Toward Transparent Coexistence for Multi-hop Secondary Cognitive Radio Networks

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***Abstract*—The dominate spectrum sharing paradigm of today is interference avoidance, where a secondary network can use the spectrum only when such a use is not interfering with the primary network. However, with the advances of physical layer technologies, the mindset of this paradigm is being challenged. This paper explores a new paradigm called "transparent coexistence" for spectrum sharing between primary and secondary nodes in a multi-hop network environment. Under this paradigm, the secondary network is allowed to use the same spectrum simultaneously with the primary network as long as their activities are "transparent" (or "invisible") to the primary network. Such transparency is accomplished through a systematic interference cancelation (IC) by the secondary nodes without any impact on the primary network. Although such a paradigm has been studied in the information theory (IT) and communications (COMM) communities, it is not well understood in the wireless networking community, particularly for multi-hop networks. This paper offers an in-depth study of this paradigm in a multi-hop network environment and addresses issues such as scheduling (both in frequency channels and time slots) and IC (to/from primary network and within the secondary network). Through a rigorous modeling and formulation, problem formulation, solution development, and simulation results, we show that transparent coexistence paradigm offers significant improvement in terms of spectrum access and throughput performance as compared to the current prevailing interference avoidance paradigm.**

***Index Terms*—Spectrum sharing; coexistence; underlay; cognitive radio; multi-hop network; MIMO; interference cancelation**

I. Introduction

Recent push by the government agencies to share federal government radio spectrum with non-government entities has fueled the development of innovative technologies for spectrum sharing [12]. The current prevailing spectrum-sharing paradigm is that secondary nodes (typically equipped with cognitive radios (CRs)) are allowed to use a spectrum channel allocated to the primary nodes only when such a use will not cause interference to the primary nodes [1], [5], [8], [13]. This is also called "interweave" paradigm in [6], which we call *interference avoidance* paradigm in this paper. Under this paradigm, the wireless networking community has invested significant research efforts in algorithm design and protocol implementation to optimize secondary CR users' performance while ensuring that their activities will not interfere with the primary users.

On the other hand, in the information theory (IT) community, there is a strong interest in exploring information theoretic limit of CR [6]. In particular, researchers have been exploring the potential of *simultaneous activation* of a secondary network with the primary network, as long as the interference produced by secondary nodes can be properly "controlled" (e.g., canceled) by the secondary nodes. Here, secondary nodes are allowed to access the spectrum as long as they can cancel their interference to the primary nodes in such a way that the primary nodes do not feel the presence of the secondary nodes. In other words, activities by the secondary nodes are made transparent (or "invisible") to the primary nodes. We call this *transparent coexistence* paradigm in this paper.[1] Under this paradigm, secondary nodes are assumed to have powerful (physical layer) capabilities to perform interference cancelation (IC), thereby, allowing them to access the spectrum in a much more aggressive manner than the interference avoidance paradigm.

Although the idea of the transparent coexistence paradigm has been explored in the IT community, results from the IT and communications (COMM) communities have mainly limited to very simple network settings, e.g., several nodes or link pairs, all for *single-hop* communications [2], [7], [11], [21], [22]. The more difficult problem of how transparent coexistence can be achieved in a *multi-hop* secondary network remains open. As shown in [8], [13], the problem complexity associated with multi-hop CR networks is much higher than single-hop CR networks. To date, there are no prior results on transparent coexistence for a multi-hop CR networks.

The goal of this paper is to advance the theoretical foundation of transparent coexistence paradigm for a multi-hop secondary CR network. We study how a multi-hop secondary CR network can co-exist with a primary network transparently. For IC, we assume that each secondary node is equipped with multiple transmit/receive antennas (MIMO).[2] For a set of

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[1] This is also called "underlay" paradigm in [6].

[2] Other IC techniques may also be employed and will be explored in our future studies.

channels owned by the primary networks, the primary nodes may use them in whatever manner to suit their needs. On the other hand, the secondary nodes are only allowed to use these channels if they can cancel their interference to the primary nodes. Further, to ensure successful transmission among the secondary nodes, the secondary nodes also need to perform IC to/from the primary nodes as well as potential interference among the secondary nodes. Simply put, all IC burden should rest solely on the secondary nodes and remain invisible to the primary nodes. For this paradigm, we offer a mathematical modeling of channel/time slot scheduling, IC between primary and secondary nodes, and IC within the secondary network. Based on this model, we study a throughput maximization problem (with the objective of maximizing the minimum throughput among all sessions in the secondary network) without any impact on the primary users. Since the problem has a mixed-integer linear program (MILP) formulation, we develop an efficient solution based on a *sequential fixing* (SF) technique. Through simulation results, we demonstrate how the transparent coexistence paradigm can offer much improved spectrum access and throughput performance than the current interference avoidance paradigm.

The remainder of this paper is organized as follows. In Section II, we give essential background on how IC may be performed by MIMO. Section III describes our problem and key challenges. In Section IV, we present a mathematical model for the transparent coexistence paradigm where both the primary and secondary networks are multi-hop. Based on this model, in Section V, we study a throughput maximization problem and presents an efficient solution algorithm. Section VI presents simulation results and demonstrates the significant improvement in spectrum access and throughput performance under the transparent coexistence paradigm. Section VII concludes this paper and discusses the further work.

II. Background and Motivation

We give a brief review of MIMO in terms of its spatial multiplexing (SM) and IC capabilities [4], [10], [17], [18]. Other capabilities such as spatial diversity [23] and interference alignment [20] are not explored in this paper and will be considered in our future work.

A simple representation of MIMO can be built upon the so-called degree-of-freedom (DoF) concept [10], [18]. Simply put, the total number of DoFs at a node (no more than the number of antenna elements) represents the available resources at the node. A DoF can be used for either data transmission/reception or IC. Typically, transmitting one data stream requires one DoF at a transmitter and one DoF at its receiver. SM refers to the scenario where multiple DoFs are used to transmit multiple data streams, thus substantially increasing data throughput between the two nodes. On the other hand, IC refers to a node's capability to use some of its DoFs to cancel interference, either as a transmitter or as a receiver. Depending on whether IC is done at a transmitter or receiver, the number of required DoF consumption may be different.

- **IC by Tx.** If a transmitter (Tx) is to cancel its interference to an unintended receiver, the number of DoFs required at this transmitter is equal to the number of data streams (or DoFs) that the unintended receiver is trying to receive from its transmitter.
- **IC by Rx.** If a receiver (Rx) is to cancel the interference from an interfering transmitter, the number of DoFs required at this receiver is equal to the number of data streams (or DoFs) that the interfering transmitter is trying to transmit to its intended receiver.

At any node, the sum of DoFs used for SM and IC cannot exceed the total number of DoFs at the node.

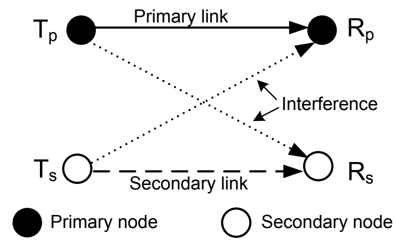


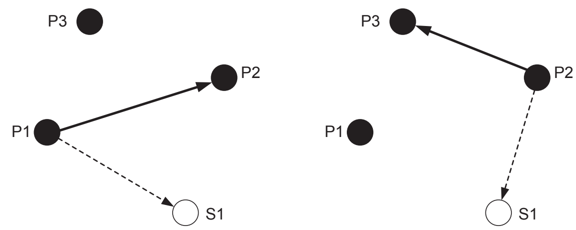
Fig. 1. A simple example illustrating the benefits of using MIMO to allow simultaneous activation of primary and secondary nodes.

A MIMO node's ability to use a subset of its DoFs to cancel interference while to use the remaining subset of DoFs for data transmission allows the possibility of simultaneous activation of the secondary nodes with the primary nodes. We use a simple example to illustrate this point. In Fig. 1, suppose T_p and R_p are a pair of transmit and receive nodes in the primary network, while T_s and R_s are a pair of transmit and receive nodes in the secondary network. Assume that all nodes share the same channel. Suppose T_p is transmitting 1 data stream to R_p. Under the interference avoidance paradigm, secondary transmit node T_s is prohibited from transmission on the same channel as it will interfere with primary receive node R_p. However, when MIMO is employed on the secondary nodes, simultaneous transmissions can be achieved. Assume secondary nodes T_s and R_s are each equipped with 4 antennas (4 DoFs). T_s can use 1 of its DoFs to cancel its interference to R_p so that R_p can receive its 1 data stream correctly from T_p. At node R_s, R_s can use 1 of its DoFs to cancel interference from T_p. After IC, both T_s and R_s still have 3 DoFs remaining, which can be used for SM of 3 data stream from T_s to R_s.

A. Channel State Information

As the above example shows, under transparent coexistence, all IC burden rests upon the secondary nodes. Specifically, a secondary transmit node needs to cancel its interference to all neighboring primary receive nodes who are interfered by this secondary transmitter; a secondary receive node needs to cancel interference from all neighboring primary transmit nodes that interfere with this secondary receiver. To achieve transparency to the primary nodes, it is important for the secondary nodes to have accurate channel state information (CSI). The problem is: how can a secondary node obtain the CSI between itself and its neighboring primary nodes while remaining transparent to the primary nodes?

We propose the following solution to resolve this problem. For each primary node, it typically sends out a pilot sequence



(a) Time slot t_1 (b) Time slot t_2

Fig. 2. CSI estimation at secondary node S_1.

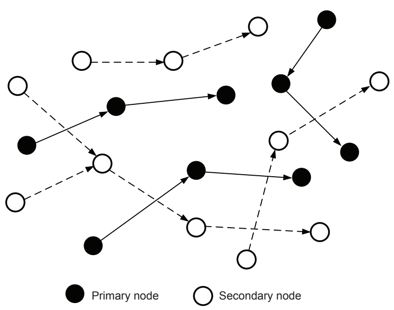


Fig. 3. A multi-hop secondary network co-located with a multi-hop primary network.

(training sequence) to its neighboring primary nodes so that those primary nodes can estimate the CSI. This is the practice for current cellular networks and we assume such a mechanism is available for a primary network. Since we consider a multi-hop network, where each node will act as a transmitter in one time slot but as a receiver in another time slot. Then, each secondary node can *overhear* the pilot sequence signal from the primary node while staying transparent. For example, in Fig. 2(a), in time slot t_1, when P_1 is transmitting the pilot sequence, a secondary node S_1 can overhear this sequence from P_1. Likewise, in Fig. 2(b), in time slot t_2, when P_2 is transmitting its pilot sequence, the secondary node S_1 can overhear this pilot sequence from P_2. Suppose the pilot sequence from the primary nodes is publicly available (as in cellular networks) and is known to the secondary nodes. Then the secondary node S_1 can use this information and the actual received pilot sequence signal from the primary nodes for channel estimation. Based on the reciprocity property of a wireless channel [16], a secondary node S_1 will be able to estimate the CSI in both directions to/from P_1 and P_2. Likewise, the CSI among the secondary nodes may be derived following a similar approach.

III. Problem Scope

We consider a primary multi-hop ad hoc network $\mathcal{P}$ shown in Fig. 3, which is co-located with a secondary multi-hop network $\mathcal{S}$ in the same geographical region. Suppose that there is a set of channels $\mathcal{B}$ owned by the primary network. For scheduling on each channel, we consider a time frame with T equal-length time slots. The primary nodes can use this set of channels and time slots freely as if they were the only nodes in the network. The primary nodes are assumed to be single-antenna nodes. For the secondary nodes, they are allowed to use a time slot t $(1 \leq t \leq T)$ on a channel only if their interference to the primary nodes are canceled properly, with complete transparency to the primary nodes. For IC, we assume that the secondary nodes are equipped with MIMO. Some key assumptions that we make in this paper are the following:

- In primary network, we assume that each primary node is a single-antenna node.[3]
- The secondary nodes need to know the primary nodes' transmission behavior (link scheduling). We assume this information can be derived by the secondary nodes through monitoring/sensing of the primary nodes' activities.
- The secondary nodes need to have CSI to perform IC (to/from the primary nodes and within the secondary nodes). A proposed solution was given in Section II-A.
- We further assume that the CSI obtained at the secondary nodes is perfect. This assumption allows us to develop an information theoretic understanding on the potential benefits of transparent coexistence paradigm. In practice, perfect CSI is hard to achieve and inaccurate CSI will cause interference leakage. This may be treated as additional noise and will degrade link quality. Just like any other system, there is a gap between what a theoretical limit is and what can actually be achieved in practice. Investigation of this gap (between theoretical limit and achievable performance in practice) and how to close this gap will be deferred for future research.
- We assume each data stream is associated with the same constant rate. In practice, the data rate of a data stream depends on channel condition and many other factors. But for tractability, we assume that we use a simple fixed rate coding and modulation scheme for a data stream. In other words, we assume that there is a minimum rate with our fixed rate coding and modulation for a data stream and we will just use this minimum rate for all data streams, despite that some streams with better channels could in fact achieve higher rates if an adaptive coding and modulation scheme is used. We agree that such a simple fixed rate coding and modulation scheme is not optimal. But this assumption allows us to keep the problem tractable when performing performance study.
- In our throughput optimization problem in the transparent coexistence paradigm, we assume to have global knowledge so that we can develop a centralized solution and use it to examine the benefits of such a paradigm.

Based on these assumptions, we explore the following challenges in the secondary network:

- **Channel/time slot scheduling** In a secondary network, an intermediate relay node is both a transmitter and a receiver. Under the half-duplex, a node cannot transmit and receive on the same channel within the same time slot. Therefore, scheduling (either in time slot or channel) is needed. Here, scheduling can be performed both in time slot and channel allocation (time and frequency

[3] The case where the primary nodes also have multiple antennas will be left for further research.

domains). Note that scheduling transmission/reception at a secondary node will lead to a particular interference relationship among the primary and secondary nodes in the underlying time slot and channel. This joint time/channel scheduling plays an integral role for IC in the network.

- **Inter-netwrok IC** We discussed this challenge in Section II (see Fig. 1), where a secondary transmitter needs to cancel its interference to its neighboring primary receivers while a secondary receiver needs to cancel the interference from its neighboring primary transmitters.
- **Intra-network IC** In addition to inter-network IC, interference from a secondary node may also interfere with another secondary node within their own network (i.e., "intra-network" interference). Such an interference must also be canceled properly (either by a secondary transmitter or receiver) to ensure successful data communications inside the secondary network.

It is important to realize that the above three key challenges are not independent, but deeply intertwined with each other. In particular, channel/time slot scheduling at a secondary node is directly tied to the interference relationship between the primary and secondary nodes as well as interference among the secondary nodes. Therefore, a mathematical modeling of transparent coexistence paradigm must capture all these components jointly.

IV. Mathematical Modeling

In this section, we develop a mathematical model for the transparent coexistence paradigm under which a multi-hop secondary network can access the same spectrum as a primary network (see Fig. 3). This mathematical model will address the challenges outlined in the last section through a joint formulation.

A. Notation

Table I lists notation in this paper. Suppose there is a set of sessions $\tilde{\mathcal{F}}$ within the primary network $\mathcal{P}$. For a given routing for each session, denote $\tilde{\mathcal{L}}$ as the set of links in the primary network that are traversed by these sessions (shown in solid arrow lines in Fig. 3). Denote $\tilde{z}^b_{(\tilde{l})}(t)$ as the number of data streams over primary link $\tilde{l} \in \tilde{\mathcal{L}}$ on channel b in time slot t. Since a primary node only has one antenna, $\tilde{z}^b_{(\tilde{l})}(t) = 1$ if link $\tilde{l}$ is active (on channel b and time slot t) and 0 otherwise.

For the secondary network, we assume MIMO capability at each node. Denote A_i as the number of antennas on a secondary node $i \in \mathcal{S}$. Suppose there is a set of multi-hop sessions $\mathcal{F}$ in $\mathcal{S}$. For a given routing for each session, denote $\mathcal{L}$ as the set of secondary links (shown in dashed arrow line in Fig. 3).

To model scheduling at a secondary node for transmission or reception, we denote $x^b_i(t)$ and $y^b_i(t)$ ($i \in \mathcal{S}, b \in \mathcal{B}$ and $1 \le t \le T$) as whether node i is a transmitter or receiver on channel b in time slot t, respectively. We have

$$x^b_i(t) = \begin{cases} 1 & \text{if node } i \text{ is a transmitter on channel } b \\ & \text{ in time slot } t; \\ 0 & \text{otherwise.} \end{cases}$$

TABLE I
NOTATION

Primary Network	
$\mathcal{P}$	The set of nodes in the primary network
T	The number of time slots in a frame
$\mathcal{B}$	The sets of channels owned by the primary network
B	The number of channels in set $\mathcal{B}$, $B = \|\mathcal{B}\|$
$\tilde{\mathcal{F}}$	The set of sessions in the primary network
$\tilde{\mathcal{I}}_i$	The set of primary nodes within the interference range of secondary node i
$\tilde{\mathcal{L}}^{\text{In}}_i$	The set of incoming links (from other primary nodes) at node $i \in \mathcal{P}$
$\tilde{\mathcal{L}}^{\text{Out}}_i$	The set of outgoing links (to other primary nodes) at node $i \in \mathcal{P}$
$\tilde{\mathcal{L}}$	The set of links in the primary network
$\tilde{z}^b_{(\tilde{l})}(t)$	The number of data streams over primary link $\tilde{l}$ on channel b in time slot t
Secondary Network	
$\mathcal{S}$	The set of nodes in the secondary network
S	The number of secondary nodes in the network, $S = \|\mathcal{S}\|$
A_i	The number of antennas at secondary node $i \in \mathcal{S}$
c	The minimum data rate carried by a data stream
$\mathcal{F}$	The set of sessions in the secondary network
$\mathcal{I}_i$	The set of node in $\mathcal{S}$ that are within the interference range of secondary node i
$\mathcal{L}^{\text{In}}_i$	The set of incoming links (from other secondary nodes) at node $i \in \mathcal{S}$
$\mathcal{L}^{\text{Out}}_i$	The set of outgoing links (to other secondary nodes) at node $i \in \mathcal{S}$
$\mathcal{L}$	The set of secondary links
$r(f)$	The data rate of the session $f \in \mathcal{F}$
$r_{\min}$	The minimum data rate among all secondary sessions
$\text{Rx}(l)$	The receiver of link $l \in \mathcal{L}$
$\text{Tx}(l)$	The transmitter of link $l \in \mathcal{L}$
$x^b_i(t)$	$= 1$ if node $i \in \mathcal{S}$ is a transmitter on channel b in time slot t, and is 0 otherwise
$y^b_i(t)$	$= 1$ if node $i \in \mathcal{S}$ is a receiver on channel b in time slot t, and is 0 otherwise
$z^b_{(l)}(t)$	The number of data streams over link $l \in \mathcal{L}$ on channel b in time slot t
$\lambda^b_{j,i}(t)$	The number of DoFs used by transmit node $i \in \mathcal{S}$ to cancel its interference to receive node $j \in \mathcal{S}$ on channel b in time slot t
$\mu^b_{j,i}(t)$	The number of DoFs used by receive node $i \in \mathcal{S}$ to cancel the interference from transmit node $j \in \mathcal{S}$ on channel b in time slot t
$\theta^b_{j,i}(t)$	Binary indicator showing the relationship between nodes i and j in ordered list on channel b in time slot t, $i, j \in \mathcal{S}$
$\pi^b(t)$	An ordering for IC among the secondary nodes on channel b in the time slot t
$\pi^b_i(t)$	The position of node $i \in \mathcal{S}$ in $\pi^b(t)$

$$y^b_i(t) = \begin{cases} 1 & \text{if node } i \text{ is a receiver on channel } b \\ & \text{ in time slot } t; \\ 0 & \text{otherwise.} \end{cases}$$

Under half-duplex (a node cannot transmit and receive on the same channel in the same time slot), we have the following constraint on $x^b_i(t)$ and $y^b_i(t)$:

$$x^b_i(t) + y^b_i(t) \le 1 \qquad (i \in \mathcal{S}, b \in \mathcal{B}, 1 \le t \le T). \quad (1)$$

B. Node ordering for IC in secondary network

Recall that the secondary network is solely responsible for "inter-network" IC (in addition to "intra-network" IC). To avoid unnecessary duplication in allocating DoFs for IC, it was shown in [14] that node-ordering based IC is very effective. Under this scheme, all secondary nodes are put into an ordered list. DoF allocation at each secondary node for IC is based on the position of the node in the list. It was shown in [14] that such disciplined approach can ensure: (i) there is no duplication in IC (and thus no waste of DoF resources), and (ii) the final DoF allocation is feasible. We will describe the specific rules for DoF allocation at a secondary node for IC (depending on whether it is a transmitter or receiver) in the following two sections. But first, we give a mathematical model for the node ordering concept.

Denote $\pi^b(t)$ as an ordered list of the secondary nodes in the network on $b \in \mathcal{B}$ and $1 \le t \le T$, and denote $\pi_i^b(t)$ as the position of node $i \in \mathcal{S}$ in $\pi^b(t)$. Therefore, $1 \le \pi_i^b(t) \le S$, where $S = |\mathcal{S}|$. For example, if $\pi_i^b(t) = 3$, then it means that node i is the third node in the list $\pi^b(t)$.

To model the relative ordering between any two secondary nodes i and j in $\pi^b(t)$, we use a binary variable $\theta_{j,i}^b(t)$ and define it as follows:

$$\theta_{j,i}^b(t) = \begin{cases} 1 & \text{if node } j \text{ is before node } i \text{ in } \pi^b(t); \\ 0 & \text{otherwise.} \end{cases}$$

It was shown in [14] that the following relationships hold among $\pi_i^b(t), \pi_j^b(t)$ and $\theta_{j,i}^b(t)$.

$$\pi_i^b(t) - S\cdot\theta_{j,i}^b(t) + 1 \le \pi_j^b(t) \le \pi_i^b(t) - S\cdot\theta_{j,i}^b(t) + S - 1 \ , \quad (2)$$

where $i, j \in \mathcal{S}, b \in \mathcal{B}$, and $1 \le t \le T$.

We point out that such a node ordering approach for DoF allocation is the most efficient approach among *all* existing DoF models that can guarantee feasibility. As pointed out in [14], an "optimal" node ordering can be found by inserting the above ordering relationship as a constraint into the overall formulation of the optimization problem, as we shall do in Section V.

C. DoF allocation at a secondary transmitter

At a secondary transmitter i, it needs to expend DoFs for (i) SM, (ii) IC to neighboring primary receivers, and (iii) IC to a subset of its neighboring secondary receivers based on their orders in the node list.

(i) DoF for SM. For SM, denote $z_{(l)}^b(t)$ and $\mathcal{L}_i^{\text{Out}}$ as the number of data streams on link $l \in \mathcal{L}$ and the set of outgoing links from secondary node i. Then the number of DoFs at secondary node $i \in \mathcal{S}$ for SM is $\sum_{l \in \mathcal{L}_i^{\text{Out}}} z_{(l)}^b(t)$ for $b \in \mathcal{B}$ and $1 \le t \le T$.

(ii) DoF for IC to neighboring primary receivers. To ensure transparent coexistence, a secondary transmitter needs to cancel its interference to neighboring primary receivers. Recall that if a primary receiver $p \in \mathcal{P}$ is within the interference range of node i, the number of DoFs at node i that is used for canceling the interference to node p is equal to the number of data stream that are received at node p. Denote $\tilde{\mathcal{L}}_p^{\text{In}}$ as the set of incoming primary links to node p. Denote $\tilde{\mathcal{I}}_i$ as the set of primary nodes that are located within the interference range of secondary transmitter i. For node $p \in \tilde{\mathcal{I}}_i$, the number of DoFs used at node i for canceling interference to node p is $\sum_{\tilde{l} \in \tilde{\mathcal{L}}_p^{\text{In}}} \tilde{z}_{(\tilde{l})}^b(t)$ for $b \in \mathcal{B}$ and $1 \le t \le T$. Now for all primary receive nodes in $\tilde{\mathcal{I}}_i$, the number of DoFs used at node i to cancel interference to these nodes is $\left(\sum_{p \in \tilde{\mathcal{I}}_i} \sum_{\tilde{l} \in \tilde{\mathcal{L}}_p^{\text{In}}} \tilde{z}_{(\tilde{l})}^b(t)\right)$ for $b \in \mathcal{B}$ and $1 \le t \le T$.

(iii) DoF for IC to secondary receivers. For IC within the secondary network, this secondary transmitter i only needs to cancel its interference to a subset (instead of all) of its neighboring secondary receivers based on the node ordering list [14]. Specifically, this secondary transmitter i only needs to expend DoFs to null its interference to neighboring secondary receivers that are *before* itself in the ordered secondary node list $\pi^b(t)$. Node i does not need to expend any DoF to null its interference to those secondary receivers that are *after* itself in the ordered node list $\pi^b(t)$. This is because the interference from node i to those secondary receivers (that are after this node in $\pi^b(t)$) will be nulled by those secondary receivers later (when we perform DoF allocation at those nodes). This is the key to avoid duplication in IC.

Recall that if a secondary receiver $j \in \mathcal{S}$ is within the interference range of secondary transmit node i, the number of DoFs required at transmit node i to cancel its interference to node j is equal to the number of data stream that are being received at node j. Denote $\mathcal{L}_j^{\text{In}}$ as the set of incoming links to node j. Denote $\mathcal{I}_i$ as the set of secondary nodes that are located within the interference range of node i. For secondary receive node $j \in \mathcal{I}_i$, the number of DoFs used at secondary transmit node i for canceling its interference to node j is $\left(\theta_{j,i}^b(t) \cdot \sum_{k \in \mathcal{L}_j^{\text{In}}}^{\text{Tx}(k) \ne i} z_{(k)}^b(t)\right)$. Note that we are using the indicator variable $\theta_{j,i}^b(t)$ to consider only those secondary receive nodes that are before node i in the ordered node list $\pi^b(t)$. Now for all secondary receive nodes in $\mathcal{I}_i$, the number of DoFs used at node i to cancel interference to these nodes is $\sum_{j \in \mathcal{I}_i} \left(\theta_{j,i}^b(t) \cdot \sum_{k \in \mathcal{L}_j^{\text{In}}}^{\text{Tx}(k) \ne i} z_{(k)}^b(t)\right)$ for $b \in \mathcal{B}$ and $1 \le t \le T$.

Total DoF consumption. Putting all these DoF consumptions together at a secondary transmitter i, we have the following constraints:

- If this secondary transmit node i is active, i.e., $x_i^b(t) = 1$, we have
$$x_i^b(t) \le \sum_{l \in \mathcal{L}_i^{\text{Out}}} z_{(l)}^b(t) + \left(\sum_{p \in \tilde{\mathcal{I}}_i} \sum_{\tilde{l} \in \tilde{\mathcal{L}}_p^{\text{In}}} \tilde{z}_{(\tilde{l})}^b(t)\right) + \sum_{j \in \mathcal{I}_i} \left(\theta_{j,i}^b(t) \cdot \sum_{k \in \mathcal{L}_j^{\text{In}}}^{\text{Tx}(k) \ne i} z_{(k)}^b(t)\right) \le A_i \ , \quad (3)$$
which means that the DoF consumption at node i cannot be more that the total number of its antennas.
- If node i is not active, i.e., $x_i^b(t) = 0$, we have
$$\sum_{l \in \mathcal{L}_i^{\text{Out}}} z_{(l)}^b(t) = 0 \ . \quad (4)$$

We can rewrite (3) and (4) into the following two mathematical constraints:

$$x_i^b(t) \le \sum_{l \in \mathcal{L}_i^{\text{Out}}} z_{(l)}^b(t) + \left(\sum_{p \in \tilde{\mathcal{I}}_i} \sum_{\tilde{l} \in \tilde{\mathcal{L}}_p^{\text{In}}} \tilde{z}_{(\tilde{l})}^b(t)\right) + \sum_{j \in \mathcal{I}_i} \left(\theta_{j,i}^b(t) \cdot \sum_{k \in \mathcal{L}_j^{\text{In}}}^{\text{Tx}(k) \ne i} z_{(k)}^b(t)\right) \le A_i x_i^b(t) + \left(1 - x_i^b(t)\right) M, \quad (5)$$

$$\sum_{l\in\mathcal{L}_i^{\rm Out}} z_{(l)}^b(t) \le x_i^b(t)\cdot A_i\ , \tag{6}$$

where M is a large constant, which is an upper bound of $\left[\sum_{p\in\tilde{\mathcal{I}}_i}\sum_{\tilde{l}\in\tilde{\mathcal{L}}_p^{\rm In}}\tilde{z}_{(\tilde{l})}^b(t)+\sum_{j\in\mathcal{I}_i}\theta_{j,i}^b(t)\cdot\sum_{k\in\mathcal{L}_j^{\rm In}}^{{\rm Tx}(k)\neq i} z_{(k)}^b(t)\right]$ when $x_i^b(t)=0$. For example, we can set $M=\sum_{j\in\mathcal{I}_i}A_j+\sum_{p\in\tilde{\mathcal{I}}_i}\sum_{\tilde{l}\in\tilde{\mathcal{L}}_p^{\rm In}}\tilde{z}_{(\tilde{l})}^b(t)$.

To see that (5) and (6) can replace (3) and (4), note that (i) when $x_i^b(t)=1$, (5) becomes (3) and (6) holds trivially; (ii) when $x_i^b(t)=0$, (4) and (6) are equivalent, and (5) holds trivially.

Reformulation. Since (5) has a nonlinear term $\left(\theta_{j,i}^b(t)\cdot\sum_{k\in\mathcal{L}_j^{\rm In}}^{{\rm Tx}(k)\neq i} z_{(k)}^b(t)\right)$, we can use *Reformulation-Linearization Technique* (RLT) [9, Chapter 6] to reformulate this nonlinear term by introducing new variables and adding new linear constraints. We define a new variable $\lambda_{j,i}^b(t)$ as follows:

$$\lambda_{j,i}^b(t)=\theta_{j,i}^b(t)\cdot\sum_{k\in\mathcal{L}_j^{\rm In}}^{{\rm Tx}(k)\neq i} z_{(k)}^b(t)\ ,$$

where $i\in\mathcal{S}, j\in\mathcal{I}_i, b\in\mathcal{B}$, and $1\le t\le T$. For binary variable $\theta_{j,i}^b(t)$, we have the following associated constraints:

$$\begin{aligned}\theta_{j,i}^b(t) &\ \ge\ 0\ ,\\ (1-\theta_{j,i}^b(t)) &\ \ge\ 0\ .\end{aligned}$$

For $\sum_{k\in\mathcal{L}_j^{\rm In}}^{{\rm Tx}(k)\neq i} z_{(k)}^b(t)$, we have:

$$\begin{aligned}\sum_{k\in\mathcal{L}_j^{\rm In}}^{{\rm Tx}(k)\neq i} z_{(k)}^b(t) &\ \ge\ 0\ ,\\ A_j-\sum_{k\in\mathcal{L}_j^{\rm In}}^{{\rm Tx}(k)\neq i} z_{(k)}^b(t) &\ \ge\ 0\ .\end{aligned}$$

We can cross-multiply the two constraints involving $\theta_{j,i}^b(t)$ with the two constraints involving $\sum_{k\in\mathcal{L}_j^{\rm In}}^{{\rm Tx}(k)\neq i} z_{(k)}^b(t)$, and replacing the product term $\left(\theta_{j,i}^b(t)\cdot\sum_{k\in\mathcal{L}_j^{\rm In}}^{{\rm Tx}(k)\neq i} z_{(k)}^b(t)\right)$ with $\lambda_{j,i}^b(t)$. Then (5) can be replaced by the following linear constraints:

$$x_i^b(t)\le\sum_{l\in\mathcal{L}_i^{\rm Out}} z_{(l)}^b(t)+\left(\sum_{p\in\tilde{\mathcal{I}}_i}\sum_{\tilde{l}\in\tilde{\mathcal{L}}_p^{\rm In}}\tilde{z}_{(\tilde{l})}^b(t)\right)+\sum_{j\in\mathcal{I}_i}\lambda_{j,i}^b(t)\le A_i x_i^b(t)+\left(1-x_i^b(t)\right)M, \tag{7}$$

$$\lambda_{j,i}^b(t)\ge 0, \tag{8}$$

$$\lambda_{j,i}^b(t)\le\sum_{k\in\mathcal{L}_j^{\rm In}}^{{\rm Tx}(k)\neq i} z_{(k)}^b(t), \tag{9}$$

$$\lambda_{j,i}^b(t)\le A_j\cdot\theta_{j,i}^b(t), \tag{10}$$

$$\lambda_{j,i}^b(t)\ge A_j\cdot\theta_{j,i}^b(t)-A_j+\sum_{k\in\mathcal{L}_j^{\rm In}}^{{\rm Tx}(k)\neq i} z_{(k)}^b(t), \tag{11}$$

where $i\in\mathcal{S}, j\in\mathcal{I}_i, b\in\mathcal{B}$, and $1\le t\le T$.

D. DoF allocation at a secondary receiver

At a secondary receiver i, it needs to expend DoFs for (i) SM, (ii) canceling interference from neighboring primary transmitters, and (iii) canceling interference from a subset of its neighboring secondary transmitters based on their orders in the node list.

(i) DoF for SM. For SM, the number of DoFs consumed at a secondary receiver $i\in\mathcal{S}$ is $\sum_{k\in\mathcal{L}_i^{\rm In}} z_{(k)}^b(t)$ for $b\in\mathcal{B}$ and $1\le t\le T$.

(ii) DoF for IC from neighboring primary transmitters. A secondary receiver needs to cancel the interference from neighboring primary transmitters. If a primary transmitter $p\in\mathcal{P}$ is within the interference range of secondary receive node $i\in\mathcal{S}$, the number of DoFs at node i required for canceling this interference from node p is equal to the number of data streams that are being transmitted by node p. Denote $\tilde{\mathcal{L}}_p^{\rm Out}$ as the set of outgoing links from primary node p. For $p\in\tilde{\mathcal{I}}_i$, the number of DoFs used at node i for canceling interference from node p is $\sum_{\tilde{l}\in\tilde{\mathcal{L}}_p^{\rm Out}}\tilde{z}_{(\tilde{l})}^b(t)$. Now for all primary transmit nodes in $\tilde{\mathcal{I}}_i$, the number of DoFs used at node i to cancel interference from these nodes is $\left(\sum_{p\in\tilde{\mathcal{I}}_i}\sum_{\tilde{l}\in\tilde{\mathcal{L}}_p^{\rm Out}}\tilde{z}_{(\tilde{l})}^b(t)\right)$ for $b\in\mathcal{B}$ and $1\le t\le T$.

(iii) DoF for IC from secondary transmitters. For IC within the secondary network, this secondary receiver i only needs to null the interference from a subset (instead of all) of its neighboring secondary transmitters based on node ordering list. Specifically, this secondary receiver i only needs to expend DoFs to null the interference from neighboring secondary transmitters that are *before* itself in the ordered secondary node list $\pi^b(t)$. Node i does not need to expend any DoF to null the interference from those secondary transmitters that are *after* itself in the ordered node list $\pi^b(t)$. This is because the interference to node i from those secondary transmitters will be nulled by those secondary transmitters later (when we perform DoF allocation at those nodes).

Recall that if node i is within the interference range of a secondary transmit node $j\in\mathcal{S}$, the number of DoFs at node i that is used for canceling the interference from node j is equal to the number of data stream that are being transmitted at node j. For a secondary transmit node $j\in\mathcal{I}_i$, the number of DoFs used at secondary receive node i for canceling interference from node j is $\left(\theta_{j,i}^b(t)\cdot\sum_{l\in\mathcal{L}_j^{\rm Out}}^{{\rm Rx}(l)\neq i} z_{(l)}^b(t)\right)$. Now for all other secondary transmit nodes in $\mathcal{I}_i$, the number of DoFs used at node i to cancel interference from those nodes is $\sum_{j\in\mathcal{I}_i}\left(\theta_{j,i}^b(t)\cdot\sum_{l\in\mathcal{L}_j^{\rm Out}}^{{\rm Rx}(l)\neq i} z_{(l)}^b(t)\right)$ for $b\in\mathcal{B}$ and $1\le t\le T$.

Total DoF consumption. We can put all DoF consumption at a secondary receiver as follows:

$$y_i^b(t) \le \sum_{k\in\mathcal{L}_i^{\text{In}}} z_{(k)}^b(t) + \left(\sum_{p\in\tilde{\mathcal{I}}_i}\sum_{\tilde{l}\in\tilde{\mathcal{L}}_p^{\text{Out}}} \tilde{z}_{(\tilde{l})}^b(t)\right) + \sum_{j\in\ \mathcal{I}_i}\left(\theta_{j,i}^b(t)\cdot\sum_{l\in\mathcal{L}_j^{\text{Out}}}^{\text{R}x(l)\neq i} z_{(l)}^b(t)\right) \le A_i y_i^b(t) + \left(1-y_i^b(t)\right)N, \quad (12)$$

$$\sum_{k\in\mathcal{L}_i^{\text{In}}} z_{(k)}^b(t) \le y_i^b(t)\cdot A_i\ , \quad (13)$$

where N is a large constant, which is an upper bound of $\left[\sum_{p\in\tilde{\mathcal{I}}_i}\sum_{\tilde{l}\in\tilde{\mathcal{L}}_p^{\text{Out}}}\tilde{z}_{(\tilde{l})}^b(t)+\sum_{j\in\ \mathcal{I}_i}(\theta_{j,i}^b(t)\cdot\sum_{l\in\mathcal{L}_j^{\text{Out}}}^{\text{R}x(l)\neq i} z_{(l)}^b(t)\right]$ when $y_i^b(t)=0$. For example, we can set $N=\sum_{j\in\mathcal{I}_i}A_j+\sum_{p\in\tilde{\mathcal{I}}_i}\sum_{\tilde{l}\in\tilde{\mathcal{L}}_p^{\text{Out}}}\tilde{z}_{(\tilde{l})}^b(t)$.

Reformulation. Following the same token as in the last section, we use RLT to linearize the nonlinear term $\left(\theta_{j,i}^b(t)\cdot\sum_{l\in\mathcal{L}_j^{\text{Out}}}^{\text{Rx}(l)\neq i} z_{(l)}^b(t)\right)$ in (12). Denote $\mu_{j,i}^b(t)$ as $\left(\theta_{j,i}^b(t)\cdot\sum_{l\in\mathcal{L}_j^{\text{Out}}}^{\text{Rx}(l)\neq i} z_{(l)}^b(t)\right)$. Then (12) can be replaced by the following linear constraints:

$$y_i^b(t) \le \sum_{k\in\mathcal{L}_i^{\text{In}}} z_{(k)}^b(t) + \left(\sum_{p\in\tilde{\mathcal{I}}_i}\sum_{\tilde{l}\in\mathcal{L}_p^{\text{Out}}}\tilde{z}_{(\tilde{l})}^b(t)\right) + \sum_{j\in\mathcal{I}_i}\mu_{j,i}^b(t) \le A_i y_i^b(t) + \left(1-y_i^b(t)\right)N, \quad (14)$$

$$\mu_{j,i}^b(t) \ge 0, \quad (15)$$

$$\mu_{j,i}^b(t) \le \sum_{l\in\mathcal{L}_j^{\text{Out}}}^{\text{Rx}(l)\neq i} z_{(l)}^b(t), \quad (16)$$

$$\mu_{j,i}^b(t) \le A_j\cdot\theta_{j,i}^b(t), \quad (17)$$

$$\mu_{j,i}^b(t) \ge A_j\cdot\theta_{j,i}^b(t) - A_j + \sum_{l\in\mathcal{L}_j^{\text{Out}}}^{\text{Rx}(l)\neq i} z_{(l)}^b(t), \quad (18)$$

where $i\in\mathcal{S}, j\in\mathcal{I}_i, b\in\mathcal{B}$, and $1\le t\le T$.

V. Case Study for A Throughput Maximization Problem

A. Problem Formulation

Using the above mathematical model for the transparent coexistence paradigm for a multi-hop secondary network, various problems can be studied. In this section, we study a throughput optimization problem in the secondary network. Denote $r(f)$ as the rate of session $f\in\mathcal{F}$. Then at any link $l\in\mathcal{L}$ in the network, the aggregate throughput rate among the flows that traverse this link cannot exceed the link's scheduling capacity (over a time frame). That is,

$$\sum_{f\in\mathcal{F}}^{f\ \text{traversing}\ l} r(f) \le c\cdot\frac{1}{T}\sum_{b\in\mathcal{B}}\sum_{t=1}^{T} z_{(l)}^b(t) \qquad (l\in\mathcal{L}), \quad (19)$$

where c is the data rate carried by a data stream.

For the throughput maximization problem, suppose we are interested in maximizing the minimum throughput rate among all secondary sessions. Then the problem can be formulated as follows:

OPT
max $r_{\min}$
s.t $r_{\min}\le r(f) \qquad (f\in\mathcal{F})$;
Half-duplex constraints: (1);
Node ordering constraints: (2);
Transmitter DoF constraints: (6)–(11);
Receiver DoF constraints: (13)–(18);
Link capacity constraints: (19).

In this formulation, $r_{\min}, r(f), x_i^b(t), y_i^b(t), z_{(l)}^b(t), \pi_i^b(t)$, $\lambda_{j,i}^b(t)$, $\mu_{j,i}^b(t)$ and $\theta_{j,i}^b(t)$ are optimization variables, and $A_i, M, N, \tilde{z}_{(\tilde{l})}^b(t)$ and c are given constants. This optimization problem is in the form of a mixed-integer linear program (MILP), which is NP-hard in general. Although commercial solvers such as CPLEX can be used, they are not scalable to address problems with moderate to large-sized networks. In this section, we develop an efficient heuristic algorithm.

B. Overview of Solution Algorithm

The algorithm that we propose is based on the so-called *sequential fixing* (SF) technique in [9, Chapter 5]. SF offers a general framework to handle integer variables in a MILP problem, and has a polynomial time complexity. The basic idea of SF is as follows. For a MILP like ours, if we were able to set the optimal values for all integer variables, then the original problem would be reduced to an LP, which can be solved in polynomial time. So the key challenge in MILP is how to determine the values for all the integer variables. Under SF, this can be done by studying the linear relaxation of the original problem, obtained by relaxing all the integer variables to continuous variables. Although the solution to this linear relaxation may not have an integer value for each integer variable, we can *fix* the values of one or more integer variables based on their *closeness* to certain integer values. Instead of determining all the integer variables in one iteration, we can fix only one or a few integer variables in each iteration. For the remaining (unfixed) integer variables, we can solve a new linear relaxation and then fix one or more remaining integer variables. This SF procedure terminates after all integer variables are fixed. At this point, the MILP becomes an LP. Any remaining continuous variable in the LP can be solved efficiently.

Although the idea of SF is straightforward, it requires a careful design to ensure its performance. A naive application of SF, as we have experienced, may lead to either infeasible solution or poor performance. This is because that fixing relaxed variables solely based on their closeness to integers do not take into consideration of the physical significance of different variables in the particular problem and their intricate relationships. In our design, we propose to classify integer variables into three groups: (π,θ), (x,y), and z. The first group (π,θ), determines the ordering among the secondary nodes in DoF allocation and is considered the structural foundation of all integer variables. Therefore, we will determine

(π, θ) first in our SF algorithm. For the remaining (x, y) and z variables, (x, y) can be determined if we know the link status for the corresponding z. Therefore, we will determine the link status (i.e., whether $z = 0$ or $z \geq 1$) first and then we can fix the corresponding (x, y). Note that in this step, we only determine whether $z = 0$ (link inactive) or $z \geq 1$ (link active). In the last step, we will fix those z's with $z \geq 1$ to exact integer values iteratively. Some important details of each step are given in the following section.

C. Algorithm Details

Phase I: Fixing π and θ variables. In this phase, for $b \in \mathcal{B}$ and $1 \leq t \leq T$, we will fix one $\pi_i^b(t)$ variable, and further fix related $\theta_{i,j}^b(t)$ (or $\theta_{j,i}^b(t)$) variables during an iteration. Since there are a total of S of $\pi_i^b(t)$'s ($i \in \mathcal{S}$) for $b \in \mathcal{B}$ and $1 \leq t \leq T$, there are S iterations in Phase I.

Specifically, in the first iteration, for $b \in \mathcal{B}$ and $1 \leq t \leq T$, we identify node i with the smallest value of $\pi_i^b(t)$ among all $\pi_j^b(t)$'s ($j \in \mathcal{S}$). We set $\pi_i^b(t) = 1$. Since this is the first node on channel b in time slot t, we set $\theta_{i,j}^b(t) = 1$ and $\theta_{j,i}^b(t) = 0$ for $j \neq i$. In the second iteration, another node k with the smallest value $\pi_k^b(t)$ among all un-fixed $\pi_j^b(t)$'s will be chosen and we set $\pi_k^b(t) = 2$. Likewise, we set $\theta_{k,j}^b(t) = 1$ and $\theta_{j,k}^b(t) = 0$ for $j \neq i$, $j \neq k$. This process continues till the end of S-th iteration, when all $\pi_i^b(t)$ and $\theta_{i,j}^b(t)$ $(i, j \in \mathcal{S})$ are fixed for $b \in \mathcal{B}, 1 \leq t \leq T$.

Phase II: Fixing x and y variables. In this phase, we will determine each link l's status (i.e., active or inactive) and fix $x_i^b(t)$ and $y_i^b(t)$ variables. In the case of an inactive link l, we set $z_{(l)}^b(t) = 0$; in the case of an active link l, we will leave the determination of $z_{(l)}^b(t)$ to Phase III.

Specifically, in each iteration, we choose the largest $z_{(l)}^b(t)$ on channel b in time slot t and determine the status of the corresponding link l (i.e., active or inactive). This link l is determined to be active for $b \in \mathcal{B}$ and $1 \leq t \leq T$ if it satisfies the following conditions:

- (1) is satisfied, which means that the transmitter and receiver of this link each meets half-duplex constraint.
- Link l's transmitter should satisfy (5) and its receiver should satisfy (12), i.e., not exceeding DoF resources at both transmitter and receiver. In the case that the status of another associated link k is yet to be determined, we assume its $z_{(k)}^b(t) = 0$. Similarly, in the case that the status of another associated link k is active, we assume $z_{(k)}^b(t) = 1$. Note that in either case, we do not set the values for these $z_{(k)}^b(t)$'s permanently, but rather, only a lower bound value so that we can test whether (5) and (12) can hold.

If link l does not meet the above two conditions, it is considered inactive. Depending on whether link l is active or inactive, we can fix $(x_i^b(t), y_i^b(t))$ and possibly some other $z_{(k)}^b(t)$ variables based on the following three rules:

(a) If link l is active for $b \in \mathcal{B}$ and $1 \leq t \leq T$, we can fix $x_{\mathrm{Tx}(l)}^b(t) = 1$ and $y_{\mathrm{Rx}(l)}^b(t) = 1$. As a result of this fixing, we can also fix $y_{\mathrm{Tx}(l)}^b(t) = 0$ and $x_{\mathrm{Rx}(l)}^b(t) = 0$ by (1). Otherwise (i.e., link l is inactive for $b \in \mathcal{B}$ and $1 \leq t \leq T$), we can fix $z_{(l)}^b(t) = 0$. Further, if all links in $\mathcal{L}_{\mathrm{Tx}(l)}^{\mathrm{Out}}$ are inactive for $b \in \mathcal{B}$ and $1 \leq t \leq T$, we set $x_{\mathrm{Tx}(l)}^b(t) = 0$. Similarly, if all links in $\mathcal{L}_{\mathrm{Rx}(l)}^{\mathrm{In}}$ are inactive for $b \in \mathcal{B}$ and $1 \leq t \leq T$, we set $y_{\mathrm{Rx}(l)}^b(t) = 0$.

(b) If $x_i^b(t) = 0$, i.e., node i does not transmit data for $b \in \mathcal{B}$ and $1 \leq t \leq T$, then we set all links $k \in \mathcal{L}_i^{\mathrm{Out}}$ to be inactive. Further, we set $z_{(k)}^b(t) = 0$ on these links.

(c) If $y_i^b(t) = 0$, i.e., node i does not receive data for $b \in \mathcal{B}$ and $1 \leq t \leq T$, then we set all link $k \in \mathcal{L}_i^{\mathrm{In}}$ to be inactive. Further, we set $z_{(k)}^b(t) = 0$ on these links.

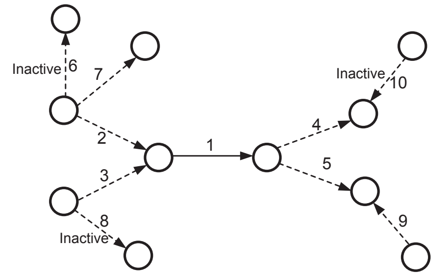


Fig. 4. An example illustrating how to fix x, y, and some z variables in Phase II.

We use an example to illustrate the case when a link is determined to be active. Referring to Fig. 4, suppose the status on links 6, 8, and 10 are determined to be inactive on b and t in the last iteration. In this iteration, suppose link 1's status is determined to be active. Then, we can set $x_{\mathrm{Tx}(1)}^b(t) = 1$ and $y_{\mathrm{Rx}(1)}^b(t) = 1$. Since $x_{\mathrm{Tx}(1)}^b(t) = 1$, we can set $y_{\mathrm{Tx}(1)}^b(t) = 0$ and $z_{(2)}^b(t) = 0$, $z_{(3)}^b(t) = 0$. The link status of 2 and 3 can be set to be inactive. Since all outgoing links from node $\mathrm{Tx}(3)$ are inactive, we can set $x_{\mathrm{Tx}(3)}^b(t) = 0$. Similarly, since $y_{\mathrm{Rx}(1)}^b(t) = 1$, we can set $x_{\mathrm{Rx}(1)}^b(t) = 0$ and $z_{(4)}^b(t) = 0$, $z_{(5)}^b(t) = 0$. The link status of 4 and 5 can be set to be inactive. Since all incoming links to $\mathrm{Rx}(4)$ are inactive, we can set $x_{\mathrm{Rx}(4)}^b(t) = 0$.

Phase III: Fixing z variables. In Phase II, we have fixed $z_{(l)}^b(t)$'s to 0 for those inactive links. For those links that are active, we have not yet determined the exact integer values for $z_{(l)}^b(t)$'s. In Phase III, we will fix these integer values.

On all active links l, if there exists some $z_{(l)}^b(t)$'s that are not yet integer, we use SF to fix these $z_{(l)}^b(t)$'s iteratively until they are all integers. In particular, during each iteration, we identify link l with the $\min_l\{z_{(l)}^b(t) - \lfloor z_{(l)}^b(t) \rfloor\}$ for each for $b \in \mathcal{B}$ and $1 \leq t \leq T$ and set $z_{(l)}^b(t) = \lfloor z_{(l)}^b(t) \rfloor$.

VI. Performance Evaluation

The goal of this section is twofold. First, we want to use numerical results to illustrate how transparent coexistence can be achieved for a multi-hop secondary network. Note that we cannot compare our heuristic solution to the global optimal solution because a global optimal solution is not available due to the exponential complexity of an MILP formulation. But this limitation does not prevent us from demonstrating the potential benefits of the transparent coexistence paradigm. Therefore, our second goal in this section is to show the

TABLE II
LOCATION OF EACH NODE FOR THE 20-NODE PRIMARY NETWORK AND 30-NODE SECONDARY NETWORK.

Primary Network					
Node	Location	Node	Location	Node	Location
P_1	(10, 10)	P_8	(15, 50)	P_{15}	(20, 80)
P_2	(30, 30)	P_9	(40, 70)	P_{16}	(31, 48)
P_3	(50, 30)	P_{10}	(60, 90)	P_{17}	(35, 85)
P_4	(75, 50)	P_{11}	(85, 90)	P_{18}	(90, 80)
P_5	(90, 20)	P_{12}	(40, 10)	P_{19}	(3, 35)
P_6	(90, 45)	P_{13}	(70,10)	P_{20}	(6, 97)
P_7	(75, 65)	P_{14}	(55, 55)		
Secondary Network					
Node	Location	Node	Location	Node	Location
S_1	(23, 66)	S_{11}	(55, 60)	S_{21}	(88, 62)
S_2	(3, 89)	S_{12}	(8, 56)	S_{22}	(70, 20)
S_3	(42, 41)	S_{13}	(3, 78)	S_{23}	(76, 74)
S_4	(19, 37)	S_{14}	(62, 2)	S_{24}	(84, 30)
S_5	(10, 70)	S_{15}	(92, 92)	S_{25}	(22, 92)
S_6	(29, 6)	S_{16}	(36, 94)	S_{26}	(60, 40)
S_7	(8, 25)	S_{17}	(82, 4)	S_{27}	(28, 16)
S_8	(51, 10)	S_{18}	(35, 60)	S_{28}	(99, 3)
S_9	(63, 75)	S_{19}	(76, 40)	S_{29}	(98, 38)
S_{10}	(65, 98)	S_{20}	(48, 21)	S_{30}	(47, 85)

TABLE III
SOURCE AND DESTINATION NODES OF EACH SESSION IN THE PRIMARY AND SECONDARY NETWORKS.

Primary Network		
Session	Source Node	Destination Node
1	P_1	P_{14}
2	P_5	P_7
3	P_{11}	P_{15}
Secondary Network		
Session	Source Node	Destination Node
1	S_7	S_{25}
2	S_{21}	S_{17}
3	S_{14}	S_3
4	S_{30}	S_{23}

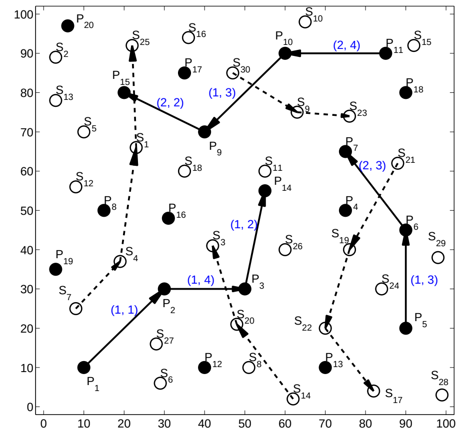


Fig. 5. Active sessions in the primary and secondary networks.

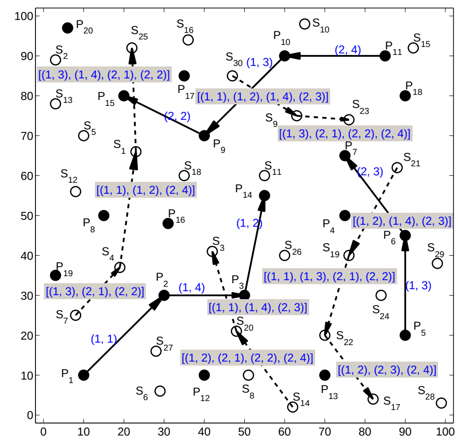


Fig. 6. Channel and time slot scheduling on each link for the secondary sessions by our solution algorithm. Channel and time slot scheduling on each link for the primary sessions are given in Fig. 5.

tremendous benefits (in terms of spectrum access and throughput gain) of the transparent coexistent paradigm over the existing interference avoidance paradigm.

A. An Example

Consider a 20-node primary network and a 30-node secondary network randomly deployed in the same 100×100 area. For the ease of scalability and generality, we normalize all units for distance, bandwidth, and throughput with appropriate dimensions. The location for each node (both primary and secondary) is generated at random and is listed in Table II. We assume that there are four antennas on each secondary node, and all nodes' transmission range and interference range are 30 and 50, respectively.[4] There are two channels owned by the primary network ($B = 2$). A time frame is divided into four time slots ($T = 4$). For simplicity, we assume the data rate of one data stream in a time slot is 1 unit ($c = 1$).

We assume there are three active sessions in the primary network and four active sessions in the secondary network (see Table III). For simplicity, we assume that minimum-hop routing is used for the primary and secondary sessions, although other routing methods will also work here. Further, the channel and time slot allocation on each hop for each primary session is known *a priori* and is shown in Fig. 5, where (b, t) means this link is transmitting on channel b in time slot t. The solid arrows represent the links in the primary network, while the dashed arrows represent the links in the secondary network.

For this network setting, we apply our solution algorithm to solve OPT. The obtained objective value is 1.0. The channel and time slot scheduling on each link for each secondary session is shown in the shaded box as in Fig. 6, where (b, t) on each secondary link represents that this link transmits on channel b in time slot t. The details of DoFs used for SM on each channel in each time slot on each link in the secondary network are shown in Table IV. The link rate (i.e., total number of DoFs used for SM averaged over a 4-time-slot frame) on a link is also shown in this table.

To see how the secondary node can be active simultaneously with the primary nodes while remain transparent, consider $(b, t) = (1, 2)$ (channel 1, time slot 2) in Fig. 6. Here, link $P_3 \to P_{14}$ in the primary network is active; links $S_{14} \to S_{20}$, $S_{22} \to S_{17}$, $S_{21} \to S_{19}$, $S_{30} \to S_9$ and $S_4 \to S_1$ in the secondary network are also active. Based on a node's

[4]For an indepth study on how to set interference range, we refer readers to our previous work in [15].

TABLE IV
CHANNEL AND TIME SLOT SCHEDULING ON EACH LINK, DOF ALLOCATION FOR SM, AND THROUGHPUT ON EACH LINK FOR THE SECONDARY SESSIONS.

Session	Link	(channel, time slot) scheduling	DoF for SM	Link rate
1	$S_7 \longrightarrow S_4$	(1, 3)	2	1.0
		(2, 1)	1.	
		(2, 2)	1	
	$S_4 \longrightarrow S_1$	(1, 1)	1	1.0
		(1, 2)	1	
		(2, 4)	2	
	$S_1 \longrightarrow S_{25}$	(1, 3)	1	1.0
		(1, 4)	1	
		(2, 1)	1	
		(2, 2)	1	
2	$S_{21} \longrightarrow S_{19}$	(1, 2)	1	1.0
		(1, 4)	2	
		(2, 3)	1	
	$S_{19} \longrightarrow S_{22}$	(1, 1)	1	1.0
		(1, 3)	1	
		(2, 1)	1	
		(2, 2)	1	
	$S_{22} \longrightarrow S_{17}$	(1, 2)	1	1.0
		(2, 3)	1	
		(2, 4)	2	
3	$S_{14} \longrightarrow S_{20}$	(1, 2)	1	1.0
		(2, 1)	1	
		(2, 2)	1	
		(2, 4)	1	
	$S_{20} \longrightarrow S_3$	(1, 1)	2	1.0
		(1, 4)	2	
		(2, 3)	1	
4	$S_{30} \longrightarrow S_9$	(1, 1)	1	1.0
		(1, 2)	1	
		(1, 4)	1	
		(2, 3)	1	
	$S_9 \longrightarrow S_{23}$	(1, 3)	1	1.0
		(2, 1)	1	
		(2, 2)	1	
		(2, 4)	1	

TABLE V
DOF ALLOCATION FOR SM AND IC ON $(b, t) = (1, 2)$ AT EACH NODE IN THE SECONDARY NETWORK.

Node i	TX/RX	$\pi_i^1(2)$	DoF for SM	DoF for IC to/from primary nodes	DoF for IC within secondary network
S_{19}	RX	1	1	1 from P_3	0
S_{14}	TX	2	1	0	1 to S_{19}
S_{22}	TX	4	1	1 to P_{14}	1 to S_{19}
S_{21}	TX	5	1	1 to P_{14}	0
S_{17}	RX	6	1	1 from P_3	1 from S_{14}
S_{20}	RX	8	1	1 from P_3	1 from S_{22}
S_{30}	TX	9	1	1 to P_{14}	0
S_9	RX	11	1	1 from P_3	1 from S_{21}
S_4	TX	12	1	1 to P_{14}	1 to S_{20}
S_1	RX	13	1	1 from P_3	1 from S_{30}

interference range, the interference relationships among the nodes associated with these active links are shown in Fig. 7, where the dotted arrow lines show the interference from a (primary or secondary) transmitter to an unintended (primary or secondary) receiver. Table V shows the DoF allocation at each secondary node for SM, IC to/from primary nodes, and IC within the secondary network for $(b, t) = (1, 2)$.

- First, we check whether there is any interference to primary receiver P_{14}. Note that there are four potential interference from secondary transmitters, i.e., S_4, S_{21}, S_{22} and S_{30}. Since each of these secondary transmitter uses one DoF to cancel its interference to primary receiver P_{14} (fifth column in Table V), all interference on the primary receiver P_{14} is effectively nulled. Therefore, the primary receiver P_{14} is not interfered by the simultaneous activation of its neighboring secondary transmitters.
- Next, we check whether the interference from the primary transmitter is nulled properly at its neighboring secondary receivers ("inter-network" interference). Note that primary transmit node P_3 is interfering its neighboring secondary receive nodes S_1, S_{20}, S_{17}, S_{19} and S_9. Since each of these secondary receive nodes uses one DoF to cancel this interference (fifth column in Table V), this interference from primary transmit node P_3 is effectively nulled at these secondary receive nodes.
- Finally, we check whether the interference within the secondary network ("intra-network" interference) is nulled properly by the secondary nodes themselves. The IC within the secondary network follows the node ordering, which is shown in the third column of Table V. The number of DoFs used for IC to/from other secondary nodes is shown in the last column of Table V. As an example, consider node S_{22}, which is a transmit node. Referring to Table V, S_{22} only needs to cancel its interference to those receive nodes that are before itself in the ordered node list and within S_{22}'s interference range, i.e., node S_{19}. Table V (last column) shows that S_{22} indeed uses one DoF to cancel its interference to S_{19}. For its interference to the secondary receive node S_{20} which is also in S_{22}'s interference range, S_{22} does not need to do anything as S_{20} is after node S_{22} in the ordered list. This interference to S_{20} will be canceled by S_{20} (as shown in Table V, last column).

  It can be easily verified that for all interference among the active secondary nodes are properly canceled. Further, at each active secondary node, the DoFs used for SM, IC to/from the primary nodes, IC within the secondary network is not more than its total DoFs (i.e., 4).

The above illustration is for $(b, t) = (1, 2)$ (i.e., channel 1, time slot 2), the results for the other channel and time slots (i.e., $(1, 1)$, $(1, 4)$, $(1, 3)$, $(2, 2)$, $(2, 3)$ and $(2, 4)$) are similar and are omitted to conserve space.

B. Comparison to Interference Avoidance Paradigm

To see the benefits of the transparent coexistence paradigm, we compare it to the prevailing interference avoidance paradigm. Under the interference avoidance paradigm, a secondary node is not allowed to transmit (receive) on the same channel at the same time when a nearby primary receiver (transmitter) is using this channel. Therefore, the set of available channel and time slots that can be used by secondary nodes is smaller. The problem formulation for this paradigm is similar to (but simpler than) OPT. In particular, we can remove the second term ($\sum_{p \in \tilde{\mathcal{I}}_i} \sum_{\tilde{l} \in \tilde{\mathcal{L}}_p^{\text{In}}} \tilde{z}_{(\tilde{l})}^b(t)$ and $\sum_{p \in \tilde{\mathcal{I}}_i} \sum_{\tilde{l} \in \tilde{\mathcal{L}}_p^{\text{Out}}} \tilde{z}_{(\tilde{l})}^b$) in constraints (5) and (12) in OPT that are used for secondary nodes to cancel interference to/from the primary nodes. The problem formulation remains an MILP

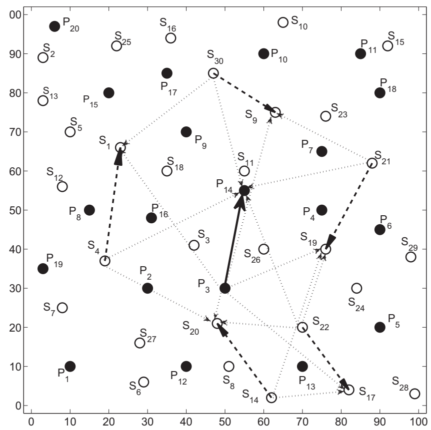


Fig. 7. Illustration of interference relationships among the primary and secondary links on channel 1 in time slot 2 in the case study.

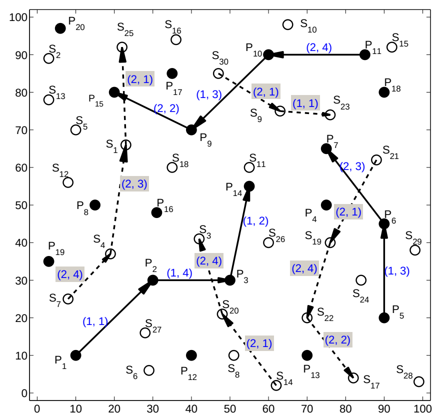


Fig. 8. Channel and time slot scheduling on each link for the secondary sessions under the interference avoidance paradigm.

and a solution algorithm similar to that in Section V-C can be used to solve it.

Following the same setting as in the case study in Section VI-A, we solve the above optimization problem under the interference avoidance paradigm. Note that the available channels and time slot resources at each node are only a subset of 2 channels and 4 time slots, versus full 2 channels and 4 time slots for each secondary node in the transparent coexistence paradigm. The obtained objective value is 0.5 (compared to 1.0 in Section VI-A). The channel and time slot scheduling on each link of each secondary session is shown in Fig. 8. Comparing Figs. 6 and 8, we find that the set of channels and time slots used by each secondary link under interference avoidance paradigm is smaller. The details for the DoF allocation for SM on each channel in each time slot and link rate are shown in Table VI. Comparing Tables VI and IV, the rates on most links are smaller under the interference avoidance paradigm.

C. Impact of Various System Parameters

The results in Sections VI-A and VI-B show the solution details for a case study in the transparent coexistence paradigm and its improvement in objective value over that in the interference avoidance paradigm. To show the robustness

TABLE VI
CHANNEL AND TIME SLOT SCHEDULING ON EACH LINK, DOF ALLOCATION FOR SM, AND LINK RATE ON EACH LINK FOR THE SECONDARY SESSIONS UNDER THE INTERFERENCE AVOIDANCE PARADIGM.

Session	Link	(channel, time slot) scheduling	DoF for SM	Link rate
1	$S_7 \longrightarrow S_4$	(2, 4)	2	0.5
	$S_4 \longrightarrow S_1$	(2, 3)	4	1.0
	$S_1 \longrightarrow S_{25}$	(2, 1)	2	0.5
2	$S_{21} \longrightarrow S_{19}$	(2, 1)	2	0.5
	$S_{19} \longrightarrow S_{22}$	(2, 4)	2	0.5
	$S_{22} \longrightarrow S_{17}$	(2, 2)	4	1.0
3	$S_{14} \longrightarrow S_{20}$	(2, 1)	2	0.5
	$S_{20} \longrightarrow S_3$	(2, 4)	2	0.5
4	$S_{30} \longrightarrow S_9$	(2, 1)	2	0.5
	$S_9 \longrightarrow S_{23}$	(1, 1)	4	1.0

of our results, we further perform numerical study for the same network under different system parameters, such as interference range setting, the number of antennas on each node, and the number of sessions in the secondary network.

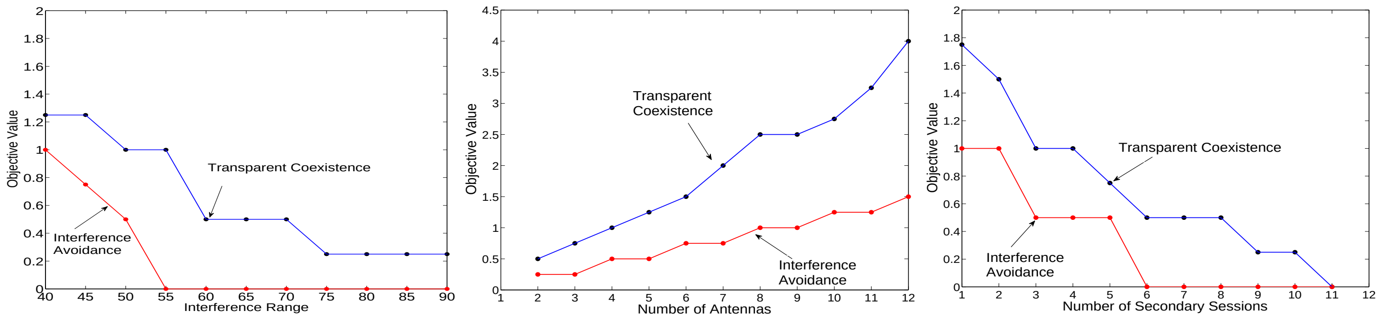
Fig. 9(a) shows the objective values under the transparent coexistence paradigm and the interference avoidance paradigm when the interference range for the secondary network is varied from 40 to 90 (while keeping the transmission range at 30). As shown in the figure, the performance under the transparent coexistence paradigm is always better than that under the interference avoidance paradigm for the same interference range, although the performance under both paradigms degrades when the interference range increases.

Fig. 9(b) shows the comparison of objective values with different antenna numbers for each secondary node under the two paradigms. Interference range for the secondary nodes is set to 50. For MIMO, the minimum number of antennas on a node is 2. As shown in the figure, the objective value under the transparent coexistence paradigm is always better than that under the interference avoidance paradigm for the same number of antennas. Further, the objective value increases under both paradigms.

Fig. 9(c) shows the comparison of objective values with different number of secondary sessions under the two paradigms. The number of antennas on each secondary node is 4. As shown in the figure, the objective value under the transparent coexistence paradigm is always better than that under the interference avoidance paradigm for the same number of secondary sessions, although the objective value decreases under both paradigms when the number of secondary sessions increases.

D. Complete Results for 50 Network Instances

Following the same setting as for the case study of one network instance in the Section VI-A, we randomly generate 50 instances, each with 20-node primary network and 30-node secondary network. For each instance, we randomly generate primary and secondary sessions, and compare the objective values obtained by the transparent coexistence paradigm and interference avoidance paradigm. Table VII shows the results from 50 network instances. The fourth column shows the percentage improvement for transparent coexistence paradigm



(a) Interference range (b) Number of antennas (c) Number of secondary sessions

Fig. 9. Impact of various system parameters on the performance of transparent coexistence and interference avoidance paradigms.

over interference avoidance paradigm. Note that some of the entries have ∞, indicating that the achievable session throughput (in DoFs) in the interference avoidance paradigm is 0. Overall, we find that the achievable session throughput under the transparent coexistence paradigm is much higher than that under the interference avoidance paradigm.

VII. Conclusions and Further Work

This paper explored the transparent coexistence paradigm for a multi-hop secondary network. This paradigm allows a secondary network to use the same spectrum simultaneously with the primary network as long as its activities are "transparent" (or "invisible") to the primary network. Such transparency is accomplished through a systematic interference cancelation (IC) by the secondary nodes without any impact on the primary network. The new technical challenges in a multi-hop network include channel/time slot scheduling, IC to/from primary network by the secondary network, and IC within the secondary network. We developed a rigorous mathematical modeling for a secondary multi-hop network in the transparent coexistence paradigm. As an application, we applied this model to study a throughput maximization problem with the objective of maximizing the minimum throughput among all secondary sessions. For the optimization problem, we developed an efficient polynomial time algorithm. Through simulation results, we show that the transparent coexistence paradigm offers significant improvement in spectrum access and throughput performance over the existing prevailing interference avoidance paradigm.

Although this work shows the potential of transparent coexistence in terms of throughput improvement for the secondary networks, much work remains to be done to transition this idea into reality. In particular, the focus of this paper has been on exploring performance gain of transparent coexistence under idealized network setting (by ignoring many details that may arise from practical operations). We briefly discuss some of the practical issues that must be addressed in future work to achieve transparent coexistence in the real world. This discussion is not meant to be exhaustive, as the transparent coexistence is a novel concept and its path to adaptation is bound to encounter many challenges, both known and unknown. The first issue is that the secondary nodes need to have accurate knowledge of the primary nodes' transmission behavior (information regarding transmitter, receiver, time slot, and channel). This issue is easier to address in a single-hop environment (cellular, TV tower, WiFi) but is a major challenge in a multi-hop ad hoc network environment. Second, we assume the schemes in Section II-A to obtain CSI would work perfectly and channel reciprocity strictly holds. But in reality, the communication channel not only consists of the physical channel, but also the antennas, RF mixers, filters, A/D converters, etc., which are not necessarily identical on all the nodes. Therefore, complex calibration among the nodes is needed to achieve channel reciprocity. Such calibration is no simple task for a pair of transmitter and receiver and is even more complicated among a network of nodes. Third, zero-forcing based IC may not be perfect even if we have perfect CSI. A consequence of non-perfect IC is interference leakage, which is undesirable for both primary and secondary receivers. How to mitigate such interference leakage to a minimal acceptable level should be a key consideration when deploying transparent coexistence for real applications. Fourth, the IC and DoF allocation algorithm that we designed for the secondary network is a centralized one. Such a centralized solution serves our purpose of introducing a new concept. It bears similar pros and cons of other centralized algorithm for a wireless network. If a centralized solution is adopted in practice, those issues must be carefully addressed. On the other hand, if a distributed solution is desired, then a different set of issues need to be addressed. These issues include partial network knowledge, limited information sharing, communication overheard, ensuring IC feasibility at each secondary node, among others. Regardless centralized or distributed solution, flow dynamics (new session initiation, existing session termination) will add additional complexity on information update and algorithm execution. Clearly, there is a large landscape for further research on these important practical operation issues. We expect to see more follow-up research in this area in the near future.

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TABLE VII
ACHIEVABLE MINIMUM SESSION THROUGHPUT UNDER TRANSPARENT COEXISTENCE PARADIGM AND INTERFERENCE AVOIDANCE PARADIGM FOR 50 CASES.

Network Instance	Transparent Coexistence	Interference Avoidance	Percentage Improvement
1	1.0	0.5	100%
2	1.0	0.5	100%
3	1.25	0.75	66.7%
4	1.0	0.5	100%
5	1.0	0	∞
6	1.0	0.75	33.3%
7	1.0	0	∞
8	1.0	0.5	100%
9	1.5	1	50%
10	1.0	0.5	50%
11	1.0	0.5	50%
12	1.0	0.75	33.3%
13	1.25	0.75	66.7%
14	1.0	0	∞
15	1.0	0.5	100%
16	1.0	0.5	100%
17	1.0	0.75	33.3%
18	0.75	0.5	50%
19	1.0	0.5	100%
20	0.75	0	∞
21	1.0	0	∞
22	0.75	0.5	50%
23	1.0	0.5	100%
24	1.25	0.75	66.7%
25	0.5	0	∞
26	0.5	0	∞
27	0.75	0.5	50%
28	1.0	0.5	100%
29	0.25	0	∞
30	1.0	0.75	33.3%
31	1.5	0.75	100%
32	1.25	0	∞
33	1.0	0.5	100%
34	1.0	0.5	100%
35	1.25	0.75	66.7%
36	0.75	0.5	50%
37	0.5	0	∞
38	1.0	0.25	300%
39	0.25	0	∞
40	1.0	0.5	100%
41	1.25	1.0	25%
42	1.0	0.5	100%
43	1.0	0.5	100%
44	0.5	0	∞
45	1.0	0.5	100%
46	1.0	0.5	100%
47	0.75	0.5	50%
48	0.25	0	∞
49	1.0	0.5	100%
50	1.0	0.5	100%

this paper are those of the authors and do not reflect the views of the NSF.

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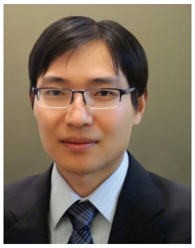

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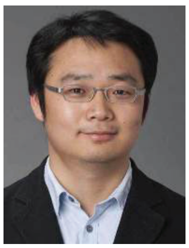

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