Optimal Base Station Placement in Wireless Sensor Networks

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Base station location has a significant impact on network lifetime performance for a sensor network. For a multihop sensor network, this problem is particularly challenging due to its coupling with data routing. This article presents an approximation algorithm that can guarantee $(1-\varepsilon)$-optimal network lifetime performance for base station placement problem with any desired error bound $\varepsilon > 0$. The proposed $(1-\varepsilon)$-optimal approximation algorithm is based on several novel techniques that makes it possible to reduce an infinite search space to a finite-element search space for base station location. The first technique used in this reduction is to discretize cost parameter (associated with energy consumption) with performance guarantee. Subsequently, the continuous search space can be broken up into a finite number of subareas. The second technique is to exploit the cost property of each subarea and represent it by a novel notion called fictitious cost point, each with guaranteed cost bounds. We give a proof that the proposed base station placement algorithm is $(1-\varepsilon)$-optimal. This approximation algorithm is simpler and faster than a state-of-the-art algorithm and represents the best known result to the base station placement problem.

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1. INTRODUCTION

An important performance metric for wireless sensor networks is the so-called network lifetime, which is highly dependent upon the physical topology of the network. This is because energy expenditure at a node to transmit data to another node not only depends on the data bit rate, but also on the physical distance between these two nodes. Consequently, it is important to understand the impact of location related issues on network lifetime performance and to optimize topology during network deployment stage.

This article considers the important base station placement problem for a given sensor network such that network lifetime can be maximized. Specifically, we consider the following problem. Given a sensor network with each node i producing sensing data at a rate of r_i, where should we place the base station in this sensor network such that all the data can be forwarded to the base station (via multihop and multipath if necessary) such that the network lifetime is maximized?

In Section 6, we give a comprehensive review of related work on network lifetime and node placement problems and contrast their differences with this work. The most relevant work on this problem, done by Efrat et al. [2005], represents the state-of-the-art result on this problem. As we shall show, the computational complexity of this algorithm is higher than the one to be presented in this paper for most cases.

The main idea of our approximation algorithm is to exploit a clever way of discretizing cost parameter associated with energy consumption into a geometric sequence with tight upper and lower bounds. As a result, we can divide a continuous search space into a finite number of subareas. By further exploiting the cost property of each subarea, we conceive a novel idea to represent each subarea with a so-called fictitious cost point, which is an N-tuple cost vector with each component representing the upper bound of cost to a sensor node in the network. Based on these ideas, we can successfully reduce an infinite search space for base station location into finite number of "points" upon which we can apply a linear program (LP) to find the achievable network lifetime and data routing solution. By comparing the achievable network lifetime among all the fictitious cost points, we show that the largest is $(1 - \varepsilon)$-optimal. We also show that placing the base station at *any physical point* in the subarea corresponding to the best fictitious cost point is $(1 - \varepsilon)$-optimal. We analyze the complexity of our approximation algorithm and show that it is practically faster than the algorithm proposed in Efrat et al. [2005] for most cases, which was the best known result on this problem. As a result, the algorithm presented in this paper represents the best known result to the base station placement problem.

The rest of this article is organized as follows. Section 2 presents the network model used in this study and describes the base station placement problem. Section 3 presents the main result of this article, which is a $(1-\varepsilon)$-approximation algorithm for the base station placement problem. In Section 4, we present some numerical results illustrating the efficacy of the proposed algorithm. In Section 5, we extend our approximation algorithm to handle bounded transmission power and multiple base stations. Section 6 reviews related work and Section 7 concludes this article.

2. NETWORK MODEL AND PROBLEM DESCRIPTION

2.1 Network Model

We consider a static sensor network consisting of a set $\mathcal{N}$ of sensor nodes deployed over a two-dimensional area. The location of each sensor node is fixed and the initial energy on sensor node i is denoted as e_i. Each sensor node i generates data at a rate r_i. For the time being, we assume there is one base station that needs to be deployed in the area to collect sensing data. Extension to multiple base station will be discussed in Section 5.

In this article, we focus on the energy consumption due to communications (i.e., data transmission and reception). Suppose sensor node i transmits data to sensor node j with a rate of f_{ij} b/s. Then we model the transmission power at sensor node i as [Heinzelman 2000]

$$u_{ij}^t = c_{ij} \cdot f_{ij}. \tag{1}$$

c_{ij} is the cost associated with link (i, j) and can be modeled as

$$c_{ij} = \beta_1 + \beta_2 \cdot d_{ij}^{\alpha}, \tag{2}$$

where β_1 and β_2 are constant coefficients, d_{ij} is the physical distance between sensor nodes i and j, α is the path loss index, and $2 \leq \alpha \leq 4$ [Heinzelman 2000].

The power consumption at the receiving sensor node i can be modeled as [Heinzelman 2000]

$$u_i^r = \rho \cdot \sum_{k \in \mathcal{N}}^{k \neq i} f_{ki}, \tag{3}$$

where f_{ki} (also in b/s) is the incoming bit-rate received by sensor i from sensor k and ρ is a constant coefficient.

In this article, we assume that the interference from simultaneous transmissions can be effectively avoided by appropriate MAC layer scheduling. For low bit rate and deterministic traffic pattern considered in this article, a contention-free MAC protocol is fairly easy to design (see, e.g., Sohrabi et al. [2000]) and its discussion is beyond the scope of this article. Table I lists all notation used in this article.

2.2 Problem Description

The focus of this article is to investigate how to optimally place a base station to collect data in a sensor network so that the network lifetime can be maximized.

Table I. Notation

Symbol	Definition
$\mathcal{A}$	The search space for the base station, which can be the smallest enclosing disk to cover all sensor nodes
$\mathcal{A}_m$	The m-th subarea in the search space
B	The base station
c_{ij} (or $c_{iB}(p)$)	Power consumption coefficient for transmitting data from sensor i to sensor j (or base station B at point p)
$C_{iB}^{\min}, C_{iB}^{\max}$	Lower and upper bounds of $c_{iB}(p)$
$C[h]$	The transmission cost for the h-th circle
d_{ij} (or d_{iB})	Distance between sensor i and sensor j (or base station B)
e_i	Initial energy at sensor i
f_{ij} (or f_{iB})	Data rate from sensor i to sensor j (or base station B)
H_i	Total number of circles for discretization at sensor node i required for a given ε
K	Total number of circles for discretization under a given ε
M	Total number of subareas for discretization under a given ε
$\mathcal{N}$	Set of sensor nodes in the network
N	Number of sensor nodes in the network, $N = \|\mathcal{N}\|$
$O_{\mathcal{A}}$	The center of the smallest enclosing disk $\mathcal{A}$
p_m	Fictitious cost point representation for the m-th subarea
p_{opt}	The best base station location
p^*	The best location among M fictitious cost points
p_ε	A point in the subarea corresponding to p^*
r_i	Sensing data rate produced at sensor i
$R_{\mathcal{A}}$	The radius of the smallest enclosing disk $\mathcal{A}$
T_m	Maximum achievable network lifetime by placing the base station at p_m
T_{opt}	Optimal network lifetime achieved by placing the base station at p_{opt}
T^*	$= \max\{T_m : m = 1, 2, \ldots, M\}$
T_ε	$(1-\varepsilon)$-optimal network lifetime achieved by p_ε
u_{ij}^t	Transmission power at sensor node i to sensor node i
u_i^r	Power consumption at the receiving sensor node i
V_{ij} (or V_{iB})	Total data volume from sensor i to sensor j (or base station B)
α	Path loss index
β_1, β_2	Constant terms in transmission power modeling
ε	Desired approximation error, $\varepsilon > 0$
ρ	Power consumption coefficient for receiving data
ψ_{opt}	The best routing solution when the base station is at p_{opt}
ψ^*	Routing solution when the base station is at p^*
ψ_ε	Routing solution when the base station is at p_ε

The network lifetime is defined as the time until any sensor node uses up its energy. To achieve optimality, the data generated by each sensor node is allowed to be routed to the base station via multihop or multipath. Also, power control at a node is allowed, as modeled in (1) and (2).

Assume that base station B is located at a point p. Denote (x_B, y_B) the position of point p and T the network lifetime. A feasible routing solution achieving this network lifetime T should satisfy both flow balance and energy constraints at each sensor node. These constraints can be formally stated as follows. Denote f_{ij} and f_{iB} the data rates from sensor node i to sensor node j and base station B, respectively (since we allow multi-path). Then the flow balance for each sensor node i is

$$\sum_{k\in\mathcal{N}}^{k\neq i} f_{ki} + r_i = \sum_{j\in\mathcal{N}}^{j\neq i} f_{ij} + f_{iB},$$

that is, the sum of total incoming flow rates plus self-generated data rate is equal to the sum of total outgoing flow rates. The energy constraint for each sensor node i is

$$\sum_{k\in\mathcal{N}}^{k\neq i} \rho \cdot f_{ki}T + \sum_{j\in\mathcal{N}}^{j\neq i} c_{ij} \cdot f_{ij}T + c_{iB}(p) \cdot f_{iB}T \le e_i,$$

that is, total consumed energy due to receiving and transmission over time T cannot exceed its initial energy e_i. By (2), we have

$$c_{iB}(p) = \beta_1 + \beta_2 \left[\sqrt{(x_B - x_i)^2 + (y_B - y_i)^2}\right]^\alpha,$$

which is a nonlinear function of base station location (x_B, y_B).

Our objective is maximizing the network lifetime T under the flow balance and energy constraints, that is,

$$\begin{array}{llll}
\text{Max} & T & & \\
\text{s.t.} & \sum_{j\in\mathcal{N}}^{j\neq i} f_{ij} + f_{iB} - \sum_{k\in\mathcal{N}}^{k\neq i} f_{ki} = r_i & (i \in \mathcal{N}) & (4)\\
& \sum_{k\in\mathcal{N}}^{k\neq i} \rho f_{ki}T + \sum_{j\in\mathcal{N}}^{j\neq i} c_{ij} f_{ij}T + c_{iB}(p) f_{iB}T \le e_i & (i \in \mathcal{N}) & (5)\\
& c_{iB}(p) - \beta_2 \left[\sqrt{(x_B - x_i)^2 + (y_B - y_i)^2}\right]^\alpha = \beta_1 & (i \in \mathcal{N}) & \\
& (x_B, y_B) \in \mathcal{A}, T, f_{ij}, f_{iB} \ge 0 & (i, j \in \mathcal{N}, i \neq j), &
\end{array}$$

where $\mathcal{A}$ is an area of possible base station locations and will be determined by Lemma 3.1. This optimization problem is in the form of *non-linear program*, which is NP-hard in general [Garey and Johnson 1979].

3. ALGORITHM DESIGN

In this article, we aim to develop an approximation algorithm to solve the base station placement problem. In particular, the algorithm that we will develop is $(1-\varepsilon)$-optimal, meaning that for any small $\varepsilon > 0$, the solution is guaranteed to be within $(1-\varepsilon)$ from the optimal, which is unknown (due to the NP-hardness of the problem).

We first give a roadmap for the design of this approximation algorithm, which also corresponds to the structure of this section. First, in Section 3.1, we show that for a given base station location, the maximum achievable network lifetime and the corresponding optimal routing can be found via a single linear program (LP). So the problem becomes how to search the best base station location. Instead of searching the entire two-dimensional plane, in Section 3.2, we narrow down the search space to the so-called smallest enclosing disk (SED), which is the smallest circular area that covers all the sensor nodes in the network. Although the SED contains a smaller area, there are still infinite number of points in this SED. In Section 3.3, we divide the continuous search space of

SED into a finite number of subareas. This is made possible by a novel idea of discretizing cost parameter associated with energy consumption with tight upper and lower bounds. By further exploiting the cost property of each subarea, we employ another novel idea of representing each subarea by a so-called fictitious cost point, which is an N-tuple cost vector with each component representing the upper bound of cost to a sensor node in the network. Based on these ideas, we are able to successfully reduce an infinite search space for base station location into finite "points" upon which we can apply a LP to find the corresponding achievable network lifetime and data routing solution for each of these points. By comparing the achievable network lifetime among all the fictitious cost points, we find the fictitious cost point corresponding to the maximum network lifetime. We show that by placing the base station at *any point* in the subarea corresponding to the best fictitious cost point will give a $(1-\varepsilon)$-optimal network lifetime. In Section 3.4, we summarize all the steps as an algorithm and give an example for illustration. In Section 3.5, we prove the correctness of the algorithm and analyze its complexity.

3.1 Optimal Routing for a Given Base Station Location

As discussed earlier, the maximum network lifetime depends on both base station location and data routing. To start with, we show that for a *given* base station location, we can find the maximum achievable network lifetime and optimal routing via a single LP as follows.

The objective function is network lifetime T and the constraints are given in (4) and (5). Multiply both sides of (4) by T and denote

$$V_{ij} = f_{ij}T \;\text{ and }\; V_{iB} = f_{iB}T, \tag{6}$$

where V_{ij} (or V_{iB}) can be interpreted as the total data volume from sensor node i to sensor node j (or base station B) over time T. We have

$$\begin{array}{lll}
\text{Max} & T & \\
\text{s.t.} & \sum_{k\in\mathcal{N}}^{k\neq i} V_{ki} + r_i T - \sum_{j\in\mathcal{N}}^{j\neq i} V_{ij} - V_{iB} = 0 & (i \in \mathcal{N}) \\
& \sum_{k\in\mathcal{N}}^{k\neq i} \rho V_{ki} + \sum_{j\in\mathcal{N}}^{j\neq i} c_{ij} V_{ij} + c_{iB}(p) V_{iB} \le e_i & (i \in \mathcal{N}) \\
& T, V_{ij}, V_{iB} \ge 0 & (i, j \in \mathcal{N}, i \neq j)
\end{array}$$

Note that for a given base station location, $c_{iB}(p)$'s are constants. Therefore, this formulation is in the form of a LP. Once we solve the LP, we can obtain optimal routing solution for f_{ij} and f_{iB} by $f_{ij} = \frac{V_{ij}}{T}$ and $f_{iB} = \frac{V_{iB}}{T}$.

3.2 Search Space

Although for a given base station location, we can find the corresponding maximum achievable network lifetime via a single LP, it is not possible to examine all points (infinite) in the two-dimensional plane and select the point with the maximum network lifetime.

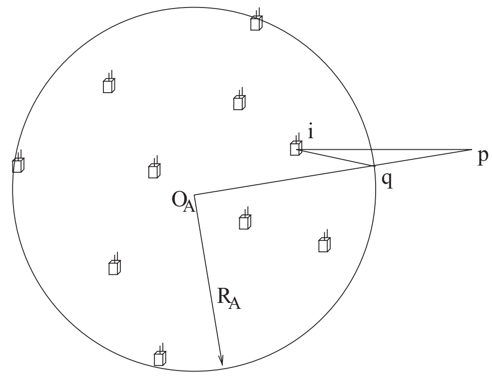

Fig. 1. A schematic diagram showing that optimal base station location must be within SED.

As a first step, we show that it is only necessary to consider points inside the so-called *smallest enclosing disk* (SED) [Welzl 1991],[1] which is a unique disk with the smallest radius that contains all the N sensor nodes in the network and can be found in $O(N)$ time [Megiddo 1983]. This is formally stated in the following lemma.

LEMMA 3.1. *To maximize network lifetime, the base station location must be within the smallest enclosing disk $\mathcal{A}$ that covers all the N sensor nodes in the network.*

PROOF. The proof is based on contradiction. That is, if the base station location is not in SED, then its corresponding network lifetime cannot be maximum.

Assume that the optimal base station location is at point p, which is outside SED $\mathcal{A}$ (see Figure 1). Denote $O_{\mathcal{A}}$ the center of SED. Let q be the intersecting point between the line segment $[p, O_{\mathcal{A}}]$ and the circle of SED. Then for any sensor node i (all in $\mathcal{A}$), we have $d_{iq} < d_{ip}$. Consequently, $c_{iq} < c_{ip}$. As a result, we can save transmission energy on every sensor node $i \in \mathcal{N}$ by relocating p to q. This saving in energy at each node increases network lifetime. This shows that point p cannot be the optimal location to maximize network lifetime. This completes the proof. □

Now we have narrowed down the search space for base station B from a two-dimensional plane to SED $\mathcal{A}$. However, the number of points in $\mathcal{A}$ remains infinite. It is tempting to divide $\mathcal{A}$ into small subareas (e.g., a gridlike structure), $\mathcal{A}_1, \mathcal{A}_2, \ldots$, up to say $\mathcal{A}_M$, that is,

$$\mathcal{A} = \bigcup_{m=1}^{M} \mathcal{A}_m.$$

When each subarea is sufficiently small (i.e., M is sufficiently large), we can use some point $q_m \in \mathcal{A}_m$ to represent $\mathcal{A}_m$, $m = 1, 2, \ldots, M$. By applying an

[1]In fact, we can consider points in an even smaller area, that is, the convex hull of all sensor nodes. However, using convex hull cannot reduce the order of complexity of our algorithm. On the other hand, the use of SED can simplify the discussion.

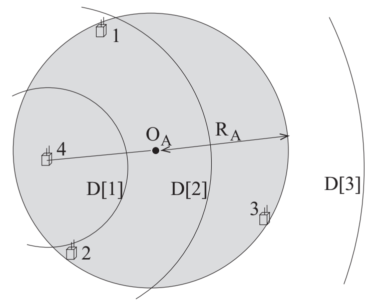

Fig. 2. A sequence of circles with increasing costs with center at node 4.

LP on each of the M points, we can select the best location among all points and obtain a good solution for base station placement. However, such approach is *heuristic* at best and does not provide any *theoretical guarantee* on performance.

The key to provide a theoretical guarantee on performance is to divide the subarea in such a way that tight bounds can be guaranteed on any point in the subarea. If this is possible, then we may be able to exploit such property and develop an approximation algorithm that yields *provably* $(1-\varepsilon)$-optimal network lifetime performance. The goal of this paper is to develop such an algorithm with performance guarantee instead of a heuristic algorithm. In the following section, we show a novel technique to divide SED $\mathcal{A}$ into subareas where each subarea can be represented by a point with a set of tight bounds. Consequently, we show how a $(1-\varepsilon)$-optimal approximation algorithm can be developed.

3.3 Subarea Division and Fictitious Cost Points

3.3.1 *Subarea Division.* The proposed subarea division (with guaranteed performance bounds) hinges upon a novel discretization of cost parameter. A close look at the energy constraint in (5) suggests that the location of the base station is *embedded* in the cost parameter $c_{iB}(p)$'s. In other words, if we can discretize these cost parameters, we may also discretize the location for the base station.

Since the search space is narrowed down to the SED $\mathcal{A}$, we can limit the range for the distance between a sensor node i to the possible location for the base station. Denote $O_{\mathcal{A}}$ and $R_{\mathcal{A}}$ the origin and radius of the SED $\mathcal{A}$. For each sensor node $i \in \mathcal{N}$, denote $D_{i,O_{\mathcal{A}}}$ the distance from sensor node i to the origin of disk $\mathcal{A}$ (see node 4 in Figure 2 as an example). Denote $D_{iB}^{\min}$ and $D_{iB}^{\max}$ the minimum and maximum distances between sensor node i and possible location for the base station B, respectively. Then we have

$$\begin{aligned} D_{iB}^{\min} &= 0, \\ D_{iB}^{\max} &= D_{i,O_{\mathcal{A}}} + R_{\mathcal{A}}. \end{aligned}$$

Corresponding to $D_{iB}^{\min}$ and $D_{iB}^{\max}$, denote $C_{iB}^{\min}$ and $C_{iB}^{\max}$ the minimum and maximum cost between sensor node i and base station B, respectively. Then by (2), we have

$$C_{iB}^{\min} = \beta_1, \tag{7}$$
$$C_{iB}^{\max} = \beta_1 + \beta_2 (D_{iB}^{\max})^\alpha = \beta_1 + \beta_2 (D_{i,O_\mathcal{A}} + R_\mathcal{A})^\alpha. \tag{8}$$

Given the range of $d_{iB} \in [D_{iB}^{\min}, D_{iB}^{\max}] = [0, D_{i,O_\mathcal{A}} + R_\mathcal{A}]$ for each sensor node i, we now show how to divide disk $\mathcal{A}$ into a finite number of subareas with the distance of each subarea to sensor node i meeting some tight bounds. Specifically, from a sensor node i, we draw a sequence of circles centered at this sensor node, each with increasing radius $D[1], D[2], \ldots, D[H_i]$ corresponding to costs $C[1], C[2], \ldots, C[H_i]$ that are defined as the following geometric sequence.

$$C[h] = C_{iB}^{\min}(1+\varepsilon)^h = \beta_1 (1+\varepsilon)^h \ \ (1 \le h \le H_i). \tag{9}$$

This geometric sequence $C[h]$ (with a factor of $(1+\varepsilon)$) is carefully chosen and will offer tight performance bounds for any point in a subarea (more on this later). The number of required circles H_i can be determined by having the last circle in the sequence (with radius $D[H_i]$) to completely contain disk $\mathcal{A}$, i.e. $D[H_i] \ge D_{iB}^{\max}$, or equivalently,

$$C[H_i] \ge C_{iB}^{\max} .$$

Thus, we can determine H_i as follows.

$$H_i = \left\lceil \frac{\ln\left(C_{iB}^{\max}/C_{iB}^{\min}\right)}{\ln(1+\varepsilon)} \right\rceil = \left\lceil \frac{\ln\left(1 + \frac{\beta_2}{\beta_1}(D_{i,O_\mathcal{A}} + R_\mathcal{A})^\alpha\right)}{\ln(1+\varepsilon)} \right\rceil . \tag{10}$$

For example, for node 4 in Figure 2, we have $H_4 = 3$, that is, $D[3]$ is the circle centered at node 4 that will completely contain the disk. As a result, with sensor node i as center, we have a total of H_i circles, each with cost $C[h]$, $h = 1, 2, \ldots, H_i$.

This partitioning of SED $\mathcal{A}$ is with respect to a specific node i. We now perform the above process for *all* sensor nodes. These intersecting circles will cut disk $\mathcal{A}$ into a finite number of *irregular* subareas, with the boundaries of each subarea being either an arc (with a center at some sensor node i and some cost $C[h]$, $1 \le h < H_i$) or an arc from SED $\mathcal{A}$. As an example, the SED $\mathcal{A}$ in Figure 3 is now divided into 28 subareas.

Now we claim that under this subarea partitioning technique, for any point in a given subarea, its cost to each sensor node in the network can be *tightly* bounded quantitatively. This is because with respect to each sensor node i, a subarea $\mathcal{A}_m$ must be enclosed within some arc centered at sensor node i. Denote the index of this arc (w.r.t. sensor node i) as $h_i(\mathcal{A}_m)$. So when the base station B is at any point $p \in \mathcal{A}_m$, we have

$$C[h_i(\mathcal{A}_m) - 1] \le c_{iB}(p) \le C[h_i(\mathcal{A}_m)], \tag{11}$$

where we define $C[0] = C_{iB}^{\min} = \beta_1$. Since $\frac{C[h_i(\mathcal{A}_m)]}{C[h_i(\mathcal{A}_m)-1]} = 1 + \varepsilon$ by (9), we have a very tight upper and lower bounds for $c_{iB}(p)$. The reader can now have a better appreciation of the benefit of the proposed discretization for cost and distance.

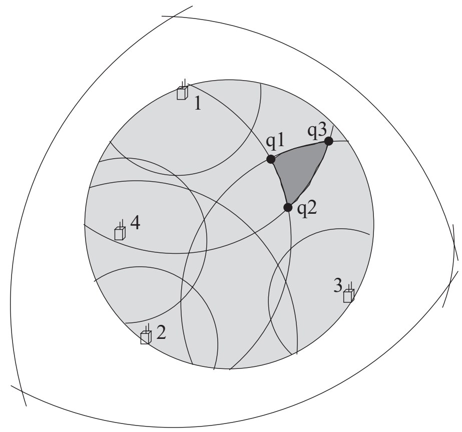

Fig. 3. An example of subareas within disk $\mathcal{A}$ that are obtained by intersecting arcs from different circles.

3.3.2 *Fictitious Cost Point.* We now introduce a novel concept called *fictitious cost point*. It will be used to represent upper bound of cost for any point in a subarea $\mathcal{A}_m$, $m = 1, 2, \ldots, M$.

Definition 3.2. Denote the fictitious cost point for subarea $\mathcal{A}_m$ ($m = 1, 2, \ldots, M$) as p_m, which is represented by an N-tuple vector with its i-th element ($i = 1, 2, \ldots, N$) being upper bound of cost for any point in subarea $\mathcal{A}_m$ to the i-th sensor node in the network.

That is, the N-tuple cost vector for fictitious cost point p_m is [$c_{1B}(p_m)$, $c_{2B}(p_m)$, ..., $c_{NB}(p_m)$], with the i-th element $c_{iB}(p_m)$ being

$$c_{iB}(p_m) = C[h_i(\mathcal{A}_m)], \tag{12}$$

where $h_i(\mathcal{A}_m)$ is determined by (11).

As an example, the fictitious cost point for subarea with corner points (q_1, q_2, q_3) in Figure 3 can be represented by 4-tuple cost vector [$c_{1B}(p_m)$, $c_{2B}(p_m)$, $c_{3B}(p_m)$, $c_{4B}(p_m)$] = [C[2], C[3], C[2], C[3]], where the first component C[2] represents an upper bound of cost for any point in this subarea to sensor node 1, the second component C[3] represents an upper bound of cost (which is loose here) for any point in this subarea to sensor node 2, and so forth.

We emphasize that the reason we use the word "fictitious" is that a fictitious cost point p_m may not be mapped to a *physical* point within the corresponding subarea $\mathcal{A}_m$. This happens when there does not exist a physical point in subarea $\mathcal{A}_m$ that has its costs to all the N sensor nodes *equal* (one by one) to the respective N-tuple cost vector embodied by p_m *simultaneously.* As an example, any point within the dark subarea bounded by corner points q_1, q_2, and q_3 cannot have its costs to the four sensor nodes in the network equal to the respective element in [C[2], C[3], C[2], C[3]] simultaneously, where [C[2], C[3], C[2], C[3]] is the cost vector of the fictitious cost point for this subarea.

Using fictitious cost points to represent subareas (and thus dividing SED into a finite search space) is a key step in designing our low complexity

Table II. Sensor Locations, Data Rate, and Initial Energy of the Example Sensor Network

Node Index	Location	Data Rate	Initial Energy
1	(0.1, 0.5)	0.8	390
2	(1.1, 0.7)	1.0	400
3	(0.4, 0.1)	0.5	130

approximation algorithm. This approach overcomes the limitation in Efrat et al. [2005], where the authors used a physical points to construct a finite search space. Under that approach, the authors were not able to discretize cost directly. Instead, they considered how to discretize transmission energy, flow rate, and network lifetime such that cost can be discretized. The number of discretized costs is the product of the numbers of discretized transmission energies, flow rates, and network lifetimes. We will show that the complexity associated with that approach in Efrat et al. [2005] is higher than ours for most cases.

The following important property for fictitious cost point p_m will be used in the proof of $(1-\varepsilon)$-optimal of the approximation algorithm.

PROPERTY 1. *For any point $p \in \mathcal{A}_m$ and the corresponding fictitious cost point p_m, we have*

$$c_{iB}(p_m) \leq (1+\varepsilon)c_{iB}(p).$$

PROOF. By (11) and definition of fictitious cost point p_m (see (12)), we have

$$c_{iB}(p_m) = C[h_i(\mathcal{A}_m)] = (1+\varepsilon)\cdot C[h_i(\mathcal{A}_m)-1] \leq (1+\varepsilon)\cdot c_{iB}(p),$$

where the inequality follows from (11). This completes the proof. □

3.4 Summary of Algorithm and Example

By discretizing the cost parameters and the corresponding distances, we have partitioned the search space (SED $\mathcal{A}$) into a finite number of M subareas. By introducing the concept of fictitious cost points (FCPs), we can represent each subarea with a point. As a result, we can now readily apply the LP approach discussed in Section 3.1 to examine each FCP and choose the FCP that offers the maximum network lifetime. The complete approximation algorithm is outlined in Algorithm 1. The correctness proof of $(1-\varepsilon)$-optimality is given in Section 3.5.

Example 1. We use a small 3-node network to illustrate the steps of the approximation algorithm. The location, data rate, and initial energy for each sensor are shown in Table II, where the units of distance, rate, and energy are all normalized. Also, we set $\alpha = 2$, $\beta_1 = 1$, $\beta_2 = 0.5$, and $\rho = 1$ under the normalized units. For illustration, we set the error bound to $\varepsilon = 0.2$.[2]

(1) We identify SED $\mathcal{A}$ with origin $O_{\mathcal{A}} = (0.61, 0.57)$ and radius $R_{\mathcal{A}} = 0.51$ (see Figure 4).

[2]This ε is used here to simplify the illustration of each step. In our numerical results in Section 4, we use $\varepsilon = 0.05$ for all computations.

Algorithm 1. (A $(1-\varepsilon)$-Approximation Algorithm)

(1) Find the smallest enclosing disk $\mathcal{A}$ that covers all the N nodes.
(2) Within $\mathcal{A}$, compute the lower and upper cost bounds $C_{iB}^{\min}$ and $C_{iB}^{\max}$ for each node $i \in \mathcal{N}$ by (7) and (8).
(3) For a given $\varepsilon > 0$, define a sequence of costs $C[1], C[2], \ldots, C[H_i]$ by (9), where H_i is calculated by (10).
(4) At each node i, draw a sequence of $H_i - 1$ circles centered at node i with increasing radius corresponding to cost $C[h]$, $h = 1, 2, \ldots, H_i - 1$. The intersection of these circles within disk $\mathcal{A}$ will divide $\mathcal{A}$ into M subareas $\mathcal{A}_1, \mathcal{A}_2, \ldots, \mathcal{A}_M$.
(5) For each subarea $\mathcal{A}_m$, $1 \le m \le M$, define a FCP p_m by an N-tuple cost vector $[c_{1B}(p_m), c_{2B}(p_m), \ldots, c_{NB}(p_m)]$, where $c_{iB}(p_m)$ is defined in (12).
(6) For each FCP p_m, $1 \le m \le M$, apply the LP in Section 3.1 and obtain the achievable network lifetime T_m.
(7) Choose the FCP p^* that offers the maximum network lifetime among these M FCPs. The base station can be placed at any point p_ε within the subarea corresponding to p^*.
(8) For the chosen point p_ε, apply the LP in Section 3.1 and obtain $(1-\varepsilon)$ optimal network lifetime T_ε.

(2) We first have $D_{i,O_{\mathcal{A}}} = R_{\mathcal{A}} = 0.51$ for each node i, $1 \le i \le 3$. We then find the lower and upper bounds of c_{iB} for each node i, $1 \le i \le 3$, as follows.

$$\begin{aligned} C_{iB}^{\min} &= \beta_1 = 1, \\ C_{iB}^{\max} &= \beta_1 + \beta_2 (D_{i,O_{\mathcal{A}}} + R_{\mathcal{A}})^\alpha = 1 + 0.5 \cdot (0.51 + 0.51)^2 = 1.52. \end{aligned}$$

(3) For each node i, $1 \le i \le 3$, we find

$$H_i = \left\lceil \frac{\ln\left(1 + \frac{\beta_2}{\beta_1}(D_{i,O_{\mathcal{A}}} + R_{\mathcal{A}})^\alpha\right)}{\ln(1+\varepsilon)} \right\rceil = \left\lceil \frac{\ln\left(1 + \frac{0.5}{1}(0.51 + 0.51)^2\right)}{\ln(1+0.2)} \right\rceil = 3,$$

and

$$\begin{aligned} C[1] &= \beta_1(1+\varepsilon) = 1 \cdot (1+0.2) = 1.20, \\ C[2] &= \beta_1(1+\varepsilon)^2 = 1 \cdot (1+0.2)^2 = 1.44, \\ C[3] &= \beta_1(1+\varepsilon)^3 = 1 \cdot (1+0.2)^3 = 1.73. \end{aligned}$$

(4) We draw circles centered at each node i, $1 \le i \le 3$, and with cost $C[h]$, $1 \le h < H_i = 3$, to divide the whole disk $\mathcal{A}$ into 16 subareas $\mathcal{A}_1, \mathcal{A}_2, \ldots, \mathcal{A}_{16}$.

(5) We define a FCP p_m for each subarea $\mathcal{A}_m$, $1 \le m \le 16$. For example, for FCP p_1, we define a 3-tuple cost vector as $[c_{1B}(p_1), c_{2B}(p_1), c_{3B}(p_1)] = [C[1], C[3], C[2]] = [1.20, 1.73, 1.44]$.

(6) We apply LP in Section 3.1 on these 16 FCPs and obtain the achievable network lifetime for each FCP.

(7) Since the FCP $p^* = p_9$ has the maximum achievable network lifetime 226.47 among all 16 FCPs, we can place the base station at any point in subarea $\mathcal{A}_9$, e.g., $p_\varepsilon = (0.6, 0.6)$.

(8) We apply LP in Section 3.1 on p_ε and obtain a $(1-\varepsilon)$-optimal network lifetime $T_\varepsilon = 227.07$. This completes the algorithm.

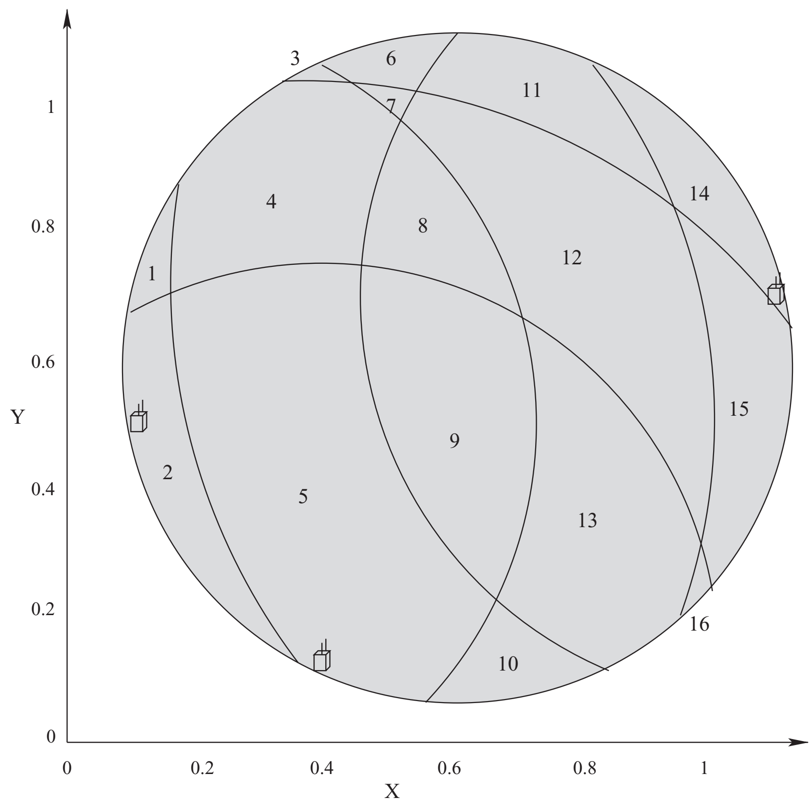

Fig. 4. The SED is divided into 16 subareas for the 3-node example.

3.5 Correctness Proof and Complexity Analysis

In this section, we give a formal proof that the solution obtained by Algorithm 1 is $(1-\varepsilon)$-optimal and analyze its complexity.

Denote as p_{opt} the optimal location for base station placement (unknown) and T_{opt} and ψ_{opt} the corresponding maximum network lifetime and data routing solution, all of which are unknown.

Denote as p^* the best FCP among the M FCPs p_m, $m = 1, 2, \ldots, M$, based on their achievable network lifetime performance. Denote T^* and ψ^* the corresponding maximum network lifetime and data routing solution, i.e., $T^* = \max\{T_m : m = 1, 2, \ldots, M\}$.

Based on (7) in Algorithm 1, we choose a physical point p_ε in the subarea corresponding to p^* for base station placement. For point p_ε, denote the maximum achievable network lifetime as T_ε and corresponding routing solution as ψ_ε.

Our roadmap for the proof is as follows. In Theorem 3.3, we prove that T^* for the best FCP p^* is within $(1-\varepsilon)$ of the optimum, that is, $T^* \geq (1-\varepsilon)T_{\text{opt}}$. Then, in Theorem 3.5, we show that for the physical point p_ε, its corresponding network lifetime T_ε is also $(1-\varepsilon)$ of the optimum, that is, $T_\varepsilon \geq (1-\varepsilon)T_{\text{opt}}$.

THEOREM 3.3. *For T^* and T_{opt} as defined, we have $T^* \geq (1-\varepsilon)T_{opt}$.*

To prove Theorem 3.3, we first present the following lemma, which is a general case for the theorem.

LEMMA 3.4. *For any given base station location p and corresponding optimal routing solution φ and achievable network lifetime T (obtained via LP), denote $\mathcal{A}_m$ the subarea that contains p for a given ε. Then for the corresponding FCP p_m, its achievable network lifetime T_m is at least $(1-\varepsilon)$ of T, i.e., $T_m \geq (1-\varepsilon)\cdot T$.*

PROOF. Instead of considering the optimal routing solution for FCP p_m, we use the same routing φ on p_m, which is clearly suboptimal. That is, denoting $\hat{T}_m$ the network lifetime for FCP p_m under φ, we have $T_m \geq \hat{T}_m$. Then we only need to show $\hat{T}_m \geq (1-\varepsilon)\cdot T$.

To show $\hat{T}_m \geq (1-\varepsilon)\cdot T$, we compute the total consumed energy on node $i \in \mathcal{N}$ under φ for FCP p_m and at time $(1-\varepsilon)\cdot T$, which is

$$\begin{aligned}
&\sum_{k\in\mathcal{N}}^{k\neq i} \rho f_{ki}\cdot(1-\varepsilon)T + \sum_{j\in\mathcal{N}}^{j\neq i} c_{ij} f_{ij}\cdot(1-\varepsilon)T + c_{iB}(p_m) f_{iB}\cdot(1-\varepsilon)T \\
<\ &\sum_{k\in\mathcal{N}}^{k\neq i} \rho f_{ki} T + \sum_{j\in\mathcal{N}}^{j\neq i} c_{ij} f_{ij} T + (1+\varepsilon)c_{iB}(p) f_{iB}\cdot(1-\varepsilon)T \\
<\ &\sum_{k\in\mathcal{N}}^{k\neq i} \rho f_{ki} T + \sum_{j\in\mathcal{N}}^{j\neq i} c_{ij} f_{ij} T + c_{iB}(p) f_{iB} T \leq e_i.
\end{aligned}$$

The first inequality holds via Property 1. The last inequality holds by the energy constraint in routing solution φ for point p. Thus, the network lifetime $\hat{T}_m$ for FCP p_m under routing solution φ is at least $(1-\varepsilon)\cdot T$. As a result, we have $T_m \geq \hat{T}_m \geq (1-\varepsilon)\cdot T$. This completes the proof. □

With Lemma 3.4, we are ready to prove Theorem 3.3.

PROOF OF THEOREM 3.3. Consider the special case of Lemma 3.4 that the given base station location p is the optimal location p_{opt}, with corresponding optimal data routing solution φ_{opt}, and maximum network lifetime T_{opt}. Following the same token in Lemma 3.4, we can find a corresponding subarea $\mathcal{A}_m$ that contains point p_{opt} with corresponding FCP p_m. As a result, for FCP p_m, we have $T_m \geq (1-\varepsilon)T_{\mathrm{opt}}$. Thus, for the best FCP p^* among all the FCPs, we have $T^* \geq T_m \geq (1-\varepsilon)T_{\mathrm{opt}}$. This completes the proof. □

Theorem 3.3 guarantees that the best network lifetime among the M FCPs is at least $(1-\varepsilon)$ of T_{opt}. Now consider a point p_ε in the subarea represented by the best FCP p^*. We have the following theorem.

THEOREM 3.5. *For T_ε and T_{opt} as defined, we have $T_\varepsilon \geq (1-\varepsilon)T_{opt}$.*

PROOF. Denote as $\hat{T}_\varepsilon$ the network lifetime for point p_ε under the same routing solution ψ^* for FCP p^*. Since ψ^* is a suboptimal routing for p_ε, we have $T_\varepsilon \geq \hat{T}_\varepsilon$. Thus, to show $T_\varepsilon \geq (1-\varepsilon)T_{\text{opt}}$, we only need to show $\hat{T}_\varepsilon \geq T^* \geq (1-\varepsilon)T_{\text{opt}}$, where the second inequality follows from Theorem 3.3.

To analyze whether $\hat{T}_\varepsilon \geq T^*$, we compute the total consumed energy on node $i \in \mathcal{N}$ under ψ^* for point p_ε and at time T^*, which is

$$\begin{aligned} &\sum_{k\in\mathcal{N}}^{k\neq i} \rho f_{ki} T^* + \sum_{j\in\mathcal{N}}^{j\neq i} c_{ij} f_{ij} T^* + c_{iB}(p_\varepsilon) f_{iB} T^* \\ \leq &\sum_{k\in\mathcal{N}}^{k\neq i} \rho f_{ki} T^* + \sum_{j\in\mathcal{N}}^{j\neq i} c_{ij} f_{ij} T^* + c_{iB}(p^*) f_{iB} T^* \leq e_i. \end{aligned}$$

The first inequality holds by (11) and (12). The second inequality holds by the energy constraint on p^* under routing solution ψ^*. Thus, the network lifetime $\hat{T}_\varepsilon$ for location p_ε under ψ^* is at least T^*. As a result, the maximum network lifetime T_ε for location p_ε is at least $\hat{T}_\varepsilon \geq T^* \geq (1-\varepsilon)\cdot T_{\text{opt}}$. This completes the proof. □

The complexity of Algorithm 1 can be measured by the number of LPs that need to be solved, which is equal to the total number of subareas M. So let us calculate M.

The boundaries of each subarea is either an arc centered at some sensor node i (with some cost $C[h]$, $1 \leq h < H_i$, with H_i being defined in (10)), or an arc of disk $\mathcal{A}$. Since there are $H_i - 1$ circles radiating from each sensor node i and one circle for disk $\mathcal{A}$, the total number of circles is $K = 1 + \sum_{i\in\mathcal{N}}(H_i - 1)$. The maximum number of subareas M that can be obtained by K circles is upper bounded by [de Berg et al. 1998]

$$M \leq K^2 - K + 2. \tag{13}$$

We have

$$M = O(K^2) = O\left(\left(\sum_{i\in\mathcal{N}} H_i\right)^2\right) = O((N/\varepsilon)^2).$$

As for comparison, the complexity of the approximation algorithm proposed in Efrat et al. [2005] is given in Section 4.1. Numerical comparison on complexity for some network topologies are also given there.

4. NUMERICAL RESULTS

In this section, we apply the approximation algorithm on various network topologies and use numerical results to demonstrate its efficacy. The units of distance, rate, and energy are all normalized appropriately. The normalized parameters in energy consumption model are $\beta_1 = \beta_2 = \rho = 1$ and we set path loss index $\alpha = 2$.

We consider four randomly generated networks consisting of 10, 20, 50, and 100 nodes deployed over a 1×1 square area. In all cases, the targeted accuracy for approximation algorithm is 0.95-optimal, that is, $\varepsilon = 0.05$.

The network setting (location, data rate, and initial energy for each node) for the 10-node network is given in Table III. By applying Algorithm 1, we

Table III. Each Node's Cartesian Coordinates, Data Generation Rate and Initial Energy for a 10-Node Network

Node Index	Location	Data Rate	Initial Energy
1	(0.81, 0.86)	0.7	390
2	(0.25, 0.71)	0.4	400
3	(0.47, 0.44)	1.0	440
4	(0.28, 0.03)	0.6	330
5	(0.25, 0.36)	0.2	440
6	(0.48, 0.22)	0.1	300
7	(0.53, 0.16)	0.8	410
8	(0.66, 0.52)	0.2	210
9	(0.91, 0.86)	0.1	320
10	(0.44, 0.21)	0.9	330

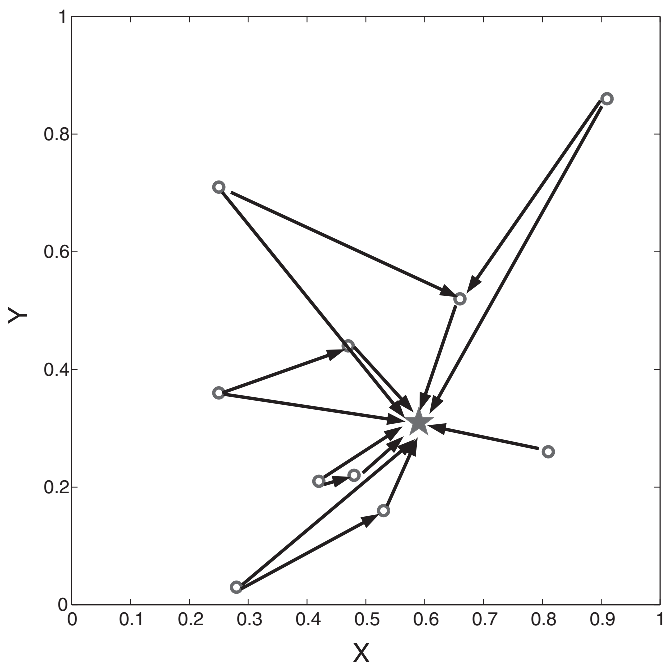

Fig. 5. A schematic showing the routing solution for the 10-node network with base station being placed at (0.59, 0.31).

find that FCP with cost vector [1.05, 1.28, 1.05, 1.22, 1.16, 1.05, 1.05, 1.05, 1.41, 1.05] has the maximum network lifetime $T^* = 357.49$, which is at least 95% of the optimum. By placing the base station at a point in the corresponding subarea, e.g., at point (0.59, 0.31), the network lifetime is $T_\varepsilon = 359.17 > T^*$. This network lifetime is also at least 95% of the optimum. The flow routing solution is shown in Figure 5, where a circle represents a sensor node and a star represents the location of the base station (0.59, 0.31).

The network setting for a 20-node network (with location, data rate, and initial energy for each of the 20 sensor nodes) is given in Table IV. By applying

Table IV. Each Node's Cartesian Coordinates, Data Generation Rate, and Initial Energy for a 20-Node Network

Node Index	Location	Data Rate	Initial Energy
1	(0.98, 0.49)	0.4	180
2	(0.44, 0.67)	0.8	320
3	(0.57, 0.52)	0.1	340
4	(0.13, 0.19)	0.6	430
5	(0.74, 0.73)	0.1	350
6	(0.24, 0.19)	0.7	310
7	(0.49, 0.38)	0.9	410
8	(0.63, 0.33)	0.7	500
9	(0.76, 0.63)	0.6	270
10	(0.92, 0.33)	0.5	180
11	(0.09, 0.84)	0.7	60
12	(0.65, 0.62)	0.1	100
13	(0.92, 0.05)	0.1	310
14	(1.00, 0.33)	0.6	280
15	(0.63, 1.00)	0.2	210
16	(0.11, 0.36)	0.3	70
17	(0.89, 0.12)	0.7	420
18	(0.52, 0.86)	0.3	270
19	(0.24, 0.91)	0.9	160
20	(0.40, 0.67)	1.0	180

Algorithm 1, we find that FCP with a cost vector [1.55, 1.05, 1.16, 1.05, 1.41, 1.22, 1.41, 1.22,1.41, 1.28, 1.63, 1.05, 1.16, 1.98, 1.71, 1.16, 1.28, 1.80, 1.05, 1.05] has the maximum network lifetime $T^* = 82.86$ among all FCPs. Subsequently, we place the base station at a point in the corresponding subarea, say at point $(0.31, 0.79)$. The corresponding network lifetime is $T_\varepsilon = 82.91 > T^*$, which is also at least 95% of the optimum. The flow routing solution is shown in Figure 6.

For the 50-node and 100-node networks, the positions of the nodes and location for the base station are shown in Figure 7 and Figure 8, respectively. We omit to list the coordinates of each node for both networks due to paper length limitation. The data rate and initial energy for each node are randomly generated between [0.1, 1] and [50, 500], respectively. For the 50-node network, we obtain a $(1-\varepsilon)$-optimal solution with $T_\varepsilon = 135.17$ when the base station is placed at $(0.51, 0.68)$; for the 100-node network, the $(1-\varepsilon)$-optimal solution is $T_\varepsilon = 61.73$ when the base station is placed at $(0.57, 0.52)$.

4.1 Complexity Comparison

We now compare the complexity of our algorithm (Section 3.4) with the approximation algorithm proposed in Efrat et al. [2005]. Similarly, the complexity of the approximation algorithm in Efrat et al. [2005] can also be measured by the number of LPs that need to be solved, which is

$$\left\lfloor \frac{4}{\varepsilon} \right\rfloor \left\lceil \frac{\alpha \ln 2}{\ln(1+\varepsilon/8)} \right\rceil \left\lceil \frac{8\alpha\pi}{\varepsilon} \right\rceil \cdot \sum_{i \in \mathcal{N}} \left\lceil \frac{\ln(8N^3 \sum_{j \in \mathcal{N}} r_j/(\varepsilon r_i))}{\ln(1+\varepsilon/8)} \right\rceil . \qquad (14)$$

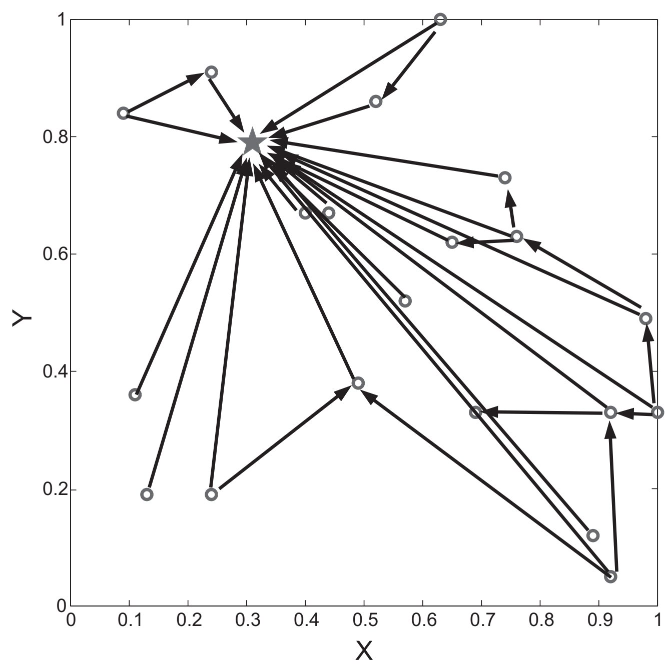

Fig. 6. A schematic showing the routing solution for the 20-node network with base station being placed at (0.31, 0.79).

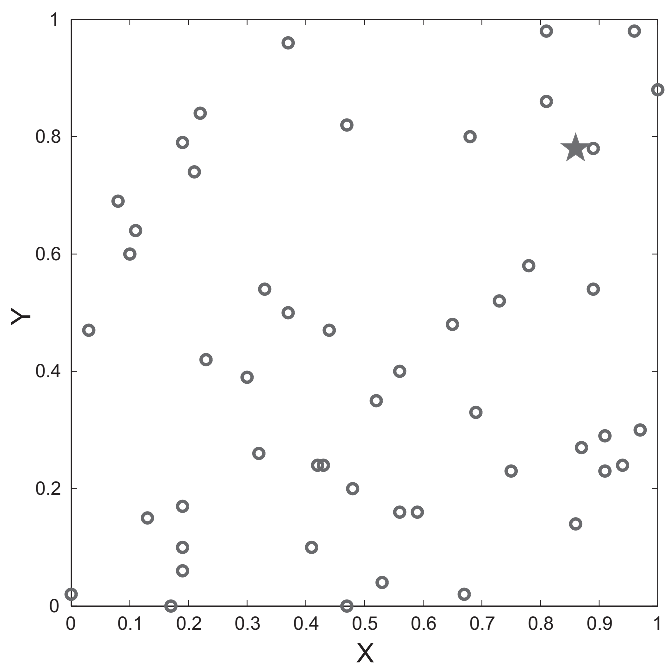

Fig. 7. A 50-node network.

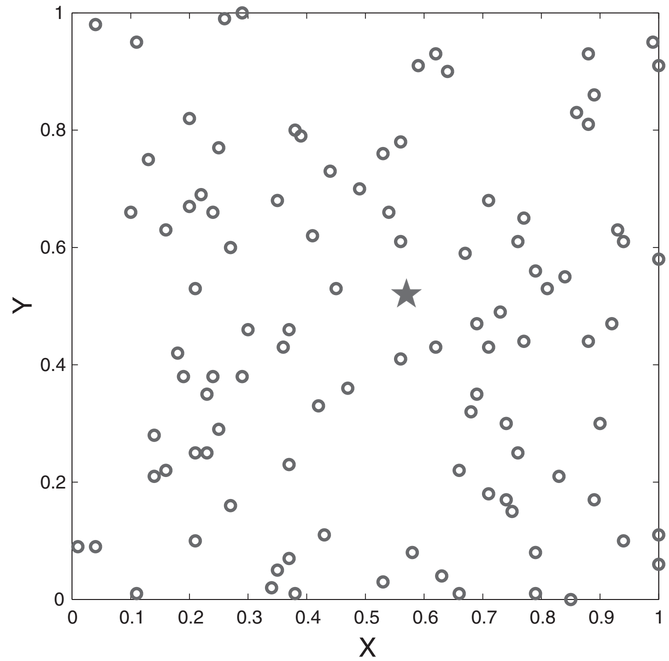

Fig. 8. A 100-node network.

To have a sense of quantitative comparison of complexity between our algorithm and the one in Efrat et al. [2005], we use (13) and (14) on the 10, 20, 50, and 100-node network considered in this section. Corresponding to each network topology, we find that the complexity of the approximation algorithm in Efrat et al. [2005] is 3.7×10^7, 1.5×10^7, 5.2×10^6, and 3.2×10^6 times of the complexity of our proposed approximation algorithm.

5. EXTENSIONS

In this section, we make two extensions for our approximation algorithm in Section 3. In the first extension, we consider the case where the transmit power at each sensor node is upper bounded, that is, there exists a limit on power level at a sensor node. In the second extension, we consider the case for multiple base stations.

5.1 Bounded Transmission Power

The approximation algorithm developed in Section 3 assumes there is no bound on the transmit power. For the case when there is a bound on transmission power, we show that our approximation algorithm can be extended without much difficulty.

Denote as U the maximum transmission power on each sensor node, that is, each sensor node can exercise power control between $[0, U]$. Given the maximum transmit power U, the maximum transmission range of a sensor node is

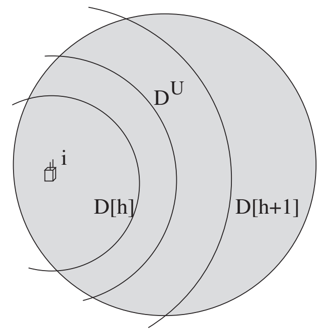

Fig. 9. A sequence of circles with increasing costs with center at node i.

limited. This maximum transmission range, denoted as D^U, can be computed from (1) and (2).

Figure 9 illustrates the impact of this maximum transmission range D^U on the discretization process. Now for a sensor node i, in addition to circles with radius $D[1], D[2], \ldots, D[H_i]$, we have one more circle with radius D^U. That is, we now have $(H_i + 1)$ circles centered around sensor node i, with radius $D[1], D[2], \ldots, D[h], D^U, D[h+1], \ldots, D[H_i]$. These circles from all the N sensor nodes will divide the SED into subareas. The definition for FCP remains the same under Definition 3.2. Now, with the newly defined upper bound cost sequence $C[1], C[2], \ldots, C[h], C^U, C[h+1], \ldots, C[H_i]$, where C^U is the transmission cost corresponding to D^U, it can be shown that Property 1 still holds.

Further, to account for the maximum transmission range, (6) and (8) in Algorithm 1 need to be slightly updated as follows.

(6) For a transmitting node i, if node j is outside node i's maximum transmission range D^U, then the traffic flow f_{ij} cannot exist. Thus, we set $f_{ij} = 0$ for such node j. Correspondingly, in the LP formulation, V_{ij} is also set to 0 since $V_{ij} = f_{ij}T$. Since $V_{ij} = 0$ for such node j that is out of transmission range, we can remove the corresponding terms from the flow balance constraints and energy constraints.

(8) When base station is at a FCP p_m or at a point p_ε, the base station may be outside the maximum transmission range of node i. In this case, the corresponding traffic flow f_{iB} cannot exist. Thus, we set f_{iB} and $V_{iB} = 0$. Therefore, we can remove the corresponding terms from the flow balance constraints and energy constraints.

After we perform this operation with respect to each sensor node, the updated LP formulation will give an optimal solution under bounded transmission power for the given base station location.

We now show that the $(1-\varepsilon)$ optimality is still maintained. Since Property 1 holds, it can be shown, rather straightforward, that both Theorem 3.3 and Theorem 3.5 also hold. As a result, the final solution obtained in the last step of Algorithm 1 is $(1-\varepsilon)$-optimal.

Regarding the impact of our extension (for bounded transmission power) on the algorithm's complexity, we now show that the complexity order remains the same. To see this, recall that the number of circles for each sensor node is increased by one. As a result, the total number of circles is increased from K to $K+N$. By (13), the number of subareas is at most $(K+N)^2-(K+N)+2$. Since $K=O(\frac{N}{\varepsilon})$ and $\frac{N}{\varepsilon}\gg N$, the total number of subareas is still $O(K^2)$.

It is worth pointing out that bounded transmission power will reduce network lifetime. This is because such bound adds more constraints on flow routing, thus reduces its solution space. As a result, the achievable network lifetime is reduced.

5.2 Multiple Base Stations

So far, the approximation algorithm we developed is for single base station. We now show how this algorithm can be extended for multiple base stations.

Denote as L the number of base stations to be placed in the sensor network. The roadmap for multiple base station follows a similar approach taken as for single base station. First, we show that given a set of L base station locations, the optimal routing solution can be solved by an LP. Then, we narrow down the search space for each base station location into a finite search space with performance guarantee. Finally, the optimal base station locations can be obtained by finding the best L locations corresponding to the maximum achievable network lifetime among all possible set of locations. In this rest of this section, we briefly elaborate the above steps.

First, similar to Section 3.1, for a set of L *given* base station locations, we can find the corresponding maximum achievable network lifetime and optimal routing via a single LP as follows.

$$\begin{array}{ll} \text{Max} & T \\ \text{s.t.} & \sum_{k\in\mathcal{N}}^{k\neq i} V_{ki} + r_i T - \sum_{j\in\mathcal{N}}^{j\neq i} V_{ij} - \sum_{l=1}^{L} V_{i,B(l)} = 0 \qquad (i\in\mathcal{N}) \\ & \sum_{k\in\mathcal{N}}^{k\neq i} \rho V_{ki} + \sum_{j\in\mathcal{N}}^{j\neq i} c_{ij} V_{ij} + \sum_{l=1}^{L} c_{i,B(l)}(p(l)) V_{i,B(l)} \le e_i \quad (i\in\mathcal{N}) \\ & T, V_{ij}, V_{i,B(l)} \ge 0 \qquad (i,j\in\mathcal{N}, i\neq j, 1\le l\le L), \end{array}$$

where $p(l)$, $1\le l\le L$, is the location of base station $B(l)$.

It is easy to prove that Lemma 3.1 still holds. We then perform the same subarea division for SED $\mathcal{A}$ and define the same set of FCPs. For multiple base stations, (7) and (8) in Algorithm 1 need to be extended as follows.

(7) Choose the FCP set $\{p(1)^*, p(2)^*, \ldots, p(L)^*\}$ that offers the maximum network lifetime among all the L-element subset of these M FCPs. The base station $B(l)$, $1\le l\le L$, can be placed at any point $p_\varepsilon(l)$ within the subarea corresponding to $p(l)^*$.

(8) For points $p_\varepsilon(l)$, $1\le l\le L$, apply an LP to obtain $(1-\varepsilon)$-optimal network lifetime T_ε.

Such approach basically enumerates all possible subsets of L locations among M FCPs and choose the best set of L points. So the complexity is $O(M^L)$, where $M=O((\frac{N}{\varepsilon})^2)$ is defined in (13).

It is worth pointing out that due to the availability of multiple base stations, each sensor has more choices to transmit its data (and thus larger solution space). As a result, the network lifetime will be larger than that under single base station.

6. RELATED WORK

Due to energy constraint, network lifetime for a wireless sensor network is limited. As a result, there is a flourish of research activities on how to prolong network lifetime. Many of these efforts (e.g., Bhardwaj and Chandrakasae [2002]; Brown et al. [2001], Kalpakis et al. [2002]; Zhang and Hou [2004]) studied lifetime problem under a given network topology and without explicit consideration of the impact of node placement on network performance.

Among the body of research on node placement, researchers have studied sensor node placement [Dhillon and Chakrabarty 2004; Wang et al. 2004; Wu and Yang 2005; Zou and Chakrabarty 2003], relay node placement [Hou et al. 2005; Xu et al. 2005], and base station placement [Bogdanov et al. 2004; Efrat et al. 2005; Pan et al. 2003]. The main focus of sensor node placement has been on coverage (in order to have either better geographical coverage of the area or better connectivity in the network). Relay node placement deals with how to place special auxiliary nodes within a sensor network so that network performance (e.g., connectivity, lifetime) can be improved. Related work on relay node placement (e.g., Hou et al. [2005]; Xu et al. [2005]) have been limited to heuristic algorithms instead of providing theoretical performance guarantee.

Related work on base station placement include [Bogdanov et al. 2004; Efrat et al. 2005; Pan et al. 2003]. Bogdanov et al. [2004] studied how to place base station so that the network flow is proportionally maximized subject to link capacity. The authors show that although it is possible to find optimal solutions for special network topology (e.g., grid), the base station placement problem for an arbitrary network is NP-complete. The authors also pointed out that an approximation algorithm with any guarantee was not known at the time of their paper and subsequently proposed two heuristic algorithms. Pan et al. [2003] studied base station placement problem to maximize network lifetime. The optimal location is only determined for the simple case where only single-hop routing is allowed. The more difficult problem involving multihop routing was not addressed.

Efrat et al. [2005] proposed the first $(1-\varepsilon)$-optimal approximation algorithm to the base station placement problem. However, since they constructed a finite search space consisting of *physical points,* the computational complexity of their algorithm is higher than the one proposed in this paper for most cases, as illustrated in Section 4.1.

7. CONCLUSION

In this article, we investigated the base station placement problem for a multi-hop sensor network. The main result is an approximation algorithm that can guarantee $(1-\varepsilon)$-optimal network lifetime performance with any desired error bound $\varepsilon > 0$. The proposed $(1-\varepsilon)$-approximation algorithm was based on several novel techniques such as discretization of cost parameters (and distances), division of search space into a finite number of subareas, and representation of subareas with fictitious points (with nice bounding properties on costs). We gave a proof that the proposed approximation algorithm is $(1-\varepsilon)$-optimal. The proposed approximation algorithm offers significant complexity reduction when compared to a state-of-the-art algorithm and represents the best known result to the base station placement problem.

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