Some Fundamental Results on Base Station Movement Problem for Wireless Sensor Networks

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***Abstract*—The benefits of using a mobile base station to prolong sensor network lifetime have been well recognized. However, due to the complexity of the problem (time-dependent network topology and traffic routing), theoretical performance limits and provably optimal algorithms remain difficult to develop. This paper fills this important gap by contributing some theoretical results regarding the optimal movement of a mobile base station. Our main result hinges upon two key intermediate results. In the first result, we show that a time-dependent joint base station movement and flow routing problem can be transformed into a location-dependent problem. In the second result, we show that, for $(1-\varepsilon)$ optimality, the infinite possible locations for base station movement can be reduced to a finite set of locations via several constructive steps [i.e., discretization of energy cost through a geometric sequence, division of a disk into a finite number of subareas, and representation of each subarea with a fictitious cost point (FCP)]. Subsequently, for each FCP, we can obtain the optimal sojourn time for the base station (as well as the corresponding location-dependent flow routing) via a simple linear program. We prove that the proposed solution can guarantee the achieved network lifetime is at least $(1-\varepsilon)$ of the maximum (unknown) network lifetime, where ε can be made arbitrarily small depending on the required precision.**

***Index Terms*—Approximation algorithm, lifetime, mobile base station, optimization, sensor networks, theory.**

I. Introduction

The benefits of using a mobile base station to prolong sensor network lifetime have been well recognized [9], [24], [25]. Since a base station is the sink node for data collected by all the sensor nodes in the network, this approach aims to alleviate the traffic burden from a fixed set of sensor nodes near the base station to other sensor nodes in the network, and thus could extend network lifetime significantly. Furthermore, given recent advances in unmanned autonomous vehicle (UAV) [4] and customized robotics for sensors [18], it is now plausible to envision an unmanned vehicle carrying a base station for sensor data collection.

Although the potential benefit of using a mobile base station to prolong sensor network lifetime is significant, the theoretical difficulty of this problem is enormous. There are two components that are tightly coupled in this problem. First, the location of the base station is time-dependent, i.e., at different time instances, the base station may be at different locations. Second, the multihop traffic (or flow) routing appears to be dependent on both time and location of the base station. As a result, an optimization problem with the objective of maximizing network lifetime needs to consider both base station location and flow routing, both of which are also time-dependent. Due to these difficulties, existing solutions to this problem remain heuristic at best (e.g., [9] and [25]) and cannot offer a *provable* performance guarantee to network lifetime.

To fill this theoretical gap, this paper offers an in-depth study on the network lifetime problem with a mobile base station. We formulate an optimization problem that incorporates base station movement and multihop flow routing. Our solution hinges upon two important intermediate results. The first result shows that as far as network lifetime objective is concerned, we can *transform* the time-dependent problem to a location (space)-dependent problem. In particular, we show that flow routing only depends on the base station location, regardless of *when* the base station visits this location. Furthermore, the specific time instances for the base station to visit a location is not important, as long as the total sojourn time for the base station to be present at this location is the same. This result allows us to focus on solving a location-dependent problem.

For the location-dependent problem, it is sufficient to consider an area within the smallest enclosing disk (SED) that covers all sensor nodes in the network [17]. Our second result shows that to obtain a $(1-\varepsilon)$-optimal solution, where ε can be made arbitrarily small depending on the required precision, we only need to consider a finite set of points within the SED for the mobile base station location. This result is obtained by several constructive steps including discretization of energy cost through a geometric sequence, division of SED into a finite number of subareas, and representation of each subarea with a fictitious cost point (FCP). As a result, we can find the optimal sojourn time for the base station to stay at each FCP (as well as the corresponding flow routing solution) such that the overall network lifetime (i.e., sum of the sojourn times) is maximized via a single linear program (LP). We prove that the proposed solution can guarantee that the achieved network lifetime is at least $(1-\varepsilon)$ of the maximum (unknown) network lifetime.

The rest of this paper is organized as follows. In Section II, we describe the network model and formally state the base station movement problem. In Section III, we transform the time-dependent problem to a location-dependent problem. In Section IV, we first develop an optimal solution for a constrained mobile base station (C-MB) problem, where the base station is allowed to be present among a set of given locations.

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Then, we present the solution for the unconstrained mobile base station (U-MB), where the base station is allowed to roam anywhere in the two-dimensional plane. Here, we give a formal proof of $(1-\varepsilon)$-optimality of the proposed algorithm. In Section V, we present some numerical results illustrating the efficacy of the proposed algorithm. Section VI reviews related work, and Section VII concludes this paper.

II. Network Model and Problem Formulation

A. Network Model

We consider a set of $\mathcal{N}$ sensor nodes deployed over a two-dimensional area, with the location of each sensor node $i \in \mathcal{N}$ being at a fixed point (x_i, y_i). We assume that each node i generates data at a fixed rate of r_i. There is a base station B for the sensor network, and it serves as the sink node for all data collected by the sensor nodes. Data generated by each sensor node should be transmitted to the base station via single or multihop.

Communication energy is assumed to be the dominant source of energy consumption at a node. We assume that each node has power control capability. That is, suppose that node i transmits data to node j with a rate of g_{ij}, then the transmission power at node i is modeled by [7]

$$p_{ij}^{\mathrm{t}} = C_{ij} \cdot g_{ij} \tag{1}$$

where C_{ij} is the energy cost (in units of Joule) for transmitting one bit of data from node i to node j and is modeled by

$$C_{ij} = \alpha + \beta \cdot d_{ij}^{n} \tag{2}$$

where α and β are two constant terms, d_{ij} is the physical distance between nodes i and j, and n is the path loss index and is typically $2 \le n \le 4$[13]. Note that the transmission energy cost is distance-dependent.

The receiving power consumption at sensor node i is modeled by [7]

$$p_i^{\mathrm{r}} = \rho \sum_{k \in \mathcal{N}}^{k \neq i} g_{ki} \tag{3}$$

where ρ is a constant and g_{ki} is the incoming bit-rate received by sensor node i from sensor node k.

In this theoretical study, we assume a contention-free MAC protocol for medium access, where physical-layer interference has been effectively avoided. Many sensor network applications (particularly those for long-term monitoring) are likely to operate at low rates. For such low-bit-rate traffic, a contention-free MAC protocol is fairly easy to design (see, e.g., [20]), and its discussion is beyond the scope of this paper.

We assume that each sensor node $i \in \mathcal{N}$ is initially provisioned with an amount of energy e_i. The base station is not constrained with energy and is free to roam in the two-dimensional plane. In this study, network lifetime is defined as the first time instance when any of the sensor nodes run out of energy. From (1)–(3), it is not hard to realize that the location of the base station and the corresponding multihop flow routing among the nodes will affect energy consumption behavior at each node and thus the network lifetime. Table I lists the notation used in this paper.

TABLE I
NOTATION

General Notation	
$\mathcal{A}$	The movement region for the base station
B	Denotes the base station
C_{ij}	Transmission energy cost from sensor i to sensor j
$c_{iB}(t)$	Transmission energy cost from sensor i to base station B at time t
$c_{iB}(p)$	Transmission energy cost from sensor i to base station B when base station B is at point p
e_i	Initial energy at sensor node i
$f_{ij}(p)$ (or $f_{iB}(p)$)	Flow rate from sensor i to sensor j (or base station B) when base station B is at point p
$g_{ij}(t)$ (or $g_{iB}(t)$)	Flow rate from sensor i to sensor j (or base station B) at time t
n	Path loss index, $2 \le n \le 4$
$\mathcal{N}$	The set of sensor nodes in the network
N	$= \lvert\mathcal{N}\rvert$, the number of sensor nodes in the network
$p(s)$	The location of the base station after it traverses a distance s along its $\mathcal{P}$
$\mathcal{P}$	The traveling path of the base station
r_i	Bit rate generated at sensor node i
$s(t)$	Cumulative distance traversed by the base station up to time t
S	Total distance traversed by the base station
$u(s)$	$= \frac{1}{\lVert v(t)\rVert}$ when the base station traverses point $p(s)$ at time t
$U(s)$	Sojourn time of the base station at distance s
$v(t)$	The velocity of the base station at time t
$w(p)$	$= \sum_{s \in \mathcal{Z}(p)} u(s)$ when the base station traverses point p but never dwells
$W(p)$	Sojourn time for the base station at point p
$(x, y)(t)$	Location of base station B at time t
(x_i, y_i)	Location of sensor node i
$\mathcal{Z}(p)$	Set of distance s with $p(s) = p$
α, β	Two constant terms in power consumption model for data transmission
ρ	Power consumption coefficient for receiving data
Additional Notation for the C-MB Problem	
M	The number of pre-determined locations
p_m	The m-th location
$T_{\text{C-MB}}^{*}$	The maximum network lifetime achieved by $\psi_{\text{C-MB}}^{*}$
$\psi_{\text{C-MB}}^{*}$	An optimal solution to the C-MB problem
Additional Notation for the U-MB Problem	
$\mathcal{A}_m$	The m-th subarea in $\mathcal{A}$
$C_{iB}^{\min}, C_{iB}^{\max}$	Lower and upper bounds of $c_{iB}(p)$ for $p \in \mathcal{A}$
$C[h]$	$=\alpha(1+\varepsilon)^h$, the transmission energy cost for the h-th circle
H_i	The required number of circles at sensor node i
M	The number of subareas under a given ε
$O_{\mathcal{A}}, R_{\mathcal{A}}$	The center and radius of $\mathcal{A}$
p_m	The FCP corresponding to $\mathcal{A}_m$
$\mathcal{S}_m$	$= \{s : p(s) \in \mathcal{A}_m, 0 \le s \le S\}$
$T_{\text{U-MB}}$	$(1-\varepsilon)$ optimal network lifetime achieved by $\psi_{\text{U-MB}}$
$W(\mathcal{A}_m)$	Sojourn time for the base station in subarea A_m
$W(p_m)$	Sojourn time for the base station at FCP p_m
ε	Targeted approximation error, $\varepsilon > 0$ and $\varepsilon \ll 1$
$\psi_{\text{U-MB}}$	$(1-\varepsilon)$ optimal solution to the U-MB problem

B. Problem Description

The focus of this paper is to investigate how to optimally move a mobile base station to collect data in a sensor network so that network lifetime can be maximized. Note that the network lifetime problem has attracted great interest even for the fixed base station problem (see, e.g., [2], [3], and [15]).

Denote $(x, y)(t)$ as the position of base station B at time t, and T the network lifetime (which is the objective function of our optimization problem). Then, a feasible flow routing solution realizing this network lifetime T must satisfy both flow conservation and energy constraint at each sensor node. These constraints can be formally stated as follows. Denote $g_{ij}(t)$ and

$g_{iB}(t)$ the data rates from node i to node j and base station B at time t, respectively. Under multihop multipath routing, the flow conservation for each node $i \in \mathcal{N}$ at any time $t \in [0, T]$ is

$$\sum_{k\in\mathcal{N}}^{k\neq i} g_{ki}(t) + r_i = \sum_{j\in\mathcal{N}}^{j\neq i} g_{ij}(t) + g_{iB}(t)$$

i.e., for node i, the sum of total incoming flow rates plus self-generated data rate is equal to the total outgoing flow rates at time t. Note that in our problem, data generated at each node should be transmitted to the base station in real time.

The energy constraint for each node $i \in \mathcal{N}$ is

$$\int_0^T \left[\sum_{k\in\mathcal{N}}^{k\neq i} \rho \cdot g_{ki}(t) + \sum_{j\in\mathcal{N}}^{j\neq i} C_{ij} \cdot g_{ij}(t) + c_{iB}(t) \cdot g_{iB}(t) \right] dt \leq e_i$$

i.e., the total consumed energy due to reception and transmission over time T cannot exceed its initial energy e_i. We have

$$c_{iB}(t) = \alpha + \beta \left[\sqrt{(x(t) - x_i)^2 + (y(t) - y_i)^2} \right]^n$$

by (2), where (x_i, y_i) is the location of node i.

Denote $\mathcal{A}$ the movement region for the base station, which can be narrowed down to the SED for all nodes in the network [17]. Note that the SED can be found in polynomial time [23]. The optimization problem that we are interested in can be formulated as follows:

$$\begin{aligned}
&\text{Max} \quad T \\
&\text{s.t.} \quad \sum_{k\in\mathcal{N}}^{k\neq i} g_{ki}(t) + r_i = \sum_{j\in\mathcal{N}}^{j\neq i} g_{ij}(t) + g_{iB}(t) \\
&\qquad\qquad\qquad\qquad\qquad\qquad\qquad (i \in \mathcal{N}, t \in [0, T]) \\
&\int_0^T \left[\sum_{k\in\mathcal{N}}^{k\neq i} \rho \cdot g_{ki}(t) + \sum_{j\in\mathcal{N}}^{j\neq i} C_{ij} \cdot g_{ij}(t) + c_{iB}(t) \cdot g_{iB}(t) \right] dt \leq e_i \\
&\qquad\qquad\qquad\qquad\qquad\qquad\qquad (i \in \mathcal{N}) \\
&c_{iB}(t) = \alpha + \beta \left[\sqrt{(x(t) - x_i)^2 + (y(t) - y_i)^2} \right]^n \\
&\qquad\qquad\qquad\qquad\qquad\qquad\qquad (i \in \mathcal{N}, t \in [0, T]) \\
&(x, y)(t) \in \mathcal{A} \qquad\qquad\qquad\qquad\quad (t \in [0, T]) \\
&T, g_{ij}(t), g_{iB}(t) \geq 0 \qquad\qquad (i, j \in \mathcal{N}, i \neq j, t \in [0, T])
\end{aligned}$$

where the base station location (i.e., $(x, y)(t)$ for $t \in [0, T]$) and the corresponding flow routing (i.e., $g_{ij}(t)$ and $g_{iB}(t)$ for $t \in [0, T]$) form a joint optimization space for the objective T. Since the left-hand side in the second constraint is not a polynomial function of optimization variables, this formulation is in the form of *nonpolynomial program*.

III. From Time Domain to Space Domain

The difficulty of the problem formulation in the last section lies in that base station location $(x, y)(t)$ and flow routing $g_{ij}(t)$ and $g_{iB}(t)$ are all functions of time. This adds considerable difficulty in the optimization problem. In this section, we show that as far as network lifetime performance is concerned, such dependency on time can be relaxed. Specifically, we will show (Lemma 1) that the flow routing only needs to be dependent on the location of the base station and can be independent of *when* the base station is present at this location. Furthermore, as long as the total sojourn time for the base station to be present at this location is the same, the specific time instance (i.e., "when") the base station visits this location is not important (Lemma 2). These results allow us to transform the problem to a location-dependent problem.

For the solutions considered in Section II, which are time-dependent, we give the following definition.

Definition 1: A time-dependent solution consists of a network lifetime T, a path $\mathcal{P} = \{(x, y)(t) : t \in [0, T]\}$ for the base station, and a flow routing $g_{ij}(t)$ and $g_{iB}(t)$ at time t, $i, j \in \mathcal{N}, i \neq j$, where $(x, y)(t)$, $g_{ij}(t)$ and $g_{iB}(t)$ are all functions of time t.

For such a solution, denote $v(t)$ the base station velocity at time t (and thus $\|v(t)\|$ is the base station speed at time t). Denote $s(t) = \int_{\tau=0}^{t} \|v(\tau)\| d\tau$ the distance traversed by the base station up to time t. Suppose the total traversed distance at the end of network lifetime T is $s(T) = S$. Then, we have $s(t) \in [0, S]$ for $t \in [0, T]$. Note that for a given path $\mathcal{P}$, we can identify the corresponding base station location for any s, which we denote as $p(s)$. Denote $U(s)$ the sojourn time at distance s. The base station may visit the same point multiple times. Then, multiple distances may correspond to the same point. Denote $\mathcal{Z}(p)$ the set of such distances that correspond to the same point p. The total sojourn time at a point p is

$$W(p) = \sum_{s\in\mathcal{Z}(p)} U(s). \tag{4}$$

Now we give the following definition.

Definition 2: A location-dependent solution consists of a path $\mathcal{P}$ for the base station, $W(p)$ at each point $p \in \mathcal{P}$, flow routing $f_{ij}(p)$ and $f_{iB}(p)$, $i, j \in \mathcal{N}, i \neq j$, when the base station is at point p, and a network lifetime T, where $W(p)$, $f_{ij}(p)$, and $f_{iB}(p)$ are all functions of location p.

The following theorem shows that for the objective of network lifetime maximization, it is sufficient to consider location-dependent solutions.

Theorem 1: The optimal location-dependent solution can achieve the same maximum network lifetime as the optimal time-dependent solution.

The proof of Theorem 1 is based on the following two lemmas.

Lemma 1: Given a feasible time-dependent solution, we can construct a location-dependent solution with the same network lifetime.

Proof: The proof is based on the following construction. For a given time-dependent solution φ, it consists of a network lifetime T, a path $\mathcal{P}$ for the base station, and a flow routing $g_{ij}(t)$ and $g_{iB}(t)$, $i, j \in \mathcal{N}, i \neq j$. To construct a location-dependent solution $\bar{\varphi}$, we let the base station follow the same path $\mathcal{P}$, and for each point $p \in \mathcal{P}$, we compute $W(p)$ by (4). For $\bar{\varphi}$, we define location-dependent flow rates $f_{ij}(p)$ and $f_{iB}(p)$ for each point $p \in \mathcal{P}$ by the average of $g_{ij}(t)$ and $g_{iB}(t)$ over all visits to p during $[0, T]$ as follows.

- If the base station dwells at p at least once (with $W(p) > 0$), we define

$$f_{ij}(p) = \frac{\int_{t\in[0,T]}^{(x,y)(t)=p} g_{ij}(t)dt}{W(p)} \tag{5}$$

$$f_{iB}(p) = \frac{\int_{t\in[0,T]}^{(x,y)(t)=p} g_{iB}(t)dt}{W(p)}. \tag{6}$$

- If the base station traverses p (maybe multiple times) but never dwells, then there is a unique time corresponding to each $s \in \mathcal{Z}(p)$. Denote such time as $t(s)$. Define

$$u(s) = \frac{1}{\|v(t(s))\|} \tag{7}$$

$$w(p) = \sum_{s\in\mathcal{Z}(p)} u(s). \tag{8}$$

Based on $u(s)$ and $w(p)$, we can define

$$f_{ij}(p) = \frac{\sum_{s\in\mathcal{Z}(p)} g_{ij}(t(s)) \cdot u(s)}{w(p)} \tag{9}$$

$$f_{iB}(p) = \frac{\sum_{s\in\mathcal{Z}(p)} g_{iB}(t(s)) \cdot u(s)}{w(p)}. \tag{10}$$

To show the data routing scheme with $f_{ij}(p)$ and $f_{iB}(p)$ is feasible and $\bar{\varphi}$ has the same network lifetime T, we need to prove that: (i) when the base station visits each point $p \in \mathcal{P}$, flow conservation holds at this node; and (ii) at time T, the energy consumption at each node is the same as that in solution φ. Both (i) and (ii) can be intuitively explained by noting that $f_{ij}(p)$ and $f_{iB}(p)$ are defined by the average of $g_{ij}(t)$ and $g_{iB}(t)$, respectively. Now we prove (i) and (ii).

(i): For flow conservation at point p, if the base station dwells at p at least once, then we have the following flow conservation:

$$\begin{aligned}\sum_{k\in\mathcal{N}}^{k\neq i} f_{ki}(p)+r_i &= \sum_{k\in\mathcal{N}}^{k\neq i} \frac{\int_{t\in[0,T]}^{(x,y)(t)=p} g_{ki}(t)dt}{W(p)} + \frac{\int_{t\in[0,T]}^{(x,y)(t)=p} r_i dt}{W(p)} \\ &= \frac{\int_{t\in[0,T]}^{(x,y)(t)=p} \left[\sum_{k\in\mathcal{N}}^{k\neq i} g_{ki}(t) + r_i\right] dt}{W(p)} \\ &= \frac{\int_{t\in[0,T]}^{(x,y)(t)=p} \left[\sum_{j\in\mathcal{N}}^{j\neq i} g_{ij}(t) + g_{iB}(t)\right] dt}{W(p)} \\ &= \sum_{j\in\mathcal{N}}^{j\neq i} \frac{\int_{t\in[0,T]}^{(x,y)(t)=p} g_{ij}(t)dt}{W(p)} + \frac{\int_{t\in[0,T]}^{(x,y)(t)=p} g_{iB}(t)dt}{W(p)} \\ &= \sum_{j\in\mathcal{N}}^{j\neq i} f_{ij}(p) + f_{iB}(p).\end{aligned}$$

The first equality holds by (5) and the fact that $W(p) = \int_{t\in[0,T]}^{(x,y)(t)=p} 1dt$. The third equality holds by the flow conservation in solution φ. The last equality holds by (5) and (6).

If the base station traverses (but never dwells at) p, then we have the following flow conservation:

$$\begin{aligned}&\sum_{k\in\mathcal{N}}^{k\neq i} f_{ki}(p) + r_i \\ &= \sum_{k\in\mathcal{N}}^{k\neq i} \frac{\sum_{s\in\mathcal{Z}(p)} g_{ki}(t(s))u(s)}{w(p)} + \frac{\sum_{s\in\mathcal{Z}(p)} r_i \cdot u(s)}{w(p)} \\ &= \frac{\sum_{s\in\mathcal{Z}(p)} \left[\sum_{k\in\mathcal{N}}^{k\neq i} g_{ki}(t(s)) + r_i\right] u(s)}{w(p)} \\ &= \frac{\sum_{s\in\mathcal{Z}(p)} \left[\sum_{k\in\mathcal{N}}^{k\neq i} g_{ij}(t(s)) + g_{iB}(t(s))\right] u(s)}{w(p)} \\ &= \sum_{j\in\mathcal{N}}^{j\neq i} \frac{\sum_{s\in\mathcal{Z}(p)} g_{ij}(t(s))u(s)}{w(p)} + \frac{\sum_{s\in\mathcal{Z}(p)} g_{iB}(t(s))u(s)}{w(p)} \\ &= \sum_{j\in\mathcal{N}}^{j\neq i} f_{ij}(p) + f_{iB}(p).\end{aligned}$$

The first equality holds by (8) and (9). The third equality holds by the flow conservation in solution φ. The last equality holds by (9) and (10).

(ii): For energy consumption at time T, we want to show that the energy consumption at each node i in the constructed location-dependent solution $\bar{\varphi}$ is the same as that in the given time-dependent solution φ, i.e.,

$$\begin{aligned}&\sum_{k\in\mathcal{N}}^{k\neq i} \rho \left[\sum_{s\in[0,S]}^{U(s)>0} f_{ki}(p(s))U(s) + \int_{s\in[0,S]}^{U(s)=0} f_{ki}(p(s))u(s)ds\right] \\ &+\sum_{j\in\mathcal{N}}^{j\neq i} C_{ij} \left[\sum_{s\in[0,S]}^{U(s)>0} f_{ij}(p(s))U(s)+\int_{s\in[0,S]}^{U(s)=0} f_{ij}(p(s))u(s)ds\right] \\ &+ \sum_{s\in[0,S]}^{U(s)>0} c_{iB}(p(s))f_{iB}(p(s))U(s) \\ &+ \int_{s\in[0,S]}^{U(s)=0} c_{iB}(p(s))f_{iB}(p(s))u(s)ds \\ &= \sum_{k\in\mathcal{N}}^{k\neq i} \rho \int_0^T g_{ki}(t)dt + \sum_{j\in\mathcal{N}}^{j\neq i} C_{ij} \int_0^T g_{ij}(t)dt + \int_0^T c_{iB}(t)g_{iB}(t)dt\end{aligned}$$

where $c_{iB}(p)$ is the energy cost from sensor i to the base station B when the base station is at point p. To show that the above equality holds, it is sufficient to show that the following three equalities hold:

$$\begin{aligned}&\sum_{s\in[0,S]}^{U(s)>0} f_{ki}(p(s))U(s) + \int_{s\in[0,S]}^{U(s)=0} f_{ki}(p(s))u(s)ds \\ &= \int_0^T g_{ki}(t)dt \qquad (k, i \in \mathcal{N}, k \neq i)\end{aligned} \tag{11}$$

$$\sum_{s\in[0,S]}^{U(s)>0} f_{ij}(p(s))U(s) + \int_{s\in[0,S]}^{U(s)=0} f_{ij}(p(s))u(s)ds$$
$$= \int_0^T g_{ij}(t)dt \qquad (i,j\in\mathcal{N}, j\neq i) \tag{12}$$
$$\sum_{s\in[0,S]}^{U(s)>0} c_{iB}(p(s))f_{iB}(p(s))U(s) + \int_{s\in[0,S]}^{U(s)=0} c_{iB}(p(s))f_{iB}(p(s))u(s)ds$$
$$= \int_0^T c_{iB}(t)g_{iB}(t)dt \qquad (i\in\mathcal{N}). \tag{13}$$

We now prove (13). The proofs for (11) and (12) are very similar (but simpler) and are thus omitted to conserve space.

On the left-hand side (LHS) of (13), we observe that the summation and integration are over distances in $[0,S]$. Recall that $\mathcal{Z}(p)$ denotes the set of total traversed distances when the base station visits $p\in\mathcal{P}$. If the base station visits p multiple times, then $\mathcal{Z}(p)$ has multiple elements. For each $s\in\mathcal{Z}(p)$, $c_{iB}(p(s))$ is the same since $p(s)$ is the same point p. Furthermore, based on the definitions in (6) and (10) for $f_{iB}(\cdot)$, we have that $f_{iB}(p(s))$ is also the same for each $s\in\mathcal{Z}(p)$. Thus, for a point $p\in\mathcal{P}$, we can group these distances in $\mathcal{Z}(p)$ together in the summation as well as integration on the LHS of (13).

Now we select one distance from each group $\mathcal{Z}(p)$, $p\in\mathcal{P}$, to represent this group. In particular, denote $\mathcal{Y}(\mathcal{P})$ as the set of these representatives for each $\mathcal{Z}(p)$. Then, $\mathcal{Y}(\mathcal{P})$ is a subset of $[0,S]$.

For the summation (first term) on the LHS of (13), we have

$$\sum_{s\in[0,S]}^{U(s)>0} c_{iB}(p(s))f_{iB}(p(s))U(s)$$
$$= \sum_{s\in\mathcal{Y}(\mathcal{P})}^{W(p(s))>0} \sum_{z\in\mathcal{Z}(p(s))}^{U(z)>0} c_{iB}(p(z))f_{iB}(p(z))U(z)$$
$$= \sum_{s\in\mathcal{Y}(\mathcal{P})}^{W(p(s))>0} c_{iB}(p(s))f_{iB}(p(s)) \sum_{z\in\mathcal{Z}(p(s))}^{U(z)>0} U(z)$$
$$= \sum_{s\in\mathcal{Y}(\mathcal{P})}^{W(p(s))>0} c_{iB}(p(s))f_{iB}(p(s)) \sum_{z\in\mathcal{Z}(p(s))} U(z)$$
$$= \sum_{s\in\mathcal{Y}(\mathcal{P})}^{W(p(s))>0} c_{iB}(p(s))f_{iB}(p(s))W(p(s)). \tag{14}$$

The first equality holds by grouping those distances corresponding to the same point $p(s)$ that the base station dwells. The second equality holds by $c_{iB}(p(z)) = c_{iB}(p(s))$ and $f_{iB}(p(z)) = f_{iB}(p(s))$ for each $z\in\mathcal{Z}(p(s))$. The last equality holds by (4).

Following the same token, for the integration (second term) on the LHS of (13), it can be shown that

$$\int_{s\in[0,S]}^{U(s)=0} c_{iB}(p(s))f_{iB}(p(s))u(s)ds$$
$$= \int_{s\in\mathcal{Y}(\mathcal{P})}^{W(p(s))=0} c_{iB}(p(s))f_{iB}(p(s))w(p(s))ds. \tag{15}$$

Thus by (14) and (15), we have

LHS of (13)
$$= \sum_{s\in\mathcal{Y}(\mathcal{P})}^{W(p(s))>0} c_{iB}(p(s))f_{iB}(p(s))W(p(s))$$
$$+ \int_{s\in\mathcal{Y}(\mathcal{P})}^{W(p(s))=0} c_{iB}(p(s))f_{iB}(p(s))w(p(s))ds$$
$$= \sum_{s\in\mathcal{Y}(\mathcal{P})}^{W(p(s))>0} c_{iB}(p(s))\frac{\int_{t\in[0,T]}^{(x,y)(t)=p(s)} g_{iB}(t)dt}{W(p(s))}W(p(s))$$
$$+ \int_{s\in\mathcal{Y}(\mathcal{P})}^{W(p(s))=0} c_{iB}(p(s))\frac{\sum_{\hat{s}\in\mathcal{Z}(p(s))} g_{iB}(t(\hat{s}))u(\hat{s})}{w(p(s))}w(p(s))ds$$
$$= \sum_{s\in\mathcal{Y}(\mathcal{P})}^{W(p(s))>0} c_{iB}(p(s)) \int_{t\in[0,T]}^{(x,y)(t)=p(s)} g_{iB}(t)dt$$
$$+ \int_{s\in\mathcal{Y}(\mathcal{P})}^{W(p(s))=0} c_{iB}(p(s)) \sum_{\hat{s}\in\mathcal{Z}(p(s))} g_{iB}(t(\hat{s}))u(\hat{s})ds \tag{16}$$

where the second equality holds by (6) and (10).

For the right-hand side (RHS) of (13), we have

$$\text{RHS of (13)} = \int_{t\in[0,T]}^{\|v(t)\|>0} c_{iB}(t)g_{iB}(t)dt$$
$$+ \int_{t\in[0,T]}^{\|v(t)\|=0} c_{iB}(t)g_{iB}(t)dt. \tag{17}$$

We now transform each of the above integrations from "t-domain" to "s-domain."

- For the case of $\|v(t)\|=0$ (i.e., the base station dwells at the current point), denote $t_1(s)$ as the time when the base station has traversed a distance s and starts this dwelling period. Denote $t_2(s)$ as the time when the base station completes this dwelling period. Then, $U(s)>0$ and for any time t during $[t_1(s),t_2(s)]$, the base station dwells and $\|v(t)\|=0$. Thus

$$\int_{t\in[0,T]}^{\|v(t)\|=0} c_{iB}(t)g_{iB}(t)dt = \sum_{s\in[0,S]}^{U(s)>0} \int_{t=t_1(s)}^{t_2(s)} c_{iB}(t)g_{iB}(t)dt$$
$$= \sum_{s\in[0,S]}^{U(s)>0} c_{iB}(p(s)) \int_{t=t_1(s)}^{t_2(s)} g_{iB}(t)dt, \tag{18}$$

  where the first equality holds as the integration $\int_{t\in[0,T]}^{\|v(t)\|=0}$ is limited to those dwelling periods, each corresponding to a distance $s\in[0,S]$ with $U(s)>0$. The second equality holds by $c_{iB}(t) = c_{iB}(p(s))$ for $t\in[t_1(s),t_2(s)]$.
- For the case of $\|v(t)\|>0$ (i.e., the base station is traversing at the current point $p(s)$), we have

$$\int_{t\in[0,T]}^{\|v(t)\|>0} c_{iB}(t)g_{iB}(t)dt$$
$$= \int_{s\in[0,S]}^{U(s)=0} c_{iB}(p(s))g_{iB}(s)\cdot\frac{1}{\|v(t(s))\|}ds$$
$$= \int_{s\in[0,S]}^{U(s)=0} c_{iB}(p(s))g_{iB}(s)u(s)ds \tag{19}$$

where $g_{iB}(s)$ is the flow rate from sensor i to the base station B when the base station is traversing at point $p(s)$. The first equality holds by $ds = \|v(t(s))\|dt$, and the second equality holds by (7).

Therefore, by (17)–(19), we have

$$\text{RHS of (13)} = \sum_{s\in[0,S]}^{U(s)>0} c_{iB}(p(s)) \int_{t=t_1(s)}^{t_2(s)} g_{iB}(t)dt + \int_{s\in[0,S]}^{U(s)=0} c_{iB}(p(s)) g_{iB}(s)u(s)ds. \quad (20)$$

For the summation (first term) on the RHS of (20), we have

$$\begin{aligned}\sum_{s\in[0,S]}^{U(s)>0} c_{iB}(p(s)) \int_{t=t_1(s)}^{t_2(s)} g_{iB}(t)dt &= \sum_{s\in\mathcal{Y}(\mathcal{P})}^{W(p(s))>0} \sum_{z\in\mathcal{Z}(p(s))}^{U(z)>0} c_{iB}(p(z)) \int_{t=t_1(z)}^{t_2(z)} g_{iB}(t)dt \\ &= \sum_{s\in\mathcal{Y}(\mathcal{P})}^{W(p(s))>0} c_{iB}(p(s)) \sum_{z\in\mathcal{Z}(p(s))}^{U(z)>0} \int_{t=t_1(z)}^{t_2(z)} g_{iB}(t)dt \\ &= \sum_{s\in\mathcal{Y}(\mathcal{P})}^{W(p(s))>0} c_{iB}(p(s)) \int_{t\in[0,T]}^{(x,y)(t)=p(s)} g_{iB}(t)dt \quad (21)\end{aligned}$$

where the first equality holds by grouping those distances corresponding to the same point $p(s)$ that the base station dwells. The second equality holds by $c_{iB}(p(z)) = c_{iB}(p(s))$ for each $z \in \mathcal{Z}(p(s))$.

Following the same token, for the integration (second term) on the RHS of (20), it can be shown that

$$\int_{s\in[0,S]}^{U(s)=0} c_{iB}(p(s)) g_{iB}(s)u(s)ds = \int_{s\in\mathcal{Y}(\mathcal{P})}^{W(p(s))=0} c_{iB}(p(s)) \sum_{\hat{s}\in\mathcal{Z}(p(s))} g_{iB}(t(\hat{s}))u(\hat{s})ds. \quad (22)$$

Therefore, by (20)–(22), we have

$$\text{RHS of (13)} = \sum_{s\in\mathcal{Y}(\mathcal{P})}^{W(p(s))>0} c_{iB}(p(s)) \int_{t\in[0,T]}^{(x,y)(t)=p(s)} g_{iB}(t)dt + \int_{s\in\mathcal{Y}(\mathcal{P})}^{W(p(s))=0} c_{iB}(p(s)) \sum_{\hat{s}\in\mathcal{Z}(p(s))} g_{iB}(t(\hat{s}))u(\hat{s})ds. \quad (23)$$

By (16) and (23), (13) is proved.

Based on our results in (i) and (ii), the constructed location-dependent solution $\bar{\varphi}$ with $\mathcal{P}, W(p)$ (and/or $w(p)$), $f_{ij}(p)$, and $f_{iB}(p)$ is feasible and has the same network lifetime T as that achieved by time-dependent solution φ. ∎

The following lemma further extends Lemma 1 and says that the ordering and specific time instances for the base station to visit a particular point p are not important.

Lemma 2: Under a location-dependent solution, as long as $W(p)$ (and/or $w(p)$) at each point p remains the same, the network lifetime T will remain unchanged regardless of the ordering and frequency of the base station's presence at each point.

Lemma 2 can be easily proved by analyzing the energy consumption behavior at each node over time T. We omit its proof here to conserve space.

Combining Lemmas 1 and 2 and considering the special case that φ is optimal, we have Theorem 1.

Based on Theorem 1, we conclude that as far as network lifetime is concerned, it is sufficient for us to study location-dependent solutions. This result allows us to develop a provably near-optimal approximation algorithm in the space domain, which we will present in the following section.

IV. A $(1-\varepsilon)$-Optimal Algorithm

Note that in the location-dependent problem formulation, there are infinite number of points in $\mathcal{P}$. In this section, we first consider the case when the base station is only allowed to be present at a finite set of M positions. We call this problem the C-MB problem. Based on this intermediate result, we devise a solution to the general problem where the base station is allowed to roam anywhere in the two-dimensional plane. We term the latter problem the U-MB problem.

A. Optimal Sojourn Time Computation for the C-MB Problem

We now show that C-MB problem can be formulated as an LP, which can be solved in polynomial time. Recall that in the C-MB problem, the location of base station is limited to a finite set of M locations $p_m, m = 1, 2, \ldots, M$. Thus, if the base station dwells at p_m at least once, then we have $W(p_m) > 0$; otherwise (i.e., base station never dwells at p_m, we have $W(p_m) = 0$. We need to find optimal $W(p_m)$ for all points and the corresponding flow routing $f_{ij}(p_m)$ and $f_{iB}(p_m)$ for each point p_m with $W(p_m) > 0$.

When the base station is at point p_m, $1 \le m \le M$, the flow conservation for node $i \in \mathcal{N}$ is

$$\sum_{k\in\mathcal{N}}^{k\neq i} f_{ki}(p_m) + r_i = \sum_{j\in\mathcal{N}}^{j\neq i} f_{ij}(p_m) + f_{iB}(p_m). \quad (24)$$

The energy constraint for node $i \in \mathcal{N}$, at the end of network lifetime T, is

$$\sum_{m=1}^{M} \left[\sum_{k\in\mathcal{N}}^{k\neq i} \rho \cdot f_{ki}(p_m) + \sum_{j\in\mathcal{N}}^{j\neq i} C_{ij} \cdot f_{ij}(p_m) + c_{iB}(p_m) \cdot f_{iB}(p_m) \right] W(p_m) \le e_i. \quad (25)$$

Note that for given i and p_m, $c_{iB}(p_m)$ is a constant.

We can formulate the C-MB problem as an LP by letting $V_{ij}(p_m) = f_{ij}(p_m)\cdot W(p_m)$ and $V_{iB}(p_m) = f_{iB}(p_m)\cdot W(p_m)$, where $V_{ij}(p_m)$ (or $V_{iB}(p_m)$) can be interpreted as the total data volume from sensor node i to sensor node j (or base station B) when the base station is at p_m. We have

LP(C-MB)

$$\begin{aligned}
\text{Max} \quad & T \\
\text{s.t.} \quad & \sum_{m=1}^{M} W(p_m) - T = 0 \\
& \sum_{k\in\mathcal{N}}^{k\neq i} V_{ki}(p_m) + r_i \cdot W(p_m) - \sum_{j\in\mathcal{N}}^{j\neq i} V_{ij}(p_m) - V_{iB}(p_m) = 0 \\
& \qquad (i \in \mathcal{N}, 1 \leq m \leq M) \qquad (26) \\
& \sum_{m=1}^{M} \left[\sum_{k\in\mathcal{N}}^{k\neq i} \rho \cdot V_{ki}(p_m) + \sum_{j\in\mathcal{N}}^{j\neq i} C_{ij} \cdot V_{ij}(p_m) + c_{iB}(p_m) \cdot V_{iB}(p_m) \right] \leq e_i \qquad (i \in \mathcal{N}) \\
& T, W(p_m), V_{ij}(p_m), V_{iB}(p_m) \geq 0 \\
& \qquad (i, j \in \mathcal{N}, i \neq j, 1 \leq m \leq M) \qquad (27)
\end{aligned}$$

where (26) and (27) follow from (24) and (25), respectively. Once we solve the above LP, we have $W(p_m)$ for $1 \leq m \leq M$. For each point p_m with $W(p_m) > 0$, we can obtain $f_{ij}(p_m)$ and $f_{iB}(p_m)$ by $f_{ij}(p_m) = \frac{V_{ij}(p_m)}{W(p_m)}$ and $f_{iB}(p_m) = \frac{V_{iB}(p_m)}{W(p_m)}$. Recall that for those points with $W(p_m) = 0$, it means that the base station will not visit those points in this solution.

We summarize the result in this section with the following proposition.

Proposition 1: The C-MB problem can be solved via a single LP in polynomial time.

The solution to the above LP problem yields the sojourn time for the base station at each location p_m, $m = 1, 2, \ldots, M$, and the optimal flow routing when the base station is present at p_m. So far, we assume that base station B can move from one point to another in zero time. We will discuss how to relax this assumption in Section IV-E.

B. *From Infinite to Finite Locations*

We now show how to convert a U-MB problem to a C-MB problem with $(1 - \varepsilon)$ network lifetime performance guarantee. Our approach is to exploit the energy cost function and how the location of the base station affects the energy cost. Note that the location of the base station is *embedded* in the cost parameter c_{iB}, and these cost parameters directly affect network lifetime. Thus, to design a $(1-\varepsilon)$ optimal algorithm, we consider dividing disk $\mathcal{A}$ into subareas, with each subarea to be associated with some nice properties on c_{iB}'s that can be used to prove $(1 - \varepsilon)$ optimality. The key idea is to discretize the energy cost through a geometric sequence [16].

Under the U-MB problem, denote $O_{\mathcal{A}}$ and $R_{\mathcal{A}}$ as the origin and radius of the SED $\mathcal{A}$ (see Fig. 1). For each sensor node $i \in \mathcal{N}$, denote $D_{i,O_{\mathcal{A}}}$ as the distance from sensor node i to the origin of disk $\mathcal{A}$. Denote $D_{iB}^{\min}$ and $D_{iB}^{\max}$ as the minimum and maximum possible distance between sensor node i and base station B, respectively; denote $C_{iB}^{\min}$ and $C_{iB}^{\max}$ as the corresponding minimum and maximum cost between sensor node i and base station B, respectively. Then, since the movement region for base station B is within disk $\mathcal{A}$[17], we have

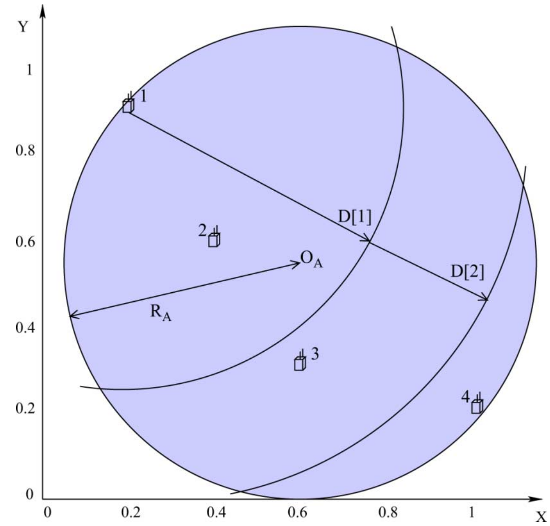

Fig. 1. Example four-node network that shows a sequence of circles centered at node 1 with increasing costs.

$$D_{iB}^{\min} = 0$$
$$D_{iB}^{\max} = D_{i,O_{\mathcal{A}}} + R_{\mathcal{A}}.$$

By (2), we have

$$C_{iB}^{\min} = \alpha \qquad (28)$$
$$C_{iB}^{\max} = \alpha + \beta \cdot (D_{iB}^{\max})^n = \alpha + \beta \cdot (D_{i,O_{\mathcal{A}}} + R_{\mathcal{A}})^n. \qquad (29)$$

Given $d_{iB} \in [D_{iB}^{\min}, D_{iB}^{\max}]$, for each sensor node $i \in \mathcal{N}$, we now show how to divide disk $\mathcal{A}$ into a set of *nonuniform* subareas with the distance of each subarea to sensor node i meeting some properties that can be used to design a $(1 - \varepsilon)$-optimal algorithm.

In the first step, we discretize the distance and energy cost following a geometric sequence with a factor of $(1 + \varepsilon)$. Specifically, for each sensor node $i \in \mathcal{N}$, we draw a sequence of circles centered at sensor node i, each with increasing radius $D[1], D[2], \ldots, D[H_i]$ (see Fig. 1) corresponding to costs $C[1], C[2], \ldots, C[H_i]$ that are defined as follows:

$$C[h] = C_{iB}^{\min}(1+\varepsilon)^h = \alpha(1+\varepsilon)^h \qquad (1 \leq h \leq H_i). \quad (30)$$

The number of required circles H_i can be determined by having the last circle in the sequence (with radius $D[H_i]$) to completely cover disk $\mathcal{A}$, i.e., $D[H_i] > D_{iB}^{\max}$, or equivalently

$$C[H_i] \geq C_{iB}^{\max}.$$

That is, we have a total of H_i circles with a common center at sensor node i, each with cost $C[h]$, $h = 1, 2, \ldots, H_i$. H_i can be easily found by the following expression:

$$H_i = \left\lceil \frac{\ln\left(C_{iB}^{\max}/C_{iB}^{\min}\right)}{\ln(1+\varepsilon)} \right\rceil = O\left(\left\lceil \frac{1}{\varepsilon} \right\rceil\right) = O\left(\frac{1}{\varepsilon}\right) \qquad (31)$$

where the second equality holds by $\ln(1+\varepsilon) \approx \varepsilon$ for small ε and $\ln(C_{iB}^{\max}/C_{iB}^{\min})$ is a constant. These H_i circles provide H_i nonoverlapping rings. Now suppose base station B is moved

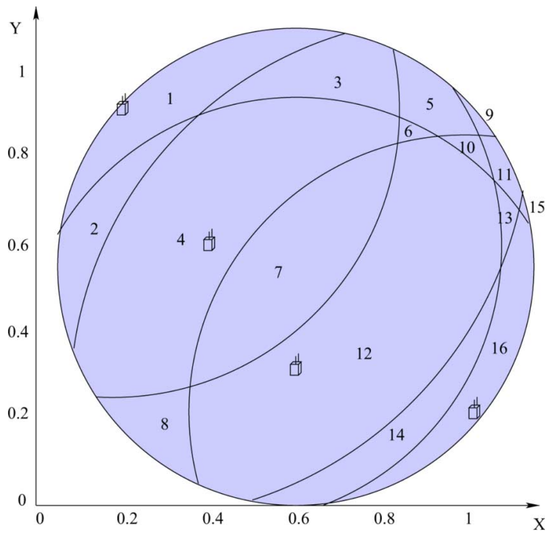

Fig. 2. Illustration of subareas for the example four-node sensor network.

to any point between the $(h-1)$th circle and the hth circle, $h = 1, 2, \ldots, H_i$. Then, we have

$$C[h-1] \le c_{iB} \le C[h], \tag{32}$$

where we define $C[0] = C_{iB}^{\min} = \alpha$.

In the second step, we divide disk $\mathcal{A}$ into subareas by performing the above process for each sensor node $i \in \mathcal{N}$. The intersecting circles will divide disk $\mathcal{A}$ into a number of irregular subareas, with the boundaries of each subarea being either an arc centered at some sensor node $i \in \mathcal{N}$ (with some cost $C[h]$, $1 \le h < H_i$) or an arc of disk $\mathcal{A}$. As an example, disk $\mathcal{A}$ in Fig. 2 is now divided into 16 irregular subareas.

We now show that for a point in each of these subareas, its cost to each sensor node can be tightly bounded from both above and below. As a result, this property can be exploited in the design of a $(1-\varepsilon)$-optimal algorithm. Note that for each sensor node $i \in \mathcal{N}$, any subarea $\mathcal{A}_m$ must be within a ring with its center at sensor node i. Denote the index of this ring as $h_i(\mathcal{A}_m)$. That is, when the base station B is at any point $p \in \mathcal{A}_m$, we have

$$C\left[h_i(\mathcal{A}_m) - 1\right] \le c_{iB}(p) \le C\left[h_i(\mathcal{A}_m)\right] \tag{33}$$

by (32).

Since $\frac{C[h_i(\mathcal{A}_m)]}{C[h_i(\mathcal{A}_m)-1]} = 1 + \varepsilon$ by (30), these two bounds for $c_{iB}(p)$ are very tight.

In the third step, we introduce the notion of FCP to represent each subarea in disk $\mathcal{A}$. That is, each subarea $\mathcal{A}_m$, $m = 1, 2, \ldots, M$, will be represented by an FCP p_m, $m = 1, 2, \ldots, M$, which is an N-tuple vector embodying an upper bound of cost for any point within this subarea $\mathcal{A}_m$ to all the N sensor nodes in the network. Specifically, denote the N-tuple cost vector for FCP p_m as $[c_{1B}(p_m), c_{2B}(p_m), \ldots, c_{NB}(p_m)]$, with the ith component $c_{iB}(p_m)$ being

$$c_{iB}(p_m) = C[h_i(\mathcal{A}_m)] \tag{34}$$

where $h_i(\mathcal{A}_m)$ is determined by (33).

As an example, the FCP p_1 for subarea 1 in Fig. 2 can be represented by a 4-tuple cost vector $[c_{1B}(p_1), c_{2B}(p_1), c_{3B}(p_1), c_{4B}(p_1)] = [C[1], C[1], C[2], C[3]]$, where the first component $C[1]$ represents an upper bound of cost for any point in this subarea to sensor node 1, the second component $C[1]$ represents an upper bound of cost for any point in this subarea to sensor node 2, and so forth.

In our design, we use the word "fictitious" to suggest that points p_m, $m = 1, 2, \ldots, M$, may only be used as a bound for the purpose of developing the $(1-\varepsilon)$-optimal algorithm. In reality, p_m may not be mapped to any *physical* point within subarea $\mathcal{A}_m$. This occurs when there is no *physical* point in subarea $\mathcal{A}_m$ that has its costs to all the N sensor nodes equal (one by one) to the respective N-tuple cost vector embodied by p_m *simultaneously*. As an example, there does not exist a physical point within subarea 1 that has its costs to the four sensor nodes being the same as those elements in the 4-tuple vector embodied by p_1. We have the following property for an FCP.

Property 1: Denote p_m as the FCP for $\mathcal{A}_m$, $m = 1, 2, \ldots, M$. Then, for any physical point $p \in \mathcal{A}_m$, we have

$$c_{iB}(p) \le c_{iB}(p_m) \le (1+\varepsilon) \cdot c_{iB}(p).$$

Proof: By (33) and the definition of FCP p_m [see (34)], we have $c_{iB}(p) \le c_{iB}(p_m)$. Furthermore, we have

$$\begin{aligned} c_{iB}(p_m) &= C[h_i(\mathcal{A}_m)] = (1+\varepsilon) \cdot C[h_i(\mathcal{A}_m) - 1] \\ &\le (1+\varepsilon) \cdot c_{iB}(p) \end{aligned}$$

where the last inequality follows from (33). ■

Now, the set of M nonuniform subareas are represented by the M FCPs, with each FCP having an N-tuple cost vector to all the N sensor nodes in the network. We will show that these FCPs will facilitate the design of a $(1-\varepsilon)$-optimal algorithm. Note that for the network lifetime problem, we only need to consider the cost terms c_{iB} for $i = 1, 2, \ldots, N$, which is precisely captured by the N-tuple representation for each FCP. As a result, we can readily apply the LP approach discussed in Section IV-A to formulate an optimization problem on these M FCPs. In the next section, we will show how to construct a $(1-\varepsilon)$-optimal solution to the U-MB problem by solving the C-MB problem on the FCPs.

C. Solution to the U-MB Problem and Proof of $(1-\varepsilon)$ *Optimality*

Denote $\psi^*_{\text{U-MB}}$ as an optimal solution to the U-MB problem and $T^*_{\text{U-MB}}$ as the maximum network lifetime, both of which are unknown. Our objective is to find a solution to the U-MB problem that has provable $(1-\varepsilon)$-optimal network lifetime. Denote $\psi^*_{\text{C-MB}}$ as an optimal solution to the C-MB problem obtained by applying an LP on the M FCP p_m, $m = 1, 2, \ldots, M$, and $T^*_{\text{C-MB}}$ as the corresponding network lifetime.

Our roadmap to construct a solution to the U-MB problem and to prove its $(1-\varepsilon)$ optimality is as follows. In Theorem 2, we will prove that $T^*_{\text{C-MB}} \ge (1-\varepsilon)T^*_{\text{U-MB}}$ (see Fig. 3). Since the optimal solution $\psi^*_{\text{C-MB}}$ corresponding to $T^*_{\text{C-MB}}$ is based on the M FCPs instead of physical points, in Theorem 3 we will further show how to construct a solution $\psi_{\text{U-MB}}$ to the U-MB

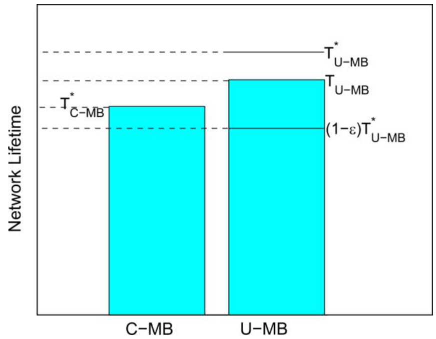

Fig. 3. Comparison of network lifetimes under different solutions that are used to construct a $(1-\varepsilon)$ optimal solution.

problem based on $\psi^*_{\text{C-MB}}$ and prove that the corresponding network lifetime is $(1-\varepsilon)$-optimal, i.e., $T_{\text{U-MB}} \geq (1-\varepsilon)T^*_{\text{U-MB}}$ (see Fig. 3).

Theorem 2: For a given $\varepsilon > 0$, define subareas $\mathcal{A}_m$ and FCPs p_m, $m = 1, 2, \ldots, M$, as in Section IV-B. Then, we have $T^*_{\text{C-MB}} \geq (1-\varepsilon) \cdot T^*_{\text{U-MB}}$.

To prove Theorem 2, we need the following lemma.

Lemma 3: Given a feasible solution $\pi_{\text{U-MB}}$ to the U-MB problem with a network lifetime $T_{\text{U-MB}}$, we can construct a solution $\pi_{\text{C-MB}}$ to the C-MB problem with a network lifetime $T_{\text{C-MB}} \geq (1-\varepsilon) \cdot T_{\text{U-MB}}$.

The proof to this lemma is based on the following construction. Solution $\pi_{\text{U-MB}}$ consists of a specific path $\mathcal{P}$ for the base station, $W(p)$ (and/or $w(p)$), $f_{ij}(p)$, $f_{iB}(p)$ values for each point $p \in \mathcal{P}$, and a network lifetime $T_{\text{U-MB}}$. For each subarea $\mathcal{A}_m$, $m = 1, 2, \ldots, M$, denote $W(\mathcal{A}_m)$ as the total sojourn time during $[0, T_{\text{U-MB}}]$ when the base station B is present within this subarea. We have

$$W(\mathcal{A}_m) = \sum_{s\in\mathcal{S}_m}^{U(s)>0} U(s) + \int_{s\in\mathcal{S}_m}^{U(s)=0} u(s)ds \tag{35}$$

where $\mathcal{S}_m = \{s : p(s) \in \mathcal{A}_m, 0 \leq s \leq S\}$. To construct solution $\pi_{\text{C-MB}}$, we can let the base station stay $W(p_m)$ amount of time on FCP p_m, $m = 1, 2, \ldots, M$, where

$$W(p_m) = (1-\varepsilon) \cdot W(\mathcal{A}_m)$$

and for each point p_m with $W(p_m) > 0$, set the flow routing when the base station is at p_m as

$$f_{ij}(p_m) = \frac{1}{W(\mathcal{A}_m)} \left(\sum_{s\in\mathcal{S}_m}^{U(s)>0} f_{ij}(p(s))U(s) + \int_{s\in\mathcal{S}_m}^{U(s)=0} f_{ij}(p(s))u(s)ds \right)$$

$$f_{iB}(p_m) = \frac{1}{W(\mathcal{A}_m)} \left(\sum_{s\in\mathcal{S}_m}^{U(s)>0} f_{iB}(p(s))U(s) + \int_{s\in\mathcal{S}_m}^{U(s)=0} f_{iB}(p(s))u(s)ds \right).$$

The details of the proof of Lemma 3 are similar to the proof of Lemma 1 and are omitted to conserve space.

Lemma 3 is a powerful result. With this lemma, we are now ready to prove Theorem 2.

Proof: Consider the special case of Lemma 3 where the given solution to U-MB problem is an optimal solution $\pi^*_{\text{U-MB}}$ with network lifetime $T^*_{\text{U-MB}}$. By Lemma 3, we can transform it into a solution to the C-MB problem with network lifetime at least $(1-\varepsilon)T^*_{\text{U-MB}}$, i.e., there is a solution to the C-MB problem on the FCPs with a network lifetime at least $(1-\varepsilon)T^*_{\text{U-MB}}$. As a result, the optimal solution $\psi^*_{\text{C-MB}}$ to the C-MB problem must have a network lifetime $T^*_{\text{C-MB}} \geq (1-\varepsilon)T^*_{\text{U-MB}}$. ■

Theorem 2 guarantees that the network lifetime obtained by the LP solution based on the M FCPs is at least $(1-\varepsilon)$ of $T^*_{\text{U-MB}}$. However, an FCP may not be mapped to a physical point, which is required in the final solution. In the following theorem, we construct a solution with each point being physically realizable. Furthermore, the network lifetime for this constructed solution is greater than or equal to the maximum network lifetime for the C-MB problem, i.e., $T_{\text{U-MB}} \geq T^*_{\text{C-MB}}$. As a result, this new solution is $(1-\varepsilon)$-optimal.

Theorem 3: For a given $\varepsilon > 0$, define subareas $\mathcal{A}_m$ and FCPs p_m, $m = 1, 2, \ldots, M$, as discussed in Section IV-B. Given an optimal solution $\psi^*_{\text{C-MB}}$ on these M FCPs with $W^*(p_m)$, $f^*_{ij}(p_m)$, $f^*_{iB}(p_m)$, and a network lifetime $T^*_{\text{C-MB}}$, a $(1-\varepsilon)$-optimal solution $\psi_{\text{U-MB}}$ to U-MB problem can be constructed by having the base station stay in $\mathcal{A}_m$ for

$$W(\mathcal{A}_m) = W^*(p_m) \tag{36}$$

amount of time and by having a corresponding flow routing for any physical point $p \in \mathcal{A}_m$ as

$$f_{ij}(p) = f^*_{ij}(p_m) \tag{37}$$

$$f_{iB}(p) = f^*_{iB}(p_m). \tag{38}$$

In Theorem 3, note that in the constructed solution to the U-MB problem, as long as the base station is within $\mathcal{A}_m$ (any point in this subarea), the flow routing is the same. The proof is similar to that for Lemma 1 and thus is omitted to conserve space.

D. Summary of Algorithm and Example

The design of a $(1-\varepsilon)$-optimal algorithm is described in Sections IV-B and IV-C. We now summarize it in Algorithm 1.

In the algorithm, Step 5 has the highest complexity (solving an LP) among all steps. Since there are $(H_i - 1)$ circles radiating from sensor node $i \in \mathcal{N}$ and one circle for disk $\mathcal{A}$, the total number of subareas M obtained through the intersection of these circles is upper-bounded by $O([1 + \sum_{i=1}^{N}(H_i - 1)]^2) = O((\frac{N}{\varepsilon})^2)$. Thus, the LP in Step 5 has polynomial size, and the complexity of the above algorithm is polynomial.

Example 1: To illustrate the steps in Algorithm 1, we solve a small four-sensor network problem as an example. The location, data rate, and initial energy for each sensor are shown in Table II, where the units of distance, rate, and energy are all normalized with appropriate dimensions. We use $n = 2$ in this example and set $\alpha = 1$, $\beta = 0.5$, and $\rho = 1$ under normalized units. For illustration, we set $\varepsilon = 0.2$.[1]

[1]In Section V, we use $\varepsilon = 0.05$ for all numerical results.

TABLE II
SENSOR LOCATION, DATA RATE, AND INITIAL ENERGY OF THE EXAMPLE FOUR-NODE SENSOR NETWORK

Node Index	(x_i, y_i)	r_i	e_i
1	$(0.2, 0.9)$	0.6	170
2	$(0.4, 0.6)$	1.0	420
3	$(0.6, 0.3)$	0.8	460
4	$(1.0, 0.2)$	0.4	230

Algorithm 1: A $(1-\varepsilon)$-Optimal Algorithm

1) Within $\mathcal{A}$, compute $C_{iB}^{\min}$ and $C_{iB}^{\max}$ for each node $i \in \mathcal{N}$ by (28) and (29).
2) For a given $\varepsilon > 0$, define a sequence of costs $C[1]$, $C[2], \ldots, C[H_i]$ by (30), where H_i is defined by (31).
3) For each node $i \in \mathcal{N}$, draw a sequence of $(H_i - 1)$ circles corresponding to cost $C[h]$ centered at node i, $1 \le h < H_i$. The intersection of these circles within disk $\mathcal{A}$ will divide $\mathcal{A}$ into M subareas $\mathcal{A}_1, \mathcal{A}_2, \ldots, \mathcal{A}_M$.
4) For each subarea $\mathcal{A}_m$, $1 \le m \le M$, define an FCP p_m, which is represented by an N-tuple cost vector $[c_{1B}(p_m), c_{2B}(p_m), \ldots, c_{NB}(p_m)]$, where $c_{iB}(p_m)$ is defined by (34).
5) For the C-MB problem on these M FCPs, apply the LP formulation in Section IV-A and obtain an optimal solution $\psi^*_{\text{C-MB}}$ with $W^*(p_m)$, $f^*_{ij}(p_m)$, and $f^*_{iB}(p_m)$.
6) Construct a $(1-\varepsilon)$-optimal solution $\psi_{\text{U-MB}}$ to the U-MB problem based on $\psi^*_{\text{C-MB}}$ using the procedure in Theorem 3.

In Step 1, we first identify SED $\mathcal{A}$ with origin $O_\mathcal{A} = (0.60, 0.55)$ and radius $R_\mathcal{A} = 0.53$ (see Fig. 2). Then, we have $D_{1,O_\mathcal{A}} = 0.53$, $D_{2,O_\mathcal{A}} = 0.21$, $D_{3,O_\mathcal{A}} = 0.25$, and $D_{4,O_\mathcal{A}} = 0.53$. We then find the lower and upper bounds of c_{iB} for each node i as follows:

$$C_{iB}^{\min} = \alpha = 1$$
$$C_{iB}^{\max} = \alpha + \beta(D_{i,O_\mathcal{A}} + R_\mathcal{A})^n.$$

Thus, we have

$$C_{1B}^{\max} = 1 + 0.5 \cdot (0.53 + 0.53)^2 = 1.56$$
$$C_{2B}^{\max} = 1 + 0.5 \cdot (0.21 + 0.53)^2 = 1.27$$
$$C_{3B}^{\max} = 1 + 0.5 \cdot (0.25 + 0.53)^2 = 1.30$$
$$C_{4B}^{\max} = 1 + 0.5 \cdot (0.53 + 0.53)^2 = 1.56.$$

In Step 2, for $\varepsilon = 0.2$, since

$$H_i = \left\lceil \frac{\ln(1 + \frac{\beta}{\alpha}(D_{i,O_\mathcal{A}} + R_\mathcal{A})^n)}{\ln(1+\varepsilon)} \right\rceil$$

we have $H_1 = 3, H_2 = 2, H_3 = 2, H_4 = 3$, and

$$C[1] = \alpha(1+\varepsilon) = 1 \cdot (1+0.2) = 1.20$$
$$C[2] = \alpha(1+\varepsilon)^2 = 1 \cdot (1+0.2)^2 = 1.44$$
$$C[3] = \alpha(1+\varepsilon)^3 = 1 \cdot (1+0.2)^3 = 1.73.$$

In Step 3, we draw a sequence of circles centered at each node i, $1 \le i \le 4$, and with cost $C[h]$, $1 \le h < H_i$, to divide the SED $\mathcal{A}$ into 16 subareas $\mathcal{A}_1, \mathcal{A}_2, \ldots, \mathcal{A}_{16}$ (see Fig. 2).

In Step 4, we define an FCP p_m for each subarea $\mathcal{A}_m$, $1 \le m \le 16$. For example, for FCP p_1, we define a 4-tuple cost vector as $[c_{1B}(p_m), c_{2B}(p_m), c_{3B}(p_m), c_{4B}(p_m)] = [C[1], C[1], C[2], C[3]] = [1.20, 1.20, 1.44, 1.73]$.

In Step 5, we obtain an optimal solution $\psi^*_{\text{C-MB}}$ to C-MB problem on these 16 FCPs by the LP approach discussed in Section IV-A. We obtain the network lifetime $T^*_{\text{C-MB}} = 247.76$, $W^*(p_7) = 144.22$, $W^*(p_{12}) = 82.50$, $W^*(p_{16}) = 21.04$, and for all other 13 FCPs, we have $W^*(p_m) = 0$ (meaning the base station will not visit these 13 subareas). When the base station is at FCP p_7, the routing is $f^*_{1B}(p_7) = 0.60$, $f^*_{2B}(p_7) = 0.51$, $f^*_{23}(p_7) = 0.49$, $f^*_{3B}(p_7) = 1.29$, and $f^*_{4B}(p_7) = 0.40$. When the base station is at FCP p_{12}, the routing is $f^*_{12}(p_{12}) = 0.60$, $f^*_{2B}(p_{12}) = 1.60$, $f^*_{3B}(p_{12}) = 0.80$, and $f^*_{4B}(p_{12}) = 0.40$. When the base station is at FCP p_{16}, the routing is $f^*_{12}(p_{16}) = 0.60$, $f^*_{23}(p_{16}) = 1.60$, $f^*_{34}(p_{16}) = 2.40$, and $f^*_{4B}(p_{16}) = 2.80$.

In Step 6, we obtain a $(1-\varepsilon)$-optimal solution $\psi_{\text{U-MB}}$ to the U-MB problem as follows. Let the base station stay at any point in subarea $\mathcal{A}_7$ for 144.22 units of time, stay at any point in subarea $\mathcal{A}_{12}$ for 82.50 units of time, and stay at any point in subarea $\mathcal{A}_{16}$ for 21.04 units of time. When the base station is at a point p in subarea $\mathcal{A}_7$, the routing is $f_{1B}(p) = 0.60$, $f_{2B}(p) = 0.51$, $f_{23}(p) = 0.49$, $f_{3B}(p) = 1.29$, and $f_{4B}(p) = 0.40$. When the base station is at a point p in subarea $\mathcal{A}_{12}$, the routing is $f_{12}(p) = 0.60$, $f_{2B}(p) = 1.60$, $f_{3B}(p) = 0.80$, and $f_{4B}(p) = 0.40$. When the base station is at a point p in subarea $\mathcal{A}_{16}$, the routing is $f_{12}(p) = 0.60$, $f_{23}(p) = 1.60$, $f_{34}(p) = 2.40$, and $f_{4B}(p) = 2.80$. The network lifetime for $\psi_{\text{U-MB}}$ is greater than or equal to 247.76 and is $(1-\varepsilon)$ optimal.

E. Discussions

We now discuss the design of a path $\mathcal{P}$ based on $W^*(p_m)$ values. Such a path is certainly not unique. In Example 1, the base station can move from subarea 7 to 12 and to 16 (denote as $(\mathcal{A}_7, \mathcal{A}_{12}, \mathcal{A}_{16})$), or, another path can be $(\mathcal{A}_{16}, \mathcal{A}_{12}, \mathcal{A}_7)$. Note that any path, as long as the total sojourn time at each subarea $\mathcal{A}_m$ is $W^*(p_m)$, the achieved network lifetime is $(1-\varepsilon)$-optimal. Thus, all of these paths are equally good under network lifetime objective. It may be arguable that one path is better than another under some other objective, e.g., minimizing the total traveled distance. However, such an objective can be formulated in a separate problem, and its discussion is beyond the scope of this paper. We will discuss it as a future work item in Section VII.

Along a path $\mathcal{P}$, it is possible that for one subarea and the next subarea that the base station visits are not adjacent. We argue that the traveling time between two subareas (e.g., minutes) is likely on a much smaller timescale than network lifetime (e.g., months). It can be shown that if buffering is available at sensor nodes when base station is in transition from one subarea to the next subarea, then the $(1-\varepsilon)$-optimal network lifetime can still be achieved. In this case, a node only needs to slightly delay its transmission until the base station arrives at the next subarea and then empties the buffer with a higher rate for a brief time.

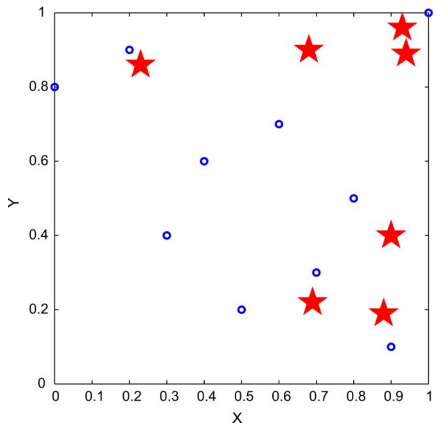

Fig. 4. Network topology and optimal locations for base station movement for a 10-node network.

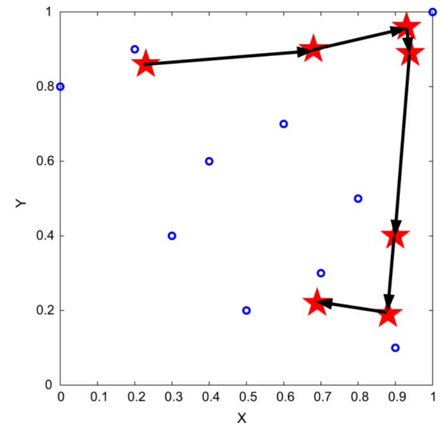

Fig. 5. Possible base station moving path for the 10-node network.

TABLE III
EACH NODE'S LOCATION, DATA GENERATION RATE, AND INITIAL ENERGY FOR A 10-NODE NETWORK

(x_i, y_i)	r_i	e_i	(x_i, y_i)	r_i	e_i
$(0.0, 0.8)$	0.8	150	$(0.6, 0.7)$	0.6	370
$(1.0, 1.0)$	1.0	200	$(0.4, 0.6)$	0.2	420
$(0.3, 0.4)$	0.6	130	$(0.2, 0.9)$	0.6	100
$(0.8, 0.5)$	0.8	460	$(0.9, 0.1)$	0.4	80
$(0.7, 0.3)$	0.3	170	$(0.5, 0.2)$	1.0	150

TABLE IV
SOJOURN TIME AT EACH OPTIMAL LOCATION FOR A 10-NODE NETWORK

$\mathcal{A}_m(x, y)$	$W(\mathcal{A}_m)$
$(0.93, 0.96)$	4.28
$(0.88, 0.19)$	3.54
$(0.68, 0.90)$	0.04
$(0.94, 0.89)$	50.36
$(0.69, 0.22)$	44.00
$(0.90, 0.40)$	27.15
$(0.23, 0.86)$	13.49

V. NUMERICAL RESULTS

Now we apply the $(1-\varepsilon)$ optimal algorithm for different-sized networks and use numerical results to demonstrate the efficacy of the algorithm. We consider four randomly generated networks consisting of 10, 20, 50, and 100 nodes deployed over a unit square area, respectively. The data rate and initial energy for each node are randomly generated between $[0.1, 1]$ and $[50, 500]$, respectively. The units of distance, rate, and energy are all normalized appropriately. The normalized parameters in the energy consumption model are $\alpha = \beta = \rho = 1$. We assume the path loss index $n = 2$ and set $\varepsilon = 0.05$.

The network setting (location, data rate, and initial energy for each node) for the 10-node network is given in Table III. By applying Algorithm 1 , we obtain a $(1-\varepsilon)$-optimal network lifetime 142.86, which is guaranteed to be at least 95% of the optimum. In Table IV, we have seven subareas that are to be visited by the base station in the $(1-\varepsilon)$-optimal solution (also shown in Fig. 4). For illustration purposes, we use a star to represent the corresponding subarea that the base station will visit in the solution. For example, we put a star on location $(0.93, 0.96)$ to represent the subarea that contains this point. Table IV also lists the corresponding sojourn time for the base station to stay in each of these seven subareas. The flow routing solution when the base station is in each of the seven subareas is different as expected. Fig. 5 shows a possible path for the 10-node network. Note that, as we discussed in Section IV-E, such a path is not unique.

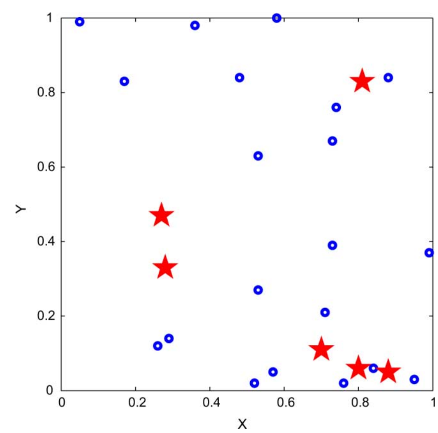

Fig. 6. Network topology and optimal locations for base station movement for a 20-node network.

TABLE V
EACH NODE'S LOCATION, DATA GENERATION RATE, AND INITIAL ENERGY FOR A 20-NODE NETWORK

(x_i, y_i)	r_i	e_i	(x_i, y_i)	r_i	e_i
$(0.52, 0.02)$	0.6	480	$(0.29, 0.14)$	0.6	120
$(0.74, 0.76)$	0.3	310	$(0.05, 0.99)$	0.4	60
$(0.95, 0.03)$	0.8	150	$(0.84, 0.06)$	1.0	180
$(0.53, 0.63)$	0.6	220	$(0.99, 0.37)$	0.4	340
$(0.58, 1.00)$	0.4	230	$(0.73, 0.67)$	0.8	220
$(0.48, 0.84)$	0.7	160	$(0.53, 0.27)$	0.5	380
$(0.17, 0.83)$	0.1	380	$(0.57, 0.05)$	0.7	250
$(0.73, 0.39)$	0.1	500	$(0.88, 0.84)$	0.2	240
$(0.36, 0.98)$	0.1	430	$(0.26, 0.12)$	0.9	440
$(0.76, 0.02)$	0.7	500	$(0.71, 0.21)$	0.3	70

It is worth noting that for 95% optimality, only seven subareas need to be visited by the base station. It turns out that for 20-, 50-, and 100-node networks, the number of subareas that need to be visited by the base station is also very small (6 subareas for 20-node network, 8 subareas for 50-node network, and 12 subareas for 100-node network). This new observation is not obvious. However, it is a good news as it hints that the base

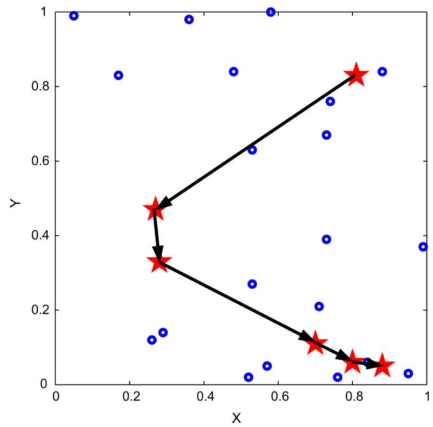

Fig. 7. Possible base station moving path for the 20-node network.

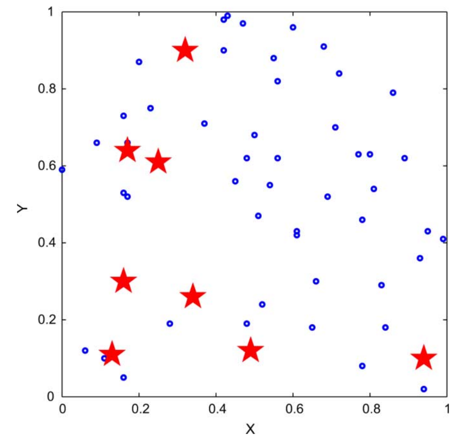

Fig. 8. Network topology and optimal locations for base station movement for a 50-node network.

TABLE VI
SOJOURN TIME AT EACH OPTIMAL LOCATION FOR A 20-NODE NETWORK

$\mathcal{A}_m(x,y)$	$W(\mathcal{A}_m)$
$(0.28, 0.33)$	2.86
$(0.27, 0.47)$	8.44
$(0.95, 0.86)$	9.62
$(0.80, 0.06)$	5.34
$(0.70, 0.11)$	108.05
$(0.88, 0.05)$	9.92

station may not need to move frequently to many different locations to achieve near-optimal solution.

The network setting for a small 20-node network (with location, data rate, and initial energy for each of the 20 sensor nodes) is given in Table V. By applying Algorithm 1, we obtain a $(1-\varepsilon)$-optimal network lifetime 144.23. Again, we use a star to represent the subarea that the base station will visit in the solution. For this particular 20-node network setting, we have six subareas (see Fig. 6) that the base station will visit in the final solution, with the corresponding sojourn time in each subarea shown in Table VI. Again, we show a possible path for the 20-node network in Fig. 7.

The network setting for the 50-node network (with location, data rate, and initial energy for each of the 50 sensor nodes) is given in Table VII. By applying Algorithm 1, we obtain a $(1-\varepsilon)$-optimal network lifetime 122.30. In Table VIII, we have eight subareas (see Fig. 8) that the base station will visit in the $(1-\varepsilon)$-optimal solution, as well as the sojourn time for the base station at each of these eight subareas. We omit to show a possible path (similar to Figs. 5 and 7) due to space limitation.

Finally, we consider a 100-node network shown in Fig. 9. We omit to list the each node's coordinates, data rate, and initial energy due to paper length limitation. They are all randomly generated as we described early in this section. By applying Algorithm 1, we obtain a $(1-\varepsilon)$-optimal network lifetime 149.45. For this particular 100-node network setting, we have 12 subareas (see Fig. 9) that the base station will visit, with the corresponding sojourn time in each subarea shown in Table IX. We omit to show a possible path (similar to Figs. 5 and 7) due to space limitation.

VI. RELATED WORK

Energy-efficient routing has been an active area of research for sensor networks in recent years (see, e.g., [14], [19], [21], and [22]). It is now well understood that energy-efficient routing differs from lifetime-optimal routing as the former advocates the use of a minimum energy-cost path, which may overload nodes along some commonly shared path, leading to poor performance in network lifetime.

TABLE VII
EACH NODE'S LOCATION, DATA GENERATION RATE, AND INITIAL ENERGY FOR A 50-NODE NETWORK

(x_i, y_i)	r_i	e_i	(x_i, y_i)	r_i	e_i
$(0.52, 0.24)$	1.0	290	$(0.80, 0.63)$	0.8	260
$(0.68, 0.91)$	0.8	160	$(0.00, 0.59)$	0.3	50
$(0.32, 0.90)$	0.4	480	$(0.81, 0.54)$	0.7	150
$(0.54, 0.55)$	0.1	500	$(0.78, 0.46)$	0.8	150
$(0.78, 0.08)$	0.1	140	$(0.84, 0.18)$	0.7	160
$(0.94, 0.02)$	0.8	300	$(0.61, 0.43)$	0.7	400
$(0.06, 0.12)$	0.1	220	$(0.11, 0.10)$	0.9	300
$(0.45, 0.56)$	0.4	370	$(0.20, 0.87)$	0.5	470
$(0.83, 0.29)$	0.4	400	$(0.16, 0.05)$	0.9	140
$(0.17, 0.52)$	0.7	160	$(0.61, 0.42)$	0.2	450
$(0.42, 0.98)$	0.8	280	$(0.72, 0.84)$	0.5	480
$(0.55, 0.88)$	0.4	320	$(0.56, 0.82)$	0.9	240
$(0.95, 0.43)$	0.6	390	$(0.42, 0.90)$	0.3	490
$(0.99, 0.41)$	0.8	180	$(0.37, 0.71)$	0.1	470
$(0.16, 0.53)$	0.8	190	$(0.71, 0.70)$	0.6	220
$(0.89, 0.62)$	0.7	340	$(0.47, 0.97)$	0.6	140
$(0.69, 0.52)$	0.7	220	$(0.16, 0.73)$	0.1	150
$(0.86, 0.79)$	0.4	50	$(0.51, 0.47)$	0.1	90
$(0.23, 0.75)$	0.6	150	$(0.77, 0.63)$	1.0	390
$(0.43, 0.99)$	0.5	290	$(0.65, 0.18)$	0.4	340
$(0.60, 0.96)$	0.3	500	$(0.48, 0.19)$	0.5	70
$(0.56, 0.62)$	0.4	420	$(0.09, 0.66)$	0.8	140
$(0.50, 0.68)$	1.0	170	$(0.48, 0.62)$	0.6	300
$(0.17, 0.66)$	1.0	250	$(0.93, 0.36)$	0.5	270
$(0.66, 0.30)$	0.1	100	$(0.28, 0.19)$	0.8	160

TABLE VIII
SOJOURN TIME AT EACH OPTIMAL LOCATION FOR A 50-NODE NETWORK.

$\mathcal{A}_m(x,y)$	$W(\mathcal{A}_m)$
$(0.13, 0.11)$	0.39
$(0.17, 0.64)$	1.47
$(0.32, 0.90)$	26.23
$(0.25, 0.61)$	27.72
$(0.49, 0.12)$	3.43
$(0.94, 0.10)$	9.66
$(0.34, 0.26)$	8.37
$(0.16, 0.30)$	45.03

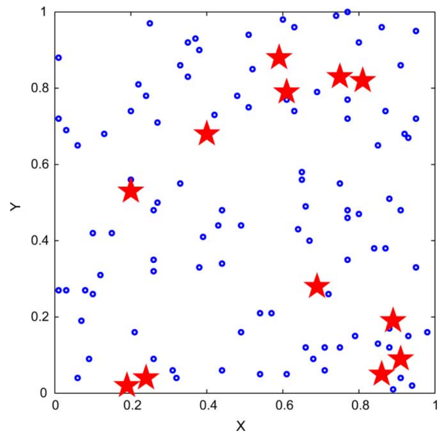

Fig. 9. Network topology and optimal locations for base station movement for a 100-node network.

TABLE IX
SOJOURN TIME AT EACH OPTIMAL LOCATION FOR A 100-NODE NETWORK

$\mathcal{A}_m(x,y)$	$W(\mathcal{A}_m)$
$(0.75, 0.83)$	0.15
$(0.86, 0.05)$	0.38
$(0.69, 0.28)$	50.14
$(0.24, 0.04)$	2.10
$(0.59, 0.88)$	0.57
$(0.81, 0.82)$	0.09
$(0.20, 0.53)$	21.21
$(0.91, 0.09)$	0.05
$(0.40, 0.68)$	41.64
$(0.89, 0.19)$	3.16
$(0.61, 0.79)$	23.82
$(0.19, 0.02)$	6.14

Routing algorithms to maximize network lifetime has been an active area of research even for a fixed base station location (see, e.g., [2], [3], [15], and references therein). The focus was mainly devoted to how to split traffic flow along different routes and how to adjust the power level at each node so that some optimal flow routing topology could be set up to maximize network lifetime. These early works laid foundation on the importance of power control and flow routing topology on network lifetime performance.

There were some efforts on optimal base station placement [5], [12], [16]. The focus of these efforts was to find an optimal *fixed* position for the base station so that network lifetime could be maximized. However, as pointed out in [9] and [25], network lifetime can be substantially increased if the optimization problem can be expanded to allow the base station to move during the course of sensor network operation.

Relevant work in the area of mobile base station for network lifetime problems includes [1], [6], [9]–[11], and [25]. In [1], [6], and [10], the locations of base stations were constrained on a set of "predetermined" locations. In [25], Younis *et al.* showed that a mobile base station could increase network lifetime. In [9], Luo and Hubaux proposed to minimize the maximum load on a node among all the nodes in the network, which could be considered as an equivalent problem to maximize network lifetime. The results in [9] and [25] were *heuristic*, and thus did not provide any theoretical bound on network lifetime performance. In [11], Luo and Hubaux extended the algorithm in [10] to remove the limitation that the locations of base stations were constrained on a set of predetermined locations. They proved that the ratio between the network lifetime achieved by their algorithm and the maximum network lifetime has a lower bound $\frac{(1-\epsilon)^2}{(1+\kappa)(3+\omega)}$, where $0 < \kappa \ll 1$, $0 < \epsilon \ll 1$ and $\omega > 0$.

Note that the mobile base station problem considered in this paper differs fundamentally from the so-called delay-tolerant network (DTN) (see, e.g., [8] and [26]). It was assumed that a DTN would experience frequent network disconnectivity and long delays. The focus was to leverage storage at intermediate nodes over a long period of time and to perform intermittent routing "over time" (i.e., delay-tolerant) so as to achieve "eventual delivery." Network lifetime is not a major performance objective in the context of DTN.

VII. CONCLUSION

This paper offered a theoretical study on how to exploit a mobile base station to prolong sensor network lifetime. Since this problem involves variables from both spatial and time domains, there were very few theoretical results available before this paper. This paper filled this theoretical gap by contributing a provably near-optimal algorithm regarding mobile base stations. We first showed a novel time-to-space transformation, which allowed us to study the problem only in the space domain instead of both time and space domains. Then, we showed that for $(1-\varepsilon)$-optimality, the infinite search space can be discretized into a finite set of subareas, with each subarea being represented by a fictitious point. Subsequently, we solved the mobile base station problem via a single LP. More importantly, we proved that the approximation algorithm can guarantee a network lifetime at least $(1-\varepsilon)$ of the maximum network lifetime (unknown), where ε can be made arbitrarily small.

Naturally, there are still many problems that deserve further research. Here, we only list one and leave out others due to space limitation. Following the results in this paper, one would be interested in looking for an optimal path $\mathcal{P}$ based on the $W^*(p_m)$ values. As discussed in Section IV-E, the optimality for such a path needs to be clearly defined. One definition could be the minimum distance traversing all subareas. Another definition, which is more interesting, is to minimize the maximum distance connecting two subsequent subareas along the path. This is important as such a path would minimize the travel time between subareas, and thus buffer size at a sensor node (see discussion in Section IV-E).

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