

Per-Node Based Optimal Power Control for Multi-Hop Cognitive Radio Networks

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Abstract—Cognitive radio network (CRN) is a promising approach to improve spectrum efficiency for wireless networking. This paper investigates how to perform optimal power control on each node (or per-node based power control) in the network so as to optimize network performance. Per-node based power control is a difficult problem due to its large design space (i.e., interaction among the powers on different nodes in the network) and the coupling relationship between power control and upper layers (scheduling and routing). In this paper, we develop a formal mathematical model for joint power control, scheduling, and routing. We formulate a cross-layer optimization problem encompassing these three layers and develop a unified solution procedure based on branch-and-bound framework and convex hull relaxation. Using numerical results, we demonstrate the efficacy of the solution procedure and offer insights on the behavior of per-node based power control.

Index Terms—Cognitive radio network (CRN), multi-hop networking, per-node based power control, interference modeling, cross-layer optimization.

I. INTRODUCTION

COGNITIVE radio (CR) is a revolution in radio technology that is enabled by recent advances in RF design, signal processing, and communications software [18]. The capability of CR has been recognized by the military and commercial sector and is now under intensive research and development by the DoD's Joint Tactical Radio System (JTRS) program [10] and wireless industry [19]. Since transmitted waveform is defined by software, a CR is capable of re-configuring RF (on the fly) and switching to newly-selected frequency bands. From networking perspective, the emergence of CR offers new challenges in algorithm and protocol design.

It is important to realize that a CR is vastly more powerful and flexible than existing *multi-channel multi-radio* (MC-MR) technology (see e.g., [1], [6], [11], [12], [17] and reference therein). Note that MC-MR remains *hardware-based* radio technology: each radio can only operate on a single channel at a time and the number of concurrent channels that can be used at a wireless node is limited by the number of radio interfaces. In addition, an MC-MR wireless network typically

works on a set of pre-assigned channels and is not able to sense available spectrum. In contrast, each node in a CRN is able to perform spectrum sensing and work on a different set of available frequency bands. These important differences warrant that algorithm design for a CRN is more complex and interesting.

A fundamental problem for a wireless network is power control. In a multi-hop wireless network, power control is challenging since it directly affects upper layers scheduling and routing. When each node is allowed to perform power control (which we call per-node based power control), the problem becomes even more difficult due to its large optimization space. Due to such difficulty, some previous work, e.g., [2], [12], considered synchronized power control, where transmission power at each node in the network is adjustable but is synchronized to be identical. Needless to say, such synchronization in power control cannot offer optimal network performance.

Recognizing the benefits of per-node based power control, there have been active research efforts in recent years. But per-node based power control is an extremely difficult problem as it is tightly coupled with scheduling and routing. Due to such coupling, a joint formulation of multiple layers is necessary, which inevitably lead to a very complex problem. Although there has been some success in the context of *asymptotic* scaling laws (e.g., [9], [12]), theoretically optimal results for a given *finite-sized* network remain unsatisfactory and many issues are still open. For example, in [3], Bhatia and Kodialam optimized power control and routing, but assumed some frequency hopping mechanism is in place for scheduling, which helps simplify joint consideration of scheduling. In [7], Elbatt and Ephremides optimized power control and scheduling, but assumed routing was given a priori. Although [4], [5] aimed to investigate joint power control, scheduling, and routing problem, both solutions followed a “de-coupled” approach, where the final solution was obtained by determining algorithm/mechanism for one layer at a time (instead of solving a unified problem, as we shall do in this paper). Due to such de-coupling approach in solution procedure, the final solution is unlikely to be cross-layer globally optimal.

In this paper, we study the per-node based power control problem for a multi-hop CRN. The benefits of per-node based power control for CRNs were discussed in [22]. This problem is both challenging and interesting as it inherits not only all the difficulties associated with per-node based power control, but also all the unique characteristics associated with a CRN. We develop a formal mathematical model for a joint per-node based power control, scheduling, and flow routing problem.

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TABLE I
NOTATION.

Symbol	Definition
\mathcal{N}	The set of nodes in the network
\mathcal{M}_i	The set of available bands at node $i \in \mathcal{N}$
\mathcal{M}	$= \sum_{i \in \mathcal{N}} \mathcal{M}_i$, the set of bands in the network
\mathcal{M}_{ij}	$= \mathcal{M}_i \cap \mathcal{M}_j$, the set of available bands at link $i \rightarrow j$
W	Bandwidth of each frequency band
\mathcal{L}	The set of user communication sessions
$s(l), d(l)$	Source and destination nodes of session $l \in \mathcal{L}$
$r(l)$	Rate requirement of session l
P	The maximum transmission power
n	Path loss index
η	Ambient Gaussian noise density
d_{ij}	Distance between nodes i and j
g_{ij}	Path attenuation loss from node i to node j
α	The threshold of minimum receiving power
β	The threshold of maximum interference power
$R_T(p), R_I(p)$	The transmission and interference ranges under transmission power p
R_T^{max}, R_I^{max}	The maximum transmission and interference ranges under full transmission power P
\mathcal{T}_i^m	The set of nodes to which node i can transmit on band m (under full transmission power P)
\mathcal{T}_i	$= \bigcup_{m \in \mathcal{M}_i} \mathcal{T}_i^m$, the set of nodes to which node i can transmit (under full transmission power P)
\mathcal{I}_j^m	The set of nodes that can interfere node j on band m (under full transmission power P)
x_{ij}^m	Binary indicator to mark whether or not band m is used by link $i \rightarrow j$
p_{ij}^m	Transmission power at node i to node j on band m
c_{ij}^m	Link capacity of link $i \rightarrow j$ under p_{ij}^m
$f_{ij}(l)$	Data rate that is attributed to session l on link $i \rightarrow j$
Q	Number of transmission power levels
q_{ij}^m	Transmission power level from node i to node j on band m
\mathbf{x}	The vector of variables $x_{ij}^m, i \in \mathcal{N}, m \in \mathcal{M}_i, j \in \mathcal{T}_i^m$
\mathbf{q}	The vector of variables $q_{ij}^m, i \in \mathcal{N}, m \in \mathcal{M}_i, j \in \mathcal{T}_i^m$
\mathbf{f}	The vector of variables $f_{ij}(l), l \in \mathcal{L}, i \in \mathcal{N}, i \neq d(l), j \in \mathcal{T}_i, j \neq s(l)$
Notation for branch-and-bound procedure	
ε	The desired accuracy in the final solution
Ω_z	The set of all possible values of (\mathbf{x}, \mathbf{q}) in problem z
LB_z, UB_z	The lower and upper bounds of problem z
ψ_z	The solution obtained by local search for problem z
LB, UB	The lower and upper bounds for the original problem
ψ_ε	The $(1 - \varepsilon)$ optimal solution

This joint formulation leads to a *mixed integer nonlinear programming* (MINLP) problem. Subsequently, we develop a unified (instead of de-coupled) solution procedure based on branch-and-bound framework and convex hull relaxation. This solution procedure guarantees a $(1 - \varepsilon)$ optimal solution, where ε is a small pre-specified error tolerance parameter. By applying the solution procedure on a random network, we validate this solution procedure and offer additional insights on the behavior of per-node based power control. The results in this paper close some long stand open issues associated with network optimization with per-node based power control.

The remainder of this paper is organized as follows. In Section II, we develop a unified mathematical model for per-node based power control, scheduling, and flow routing. In Section III, we formulate the cross-layer optimization problem. Section IV describes a solution procedure to this cross-layer optimization problem. In Section V, we use numerical results to validate the efficacy of the solution procedure. Section VI concludes this paper.

II. MATHEMATICAL MODELING

We consider a multi-hop CRN consisting of a set of \mathcal{N} nodes. The set of available frequency bands at each node depends on its location and may not be the same. For example, at node i , its available frequency bands may consist of bands I, III, and V while at a different node j , its available frequency bands may consist of bands I, IV, and VI, and so forth. More formally, denote \mathcal{M}_i the set of available frequency bands at node i . For simplicity, we assume the bandwidth of each frequency band is W (the case of heterogeneous bandwidth for each frequency band can be easily extended). Denote \mathcal{M} the set of all frequency bands present in the network, i.e., $\mathcal{M} = \bigcup_{i \in \mathcal{N}} \mathcal{M}_i$. Table I lists all notation in this paper.

A. Necessary and Sufficient Condition for Successful Transmission

Scheduling for transmission at each node in the network can be done either in time domain or frequency domain and these two scheduling schemes are equivalent in terms of achievable rate region. In this paper, we consider scheduling in the frequency domain in the form of frequency bands. Note that in this context, we can always divide bands into smaller sub-bands to ensure there are enough bands/sub-bands for scheduling.

Suppose that band m is available at both node i and node j , i.e., $m \in \mathcal{M}_{ij}$, where $\mathcal{M}_{ij} = \mathcal{M}_i \cap \mathcal{M}_j$. Denote p_{ij}^m the transmission power from node i to node j in frequency band m . For transmission from node i to node j , a simple model for path attenuation loss g_{ij} is

$$g_{ij} = d_{ij}^{-n}, \quad (1)$$

where d_{ij} is the physical distance between nodes i and j and n is the path loss index. In this context, we assume a data transmission from node i to node j is successful only if the received power at node j exceeds a threshold α . Denote the transmission range of at node i under p_{ij}^m as $R_T(p_{ij}^m)$. Then based on $g_{ij} \cdot p_{ij}^m \geq \alpha$ and (1), we can calculate the transmission range of this node as follows:

$$R_T(p_{ij}^m) = \left(\frac{p_{ij}^m}{\alpha} \right)^{1/n}. \quad (2)$$

Since the receiving node j must be physically within the transmission range of node i , we have

$$(C-1) \quad d_{ij} \leq \left(\frac{p_{ij}^m}{\alpha} \right)^{1/n}.$$

Similarly, we assume that an interference is non-negligible only if it exceeds a threshold, say β at a receiver. Denote the interference range of node k ($k \in \mathcal{N}, k \neq i$) under p_{kh}^m as $R_I(p_{kh}^m)$, where h is the intended receiving node of transmitting node k . Then following the same taken as the derivation for the transmission range, we can obtain the interference range of node k as $R_I(p_{kh}^m) = \left(\frac{p_{kh}^m}{\beta} \right)^{1/n}$. Since the receiving node j must not fall in the interference range of any other node k that is transmitting in the same band, we have

$$(C-2) \quad d_{jk} \geq \left(\frac{p_{kh}^m}{\beta} \right)^{1/n}.$$

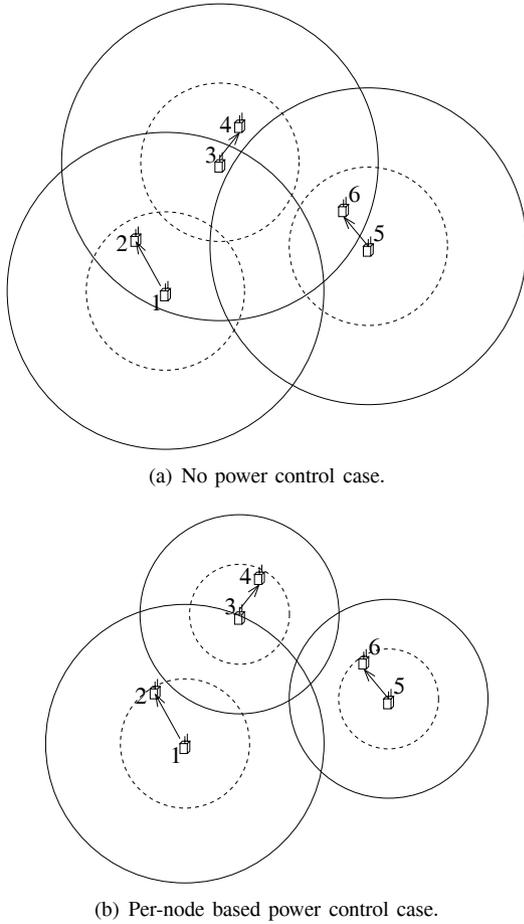


Fig. 1. A 3-link network.

As an example, Fig. 1(a) shows a network with three links ($1 \rightarrow 2, 3 \rightarrow 4$, and $5 \rightarrow 6$). For each transmitting node (1, 3, and 5), the inner circle (dashed) represents transmission range and the outer circle (solid) represents interference range. Clearly, each receiving node falls in the transmission range of its respective transmitting node. Further, we can see that both receiving nodes 2 and 6 fall in the interference range of node 3. Thus, when link $3 \rightarrow 4$ is using a frequency band m for transmission, link $1 \rightarrow 2$ and link $5 \rightarrow 6$ should not use the same band. It should also be noted that when link $3 \rightarrow 4$ is not using a frequency band m , both link $1 \rightarrow 2$ and link $5 \rightarrow 6$ may use band m . This is because that receiving node 2 does not fall in node 5's interference range and receiving node 6 does not fall in node 1's interference range. Now consider that each node can adjust its transmission power. In this setting, nodes 1, 3, and 5 can reduce their transmission powers while still maintaining data transmission to the corresponding receiving nodes (see Fig. 1(b)). Then receiving nodes 2 and 6 are no longer in node 3's interference range. As a result of this, both transmitting nodes 1 and 5 can transmit in the same frequency band m simultaneously.

B. Per-Node Based Power Control and Scheduling

In this section, we formalize a mathematical model for the joint relationship between per-node based power control and scheduling. Suppose that band m is available at both node i

and node j . Denote

$$x_{ij}^m = \begin{cases} 1 & \text{If node } i \text{ transmits data to node } j \text{ on band } m, \\ 0 & \text{otherwise.} \end{cases}$$

As mentioned earlier, we consider scheduling in the frequency domain and thus once a band $m \in \mathcal{M}_i$ is used by node i for transmission to node j , this band cannot be used again by node i to transmit to a different node. That is,

$$(C-3) \quad \sum_{j \in \mathcal{T}_i^m} x_{ij}^m \leq 1,$$

where \mathcal{T}_i^m is the set of nodes to which node i can transmit on band m under full power P .

Denote R_T^{max} the maximum transmission range of a node when it transmits at full power P . Then based on (2), we have $R_T^{max} = R_T(P) = \left(\frac{P}{\alpha}\right)^{1/n}$. Thus, we have $\alpha = \frac{P}{(R_T^{max})^n}$. Then for a node transmitting at a power $p \in [0, P]$, its transmission range is

$$R_T(p) = \left(\frac{p}{\alpha}\right)^{1/n} = \left[\frac{p(R_T^{max})^n}{P}\right]^{1/n} = \left(\frac{p}{P}\right)^{1/n} R_T^{max}. \quad (3)$$

Similarly, denote R_I^{max} the maximum interference range of a node when it transmits at full power P . Then following the same token, we have $R_I^{max} = R_I(P) = \left(\frac{P}{\beta}\right)^{1/n}$ and $\beta = \frac{P}{(R_I^{max})^n}$. For a node transmitting at a power $p \in [0, P]$, its interference range is

$$R_I(p) = \left(\frac{p}{\beta}\right)^{1/n} R_I^{max}. \quad (4)$$

Recall that \mathcal{T}_i^m denotes the set of nodes to which node i can transmit on band m under full power P . More formally, we have $\mathcal{T}_i^m = \{j : d_{ij} \leq R_T^{max}, j \neq i, m \in \mathcal{M}_j\}$. Similarly, denote \mathcal{I}_j^m the set of nodes that can make interference on node j on band m under full power P , i.e., $\mathcal{I}_j^m = \{k : d_{jk} \leq R_I^{max}, m \in \mathcal{M}_k\}$. Note that the definitions of \mathcal{T}_i^m and \mathcal{I}_j^m are both based on full transmission power P . When power level p is below P , the corresponding transmission and interference ranges will be smaller. As a result, it is necessary to keep track of the set of nodes fall in the transmission range and the set of nodes that can produce interference whenever transmission power changes at a node.

Recall the two constraints (C-1) and (C-2) for successful transmission from node i to node j and (3) and (4), respectively, we have

$$d_{ij} \leq R_T(p_{ij}^m) = \left(\frac{p_{ij}^m}{P}\right)^{1/n} R_T^{max},$$

$$d_{jk} \geq R_I(p_{kh}^m) = \left(\frac{p_{kh}^m}{P}\right)^{1/n} R_I^{max} \quad (k \in \mathcal{I}_j^m, k \neq i, h \in \mathcal{T}_k^m).$$

Based the above two constraints, we have the following requirements for the transmission link $i \rightarrow j$ and interfering link $k \rightarrow h$:

$$p_{ij}^m \begin{cases} \in \left[\left(\frac{d_{ij}}{R_T^{max}}\right)^n P, P \right] & \text{If } x_{ij}^m = 1, \\ = 0 & \text{If } x_{ij}^m = 0. \end{cases}$$

$$p_{kh}^m \leq \begin{cases} \left(\frac{d_{ki}}{R_I^{max}}\right)^n P & \text{If } x_{ij}^m = 1, \\ P & \text{If } x_{ij}^m = 0. \end{cases} \quad (k \in \mathcal{I}_j^m, k \neq i, h \in \mathcal{T}_k^m).$$

Mathematically, these requirements can be re-written as

$$(C-1') \quad p_{ij}^m \in \left[\left(\frac{d_{ij}}{R_T^{max}} \right)^n P x_{ij}^m, P x_{ij}^m \right],$$

$$(C-2') \quad p_{kh}^m \leq P - \left[1 - \left(\frac{d_{kj}}{R_I^{max}} \right)^n \right] P x_{ij}^m \\ (k \in \mathcal{I}_j^m, k \neq i, h \in \mathcal{T}_k^m).$$

In addition, for successful scheduling in frequency domain, the following two constraints must also hold:

(C-4) For a band $m \in \mathcal{M}_j$ that is available at node j , this band cannot be used for both transmission and receiving. That is, if band m is used at node j for transmission (or receiving), then it cannot be used for receiving (or transmission).

(C-5) Similar to constraint (C-3) on transmission, node j cannot use the same band $m \in \mathcal{M}_j$ to receive from two different nodes.

Note that (C-4) can be viewed as ‘‘self-interference’’ avoidance constraint where at the same node j , its transmission to another node h on band m interferes its reception from node i on the same band. It turns out that the above two constraints are mathematically *embedded* in (C-1') and (C-2'). That is, once (C-1') and (C-2') are satisfied, then both constraints (C-4) and (C-5) are also satisfied. This result is formally stated in the following lemma.

Lemma 1: If transmission powers on every transmission link and interference link satisfy (C-1') and (C-2') in the network, then (C-4) and (C-5) are also satisfied.

Proof. We first prove that (C-1') and (C-2') lead to (C-4). To do this, we let $k = j$ in (C-2'). Then (C-2') degenerates into $p_{jh}^m \leq P - P x_{ij}^m$ since $d_{jj} = 0$.

- Suppose node j is receiving from node i on band m , i.e., $x_{ij}^m = 1$, then $p_{jh}^m \leq P - P x_{ij}^m = 0$. Since $p_{jh}^m \geq \left(\frac{d_{jh}}{R_T^{max}} \right)^n P x_{jh}^m$ from (C-1'), we have that x_{jh}^m must be 0. That is, if node j is receiving from node i on band m , then node j cannot transmit to a node h in the same band.
- Now suppose node j is transmitting to node h on band m , i.e., $x_{jh}^m = 1$. We will show that this will lead to $x_{ij}^m = 0$. This can be proved by contradiction. That is, if $x_{ij}^m = 1$, then we have just proved in the above paragraph that $x_{jh}^m = 0$. But this contradicts our initial assumption that $x_{jh}^m = 1$. Therefore, x_{ij}^m must be 0. That is, if node j is transmitting to node h on band m , then node j cannot use the same band for receiving from a node i .

Combining the above two results, we have shown that (C-4) holds.

We now prove that (C-1') and (C-2') also lead to (C-5). Again the proof is based on contradiction. Suppose that (C-5) does not hold. Then node j can receive from two different nodes i and k on the same band m , i.e., $x_{ij}^m = 1$ and $x_{kj}^m = 1$. Note that link $k \rightarrow j$ can be viewed as an interfering link to link $i \rightarrow j$. This corresponds to letting $h = j$ in (C-2'). Then from (C-2'), since $x_{ij}^m = 1$, we have $p_{kj}^m \leq \left(\frac{d_{kj}}{R_I^{max}} \right)^n P$. On the other hand, by (C-1'), we have $p_{kj}^m \geq \left(\frac{d_{kj}}{R_I^{max}} \right)^n P$. However, the above two inequalities

cannot hold simultaneously (contradiction) since we have $R_I^{max} > R_T^{max}$. Thus, the initial assumption that (C-5) does not hold is incorrect and the proof is complete. \square

The significance of Lemma 1 is that since (C-4) and (C-5) are embedded in (C-1') and (C-2'), it is sufficient to consider constraints (C-1'), (C-2'), and (C-3) for scheduling and power control.

C. Flow Routing and Link Capacity Constraints

In this study, we assume there is a set of \mathcal{L} active user communication (unicast) sessions in the CRN. Denote $s(l)$ and $d(l)$ the source and destination nodes of session $l \in \mathcal{L}$ and $r(l)$ the rate requirement (in b/s) of session l . We consider the most general case of multi-path routing, i.e., we allow flow splitting between a source node and its destination node.

Mathematically, this can be easily modeled based on flow balance at each node. Denote $f_{ij}(l)$ the data rate on link (i, j) that is attributed to session l , where $i \in \mathcal{N}, i \neq d(l), j \in \mathcal{T}_i = \bigcup_{m \in \mathcal{M}_i} \mathcal{T}_i^m, j \neq s(l)$.¹ If node i is the source node of session l , i.e., $i = s(l)$, then

$$\sum_{j \in \mathcal{T}_i} f_{ij}(l) = r(l). \quad (5)$$

If node i is an intermediate relay node for session l , i.e., $i \neq s(l)$ and $i \neq d(l)$, then

$$\sum_{j \in \mathcal{T}_i, j \neq s(l)} f_{ij}(l) = \sum_{k \in \mathcal{T}_i, k \neq d(l)} f_{ki}(l). \quad (6)$$

If node i is the destination node of session l , i.e., $i = d(l)$, then

$$\sum_{k \in \mathcal{T}_i} f_{ki}(l) = r(l). \quad (7)$$

It can be easily verified that once (5) and (6) are satisfied, (7) must also be satisfied. As a result, it is sufficient to have (5) and (6) in the formulation.

In addition to the above flow balance equations at each node $i \in \mathcal{N}$ for session $l \in \mathcal{L}$, the aggregated flow rates on each radio link cannot exceed this link's capacity. Under p_{ij}^m , we have

$$\sum_{l \in \mathcal{L}, s(l) \neq j, d(l) \neq i} f_{ij}(l) \leq \sum_{m \in \mathcal{M}_{ij}} c_{ij}^m \\ = \sum_{m \in \mathcal{M}_{ij}} W \log_2 \left(1 + \frac{g_{ij}}{\eta W} p_{ij}^m \right), \quad (8)$$

where η is the ambient Gaussian noise density.

III. PROBLEM FORMULATION

Objective Function. In this paper, we consider how to minimize network resource usage to support a set of user sessions. Network resource usage can be defined in a number of ways, which typically includes bandwidth usage. However,

¹For $f_{ij}(l)$, we set $i \neq d(l)$ to ensure that the destination node $d(l)$ will be a sink node and not transmit data to a relay node i . We also set $j \neq s(l)$ to ensure that a relay node i will not transmit data back to the source node $s(l)$.

as pointed out in [13], bandwidth usage can only characterize resource usage in spectrum, but cannot characterize the impact (i.e., interference) of radio transmission in space. For example, a node transmitting with the same channel bandwidth but with different power levels will produce different interference “footprint” areas. To account for both the spectrum usage and the impact of a CR in space dimension, the so-called bandwidth-footprint-product (BFP) metric was introduced in [13] and will also be adopted in this paper. Since each node in the network will use a number of bands for transmission and each band will have a certain footprint corresponding to its transmission power, our objective will be to minimize network-wide BFP, which is the sum of BFPs among all the nodes in the network, i.e.,

$$\min \sum_{i \in \mathcal{N}} \sum_{m \in \mathcal{M}_i} \sum_{j \in \mathcal{T}_i^m} W \cdot \pi(R_I(p_{ij}^m))^2. \quad (9)$$

In some sense, minimizing network-wide BFP can be viewed as minimizing “weighted” version of bandwidth usage, where the weight is the interference footprint area. Note that (9) is equivalent to minimize $\pi(R_I^{max})^2 \sum_{i \in \mathcal{N}} \sum_{m \in \mathcal{M}_i} \sum_{j \in \mathcal{T}_i^m} W \left(\frac{p_{ij}^m}{P}\right)^{2/n}$. Since $\pi(R_I^{max})^2$ is a constant factor, we can remove it from the objective function. We also point out that although BFP is used as objective function in this paper, those traditional utility-based objective functions (e.g. [14], [16], [21]) can also be used if needed.

Discretization of Transmission Powers. For power control, we allow the transmission power to be adjusted between 0 and P . In practice, the transmission power can only be tuned into a finite number of discrete levels between 0 and P . To model this discrete version of power control, we introduce an integer parameter Q that represents the total number of power levels to which a transmitter can be adjusted, i.e., $0, \frac{1}{Q}P, \frac{2}{Q}P, \dots, P$. Denote $q_{ij}^m \in \{0, 1, 2, \dots, Q\}$ the integer power level for p_{ij}^m , i.e., $p_{ij}^m = \frac{q_{ij}^m}{Q}P$. Then (C-1’), (C-2’), and (8) can be re-written as follows:

$$q_{ij}^m \in \left[\left(\frac{d_{ij}}{R_T^{max}} \right)^n Q x_{ij}^m, Q x_{ij}^m \right], \quad (10)$$

$$q_{kh}^m \leq Q - \left[1 - \left(\frac{d_{kj}}{R_I^{max}} \right)^n \right] Q x_{ij}^m \quad (k \in \mathcal{I}_j^m, k \neq i, h \in \mathcal{T}_k^m), \quad (11)$$

$$\sum_{l \in \mathcal{L}}^{s(l) \neq j, d(l) \neq i} f_{ij}(l) \leq \sum_{m \in \mathcal{M}_i} W \log_2 \left(1 + \frac{g_{ij}P}{\eta W Q} q_{ij}^m \right).$$

We can re-formulate (11) as follows. Note that by (C-3), there is at most one $h \in \mathcal{T}_k^m$ such that $x_{kh}^m = 1$. As a result, based on (10), there is at most one $q_{kh}^m > 0$ in $\sum_{h \in \mathcal{T}_k^m} q_{kh}^m$. Thus, (11) can be rewritten as

$$\sum_{h \in \mathcal{T}_k^m} q_{kh}^m \leq Q - \left(1 - \left(\frac{d_{kj}}{R_I^{max}} \right)^n \right) Q x_{ij}^m \quad (k \in \mathcal{I}_j^m, k \neq i).$$

Problem Formulation. Putting together the objective function and all the constraints for per-node based power control, scheduling, and flow routing, we have the following formulation.

$$\text{Min} \sum_{i \in \mathcal{N}} \sum_{m \in \mathcal{M}_i} \sum_{j \in \mathcal{T}_i^m} W \left(\frac{q_{ij}^m}{Q} \right)^{2/n}$$

$$\text{s.t.} \quad \sum_{j \in \mathcal{T}_i^m} x_{ij}^m \leq 1 \quad (i \in \mathcal{N}, m \in \mathcal{M}_i)$$

$$q_{ij}^m - \left(\frac{d_{ij}}{R_T^{max}} \right)^n Q x_{ij}^m \geq 0 \quad (i \in \mathcal{N}, m \in \mathcal{M}_i, j \in \mathcal{T}_i^m) \quad (12)$$

$$q_{ij}^m - Q x_{ij}^m \leq 0 \quad (i \in \mathcal{N}, m \in \mathcal{M}_i, j \in \mathcal{T}_i^m) \quad (13)$$

$$\sum_{h \in \mathcal{T}_k^m} q_{kh}^m + \left(1 - \left(\frac{d_{kj}}{R_I^{max}} \right)^n \right) Q x_{ij}^m \leq Q \quad (i \in \mathcal{N}, m \in \mathcal{M}_i, j \in \mathcal{T}_i^m, k \in \mathcal{I}_j^m, k \neq i) \quad (14)$$

$$\sum_{l \in \mathcal{L}}^{s(l) \neq j, d(l) \neq i} f_{ij}(l) - \sum_{m \in \mathcal{M}_i} W \log_2 \left(1 + \frac{g_{ij}P}{\eta W Q} q_{ij}^m \right) \leq 0 \quad (i \in \mathcal{N}, j \in \mathcal{T}_i)$$

$$\sum_{j \in \mathcal{T}_i} f_{ij}(l) = r(l) \quad (l \in \mathcal{L}, i = s(l))$$

$$\sum_{j \in \mathcal{T}_i}^{j \neq s(l)} f_{ij}(l) - \sum_{k \in \mathcal{T}_i}^{k \neq d(l)} f_{ki}(l) = 0 \quad (l \in \mathcal{L}, i \in \mathcal{N}, i \neq s(l), d(l))$$

$$x_{ij}^m \in \{0, 1\}, q_{ij}^m \in \{0, 1, 2, \dots, Q\} \quad (i \in \mathcal{N}, m \in \mathcal{M}_i, j \in \mathcal{T}_i^m)$$

$$f_{ij}(l) \geq 0 \quad (l \in \mathcal{L}, i \in \mathcal{N}, i \neq d(l), j \in \mathcal{T}_i, j \neq s(l)),$$

where (12) and (13) come from (10), $W, g_{ij}, R_T^{max}, R_I^{max}, P, \eta, r(l)$, and Q are all constants and x_{ij}^m, q_{ij}^m , and $f_{ij}(l)$ are all optimization variables.

This optimization problem is in the form of *mixed-integer non-linear program* (MINLP), which is NP-hard in general [8]. In the next section, we develop a unified (instead of decoupled) solution procedure based on the branch-and-bound framework [15] and the convex hull relaxation to solve this problem.

IV. A SOLUTION PROCEDURE

A. Overview

Note that the difficulty of this optimization problem comes from the integer variables x_{ij}^m 's and q_{ij}^m 's. Once the values of all integer variables are determined, the values of remaining variables $f_{ij}(l)$'s can be obtained by a linear program (LP). Thus, the key step to solve our problem is to narrow down the value sets and finally determine these values for integer variables. We find that the so-called *branch-and-bound* framework [15] is most effective in solving this problem. Under this approach, we aim to provide a $(1 - \varepsilon)$ optimal solution, which is formally defined as follows.

Definition 1: For a minimization problem, denote y^* the objective value of the optimal solution. A feasible solution with objective value y is called a $(1 - \varepsilon)$ optimal solution if we have $y^* \geq (1 - \varepsilon)y$.

Initially, the value set for x_{ij}^m 's and q_{ij}^m 's are $x_{ij}^m \in \{0, 1\}$ and $q_{ij}^m \in \{0, 1, 2, \dots, Q\}$. By using some *relaxation* technique, branch-and-bound obtains a linear relaxation for the original MINLP problem and its solution provides a *lower bound (LB)* to the objective function. As we shall show shortly, this critical step is made possible by the convex hull relaxation for non-linear discrete terms. With the relaxation solution as a starting point, branch-and-bound uses a *local search* algorithm to find a feasible solution to the original

problem, which provides an upper bound (UB) for the objective function. If the obtained lower and upper bounds are within ε to each other, i.e., $LB \geq (1 - \varepsilon)UB$, we are done.

If the relaxation errors are not small, then the upper bound UB could be far away from the lower bound LB . To close this gap, we must have a tighter linear relaxation, i.e., with smaller relaxation errors. This could be achieved by further narrowing down the value sets of x_{ij}^m 's and q_{ij}^m 's. We call these variables as partition variables. Specifically, branch-and-bound selects a partition variable with the largest relaxation error and divides its value set into two sets based on its value in the relaxation solution. For example, if a q_{ij}^m with a value set $\{0, 1, 2, 3\}$ is selected and its value in the relaxation solution is 1.3, then its value set is divided into two sets $\{0, 1\}$ (the set of values no more than 1.3) and $\{2, 3\}$ (the set of values no less than 1.3). Then the original problem (denoted as problem 1) is divided into two new problems (denoted as problem 2 and problem 3). Again, branch-and-bound performs relaxation and local search on these two new problems. Now we have lower bounds LB_2 and LB_3 for problems 2 and 3, respectively. We also have feasible solutions that provide upper bounds UB_2 and UB_3 for problems 2 and 3, respectively. Since the relaxations in problems 2 and 3 are both tighter than that in problem 1, we have $\min\{LB_2, LB_3\} \geq LB_1$ and $\min\{UB_2, UB_3\} \leq UB_1$. For a minimization problem, the lower bound of the original problem is updated from $LB = LB_1$ to $LB = \min\{LB_2, LB_3\}$. Also, the upper bound of the original problem is updated from $UB = UB_1$ to $UB = \min\{UB_2, UB_3\}$, since the best feasible solution to the original problem is the solution with a smaller UB_i . As a result, we now have smaller gap between UB and LB . If $LB \geq (1 - \varepsilon)UB$, we are done. Otherwise, we choose a problem with the minimum lower bound and perform partition for this problem.

Note that during the process for branch-and-bound, if we find a problem z with $LB_z \geq (1 - \varepsilon)UB$, then we can conclude that this problem cannot contribute to find a $(1 - \varepsilon)$ optimal solution.² We can thus remove this problem from list for further consideration. Eventually, we will find $LB \geq (1 - \varepsilon)UB$ and the branch-and-bound procedure terminates. It has been shown that under very general conditions, a branch-and-bound solution procedure always converges [20]. Moreover, although the worst-case complexity of such a procedure is exponential, the actual running time could be fast when all partition variables are integers (e.g., the problem considered in this paper).

Figure 2 shows the pseudocode of branch-and-bound framework. There are several key components in this framework that are problem specific and need to be carefully designed. These components are described in the following sections.

²For such a problem z , there are two possible cases. Case 1: The global optimal solution is not in problem z . In this case, the removal of problem z will not cause the loss of the optimal solution in future iterations. Case 2: The global optimal solution is in problem z . In this case, the optimal (feasible) solution must have an objective value greater than or equal to LB_z , which means that it is also greater than or equal to $(1 - \varepsilon)UB$ (since $LB_z \geq (1 - \varepsilon)UB$). Thus, by Definition 1, the current best feasible solution with objective value UB is already a $(1 - \varepsilon)$ optimal solution. Therefore, although the removal of problem z may lead to the loss of an optimal solution, we can still guarantee $(1 - \varepsilon)$ optimality, since we have already found a $(1 - \varepsilon)$ optimal solution.

Branch-and-bound Procedure

Initialization:

1. Let the initial solution $\psi_\varepsilon = \emptyset$, the initial upper bound $UB = \infty$, and the initial problem list include only the original problem, denoted as problem 1.
2. Determine initial value set for each partition variable.
3. Build a linear relaxation for problem 1 and obtain its solution $\hat{\psi}_1$.
4. The objective value of $\hat{\psi}_1$ is a lower bound LB_1 to problem 1.

Iteration:

5. Select problem z that has the minimum LB_z among all problems in the problem list.
6. Update lower bound by $LB = LB_z$.
7. Find a feasible solution ψ_z from $\hat{\psi}_z$ via a local search algorithm and denote its objective value as UB_z .
8. If $(UB_z < UB)$ {
9. Update $\psi_\varepsilon = \psi_z$ and $UB = UB_z$.
10. If $LB \geq (1 - \varepsilon)UB$, stop with the $(1 - \varepsilon)$ optimal solution ψ_ε .
Otherwise, remove all problems z' with $LB_{z'} \geq (1 - \varepsilon)UB$ from the problem list. }
11. Select a variable with the largest relaxation error and divide its value set into two sets based on its value in $\hat{\psi}_z$.
12. Build two new problems z_1 and z_2 based on these two sets.
13. Remove problem z from the problem list.
14. Obtain LB_{z_1} and LB_{z_2} for problems z_1 and z_2 via their linear relaxations.
15. If $LB_{z_1} \leq (1 - \varepsilon)UB$, add problem z_1 into the problem list.
16. If $LB_{z_2} \leq (1 - \varepsilon)UB$, add problem z_2 into the problem list.
17. If the problem list is empty, we stop with the $(1 - \varepsilon)$ optimal solution ψ_ε . Otherwise, go to the next iteration.

Fig. 2. Pseudocode for branch-and-bound.

B. Linear Relaxation

During each iteration of branch-and-bound, we need a relaxation technique to obtain a lower bound of the objective function (see line 4 in Fig. 2). For a non-linear discrete term, we propose to use a convex hull relaxation. That is, we introduce a new variable u_{ij}^m for non-linear discrete term $(q_{ij}^m)^{2/n}$. Suppose $q_{ij}^m \in \{q_0, q_1, \dots, q_K\}$, where $(q_{ij}^m)_L = q_0 < q_1 < \dots < q_K = (q_{ij}^m)_U$. The convex hull (see Fig. 3) can be formulated as

$$u_{ij}^m - \frac{(q_K)^{2/n} - (q_0)^{2/n}}{q_K - q_0} q_{ij}^m \geq \frac{q_K (q_0)^{2/n} - (q_K)^{2/n} q_0}{q_K - q_0},$$

$$u_{ij}^m - \frac{(q_k)^{2/n} - (q_{k-1})^{2/n}}{q_k - q_{k-1}} q_{ij}^m \leq \frac{q_k (q_{k-1})^{2/n} - (q_k)^{2/n} q_{k-1}}{q_k - q_{k-1}} \quad (1 \leq k \leq K).$$

Similarly, we can introduce a new variable v_{ij}^m for non-linear discrete term $\log_2 \left(1 + \frac{g_{ij} P}{\eta W Q} q_{ij}^m\right)$ and construct convex hull constraints for v_{ij}^m .

Denote \mathbf{x} and \mathbf{q} as the vector for variables x_{ij}^m and q_{ij}^m , respectively. We have the following linear relaxation for problem z .

$$\text{Min} \quad \sum_{i \in \mathcal{N}} \sum_{m \in \mathcal{M}_i} \sum_{j \in \mathcal{T}_i^m} \frac{W}{Q^{2/n}} u_{ij}^m$$

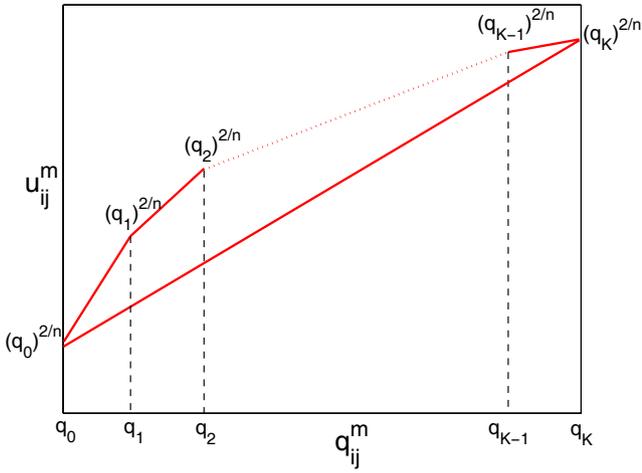


Fig. 3. Illustration of convex hull for a discrete term.

s.t. Convex hull constraints for u_{ij}^m ($i \in \mathcal{N}, m \in \mathcal{M}_i, j \in \mathcal{T}_i^m$)

$$\sum_{j \in \mathcal{T}_i^m} x_{ij}^m \leq 1 \quad (i \in \mathcal{N}, m \in \mathcal{M}_i)$$

$$q_{ij}^m - \left(\frac{d_{ij}}{R_T^{max}} \right)^n Q x_{ij}^m \geq 0 \quad (i \in \mathcal{N}, m \in \mathcal{M}_i, j \in \mathcal{T}_i^m)$$

$$q_{ij}^m - Q x_{ij}^m \leq 0 \quad (i \in \mathcal{N}, m \in \mathcal{M}_i, j \in \mathcal{T}_i^m)$$

$$\sum_{h \in \mathcal{T}_k^m} q_{kh}^m + \left(1 - \left(\frac{d_{kj}}{R_T^{max}} \right)^n \right) Q x_{ij}^m \leq Q$$

$(i \in \mathcal{N}, m \in \mathcal{M}_i, j \in \mathcal{T}_i^m, k \in \mathcal{I}_j^m, k \neq i)$

$$\sum_{l \in \mathcal{L}}^{s(l) \neq j, d(l) \neq i} f_{ij}(l) - \sum_{m \in \mathcal{M}_{ij}} W v_{ij}^m \leq 0 \quad (i \in \mathcal{N}, j \in \mathcal{T}_i)$$

Convex hull constraints for v_{ij}^m ($i \in \mathcal{N}, m \in \mathcal{M}_i, j \in \mathcal{T}_i^m$)

$$\sum_{j \in \mathcal{T}_i} f_{ij}(l) = r(l) \quad (l \in \mathcal{L}, i = s(l))$$

$$\sum_{j \in \mathcal{T}_i}^{j \neq s(l)} f_{ij}(l) - \sum_{k \in \mathcal{T}_i}^{k \neq d(l)} f_{ki}(l) = 0 \quad (l \in \mathcal{L}, i \in \mathcal{N}, i \neq s(l), d(l))$$

$$v_{ij}^m, v_{ij}^m \geq 0 \quad (i \in \mathcal{N}, m \in \mathcal{M}_i, j \in \mathcal{T}_i^m)$$

$$f_{ij}(l) \geq 0 \quad (l \in \mathcal{L}, i \in \mathcal{N}, i \neq d(l), j \in \mathcal{T}_i, j \neq s(l))$$

$(\mathbf{x}, \mathbf{q}) \in \Omega_z$,

where Ω_z is the set of all possible values of (\mathbf{x}, \mathbf{q}) in problem z . For example, Ω_1 for the original problem 1 is $\{(\mathbf{x}, \mathbf{q}) : 0 \leq x_{ij}^m \leq 1, 0 \leq q_{ij}^m \leq Q\}$.

C. Local Search Algorithm

A linear relaxation of a problem z can be solved in polynomial time. Denote the relaxation solution as $\hat{\psi}_z$, which provides a lower bound to problem z , although it may not be feasible. We now show how to obtain a feasible solution ψ_z based on $\hat{\psi}_z$ (see line 7 in Fig. 2).

For the feasible solution ψ_z , we use the same routing solution as that in $\hat{\psi}_z$, i.e., let $\mathbf{f} = \hat{\mathbf{f}}$. We then need to determine the value for \mathbf{x} and \mathbf{q} in ψ_z such that the routing solution \mathbf{f} is feasible, i.e., (8) should hold for each link $i \rightarrow j$.

Initially, each q_{ij}^m is set as the smallest value $(q_{ij}^m)_L$ in its value set and x_{ij}^m is fixed to 0 or 1 if its value set only has

Local Search Algorithm

Initialization:

1. Set $q_{ij}^m = (q_{ij}^m)_L$ and fix x_{ij}^m to 0 or 1 if its value set only has one element 0 or 1, respectively.
 2. Compute the capacity $\sum_{m \in \mathcal{M}_i} W \log_2 \left(1 + \frac{g_{ij} P}{\eta W Q} q_{ij}^m \right)$ and the requirement $\sum_{l \in \mathcal{L}}^{s(l) \neq j, d(l) \neq i} f_{ij}(l)$ for each link $i \rightarrow j$.
- Iteration:
3. If there is no link with its requirement larger than its capacity, then a feasible solution is found.
 4. Otherwise, among all the links having their requirements larger than their capacities, find link $i \rightarrow j$ that has the largest requirement.
 5. Try to increase the capacity for link $i \rightarrow j$ as follows.
 6. Increase q_{ij}^m among active bands in the non-increasing order of \hat{q}_{ij}^m and under the limitation that $q_{ij}^m \leq \min\{\lceil \hat{q}_{ij}^m \rceil, (q_{ij}^m)_U\}$.
 7. If the achieved capacity is insufficient, then try to use an available but currently unused band m in the non-increasing order of \hat{q}_{ij}^m .
 8. A band m is available if for any transmitting node k using this band, node j is not in its interference range.
 9. Increase q_{ij}^m under the constraint $q_{ij}^m \leq \min\{\lceil \hat{q}_{ij}^m \rceil, (q_{ij}^m)_U\}$.
 10. Also set $x_{ij}^m = 1$, $x_{ih}^m = 0$ for $h \in \mathcal{T}_i, h \neq j$, and let $(q_{kh})_U \leq \left\lfloor \left(\frac{d_{kj}}{R_T^{max}} \right)^n Q \right\rfloor$ for $k \in \mathcal{I}_j^m, k \neq i, h \in \mathcal{T}_k^m$.
 11. If the achieved capacity is still insufficient, increase q_{ij}^m among active bands in non-increasing order of \hat{q}_{ij}^m and under the constraint $q_{ij}^m \leq (q_{ij}^m)_U$.
 12. Finally, if the achieved capacity is still insufficient, then link $i \rightarrow j$ cannot be satisfied.

Fig. 4. An algorithm to obtain \mathbf{x} and \mathbf{q} .

one element 0 or 1, respectively. Based on these q_{ij}^m 's, we can compute the capacity $\sum_{m \in \mathcal{M}_i} W \log_2 \left(1 + \frac{g_{ij} P}{\eta W Q} q_{ij}^m \right)$ for each link $i \rightarrow j$. The requirement on a link $i \rightarrow j$ is $\sum_{l \in \mathcal{L}}^{s(l) \neq j, d(l) \neq i} f_{ij}(l)$. For each link with a requirement larger than its capacity, we will try to satisfy (8) by increasing q_{ij}^m under its value set limitation. After we do this for all links, then we can calculate the objective value of solution ψ_z . Otherwise, if the requirement of any link cannot be satisfied, then we fail to find a feasible solution and we set the objective value to ∞ . The details of this local search algorithm is shown in Fig. 4.

D. Selection of Partition Variables

If the relaxation error for a problem z is not small, the gap between its lower and upper bounds may be large. To narrow this gap, we generate two new sub-problems z_1 and z_2 from problem z . We hope that these two new problems have smaller relaxation errors. Subsequently, they may have tighter bounds for the objective function. To generate z_1 and z_2 , we identify a partition variable based on its relaxation error (line 11 in Fig. 2). As we discussed in Section IV-A, these partition variables include all \mathbf{x} and \mathbf{q} variables.

Since \mathbf{x} variables are more important than \mathbf{q} variables, we first select one of \mathbf{x} variables. In particular, for the relaxation solution $\hat{\psi}_z$, we choose an x_{ij}^m with the largest relaxation error $\min\{\hat{x}_{ij}^m, 1 - \hat{x}_{ij}^m\}$ among all \mathbf{x} variables and let its value set

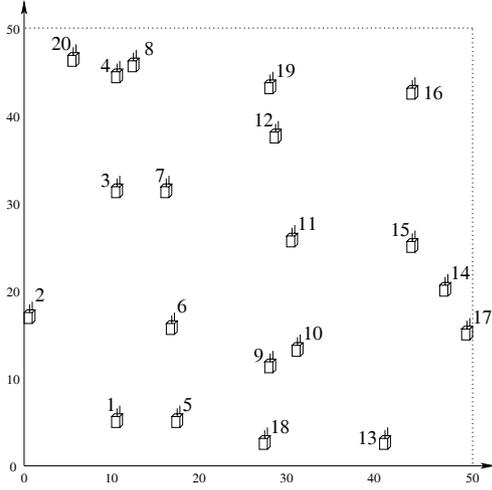


Fig. 5. A 20-node ad hoc network.

in problems z_1 and z_2 be $\{0\}$ and $\{1\}$, respectively.³ It should be noted that the new value set of x_{ij}^m imposes constraints on other variables. That is, if the new value set of x_{ij}^m is $\{0\}$, then we have $q_{ij}^m = 0$ based on (13). If the new value set of x_{ij}^m is $\{1\}$, then we have $x_{ih}^m = 0$ for $h \in \mathcal{T}_i, h \neq j$ based on (C-3), $q_{ij}^m \geq \left(\frac{d_{ij}}{R_T^{max}}\right)^n Q$ based on (12), and $q_{kh}^m \leq \left(\frac{d_{kh}}{R_T^{max}}\right)^n Q$ for $k \in \mathcal{I}_j^m, k \neq i, h \in \mathcal{T}_k$ based on (14).⁴

After we are done with all \mathbf{x} variables, i.e., all \mathbf{x} variables are fixed to either 0 or 1, then we select one of \mathbf{q} variables. In particular, for the relaxation solution $\hat{\psi}_z$, the relaxation error of a discrete term q_{ij}^m is $\min\{\hat{q}_{ij}^m - \lfloor \hat{q}_{ij}^m \rfloor, \hat{q}_{ij}^m + 1 - \hat{q}_{ij}^m\}$; the relaxation error of a non-linear discrete term $u_{ij}^m = (q_{ij}^m)^{2/n}$ is $|\hat{u}_{ij}^m - (\hat{q}_{ij}^m)^{2/n}|$; and the relaxation error of a non-linear discrete term $v_{ij}^m = \log_2\left(1 + \frac{q_{ij}^m P}{\eta W Q}\right)$ is $|\hat{v}_{ij}^m - \log_2\left(1 + \frac{q_{ij}^m P}{\eta W Q}\right)|$. Among these three types of relaxation errors, we identify the largest one and choose the corresponding q_{ij}^m as the partition variable. Assuming the value set of q_{ij}^m in problem z is $\{q_0, q_1, \dots, q_K\}$, its value set in problems z_1 and z_2 will be $\{q_0, q_1, \dots, \lfloor \hat{q}_{ij}^m \rfloor\}$ and $\{\lfloor \hat{q}_{ij}^m \rfloor + 1, \lfloor \hat{q}_{ij}^m \rfloor + 2, \dots, q_K\}$, respectively.

V. NUMERICAL RESULTS

A. Simulation Setting

In this section, we consider a randomly generated 20-node ad hoc network with each node located in a 50x50 area. For ease of exposition, we normalize all units for distance, bandwidth, rate, and power based on (1) and (8) with appropriate dimensions. An instance of network topology is given in Fig. 5 with each node's location listed in Table III. Within this network, we assume there are $|\mathcal{L}| = 5$ user sessions, with source node and destination node randomly selected and the rate of each session is randomly generated within $[10, 100]$. Table II specifies an instance of the source node, destination

³Since the value set for this x_{ij}^m only has one element, this x_{ij}^m can be replaced by a constant in the new problem. As a result, some constraints may also be removed.

⁴If the constraints make the new value set of a variable q_{kh}^m empty, then the corresponding new problem is clearly infeasible. As a result, we can remove it from the problem list.

TABLE II
SOURCE NODE, DESTINATION NODE, AND RATE REQUIREMENT OF THE 5 SESSIONS.

Session	Source Node	Destination Node	Rate Requirement
1	7	16	28
2	8	5	12
3	15	13	56
4	2	18	75
5	9	11	29

TABLE III
EACH NODE'S LOCATION AND AVAILABLE FREQUENCY BANDS FOR THE 20-NODE NETWORK.

Node Index	Location	Available Bands
1	(10.5, 4.3)	I, II, III, IV, V, VI, VII, VIII, IX, X
2	(1.7, 17.3)	II, III, IV, V, VI, VII, X
3	(10.7, 30.8)	I, III, IV, V, VI, VII, VIII, IX, X
4	(10.2, 45.3)	I, III, IV, V, VI, VII, VIII, IX, X
5	(17.8, 4)	I, II, V, VI, VII, VIII, IX
6	(17.2, 15.2)	I, II, IV, VIII
7	(16.9, 30.8)	I, II, III, IV, V, VI, VII, VIII, IX, X
8	(12.3, 47.3)	I, III, IV, V, VI, VII, VIII, IX
9	(28.2, 11.5)	I, III, V, VII
10	(32.1, 13.8)	I, II, III, IV, VI, VII, VIII, IX, X
11	(30.4, 25.6)	I, II, III, V, VI, VIII, IX, X
12	(29.7, 36)	I, II, III, IV, VI, VI
13	(41.7, 3.1)	I, II, III, V, VI, VIII, IX, X
14	(47.5, 20)	I, IV, V, VIII, IX, X
15	(43.3, 25.3)	II, III, IV, V, VI, VII, VIII, IX, X
16	(44.1, 42.7)	I, II, IV, VI, VII, VIII, IX, X
17	(49.6, 15.8)	I, II, III, IV, V, VI, VII, VIII
18	(28.7, 2.5)	I, II, III, VI, VII, VIII, IX, X
19	(28, 43.5)	II, IV, V, VI, VIII
20	(5, 46.9)	II, IV, V, VI, VII

node, and rate requirement for the five sessions in the network. We assume there are $|\mathcal{M}| = 10$ frequency bands in the network and each band has a bandwidth of $W = 50$. At each node, only a subset of these frequency bands is available. In the simulation, this is done by randomly selecting a subset of bands for each node. Table III shows the available bands for each node.

We assume $R_T^{max} = 20$, $R_T^{max} = 40$, and the path loss index $n = 4$. The threshold α is assumed to be $\alpha = \eta W = 50\eta$. Thus, we have $\beta = \left(\frac{R_T^{max}}{R_T^{max}}\right)^n \alpha W = \frac{50}{16}\eta$ and the maximum transmission power $P = (R_T^{max})^n \alpha W = 8 \cdot 10^6 \eta$. We set $\varepsilon = 0.05$, which guarantees that the obtained solution is within 5% from the optimum.

B. Results

In this set of results, we apply the solution procedure to the 20-node network described above for different level of power control granularity (Q). Note that $Q = 1$ corresponds to the case that there is no power control, i.e., a node always uses its peak power P for transmission. When Q is sufficiently large, power control approaches continuous. Figure 6 shows the results. First, we note that power control has a significant impact on objective value. Comparing the case when there is

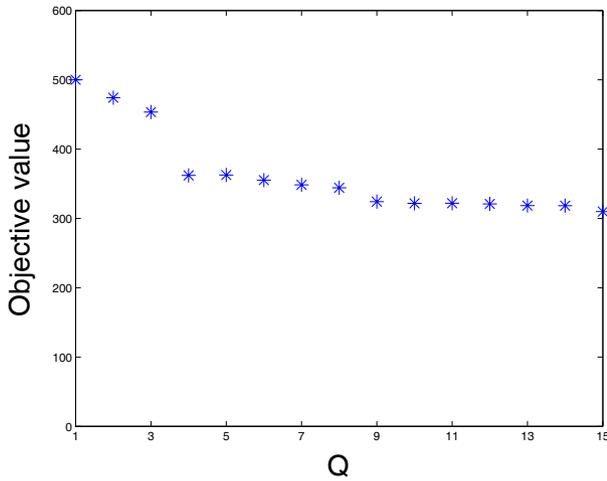


Fig. 6. Objective value as a function of Q .

no power control ($Q = 1$) and the case of $Q = 15$, we find that there is nearly a 40% reduction in the objective value. Second, although the objective value is a non-increasing function of Q , when Q becomes sufficiently large (e.g., 10 in this network setting), further increase in Q will not have much reduction on objective value. This suggests that for practical purpose, the number of required power control levels does not need to be a large number.

The rest of results are for $Q = 10$. For transmission power, we have

$$\begin{aligned} q_{9,11}^1 &= 3, & q_{12,16}^2 &= 4, & q_{7,12}^3 &= 3, \\ q_{2,1}^4 &= 4, & q_{2,1}^5 &= 4, & q_{5,18}^6 &= 2, \\ q_{17,13}^7 &= 4, & q_{8,3}^8 &= 5, & q_{14,17}^8 &= 1, & q_{1,5}^9 &= 1, & q_{15,14}^9 &= 1, \\ q_{3,2}^{10} &= 5. \end{aligned}$$

The scheduling results are

$$\begin{aligned} x_{9,11}^1 &= 1, & x_{12,16}^2 &= 1, & x_{7,12}^3 &= 1, \\ x_{2,1}^4 &= 1, & x_{2,1}^5 &= 1, & x_{5,18}^6 &= 1, \\ x_{17,13}^7 &= 1, & x_{8,3}^8 &= 1, & x_{14,17}^8 &= 1, & x_{1,5}^9 &= 1, & x_{15,14}^9 &= 1, \\ x_{3,2}^{10} &= 1. \end{aligned}$$

The flow routing topology is shown in Fig. 7. The corresponding flow rates are

$$\begin{aligned} f_{7,12}(1) &= 28, & f_{12,16}(1) &= 28, \\ f_{8,3}(2) &= 12, & f_{3,2}(2) &= 12, & f_{2,1}(2) &= 12, & f_{1,5}(2) &= 12, \\ f_{15,14}(3) &= 56, & f_{14,17}(3) &= 56, & f_{17,13}(3) &= 56, \\ f_{2,1}(4) &= 75, & f_{1,5}(4) &= 75, & f_{5,18}(4) &= 75, \\ f_{9,11}(5) &= 29. \end{aligned}$$

Note that a link may be used by multiple sessions. For example, link $2 \rightarrow 1$ is used by sessions 2 and 4. As a result, the total data rate on link $2 \rightarrow 1$ is $f_{2,1}(2) + f_{2,1}(4) = 12 + 75 = 87$.

The following observations on the numerical results show the close coupling relationships between per-node power control and upper layers. In one observation (scheduling), we can see that, links $8 \rightarrow 3$ and $14 \rightarrow 17$ are active on the same band 8. This is feasible because the interference range at node 14 is 22.49 under $q_{14,17}^8 = 1$, which is smaller than 38.35 (the distance between nodes 3 and 14). We note that if there is no power control, i.e., node 14 use the peak transmission power, then node 3 will be in the interference range of node 14,

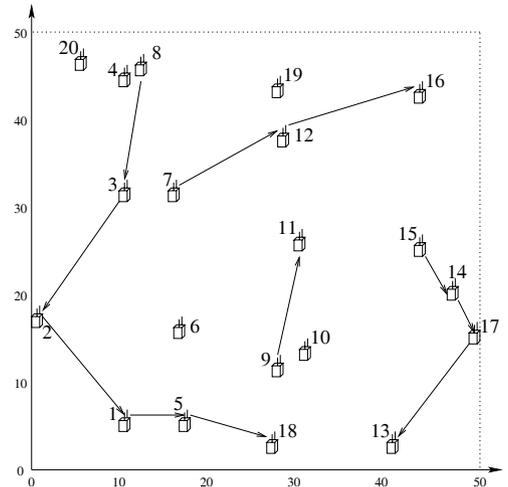


Fig. 7. Flow routing for the five sessions in the 20-node network.

which is $R_T^{max} = 40$, and will lead to a scheduling conflict. In another observation (routing), we see that, for session 2 (from node 8 to node 5), the routing path is $8 \rightarrow 3 \rightarrow 2 \rightarrow 1 \rightarrow 5$. Here, a shorter path $8 \rightarrow 3 \rightarrow 6 \rightarrow 5$ is not used. This is because that path $8 \rightarrow 3 \rightarrow 6 \rightarrow 5$ is interfered by transmissions on other paths. The optimal solution tends to choose paths that are not close to each other. Finally, for session 3 (form node 15 to node 13), the routing path is $15 \rightarrow 14 \rightarrow 17 \rightarrow 13$, while a shorter path $15 \rightarrow 17 \rightarrow 13$ is not used. This is because that node 15 can use a smaller transmission power to transmit to a closer neighboring node 14 with a smaller interference footprint. This will allow link $1 \rightarrow 5$ to be active on the same band 9 with link $15 \rightarrow 14$.

VI. CONCLUSION

In this paper, we investigated the challenging problem of per-node based power control for a multi-hop CRN. This problem is difficult due to its large design space and the coupling relationship between power control and upper layers. We developed a mathematical framework to optimize these layers jointly and formulated a mixed integer non-linear program. Subsequently, we developed a solution procedure to derive results for all three layers jointly. Our solution guarantees $(1 - \epsilon)$ optimal performance. The results in this paper close some long standing issues associated with network optimization with per-node based power control.

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