCLUSIONS

This paper advances the state-of-the-art on IA for cellular networks. We developed an IA scheme for cellular networks by relaxing a number of unrealistic assumptions made by researchers in the information theory community. Specifically, our IA scheme allows heterogeneous data streams from each user, finite number of subcarriers, different number of users for each BS, and asymmetric interference pattern between user and BS. We proved the feasibility of our IA scheme by constructing the encoding and decoding vectors for each data stream so that each data stream in the network can be

TABLE I: IA behavior at each BS in the case study.

BS j	# of users	# of data streams	Dimension of interfering streams	# of interfering streams	Interference overlapping ratio
BS 0	4	52	204	156	0.76
BS 1	8	104	152	208	1.37
BS 2	5	65	191	260	1.36
BS 3	3	39	217	247	1.14
BS 4	2	26	230	494	2.15
BS 5	12	156	100	221	2.21
BS 6	9	117	139	104	0.75
BS 7	4	52	204	325	1.59
BS 8	8	104	152	156	1.03

transported free of interference. Based on the proposed IA scheme, we studied an uplink user throughput maximization problem and demonstrated its throughput improvement over some other schemes.

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APPENDIX A PROOF OF PROPOSITION 1

We show that if the encoding vectors satisfy constraint (8), then there exists a set of decoding vectors that satisfy (4) and (5) in Definition 1. Specifically, we argue that if constraint (8) is satisfied, then the following linear system is consistent (i.e., the system has at least one feasible solution).

$$\begin{aligned} (\mathbf{v}_j^l)^T \mathbf{H}_{ji} \mathbf{u}_i^k &= 1, \\ (\mathbf{v}_j^l)^T \mathbf{H}_{ji'} \mathbf{u}_{i'}^{k'} &= 0, \quad 1 \leq k' \leq \sigma_{i'}, i' \in \mathcal{T}_j^{\text{usr}} \cup \mathcal{I}_j^{\text{usr}}, (i', k') \neq (i, k) \end{aligned}$$

where \mathbf{v}_j^l has variable elements while \mathbf{H} 's and \mathbf{u} 's are given.

Based on the definition of \mathcal{Q}_j^T and \mathcal{Q}_j^I , we know

$$\mathcal{Q}_j^T \cup \mathcal{Q}_j^I = \{\mathbf{H}_{ji'} \mathbf{u}_{i'}^{k'} : i' \in \mathcal{T}_j^{\text{usr}} \cup \mathcal{I}_j^{\text{usr}}, 1 \leq k' \leq \sigma_{i'}\}.$$

It is easy to see that $\mathcal{Q}_j^T \cup \mathcal{Q}_j^I$ is the set of coefficient-vectors of this linear system. Moreover, this system has K free variables and at most K linearly independent equations. If we can show that vector $\mathbf{H}_{ji} \mathbf{u}_i^k$ is not a linear combination of other vectors in $\mathcal{Q}_j^T \cup \mathcal{Q}_j^I$, then this system is consistent. We prove this point by contradiction as follows.

Suppose that $\mathbf{H}_{ji} \mathbf{u}_i^k$ is a linear combination of the other vectors in $\mathcal{Q}_j^T \cup \mathcal{Q}_j^I$. Since $\mathbf{H}_{ji} \mathbf{u}_i^k \in \mathcal{Q}_j^T$, we have

$$\dim(\mathcal{Q}_j^T \cup \mathcal{Q}_j^I) < |\mathcal{Q}_j^T| + \dim(\mathcal{Q}_j^I) = \sum_{i \in \mathcal{T}_j^{\text{usr}}} \sigma_i + \dim(\mathcal{Q}_j^I).$$

But this contradicts (8), which is given *a priori*. Therefore, we conclude that the linear system is consistent. This completes the proof.

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