# On the Throughput of MIMO-Empowered Multi-hop Cognitive Radio Networks

Cunhao Gao, *Student Member, IEEE,* Yi Shi, *Member, IEEE,* Y. Thomas Hou, *Senior Member, IEEE,* and Sastry Kompella, *Member, IEEE*

**Abstract**—Cognitive radio (CR) and multiple-input multiple-output (MIMO) are two independent physical layer technologies that have made significant impact on wireless networking. CR operates on the channel/band level to exploit white space across spectrum dimension while MIMO operates within the same channel to improve spectral efficiency within the same band. In this paper, we explore MIMO-empowered CR network, which we call $\text{CRN}^{\text{MIMO}}$, to achieve the ultimate flexibility and efficiency in dynamic spectrum access and spectrum utilization. Given that CR and MIMO handle interference at different levels (across channels vs. within a channel), we are interested in how to jointly optimize both so as to maximize user throughput in a multi-hop network. To answer this question, we develop a tractable mathematical model for $\text{CRN}^{\text{MIMO}}$, which captures the essence of channel assignment (for CR) and degree-of-freedom (DoF) allocation (for MIMO) within a channel. Based on this mathematical model, we use numerical results to show how channel assignment in CRN and DoF allocation in MIMO can be jointly optimized to maximize throughput. More important, for a $\text{CRN}^{\text{MIMO}}$ with $A_{\text{MIMO}}$ antennas at each node, we show that *joint optimization of CR and MIMO offers more than $A_{\text{MIMO}}$-fold throughput increase than a CRN (without MIMO)*.

**Index Terms**—Cognitive radio networks, MIMO, multi-hop ad hoc network, optimization, throughput.

## 1 Introduction

Since its inception, cognitive radio (CR) has quickly been accepted as the enabling radio technology for next-generation wireless communications [9], [32]. A CR promises unprecedented flexibility in radio functionalities via programmability at the lowest layer, which was once done in hardware. Due to its spectrum sensing, learning, and adaptation capabilities, CR is able to address the heart of the problem associated with spectrum scarcity (via dynamic spectrum access (DSA)) and interoperability (via channel switching). Already, CR (or its predecessor, software defined radio) has been implemented for cellular communications [30], the military [11], and public safety communications [20]. It is envisioned that CR will be employed as a general radio platform upon which numerous wireless applications can be implemented.

In parallel to the development of CR for DSA, MIMO [2], [29] has widely been implemented and deployed in commercial wireless products to increase throughput. To date, the research and development of MIMO are largely independent and orthogonal to CR. Instead of exploiting idle channels for wireless communications, MIMO attempts to increase throughput within the same channel via space-time processing [6]. In particular, by employing multiple antennas on both the transmitting and receiving nodes, wireless channel capacity can scale almost linearly with the number of antennas (via spatial multiplexing) [5], [27]. Further, with zero-forcing beamforming (ZFBF) [3], [31], a node may use its degrees of freedom (DoFs) to mitigate interference from other nodes or its own interference to other nodes.

Currently, the advances of CR (see, e.g., [8], [10], [16], [17], [18], [21], [23]) and MIMO (see, e.g., [1], [4], [7], [12], [13], [14], [15], [22], [26], [28]) are largely independent and parallel to each other. Recognizing the joint potential of CR (across spectrum bands) and MIMO (within the same spectrum band), S. Haykin pointed out that *"... it seems logical to explore building the MIMO antenna architecture in the design of cognitive radio. The end-result is a cognitive MIMO radio that offers the ultimate in flexibility"* [9]. Assuming that CR and MIMO will ultimately marry each other and offer the ultimate flexibility in DSA and spectral efficiency, we would like to inquire the potential throughput gain in this marriage. In particular, we are interested in how such marriage will affect the throughput of each user communication session in a *multi-hop* CR network (or CRN), where a user communication session is defined as an information flow from a source node to its destination node (likely via multi-hop). If we assume that each node in a CRN is equipped with $A_{\text{MIMO}}$ antennas, then one would expect at least $A_{\text{MIMO}}$-fold throughput increase when compared to a CRN with only a single antenna at each node, due to spatial multiplexing gain from MIMO. Now observing that CR and MIMO handle interference differently (with

- *This work was performed while C. Gao was with the Bradley Department of Electrical and Computer Engineering, Virginia Polytechnic Institute and State University. He is current with the Department of Electrical and Computer Engineering at Stony Brook University, Stony Brook, NY 11794. E-mail: cunhaogao@gmail.com.*
- *Y. Shi and Y.T. Hou are with the Bradley Department of Electrical and Computer Engineering, Virginia Polytechnic Institute and State University, Blacksburg, VA, 24061. E-mail: {yshi,thou}@vt.edu.*
- *S. Kompella is with the Information Technology Division, U.S. Naval Research Laboratory, Washington, DC, 20375. E-mail: sastry.kompella@nrl.navy.mil.*

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14. ABSTRACT

**Cognitive radio (CR) and multiple-input multiple-output (MIMO) are two independent physical layer technologies that have made significant impact on wireless networking. CR operates on the channel/band level to exploit white space across spectrum dimension while MIMO operates within the same channel to improve spectral efficiency within the same band. In this paper, we explore MIMO-empowered CR network, which we call CRNMIMO, to achieve the ultimate flexibility and efficiency in dynamic spectrum access and spectrum utilization. Given that CR and MIMO handle interference at different levels (across channels vs. within a channel), we are interested in how to jointly optimize both so as to maximize user throughput in a multi-hop network. To answer this question, we develop a tractable mathematical model for CRNMIMO, which captures the essence of channel assignment (for CR) and degree-of-freedom (DoF) allocation (for MIMO) within a channel. Based on this mathematical model, we use numerical results to show how channel assignment in CRN and DoF allocation in MIMO can be jointly optimized to maximize throughput. More important, for a CRNMIMO with AMIMO antennas at each node, we show that joint optimization of CR and MIMO offers more than AMIMO-fold throughput increase than a CRN (without MIMO).**

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CR on the channel level and MIMO within a channel), we ask the following fundamental question: *Will joint optimization of CR (via channel assignment) and MIMO (via DoF allocation) offers more than $A_{\text{MIMO}}$-fold throughput increase?*

In this paper, we investigate this fundamental problem. The answer to this question is important as it will show whether or not joint optimization of both technologies is necessary, given that one already can achieve $A_{\text{MIMO}}$-fold throughput increase by employing MIMO's spatial multiplexing capability. We consider a multi-hop MIMO-empowered CRN, which we call CRN$^{\text{MIMO}}$. We develop a tractable mathematical model for CRN$^{\text{MIMO}}$, which captures the essence of channel assignment (for CR) and DoF allocation (for MIMO). We formulate this joint optimization problem into a mathematical program with the goal of maximizing the minimum throughput among user sessions. Based on this mathematical model, we use numerical results to show how channel assignment in CRN and DoF allocation in MIMO can be jointly optimized to maximize throughput. We show that the joint optimization of CR and MIMO can indeed offer *more than* $A_{\text{MIMO}}$-fold throughput increase than a CRN without MIMO.

The remainder of this paper is organized as follows. In Section 2, we offer some basic understanding of CR and MIMO, thus laying the foundation for mathematical modeling. In Section 3, we present mathematical models for joint optimization of CR and MIMO. In Section 4, we present numerical results and validate the throughput increase with joint optimization. Section 5 concludes this paper.

## 2 UNDERSTANDING CRN$^{\text{MIMO}}$

In this section, we review some important characteristics associated with MIMO-empowered CRN, or CRN$^{\text{MIMO}}$. Our discussion is organized into the following two levels. The first is on channel level, i.e., how does a CRN exploit available spectrum and handle interference via the use of different channels. The second is within a channel, i.e., how does MIMO mitigate co-channel interference via ZFBF.

### 2.1 Transmission/Reception and Interference in a CRN

We consider a multi-hop CR ad hoc network with multiple antennas at each node. Each CR node is able to sense its environment and identify a set of available frequency bands for wireless communications. In general, the set of available frequency bands at a node may be different from those at another node in the network [10], [23].

For a node to communicate with another node, these two nodes must have at least one available band in common. For example, in Fig. 1, suppose node $1$ has available bands $\{a, b\}$ and node $2$ has available bands $\{a, c\}$ and they are within each other's transmission range. When node $1$ wants to transmit to node $2$, only band $a$ may be used. Neither bands $b$ nor $c$ will be useful since neither of them is a common band between the two nodes.

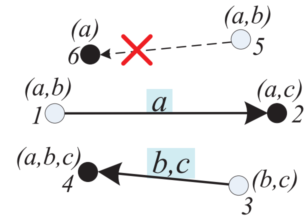



Fig. 1. Transmission/reception and interference among the nodes.

The common band condition for successful transmission/reception between two neighboring nodes also extends to interference relationship in a CRN. Given that each node may have a different set of available bands, a link can interfere another link only if these two links operate on the same band. For example, in Fig. 1, we have three links $1 \to 2$, $3 \to 4$, and $5 \to 6$, with usable bands on each link being $\{a\}$, $\{b, c\}$, $\{a\}$, respectively. Links $1 \to 2$ and $3 \to 4$ can be active at the same time since they operate on different bands (no interference). However, link $5 \to 6$ cannot be active if link $1 \to 2$ is active on band $a$, since the transmission of node $1$ interferes the reception at node $6$.

### 2.2 Co-Channel Interference Cancellation with MIMO DoFs

Complementary to a CR's ability to handle interference at the channel level, MIMO can further mitigate potential interference within a channel. The total number of antennas at a node is called *degrees of freedom* (or DoFs) [19] at the node. A node can use some or all of its DoFs for either spatial multiplexing (to achieve multiple concurrent data streams over a link) or co-channel interference cancellation (to enable multiple links on the same band), as long as the number of DoFs being used does not exceed the number of antennas at the node.

The allocation of DoFs at a node for data transmission or interference cancellation depends on how the nodes in the network are "ordered" [24], [25]. For a given ordered node list, the DoFs at a node can be used as follows.

- *Transmitting Node Behavior.* A transmitting node only needs to ensure that its transmissions do not interfere with those receiving nodes that are before this node in the ordered list. This transmitting node does not need to expend precious DoF resources to null its interference to those receiving nodes that are after this node in the ordered list. Interference from this transmitting node to those receiving nodes will be suppressed by those receiving nodes later.
  In particular, for data transmission, the number of DoFs to be used equals to the number of data streams to be transmitted. To cancel its interference to those receiving nodes that are before this node in the ordered list, this transmitting node needs to use a number of DoFs that is equal to the received data streams by those nodes.
- *Receiving Node Behavior.* A receiving node only needs to suppress interference from those transmit-

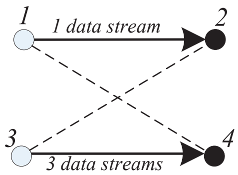



Fig. 2. An example illustrating DoF allocation in MIMO.

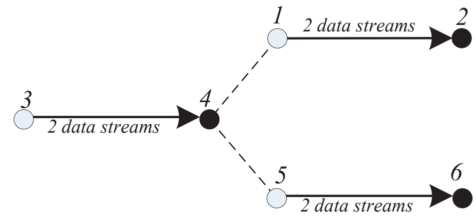



Fig. 3. A 6-node 3-link example illustrating the importance of node ordering in DoF allocation.

ting nodes that are before this node in the ordered list. It does not need to concern itself with interfering transmitting nodes that are after this node in the ordered list. Interference from those transmitting nodes will be nulled by those nodes later.

In particular, for data reception, the number of DoFs to be used equals to the number of data streams to be received. To cancel the interference from these transmitting nodes that are before this node in the ordered list, this node needs to use a number of DoFs that is equal to the transmitted data streams by those nodes.

An example is given in Fig. 2, where there are four nodes, each equipped with four antennas. All nodes operate on the same band and there are two mutually interfering links in the network: $1 \rightarrow 2$ and $3 \rightarrow 4$. Suppose the ordered node list is 1, 2, 3, and 4. Further, node 1 is transmitting to node 2 with 1 data stream. Now we show how the DoFs at each node are used for interference cancellation and spatial multiplexing.

- Starting with node 1, it is the first node in the list and it is a transmitting node. Then it uses 1 DoF for its transmission of 1 data stream. It does not need to use any DoF to cancel potential interference to other receiving nodes that are after itself in the ordered node list.
- The next node in the list is node 2. As a receiving node, it uses 1 DoF for receiving 1 data stream from node 1. It does not need to consider allocating any DoF to cancel interference from other transmitting nodes that are after itself in the ordered node list.
- The next node in the list is node 3. As a transmitting node, it needs to ensure that its transmission does not interfere with any receiving node before itself in the list, i.e., node 2. Thus, node 3 uses 1 DoF (equals to the number of received data streams by node 2) to cancel its interference to node 2. Now it has 3 remaining DoFs, which can all be used to transmit data streams (up to 3) to node 4.
- The last node in the list is node 4. As a receiving node, node 4 needs to use 3 of its DoFs for receiving 3 data streams from node 3. Node 4 also needs to use its remaining 1 DoF to cancel interference from node 1. This completes the DoF allocation at each node.

**Why Ordering Is Important** The above example for DoF allocation in Fig. 2 is for a given node order of 1, 2, 3, and 4. Now we show that the ordering of nodes in DoF allocation is important, in the sense that an ordering directly affects the final solution. This affirms that ordering should be part of the problem formulation.

The importance of node ordering for DoF allocation is best explained with an example. Consider a 6-node 3-link example in Fig. 3, where links $1 \rightarrow 2$, $3 \rightarrow 4$ and $5 \rightarrow 6$ all operate on the same band. The interference relationships are indicated as dashed lines, i.e., both nodes 1 and 5 interfere node 4. There are four antennas at each node. The goal is to transmit 2 data streams on each of these 3 links. We now show that different node ordering will lead to different result.

- **Node Order: 1, 3, 5, 4, 2 and 6.** Starting with node 1, it is the first node in the list and it is a transmitting node. Then it uses 2 DoFs to transmit 2 data streams to node 2. The next node in the list is node 3. As a transmitting node, it uses 2 DoFs to transmit 2 data streams to node 4. The next node in the list is node 5. As a transmitting node, it uses 2 DoFs to transmit 2 data streams to node 6. The next node in the list is node 4. It needs to cancel interference from transmitting nodes before itself in the list, i.e., node 1 and node 5. For each of these transmitting nodes, node 4 needs to use 2 DoFs. Now node 4 has already used up all 4 of its DoFs. But to receive 2 data streams from node 3, node 4 needs to use another 2 DoFs, which is not available. This leads to an infeasible solution.
- **Node Order: 1, 2, 4, 3, 6 and 5.** Now consider this node ordering for DoF allocation. Starting with node 1, it is a transmitting node and it uses 2 DoFs to transmit 2 data streams to node 2. The next node in the list is node 2. As a receiving node, node 2 uses 2 DoFs for receiving 2 data streams from node 1. The next node in the list is node 4. As a receiving node, node 4 needs to use 2 DoFs for receiving 2 data streams from node 3. Node 4 also needs to use its remaining 2 DoFs to cancel interference from node 1, which is before itself in the node list. The next node in the list is node 3. As a transmitting node, node 3 uses 2 DoFs to transmit 2 data streams to node 4. The next node in the list is node 6. As a receiving node, node 6 uses 2 DoFs for receiving 2 data streams from node 5. The last node in the list is node 5. As a transmitting node, node 5 needs to ensure that its transmission does not interfere with any receiving node before itself in the list, i.e., node 4. Thus node 5 uses 2 DoFs to cancel its interference on node 4. Now it has 2 remaining DoFs, which it uses to transmit 2 data streams to node 6. This

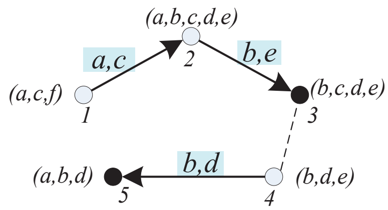



Fig. 4. An example of CRN$^{\text{MIMO}}$.

completes the DoF allocation at each node. Now we have a feasible DoF allocation under this node ordering list.

The above two examples show the importance of node ordering in DoF allocation. Consequently, such node ordering must be part of the formulation in our optimization problem.

### 2.3 An Example CRN$^{\text{MIMO}}$

We now offer an example to illustrate how interference can be jointly handled at channel level (via CR) and within a channel (via MIMO). Figure 4 shows a 5-node CRN$^{\text{MIMO}}$ with each node equipped with four antennas. The available bands at nodes 1 to 5 are $\{a, c, f\}$, $\{a, b, c, d, e\}$, $\{b, c, d, e\}$, $\{b, d, e\}$, and $\{a, b, d\}$, respectively. There are two communication sessions in the network: $1 \rightarrow 2 \rightarrow 3$ and $4 \rightarrow 5$. The usable bands on links $1 \rightarrow 2$, $2 \rightarrow 3$, and $4 \rightarrow 5$ are $\{a, c\}$, $\{b, c, d, e\}$, and $\{b, d\}$, respectively.

Suppose our objective is to maximize the minimum throughput for sessions $1 \rightarrow 2 \rightarrow 3$ and $4 \rightarrow 5$. The mathematical formulation for such type of problems will be presented in the next section. It can be shown that by solving the optimization problem, 6 data streams can be transported on each of the two sessions. An optimal band usage on each link and DoF allocation at each node is the following.

- On link $1 \rightarrow 2$, bands $a$ and $c$ are used, where on band $a$, node 1 uses 4 DoFs for transmitting 4 data streams to node 2; and on band $c$, node 1 uses 2 DoFs for transmitting 2 data streams to node 2. Correspondingly, on band $a$, node 2 uses 4 DoFs for receiving 4 data streams from node 1; and on band $c$, node 2 uses 2 DoFs for receiving 2 data streams from node 1.
- On link $2 \rightarrow 3$, bands $b$ and $e$ are used, where on band $b$, node 2 uses 2 DoFs for transmitting 2 data streams to node 3; and on band $e$, node 2 uses 4 DoFs for transmitting 4 data streams to node 3. Correspondingly, at node 3, on band $b$, 2 DoFs are used for receiving 2 data streams from node 2; and on band $e$, node 3 uses 4 DoFs for receiving 4 data streams from node 2.
- On link $4 \rightarrow 5$, bands $b$ and $d$ are used. Node 4 first uses 2 DoFs for transmitting 2 data streams to node 5 on band $b$. Since node 3 is active on band $b$ and will be interfered, and the ordered node list on band $b$ is 2, 3, 4, 5, node 4 thus uses the remaining 2 DoFs on band $b$ to cancel its interference to node 3. On band $d$, node 4 uses 4 DoFs for transmitting 4 data streams to node 5. Correspondingly, at node 5, on band $b$, 2 DoFs are used for receiving 2 data streams from node 4; and on band $d$, node 5 uses 4 DoFs for receiving 4 data streams from node 4.

TABLE 1
Notation

| Symbol | Definition |
|---|---|
| $A_i$ | The number of antennas at node $i \in \mathcal{N}$ |
| $A_{\text{MIMO}}$ | The number of antennas at each node (when each node has the same number of antennas) |
| $\mathcal{B}_i$ | The set of available bands at node $i$ |
| $\mathcal{B}_{ij}$ | The set of common available bands at nodes $i$ and $j$ |
| $c$ | The throughput when one DoF is used for data transmission on a band over a link |
| $d(q)$ | Destination node of session $q \in \mathcal{Q}$ |
| $f(q)$ | Throughput of session $q$ |
| $f_{\min}$ | The minimum throughput among all sessions |
| $g_i^b$ | A binary indicator. If node $i$ is transmitting, $g_i^b$ is 1, otherwise $g_i^b$ is 0. |
| $h_i^b$ | A binary indicator. if node $i$ is receiving, $h_i^b$ is 1, otherwise $h_i^b$ is 0. |
| $\mathcal{I}_i^b$ | The set of nodes in the interference range of node $i$ on band $b$ |
| $\mathcal{L}_{i,b}^{\text{Out}}$ | The set of outgoing links on band $b$ at node $i$ |
| $\mathcal{L}_{i,b}^{\text{In}}$ | The set of incoming links on band $b$ at node $i$ |
| $\mathcal{L}_{\text{Active}}$ | The set of links used for routing |
| $\mathcal{N}$ | The set of all nodes in the network |
| $\mathcal{Q}$ | The set of active sessions in the network |
| $\text{Rx}(l)$ | Receiving node of link $l \in \mathcal{L}_{\text{Active}}$ |
| $s(q)$ | Source node of session $q$ |
| $\text{Tx}(l)$ | Transmitting node of link $l$ |
| $z_l^b$ | The number of data streams over link $l$ on band $b$ |
| $\theta_{ji}^b$ | Binary indicator showing the relationship between nodes $i$ and $j$ in the ordered list on band $b$ |
| $\lambda_{ji}^b$ | The number of DoFs on band $b$ used by a transmitting node $i$ to cancel its interference to node $j$ |
| $\mu_{ji}^b$ | The number of DoFs on band $b$ used by a receiving node $i$ to cancel the interference from node $j$ |

It is important to realize that a node's DoFs are available for allocation on each channel. The use of DoF for interference cancellation and spatial multiplexing only has significance within the same channel. Given that there are multiple bands at each node and that there are multiple DoFs within each band, the potential optimization space for throughput is large. In fact, we shall show that the optimal objective under CRN$^{\text{MIMO}}$ is greater than $A_{\text{MIMO}}$, the number of antennas at each node (assuming same number at each node), times the optimal objective under a CRN (without MIMO). For example, it is easy to verify that for the later network (i.e., a CRN without MIMO), one of the two sessions in Fig. 4 can only have a throughput of 1 data stream. Comparing to 6 data streams under CRN$^{\text{MIMO}}$, we have 6 ($> 4$) fold increase in minimum session throughput. This result will be further discussed in Section 3 and substantiated in Section 4.

## 3 Mathematical Modeling

### 3.1 Modeling of CRN$^{\text{MIMO}}$

We consider a CRN$^{\text{MIMO}}$ consisting of a set of $\mathcal{N}$ nodes. At each node $i \in \mathcal{N}$, there is a set of $\mathcal{B}_i$ available frequency bands that can be used for communications.

As discussed, $\mathcal{B}_i$ may represent the set of bands that are unused by primary users and may be different at each node due to geographical difference. Denote the set of commonly available bands between nodes $i$ and $j$ as $\mathcal{B}_{ij} = \mathcal{B}_i \bigcap \mathcal{B}_j$. Also, denote $A_i$ as the number of antennas at node $i$. Denote $\mathcal{Q}$ the set of sessions in the network. For a session $q \in \mathcal{Q}$, denote $s(q)$ the source node, $d(q)$ the destination node, and $f(q)$ the throughput (in bps). Table 1 lists notation used in this paper.

**Scheduling Constraints.** To model the scheduling behavior of each node on a band, we use two binary variables $g_i^b$ and $h_i^b$ to indicate node $i$'s transmission/reception status on band $b$, i.e.,

$$g_i^b = \begin{cases} 1 & \text{if node } i \text{ is transmitting on band } b, \\ 0 & \text{otherwise,} \end{cases}$$

$$h_i^b = \begin{cases} 1 & \text{if node } i \text{ is receiving on band } b, \\ 0 & \text{otherwise,} \end{cases}$$

where $i \in \mathcal{N}, b \in \mathcal{B}_i$. Then the half-duplex constraint (i.e., a node cannot transmit and receive at the same time on the same band) can be represented as follows.

$$g_i^b + h_i^b \le 1 \qquad (i \in \mathcal{N}, b \in \mathcal{B}_i) \ . \tag{1}$$

**Ordering Constraints.** As discussed in Section 2.2, the DoF allocation (for transmission/reception and interference cancellation) at each node is determined sequentially based on an ordered node list. This ordering directly affects DoF allocation in the final solution and should be part of the optimization problem. To model the ordering relationship among the nodes, we define the following variable.

$$\theta_{ji}^b = \begin{cases} 1 & \text{Node } i \text{ is after node } j \text{ in the node list on band } b, \\ 0 & \text{Node } i \text{ is before node } j \text{ in the node list on band } b. \end{cases}$$

where $i, j \in \mathcal{N}, j \ne i, b \in \mathcal{B}_{ij}$. Based on the definition of $\theta$-variable, we have

$$\theta_{ji}^b + \theta_{ij}^b = 1 \quad (i, j \in \mathcal{N}, b \in \mathcal{B}_{ij}) \ . \tag{2}$$

Also, the transitivity property should hold for the $\theta$-variables. That is, for any three nodes $i, j$ and $k$ on band $b$, if node $i$ is after node $j$ and node $j$ is after node $k$ (i.e., $\theta_{ji}^b = 1$ and $\theta_{kj}^b = 1$), then node $i$ is after node $k$ (i.e., $\theta_{ki}^b = 1$). This transitivity can be formulated by the following two inequalities.

$$\theta_{kj}^b + \theta_{ji}^b - 1 \le \theta_{ki}^b \le \theta_{kj}^b + \theta_{ji}^b \ ,$$

where $i, j, k \in \mathcal{N}, b \in \mathcal{B}_i \bigcap \mathcal{B}_j \bigcap \mathcal{B}_k$. The correctness of the above two inequalities can be easily verified by trying out all possible sums of $\theta_{kj}^b$ and $\theta_{ji}^b$ and comparing with possible values of $\theta_{ki}^b$. Note that by (2), we have $\theta_{ki}^b = 1 - \theta_{ik}^b$, then the above two inequalities can be rewritten in the following form.

$$1 \le \theta_{ik}^b + \theta_{kj}^b + \theta_{ji}^b \le 2 \quad (i, j, k \in \mathcal{N}, b \in \mathcal{B}_i \textstyle\bigcap \mathcal{B}_j \bigcap \mathcal{B}_k). \tag{3}$$

**MIMO Model.** As we discussed in Section 2.2, on any given band, the total number of data streams for transmission or reception at a node is limited by its number of antennas. Denote $z_l^b$ the number of data streams over link $l$ on band $b$. Then we have the following two constraints.

$$g_i^b \le \sum_{l \in \mathcal{L}_{i,b}^{\text{Out}}} z_l^b \le g_i^b A_i \qquad (i \in \mathcal{N}, b \in \mathcal{B}_i) \ , \tag{4}$$

$$h_i^b \le \sum_{l \in \mathcal{L}_{i,b}^{\text{In}}} z_l^b \le h_i^b A_i \qquad (i \in \mathcal{N}, b \in \mathcal{B}_i) \ , \tag{5}$$

where $\mathcal{L}_{i,b}^{\text{Out}}$ and $\mathcal{L}_{i,b}^{\text{In}}$ represent the sets of outgoing and incoming links at node $i$ on band $b$, respectively.

Now we consider DoF allocation at a node, which includes DoFs used for transmission/reception and interference cancellation. For the case when node $i$ is a transmitting node, the number of required DoFs for transmission is $\sum_{l \in \mathcal{L}_{i,b}^{\text{Out}}} z_l^b$. For interference cancellation, as discussed in Section 2.2, a transmitting node needs to use its DoFs to cancel its interference to all receiving nodes before itself in the ordered node list. Denote $\mathcal{I}_i^b$ the set of nodes to which a transmission node $i$ can interfere on band $b$. Then the number of DoFs that node $i$ uses for interference cancellation can be computed as $\sum_{j \in \mathcal{I}_i^b} (\theta_{ji}^b \cdot \sum_{m \in \mathcal{L}_{j,b}^{\text{In}}}^{\text{Tx}(m) \ne i} z_m^b)$, where $\text{Tx}(m)$ is the transmitter of link $m$, the inner summation $\sum_{m \in \mathcal{L}_{j,b}^{\text{In}}}^{\text{Tx}(m) \ne i} z_m^b$ gives the number of data streams for a given receiving node $j$, and the outer summation is taken only over those receiving nodes that are before node $i$ in the ordered node list. Now considering both the DoFs at a node used for transmission and interference cancellation, we have the following constraint.

$$\sum_{l \in \mathcal{L}_{i,b}^{\text{Out}}} z_l^b + \sum_{j \in \mathcal{I}_i^b} \left( \theta_{ji}^b \cdot \sum_{m \in \mathcal{L}_{j,b}^{\text{In}}}^{\text{Tx}(m) \ne i} z_m^b \right) \le A_i \ . \tag{6}$$

On the other hand, for the case when node $i$ is not a transmitting node, we do not have constraint (6) on node $i$. To characterize both cases, we introduce a large constant $M$ (e.g., $M = \sum_{j \in \mathcal{I}_i^b} A_j$) and then have

$$\sum_{l \in \mathcal{L}_{i,b}^{\text{Out}}} z_l^b + \sum_{j \in \mathcal{I}_i^b} \left( \theta_{ji}^b \cdot \sum_{m \in \mathcal{L}_{j,b}^{\text{In}}}^{\text{Tx}(m) \ne i} z_m^b \right) \le A_i g_i^b + (1 - g_i^b) M \ , \tag{7}$$

where $i \in \mathcal{N}, b \in \mathcal{B}_i$. That is, when node $i$ is a transmitting node, (7) becomes (6); when node $i$ is not a transmitting node, (7) becomes $\sum_{j \in \mathcal{I}_i^b} \left( \theta_{ji}^b \cdot \sum_{m \in \mathcal{L}_{j,b}^{\text{In}}}^{\text{Tx}(m) \ne i} z_m^b \right) \le M$, which is always true (i.e., there is no constraint on node $i$).

Similarly, we have the following constraint for a potential receiving node's DoF allocation.

$$\sum_{l \in \mathcal{L}_{i,b}^{\text{In}}} z_l^b + \sum_{j \in \mathcal{I}_i^b} \left( \theta_{ji}^b \cdot \sum_{m \in \mathcal{L}_{j,b}^{\text{Out}}}^{\text{Rx}(m) \ne i} z_m^b \right) \le A_i h_i^b + (1 - h_i^b) M \ , \tag{8}$$

where $i \in \mathcal{N}, b \in \mathcal{B}_i$, and $\text{Rx}(m)$ is the receiver of link $m$.

**Link Capacity Constraints.** For a given route for each session, we can identify the set of links on this route. Denote $\mathcal{L}_{\text{Active}}$ the set of links that are used by all these routes in the network. Then we have the following constraint on link $l \in \mathcal{L}_{\text{Active}}$.

$$\sum_{q \in \mathcal{Q}}^{l \text{ traversed by } q} f(q) \le c \sum_{b \in \mathcal{B}_{\text{Tx}(l),\text{Rx}(l)}} z_l^b \quad (l \in \mathcal{L}_{\text{Active}}) \ , \qquad (9)$$

where $f(q)$ is the throughput (in bps) of session $q \in \mathcal{Q}$ and $c$ is the throughput (in bps) when one DoF is used for data transmission on a band over link $l$.

**Problem Formulation.** For the $\text{CRN}^{\text{MIMO}}$ under investigation, suppose we want to maximize the minimum throughput among the sessions,[1] then the optimization problem (denoted as OPT) can be formulated as follows.

**OPT**

$$\begin{array}{lll} \max & f_{\min} & \\ \text{s.t.} & f_{\min} \le f(q) & (q \in \mathcal{Q}) \\ & \text{Constraints (1)–(5), (7)–(9)} & \\ & f_{\min}, f(q) \ge 0 & (q \in \mathcal{Q}) \\ & g_i^b, h_i^b \in \{0,1\} & (i \in \mathcal{N}, b \in \mathcal{B}_i) \\ & z_l^b \ge 0 & (l \in \mathcal{L}_{\text{Active}}, b \in \mathcal{B}_{\text{Tx}(l),\text{Rx}(l)}) \\ & \theta_{ji}^b \in \{0,1\} & (i, j \in \mathcal{N}, j \ne i, b \in \mathcal{B}_{ij}) \ . \end{array}$$

In this formulation, $f_{\min}$ and $f(q)$ are continuous variables, $g_i^b$, $h_i^b$, and $\theta_{ji}^b$ are binary variables, $z_l^b$ are integer variables, and $A_i$, $M$ and $c$ are given constants. Due to the nonlinear product terms $\sum_{j \in \mathcal{I}_i^b} (\theta_{ji}^b \cdot \sum_{m \in \mathcal{L}_{j,b}^{\text{In}}}^{\text{Tx}(m) \ne i} z_m^b)$ in (7), $\sum_{j \in \mathcal{I}_i^b} (\theta_{ji}^b \cdot \sum_{m \in \mathcal{L}_{j,b}^{\text{Out}}}^{\text{Rx}(m) \ne i} z_m^b)$ in (8), and integer variables, the problem is in the form of mixed-integer non-linear program (MINLP).

### 3.2 Mathematical Reformulation

Note that the constraints in (7) and (8) have nonlinear terms (product of variables), which bring in extra complexity in problem formulation. We now show how these nonlinear terms can be removed via linearization. For the nonlinear term in (7), we define a new variable $\lambda_{ji}^b$ as follows.

$$\lambda_{ji}^b = \theta_{ji}^b \cdot \sum_{m \in \mathcal{L}_{j,b}^{\text{In}}}^{\text{Tx}(m) \ne i} z_m^b \qquad (i \in \mathcal{N}, b \in \mathcal{B}_i, j \in \mathcal{I}_i^b) \ , \qquad (10)$$

which is the number of DoFs that transmitting node $i$ uses to cancel the interference to receiving node $j$. With $\lambda_{ji}^b$, (7) can be rewritten as:

$$\sum_{l \in \mathcal{L}_{i,b}^{\text{Out}}} z_l^b + \sum_{j \in \mathcal{I}_i^b} \lambda_{ji}^b \le A_i g_i^b + (1 - g_i^b) M \ , \qquad (11)$$

1. Problems with other objectives, e.g., maximizing the sum of throughput or maximizing a weighted sum of throughput, can be formulated and solved similarly.

where $i \in \mathcal{N}, b \in \mathcal{B}_i$. Now, we need to add some constraints for $\lambda_{ji}^b$. This can be done by examining the definition of $\lambda_{ji}^b$ in (10). For binary variable $\theta_{ji}^b$, we have the following relaxed constraints: $\theta_{ji}^b \ge 0$, $1 - \theta_{ji}^b \ge 0$. For $\sum_{m \in \mathcal{L}_{j,b}^{\text{In}}}^{\text{Tx}(m) \ne i} z_m^b$, we have $\sum_{m \in \mathcal{L}_{j,b}^{\text{In}}}^{\text{Tx}(m) \ne i} z_m^b \ge 0$ and $A_j - \sum_{m \in \mathcal{L}_{j,b}^{\text{In}}}^{\text{Tx}(m) \ne i} z_m^b \ge 0$. Multiplying each constraint involving $\theta_{ji}^b$ by one of the two constraints involving $\sum_{m \in \mathcal{L}_{j,b}^{\text{In}}}^{\text{Tx}(m) \ne i} z_m^b$, and replacing the product term $\theta_{ji}^b \cdot \sum_{m \in \mathcal{L}_{j,b}^{\text{In}}}^{\text{Tx}(m) \ne i} z_m^b$ with the new variable $\lambda_{ji}^b$, we obtain the following four constraints:

$$\lambda_{ji}^b \ge 0 \qquad (12)$$

$$\lambda_{ji}^b \le \sum_{m \in \mathcal{L}_{j,b}^{\text{In}}}^{\text{Tx}(m) \ne i} z_m^b \qquad (13)$$

$$\lambda_{ji}^b \le A_j \cdot \theta_{ji}^b \qquad (14)$$

$$\lambda_{ji}^b \ge A_j \cdot \theta_{ji}^b - A_j + \sum_{m \in \mathcal{L}_{j,b}^{\text{In}}}^{\text{Tx}(m) \ne i} z_m^b \ , \qquad (15)$$

where $i \in \mathcal{N}, b \in \mathcal{B}_i, j \in \mathcal{I}_i^b$. Note that due to the relaxation of integer variable $\theta_{ji}^b$, $\sum_{m \in \mathcal{L}_{j,b}^{\text{In}}}^{\text{Tx}(m) \ne i} z_m^b$, and product operations, the above four constraints for $\lambda_{ji}^b$ might be looser than (10). However, for the special case when $\theta_{ji}^b$ is a binary variable, it can be easily verified that (10) is equivalent to the four constraints in (12)–(15). Therefore, to replace (7), it is sufficient to have linear constraints (11)–(15).

Similarly, to remove the nonlinear term in (8), we define $\mu_{ji}^b$ as the number of DoFs that receiving node $i$ uses to cancel the interference from transmitting node $j$. Following the same token, (8) can be replaced by the following linear constraints.

$$\sum_{l \in \mathcal{L}_{i,b}^{\text{In}}} z_l^b + \sum_{j \in \mathcal{I}_i^b} \mu_{ji}^b \le A_i h_i^b + (1 - h_i^b) M$$

$$\mu_{ji}^b \ge 0$$

$$\mu_{ji}^b \le \sum_{m \in \mathcal{L}_{j,b}^{\text{Out}}}^{\text{Rx}(m) \ne i} z_m^b$$

$$\mu_{ji}^b \le A_j \cdot \theta_{ji}^b$$

$$\mu_{ji}^b \ge A_j \cdot \theta_{ji}^b - A_j + \sum_{m \in \mathcal{L}_{j,b}^{\text{Out}}}^{\text{Rx}(m) \ne i} z_m^b \ ,$$

where $i \in \mathcal{N}, b \in \mathcal{B}_i, j \in \mathcal{I}_i^b$.

With the above linearization, we have a revised optimization problem formulation (denoted as OPT-R).

**OPT-R**

$$\begin{array}{lll} \max & f_{\min} & \\ \text{s.t.} & g_i^b + h_i^b \le 1 & (i \in \mathcal{N}, b \in \mathcal{B}_i) \\ & g_i^b \le \sum_{l \in \mathcal{L}_{i,b}^{\text{Out}}} z_l^b \le g_i^b A_i & (i \in \mathcal{N}, b \in \mathcal{B}_i) \\ & h_i^b \le \sum_{l \in \mathcal{L}_{i,b}^{\text{In}}} z_l^b \le h_i^b A_i & (i \in \mathcal{N}, b \in \mathcal{B}_i) \\ & \theta_{ji}^b + \theta_{ij}^b = 1 & (i, j \in \mathcal{N}, b \in \mathcal{B}_{ij}) \\ & 1 \le \theta_{ik}^b + \theta_{kj}^b + \theta_{ji}^b \le 2 & (i, j, k \in \mathcal{N}, b \in \mathcal{B}_i \bigcap \mathcal{B}_j \bigcap \mathcal{B}_k) \\ & \sum_{l \in \mathcal{L}_{i,b}^{\text{Out}}} z_l^b + \sum_{j \in \mathcal{I}_i^b} \lambda_{ji}^b \le A_i g_i^b + (1 - g_i^b) M & (i \in \mathcal{N}, b \in \mathcal{B}_i) \end{array}$$

$$\lambda_{ji}^b \leq \sum_{m \in \mathcal{L}_{j,b}^{\text{In}}}^{\text{Tx}(m) \neq i} z_m^b \quad (i \in \mathcal{N}, b \in \mathcal{B}_i, j \in \mathcal{I}_i^b)$$

$$\lambda_{ji}^b \leq A_j \cdot \theta_{ji}^b \quad (i \in \mathcal{N}, b \in \mathcal{B}_i, j \in \mathcal{I}_i^b)$$

$$\lambda_{ji}^b \geq A_j \cdot \theta_{ji}^b - A_j + \sum_{m \in \mathcal{L}_{j,b}^{\text{In}}}^{\text{Tx}(m) \neq i} z_m^b \quad (i \in \mathcal{N}, b \in \mathcal{B}_i, j \in \mathcal{I}_i^b)$$

$$\sum_{l \in \mathcal{L}_{i,b}^{\text{In}}} z_l^b + \sum_{j \in \mathcal{I}_i^b} \mu_{ji}^b \leq A_i h_i^b + (1 - h_i^b) M \quad (i \in \mathcal{N}, b \in \mathcal{B}_i)$$

$$\mu_{ji}^b \leq \sum_{m \in \mathcal{L}_{j,b}^{\text{Out}}}^{\text{Rx}(m) \neq i} z_m^b \quad (i \in \mathcal{N}, b \in \mathcal{B}_i, j \in \mathcal{I}_i^b)$$

$$\mu_{ji}^b \leq A_j \cdot \theta_{ji}^b \quad (i \in \mathcal{N}, b \in \mathcal{B}_i, j \in \mathcal{I}_i^b)$$

$$\mu_{ji}^b \geq A_j \cdot \theta_{ji}^b - A_j + \sum_{m \in \mathcal{L}_{j,b}^{\text{Out}}}^{\text{Rx}(m) \neq i} z_m^b \quad (i \in \mathcal{N}, b \in \mathcal{B}_i, j \in \mathcal{I}_i^b)$$

$$\sum_{q \in \mathcal{Q}}^{l \text{ traversed by } q} f(q) \leq c \cdot \sum_{b \in \mathcal{B}_{\text{Tx}(l),\text{Rx}(l)}} z_l^b \quad (l \in \mathcal{L}_{\text{Active}})$$

$$f_{\min} \leq f(q) \quad (q \in \mathcal{Q})$$

$$f_{\min}, f(q) \geq 0 \quad (q \in \mathcal{Q})$$

$$g_i^b, h_i^b \in \{0, 1\} \quad (i \in \mathcal{N}, b \in \mathcal{B}_i)$$

$$z_l^b \geq 0 \quad (l \in \mathcal{L}_{\text{Active}}, b \in \mathcal{B}_{\text{Tx}(l),\text{Rx}(l)})$$

$$\theta_{ji}^b \in \{0, 1\} \quad (i, j \in \mathcal{N}, j \neq i, b \in \mathcal{B}_{ij})$$

$$\lambda_{ji}^b, \mu_{ji}^b \geq 0 \quad (i, j \in \mathcal{N}, j \neq i, b \in \mathcal{B}_{ij}) .$$

In this formulation, $f_{\min}$, $f(q)$, $g_i^b$, $h_i^b$, $z_l^b$, $\theta_{ji}^b$, $\lambda_{ji}^b$, and $\mu_{ji}^b$ are optimization variables and $A_i$, $M$ and $c$ are given constants. The problem is in the form of mixed-integer linear program (MILP), which can be solved by CPLEX solver. Although the theoretical worst case complexity to solve a MILP is exponential (due to the NP-hardness of a general MILP), CPLEX can efficiently solve our problem for all network instances considered in Section 4 (with up to 50 nodes).

### 3.3 Anticipated Results

Before we present numerical results, we offer the following discussion on the possible solution to our problem. Consider a CRN with only a single transmit/receive antenna at each node (i.e., $A_i = 1$, $i \in \mathcal{N}$). Denote $f_{\text{CRN}}$ the optimal objective value for this CRN with our problem formulation. Now consider a $\text{CRN}^{\text{MIMO}}$ with the same topology as the above CRN, but with $A_{\text{MIMO}}$ transmit/receive antennas at each node. This $\text{CRN}^{\text{MIMO}}$ is a special case of our $\text{CRN}^{\text{MIMO}}$ network with all $A_i = A_{\text{MIMO}}$, $i \in \mathcal{N}$. Denote $f_{\text{CRN}^{\text{MIMO}}}$ the optimal objective value for this $\text{CRN}^{\text{MIMO}}$ under our problem formulation. Comparing $f_{\text{CRN}^{\text{MIMO}}}$ and $f_{\text{CRN}}$, we have the following observation.

*Fact 1:*

$$f_{\text{CRN}^{\text{MIMO}}} \geq A_{\text{MIMO}} \times f_{\text{CRN}} \tag{16}$$

*Proof:* To show $A_{\text{MIMO}} \times f_{\text{CRN}}$ is a lower bound of $f_{\text{CRN}^{\text{MIMO}}}$, we only need to consider spatial multiplexing. That is, for an optimal solution to CRN with optimal objective value $f_{\text{CRN}}$, we can always construct a solution to $\text{CRN}^{\text{MIMO}}$ by using the same multi-hop routing paths in $\text{CRN}^{\text{MIMO}}$ as that in the CRN but with $A_{\text{MIMO}}$ data streams on each link (by spatial multiplexing of MIMO) on these paths. Thus, link capacity on each link of these paths is increased by $A_{\text{MIMO}}$ times and throughput $f(q)$ for each session can also be increased by $A_{\text{MIMO}}$ times. This gives a lower bound for $f_{\text{CRN}^{\text{MIMO}}}$. By exploiting spatial reuse in addition to spatial multiplexing, we have larger optimization space and may do even better. This explains "$\geq$" in (16). □

We are more interested in exploring the possible *inequality* part in (16). That is, with joint channel level (CR) and co-channel level (MIMO DoF) optimization within a $\text{CRN}^{\text{MIMO}}$, we anticipate more than $A_{\text{MIMO}}$-fold increase in the optimal solution. The greater the gap is in this inequality, the more the need of joint optimization of CR and MIMO. We shall look into this potential gain via numerical results on various networks in the next section.

## 4 Numerical Results

In this section, we present some numerical results for various network configurations. The goals of this section are two-fold. First, in Section 4.1, we examine how the inter-channel interference and co-channel interference are jointly handled by CR and MIMO, respectively, in an optimal solution for an example network. Then, in Section 4.2, we validate the claim in Fact 1, particularly the inequality part, thus demonstrating the importance of joint optimization of CR and MIMO.

### 4.1 Results for An Example Network

Before we present complete results for all network instances, we use a 30-node 4-session network as a case study to explain the details of an optimal solution. This will offer us thorough understanding when we present results for the other network instances.

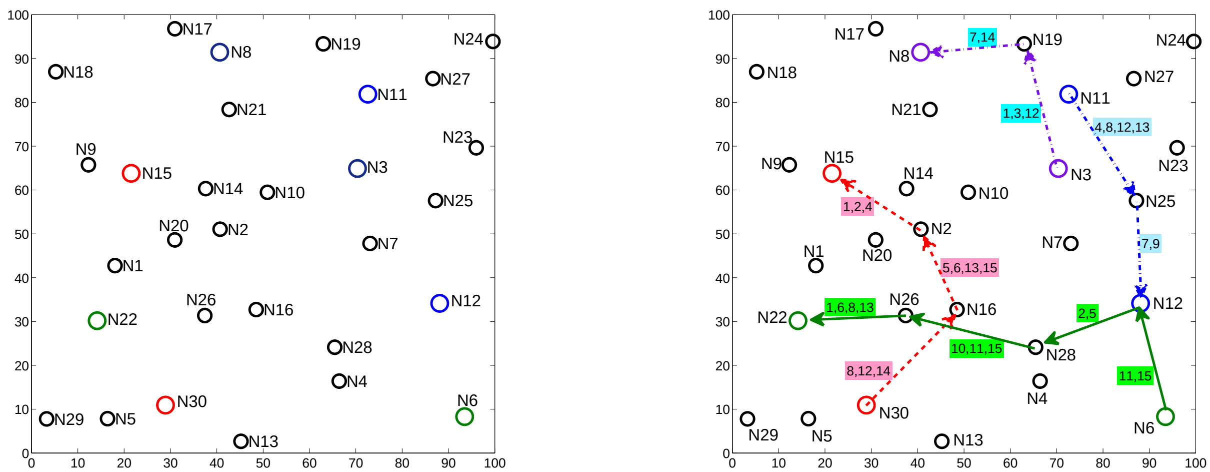
The 30-node network is randomly generated in a $100 \times 100$ area (see Table 2 and Fig. 5(a)). Table 3 specifies the source and destination nodes for each session. For ease of scalability and generality, we normalize all units for distance, bandwidth, and throughput with appropriate dimensions. There are $|\mathcal{B}| = 15$ frequency bands available in the network. The set of available bands at each node is randomly selected from the 15-band pool. The available bands and location for each node are listed in Table 2. The throughput achieved by one band and one DoF is normalized to 1. We assume that the transmission range is 30 and the interference range is 60. For MIMO, we assume each node is equipped with four antennas. We assume minimum-hop routing is used in the network.

Using CPLEX, we can obtain an optimal solution to the OPT-R problem. The optimal objective value for this 30-node network is 6, which means each session can send at least 6 data streams from its source to its destination.

In addition to the optimal objective value, we show channel level and co-channel level solution to achieve

TABLE 2
Each node's location and available frequency bands for a 30-node network

| Node | Location | Available Bands | Node | Location | Available Bands |
|---|---|---|---|---|---|
| N1 | (18.0, 42.7) | 1,2,4,5,6,8,9,10,11,12,13,14 | N16 | (48.5, 32.7) | 2,4,5,6,8,10,11,12,13,14,15 |
| N2 | (40.7, 51.0) | 1,2,4,5,6,7,8,9,10,11,13,14,15 | N17 | (31.0, 96.8) | 4,6,7,12,15 |
| N3 | (70.4, 64.9) | 1,2,3,4,5,7,8,9,12,13,14,15 | N18 | (5.3 , 87.0) | 6,7,15 |
| N4 | (66.4, 16.4) | 2,7,10 | N19 | (63.0, 93.3) | 1,3,4,7,12,14 |
| N5 | (16.4, 7.8 ) | 5,6,9,10,12,13,14 | N20 | (30.9, 48.6) | 1,2,4,6,8,9,10,11,12,13,14 |
| N6 | (93.5, 8.3 ) | 11,15 | N21 | (42.7, 78.4) | 1,3,4,7,12,14 |
| N7 | (73.1, 47.8) | 1,2,3,4,5,7,8,9,13,15 | N22 | (14.2, 30.2) | 1,2,5,6,8,9,10,11,12,13,14 |
| N8 | (40.6, 91.4) | 4,6,7,12,14 | N23 | (99.0, 69.6) | 1,2,3,4,7,9,12,13,15 |
| N9 | (12.3, 65.8) | 1,2,7,14 | N24 | (99.6, 93.9) | 3,9,12 |
| N10 | (50.9, 59.5) | 1,2,3,4,5,6,7,8,11,14,15 | N25 | (87.2, 57.6) | 1,2,4,5,7,8,9,12,13,15 |
| N11 | (72.6, 81.9) | 1,2,3,4,5,7,8,9,12,13,14,15 | N26 | (37.4, 31.4) | 1,2,4,5,6,8,9,10,11,12,13,14,15 |
| N12 | (88.1, 34.1) | 2,5,7,9,11,15 | N27 | (86.6, 85.4) | 1,2,3,4,7,9,12,13 |
| N13 | (45.2, 2.7 ) | 10,12,13,14 | N28 | (65.5, 24.1) | 2,5,7,10,11,15 |
| N14 | (37.6, 60.3) | 1,2,4,5,6,7,8,10,11,13,14,15 | N29 | (3.3 , 7.8 ) | 5,6,9,13 |
| N15 | (21.5, 63.8) | 1,2,4,7,8,10,14 | N30 | (28.9, 10.9) | 5,6,8,9,10,11,12,13,14 |




(a) Topology for the 30-node network.

(b) An optimal solution for the 30-node network.

Fig. 5. A 30-node network.

TABLE 3
Source and destination nodes of each session in a 30-node network

| Session $q$ | Source Node $s(q)$ | Destination Node $d(q)$ |
|---|---|---|
| 1 | N30 | N15 |
| 2 | N6 | N22 |
| 3 | N11 | N12 |
| 4 | N3 | N8 |

this objective. Figure 5(b) shows the optimal band assignment on each link for each session. The bands assigned on each link are shown in a shaded box. This result is also shown in Table 4 (first 3 columns). Also shown in column 4 of Table 4 is the throughput on each band under the optimal solution. In column 5, we show the link throughput (i.e., sum of throughput on each band at this link). Note that the minimum throughput is 6.

We now examine co-channel DoF allocation in the optimal solution. Recall that DoF allocation is performed within the same band. Given that we have a total of 15 bands in the network, we shall have DoF allocation within each of the 15 bands. Let's first show DoF allocation in one particular band, say band 1. Note that band 1 is used by links N2 → N15, N3 → N19, N26 → N22 in Fig. 5(b). The DoF allocation on these 6 nodes are given in Fig. 6 and Table 5. As shown in Fig. 6, there are 2 data streams on each of these 3 links on band 1. The dashed lines in Fig. 6 show the interference relationships among the nodes, i.e., node N2 interferes N19 and N22, node N3 interferes N15, and node N26 interferes N15. These transmission links and interference relationships are also listed in Table 5 (row 1), where "N2 → N15 (N19, N22)" denotes that N2 transmits to N15 and interferes N19 and N22, etc. Also shown in the first column of Table 5 is the optimal order for the 6 nodes for DoF allocation in the optimal solution, i.e., N2, N3, N15, N19, N26, N22. Based on this order, the DoFs at each node are used as follows (also see Fig. 6).

- Starting with node N2, it is the first node in the ordered node list and it is a transmitting node. Then

TABLE 5
The DoF allocation on band 1 in the optimal solution for the 30-node network

| Transmission and Interference | N2 → N15 (N19, N22), N3 → N19 (N15), N26 → N22 (N15) | |
|---|---|---|
| Ordered Node List | Interference Cancellation (# of DoFs, To/From, Node) | Spatial Multiplexing (# of DoFs, Transmit/Receive, Node) |
| N2 | — | (2, Transmit, N15) |
| N3 | — | (2, Transmit, N19) |
| N15 | (2, From, N3) | (2, Receive, N2) |
| N19 | (2, From, N2) | (2, Receive, N3) |
| N26 | (2, To, N15) | (2, Transmit, N22) |
| N22 | (2, To, N2) | (2, Receive, N26) |

TABLE 4
Details of band assignment, throughput on each band, and throughput on each link in the optimal solution for the 30-node network

| Session | Link | Assigned Band | Throughput on Band | Throughput on Link |
|---|---|---|---|---|
| 1 | N30 → N16 | 8 | 1 | 6 |
| | | 12 | 1 | |
| | | 14 | 4 | |
| | N16 → N2 | 5 | 1 | 6 |
| | | 6 | 3 | |
| | | 13 | 1 | |
| | | 15 | 1 | |
| | N2 → N15 | 1 | 2 | 6 |
| | | 2 | 1 | |
| | | 4 | 3 | |
| 2 | N6 → N12 | 11 | 3 | 6 |
| | | 15 | 3 | |
| | N12 → N28 | 2 | 3 | 6 |
| | | 5 | 3 | |
| | N28 → N26 | 10 | 4 | 6 |
| | | 11 | 1 | |
| | | 15 | 1 | |
| | N26 → N22 | 1 | 2 | 7 |
| | | 6 | 1 | |
| | | 8 | 1 | |
| | | 13 | 3 | |
| 3 | N11 → N25 | 4 | 1 | 6 |
| | | 8 | 2 | |
| | | 12 | 2 | |
| | | 13 | 1 | |
| | N25 → N12 | 7 | 2 | 6 |
| | | 9 | 4 | |
| 4 | N3 → N19 | 1 | 2 | 7 |
| | | 3 | 4 | |
| | | 12 | 1 | |
| | N19 → N8 | 7 | 2 | 6 |
| | | 14 | 4 | |

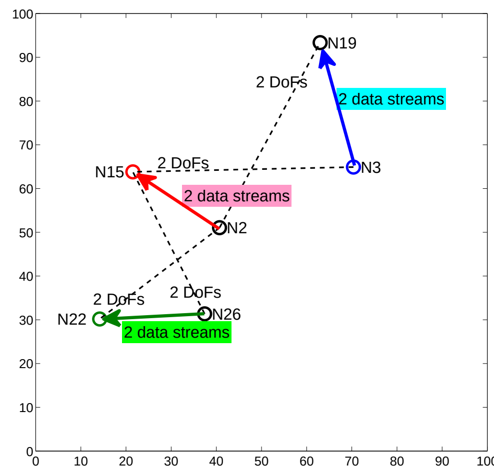



Fig. 6. The DoF allocation on band 1 in the optimal solution for the 30-node network.

it uses 2 DoFs to transmit 2 data streams to node N15. It does not need to use any DoF to cancel potential interference to other receiving nodes after itself in the node list.

- The next node in the list is N3. As a transmitting node, it uses 2 DoFs for transmitting 2 data streams to node N19. It does not need to use any DoF to cancel potential interference to receiving node N15, which is after itself in the ordered node list.
- The next node in the list is N15. As a receiving node, it needs to use 2 DoFs for receiving 2 data streams from node N2. In addition, it must ensure that its reception is not interfered by any transmitting node before itself in the list, i.e., N3. Thus it uses the remaining 2 DoFs to cancel the interference from node N3.
- The next node in the list is N19. As a receiving node, it uses 2 DoFs for receiving 2 data streams from node N3. In addition, it uses the remaining 2 DoFs to cancel interference from transmitting node N2 which is before itself in the list.
- The next node in the list is N26. As a transmitting node, it needs to ensure that its transmission does not interfere with any receiving node before itself in the list, i.e., N15. For this purpose, it uses 2 DoFs to cancel its interference to node N15. Then it uses the remaining 2 DoFs to transmit 2 data streams to node N22.
- The last node in the list is N22. As a receiving node, it uses 2 DoFs for receiving 2 data streams from node N22. In addition, it must ensure that its reception is not interfered by any transmitting node before itself in the list, i.e., N2. Thus, it uses the remaining 2 DoFs to cancel this interference from node N2.

This completes the DoF allocation for each node in the list on band 1. The DoF allocation for the 6 nodes is also listed in Table 5, where we employ the following two abbreviated notations.

- We use the tuple (# of DoFs, From/To, Node) to

TABLE 6
The DoFs allocation on band 2–15 in the optimal solution for the 30-node network

| Band 2 | | |
|---|---|---|
| Transmission and Interference | N2 → N15 (N28), N12 → N28 | |
| Ordered Node List | Interference Cancellation | Spatial Multiplexing |
| N15 | — | (1, Receive, N2) |
| N28 | — | (3, Receive, N12) |
| N2 | (3, To, N28) | (1, Transmit, N15) |
| N12 | | (3, Transmit, N28) |

| Band 3 | | |
|---|---|---|
| Transmission and Interference | N3 → N19 | |
| Ordered Node List | Interference Cancellation | Spatial Multiplexing |
| N3 | — | (4, Transmit, N19) |
| N19 | — | (4, Receive, N3) |

| Band 4 | | |
|---|---|---|
| Transmission and Interference | N2 → N15 (N25), N11 → N25 (N15) | |
| Ordered Node List | Interference Cancellation | Spatial Multiplexing |
| N25 | — | (1, Receive, N11) |
| N2 | (1, To, N25) | (3, Transmit, N15) |
| N11 | — | (1, Transmit, N25) |
| N15 | (1, From, N11) | (3, Receive, N2) |

| Band 5 | | |
|---|---|---|
| Transmission and Interference | N12 → N28 (N2), N16 → N2 (N28) | |
| Ordered Node List | Interference Cancellation | Spatial Multiplexing |
| N16 | — | (1, Transmit, N2) |
| N28 | (1, From, N16) | (3, Receive, N12) |
| N2 | — | (1, Receive, N16) |
| N12 | (1, To, N2) | (3, Transmit, N28) |

| Band 6 | | |
|---|---|---|
| Transmission and Interference | N16 → N2 (N22), N26 → N22 (N2) | |
| Ordered Node List | Interference Cancellation | Spatial Multiplexing |
| N2 | — | (3, Receive, N16) |
| N26 | (3, To, N2) | (1, Transmit, N22) |
| N16 | — | (3, Transmit, N2) |
| N22 | (3, From, N16) | (1, Receive, N26) |

| Band 7 | | |
|---|---|---|
| Transmission and Interference | N19 → N8, N25 → N12 (N8) | |
| Ordered Node List | Interference Cancellation | Spatial Multiplexing |
| N8 | — | (2, Receive, N19) |
| N12 | — | (2, Receive, N25) |
| N19 | — | (2, Transmit, N8) |
| N25 | (2, To, N8) | (2, Transmit, N12) |

| Band 8 | | |
|---|---|---|
| Transmission and Interference | N11 → N25 (N16), N26 → N22 (N16, N25), N30 → N16 (N22) | |
| Ordered Node List | Interference Cancellation | Spatial Multiplexing |
| N25 | — | (2, Receive, N11) |
| N26 | (2, To, N25) | (1, Transmit, N22) |
| N30 | — | (1, Transmit, N16) |
| N11 | — | (2, Transmit, N25) |
| N22 | (1, From, N30) | (1, Receive, N26) |
| N16 | (1, From, N26)<br>(2, From, N11) | (1, Receive, N30) |

| Band 9 | | |
|---|---|---|
| Transmission and Interference | N25 → N12 | |
| Ordered Node List | Interference Cancellation | Spatial Multiplexing |
| N12 | — | (4, Receive, N25) |
| N25 | — | (4, Transmit, N12) |

| Band 10 | | |
|---|---|---|
| Transmission and Interference | N28 → N26 | |
| Ordered Node List | Interference Cancellation | Spatial Multiplexing |
| N26 | — | (4, Receive, N28) |
| N28 | — | (4, Transmit, N26) |

TABLE 6 (Continued)

| Band 11 | | |
|---|---|---|
| Transmission and Interference | N6 → N12, N28 → N26 (N12) | |
| Ordered Node List | Interference Cancellation | Spatial Multiplexing |
| N6 | — | (3, Transmit, N12) |
| N12 | — | (3, Receive, N6) |
| N28 | (3, To, N12) | (1, Transmit, N26) |
| N26 | — | (1, Receive, N28) |
| Band 12 | | |
| Transmission and Interference | N3 → N19 (N16, N25), N11 → N25 (N16, N19), N30 → N16 | |
| Ordered Node List | Interference Cancellation | Spatial Multiplexing |
| N30 | — | (1, Transmit, N16) |
| N11 | — | (2, Transmit, N25) |
| N16 | (2, From, N11) | (1, Receive, N30) |
| N25 | — | (2, Receive, N11) |
| N3 | (1, To, N16)<br>(2, To, N25) | (1, Transmit, N19) |
| N19 | (2, From, N11) | (1, Receive, N3) |
| Band 13 | | |
| Transmission and Interference | N11 → N25 (N2), N16 → N2 (N22, N25), N26 → N22 (N2, N25) | |
| Ordered Node List | Interference Cancellation | Spatial Multiplexing |
| N2 | — | (1, Receive, N16) |
| N26 | (1, To, N2) | (3, Transmit, N22) |
| N25 | (3, From, N26) | (1, Receive, N11) |
| N16 | (1, To, N25) | (1, Transmit, N2) |
| N22 | (1, From, N16) | (3, Receive, N26) |
| N11 | (1, To, N2) | (1, Transmit, N25) |
| Band 14 | | |
| Transmission and Interference | N19 → N8, N30 → N16 | |
| Ordered Node List | Interference Cancellation | Spatial Multiplexing |
| N30 | — | (4, Transmit, N16) |
| N8 | — | (4, Receive, N19) |
| N16 | — | (4, Receive, N30) |
| N19 | — | (4, Transmit, N8) |
| Band 15 | | |
| Transmission and Interference | N6 → N12, N16 → N2 (N12, N26), N28 → N26 (N2, N12) | |
| Ordered Node List | Interference Cancellation | Spatial Multiplexing |
| N2 | — | (1, Receive, N16) |
| N6 | — | (3, Transmit, N12) |
| N28 | (1, To, N2) | (1, Transmit, N26) |
| N12 | (1, From, N28) | (3, Receive, N6) |
| N16 | (3, To, N12) | (1, Transmit, N2) |
| N26 | (1, From, N16) | (1, Receive, N28) |

denote the interference cancellation relationship between nodes. For example, (2, From, N3) denotes current node (in the first column of the same row) uses 2 DoFs to cancel the interference from N3, whereas (2, To, N15) denotes current node uses 2 DoFs to cancel its interference to N15.

- We use the tuple (# of DoFs, Transmit/Receive, Node) to denote data transmission relationship between the nodes. For example, (2, Transmit, N15) denotes the current node (in the first column of the same row) uses 2 DoFs to transmit data streams to N15, whereas (2, Receive, N2) denotes the current node uses 2 DoFs to receive data streams from N2.

Given the above explanation of DoF allocation on band 1, we now present DoF allocations on bands 2 to 15, which are listed in Table 6.

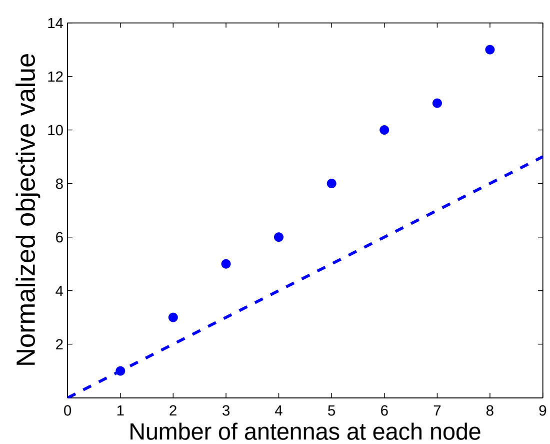



Fig. 7. Normalized objective value under different antennas for the 30-node network.

### 4.2 $f_{\text{CRN}^{\text{MIMO}}}$ vs. $A_{\text{MIMO}} \times f_{\text{CRN}}$

The results in the last section give details in an optimal solution for a 30-node network with $A_{\text{MIMO}} = 4$ antennas at each node. We have that the maximum $f_{\min}$ is 6. We now validate the result in (16) under different number of antennas at each node. That is, we obtain the optimal objective values (the maximum $f_{\min}$) under different $A_{\text{MIMO}}$ for the same 30-node network discussed in the

TABLE 7
Each node's location and available frequency bands for a 20-node network

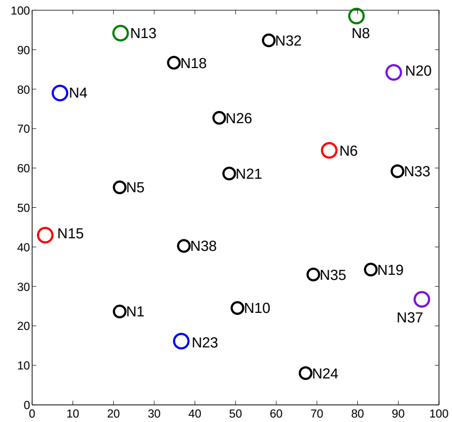
| Node | Location | Available Bands | Node | Location | Available Bands |
|---|---|---|---|---|---|
| N1 | (21.5, 23.6) | 6,7,8,11,12,13,14,15 | N11 | (48.4, 58.6) | 1,3,5,7,8,11,13,15 |
| N2 | (6.8, 79.0) | 3,8,14 | N12 | (36.7, 16.1) | 6,7,8,9,11,12,13,14 |
| N3 | (21.5, 55.1) | 1,3,5,7,8,12,13,14,15 | N13 | (67.2, 8.0 ) | 6,9,11,12,15 |
| N4 | (73.0, 64.5) | 1,3,5,7,8 | N14 | (46.0, 72.7) | 1,2,3,4,5,8,10,11,13,14,15 |
| N5 | (79.7, 98.5) | 1,2,3,4,8,9,10 | N15 | (58.2, 92.4) | 1,2,3,4,5,8,10,11,15 |
| N6 | (50.5, 24.5) | 6,7,8,9,11,12,13,15 | N16 | (89.8, 59.2) | 1,5,7,8,10,12 |
| N7 | (21.7, 94.2) | 2,3,11,14 | N17 | (69.1, 33.0) | 6,7,8,9,12,15 |
| N8 | (3.2, 43.0) | 3,7,8,13,14,15 | N18 | (95.8, 26.7) | 4,10,15 |
| N9 | (34.8, 86.7) | 2,3,4,5,10,11,14,15 | N19 | (37.3, 40.3) | 1,3,5,6,7,8,12,13,14,15 |
| N10 | (83.2, 34.3) | 4,6,7,10,15 | N20 | (88.9, 84.2) | 1,2,3,5,8,9,10,12 |

TABLE 9
Each node's location and available frequency bands for a 40-node network

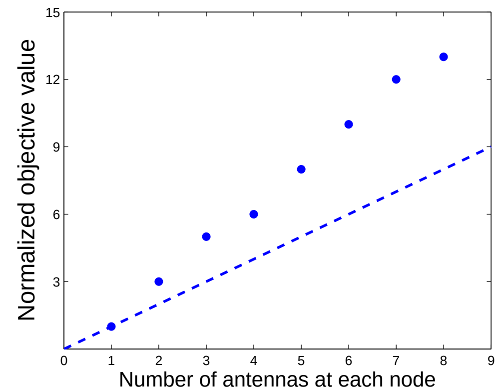
| Node | Location | Available Bands | Node | Location | Available Bands |
|---|---|---|---|---|---|
| N1 | (26.0, 60.4) | 1,2,4,6,7,8,10,11,12,13,15 | N21 | (85.8, 9.2 ) | 3,5,6,8,9,14,15 |
| N2 | (57.1, 33.6) | 2,3,5,7,8,13 | N22 | (76.3, 13.1) | 2,3,5,6,8,9,14,15 |
| N3 | (69.5, 21.2) | 2,3,5,6,8,9,13,14 | N23 | (33.4, 15.7) | 10,11,13,14 |
| N4 | (2.2 , 75.9) | 2,4,8,13,15 | N24 | (63.6, 7.4 ) | 3,5,6,8 |
| N5 | (84.6, 88.2) | 1,3,5,9,10,11,12 | N25 | (44.1, 50.6) | 1,2,7,8,10,11,12,13,15 |
| N6 | (54.1, 2.7 ) | 3,9 | N26 | (51.8, 27.2) | 2,3,8,10,13 |
| N7 | (72.9, 98.5) | 1,2,3,5,9,10,11 | N27 | (73.0, 64.7) | 3,7,12,13,15 |
| N8 | (58.3, 16.3) | 2,3,5,6,8,13 | N28 | (75.9, 72.6) | 1,3,5,7,10,11,12,15 |
| N9 | (43.6, 27.5) | 2,3,10,13 | N29 | (40.7, 38.2) | 1,2,7,10,11,12,13 |
| N10 | (56.0, 87.6) | 1,2,3,6,8,9,10,12,15 | N30 | (25.1, 88.9) | 1,2,4,6,7,8,10,12,13,15 |
| N11 | (85.3, 30.1) | 2,5,6,7,8,9,11,13,14,15 | N31 | (19.8, 7.2 ) | 10,14 |
| N12 | (23.7, 32.4) | 4,10,11,13,14 | N32 | (8.5 , 37.1) | 4,10,11,13,14 |
| N13 | (77.6, 47.9) | 2,5,7,8,9,11,12,13 | N33 | (7.5 , 64.2) | 1,2,4,7,8,11,13 |
| N14 | (16.6, 30.1) | 4,10,11,13,14 | N34 | (61.5, 52.4) | 2,3,7,12,13,15 |
| N15 | (19.7, 23.9) | 10,11,14 | N35 | (60.1, 80.7) | 1,2,3,6,7,8,9,10,12,15 |
| N16 | (1.4 , 95.5) | 4,15 | N36 | (44.8, 69.1) | 1,2,3,4,6,7,8,10,12,13,15 |
| N17 | (8.2 , 9.5 ) | 10,14 | N37 | (9.0 , 80.7) | 1,2,4,7,8,13,15 |
| N18 | (19.5, 77.7) | 1,2,4,6,7,8,12,13,15 | N38 | (90.9, 69.1) | 3,5,7,11,12 |
| N19 | (49.6, 90.1) | 2,3,6,8,9,10,12,15 | N39 | (86.1, 53.4) | 7,9,11,12,13 |
| N20 | (28.6, 41.5) | 1,4,7,10,11,12,13,14 | N40 | (87.0, 41.3) | 2,5,6,7,8,9,11,12,13,15 |

TABLE 11
Each node's location and available frequency bands for a 50-node network

| Node | Location | Available Bands | Node | Location | Available Bands |
|---|---|---|---|---|---|
| N1 | (80.5, 12.9) | 1,9 | N26 | (21.9, 130.2) | 4,8,10,14 |
| N2 | (3.5, 19.1) | 11,15 | N27 | (128.6, 105.1) | 2,4,5,6,7,8,14 |
| N3 | (100.7, 127.0) | 2,5,6,8,12,13,14 | N28 | (5.7, 55.9) | 10,11,12,15 |
| N4 | (128.8, 116.9) | 2,4,5,6,7,8,12,14 | N29 | (141.9, 47.0) | 1,2,3,7,10,13 |
| N5 | (83.5, 114.9) | 2,3,5,8,11,12,13,14,15 | N30 | (78.3, 52.6) | 1,3,4,5,6,13,15 |
| N6 | (29.1, 89.9) | 3,4,9,10,11,12,14 | N31 | (43.0, 117.7) | 3,4,8,9,10,11,12,13,14 |
| N7 | (89.9, 94.4) | 2,3,4,5,6,11,13,14,15 | N32 | (137.6, 81.6) | 2,4,5,6,7,10,13 |
| N8 | (25.3, 19.9) | 11,15 | N33 | (109.4, 145.7) | 2,5,6,8,12,13,14 |
| N9 | (49.4, 131.2) | 3,4,5,8,9,10,11,12,13,14 | N34 | (78.6, 3.6) | 1,9 |
| N10 | (32.0, 38.0) | 1,11,15 | N35 | (126.2, 23.4 ) | 1,3,9,10,13 |
| N11 | (85.2, 76.3) | 1,2,3,4,5,6,11,13,15 | N36 | (103.8, 16.1) | 1,3,9,13 |
| N12 | (65.9, 137.6) | 5,8,12,13,14 | N37 | (35.7, 130.4) | 3,4,8,9,10,13,14 |
| N13 | (148.6, 59.8) | 2,3,4,7,10,13 | N38 | (23.8, 78.4) | 3,4,9,10,11,12,15 |
| N14 | (41.1, 75.4) | 3,4,5,9,10,11,12,15 | N39 | (31.8, 3.0) | 11,15 |
| N15 | (142.9, 85.5) | 2,4,5,6,7,10 | N40 | (98.2, 31.8) | 1,3,4,6,9,13,15 |
| N16 | (109.1, 118.4) | 2,4,5,6,7,8,12,13,14 | N41 | (73.5, 29.0) | 1,3,9,11,13,15 |
| N17 | (31.8, 109.6) | 3,4,8,9,10,11,12,14 | N42 | (83.6, 135.9) | 2,5,8,12,13,14 |
| N18 | (40.8, 66.8) | 3,4,5,9,10,11,12,15 | N43 | (50.7, 11.1) | 1,11,15 |
| N19 | (63.5, 123.6) | 3,4,5,8,9,10,11,12,13,14 | N44 | (48.3, 24.6) | 1,11,15 |
| N20 | (124.0, 30.0) | 1,3,4,9,10,13 | N45 | (110.5, 51.9) | 1,3,4,6,9,13,15 |
| N21 | (61.2, 73.0) | 3,4,5,6,9,10,11,12,13,15 | N46 | (22.0, 101.3) | 3,4,8,9,10,11,12,14 |
| N22 | (102.5, 82.2) | 1,2,3,4,5,6,13,14,15 | N47 | (29.0, 100.0) | 3,4,8,9,10,11,12,14 |
| N23 | (24.7, 45.9) | 10,11,12,15 | N48 | (62.6, 97.8) | 3,4,5,9,10,11,12,13,14,15 |
| N24 | (44.6, 46.8) | 1,3,5,10,11,12,15 | N49 | (46.4, 57.6) | 1,3,5,10,11,12,15 |
| N25 | (147.1, 127.4) | 2,4,5,6,7,8 | N50 | (65.5, 104.6) | 3,4,5,8,9,10,11,12,13,14,15 |

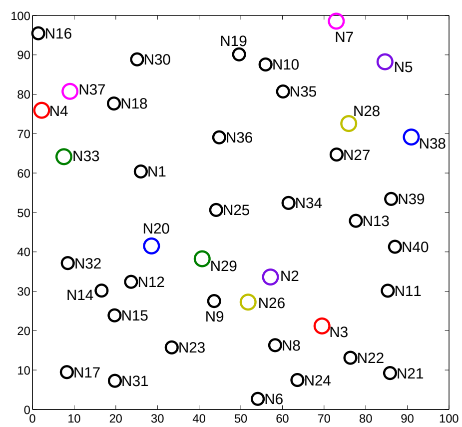



(a) Topology for a 20-node network.

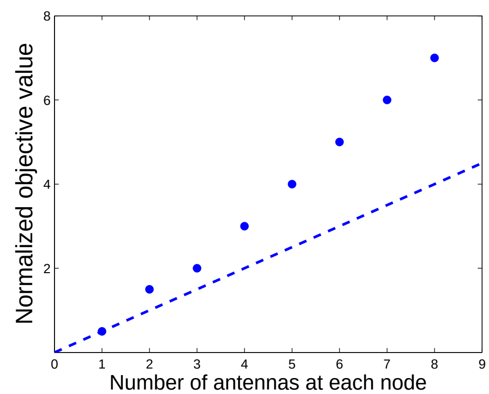



(b) Normalized objective value under different antennas for a 20-node network.

Fig. 8. Comparison of $f_{\text{CRN}^{\text{MIMO}}}$ vs $A_{\text{MIMO}} \times f_{\text{CRN}}$ for a $20$-node network.

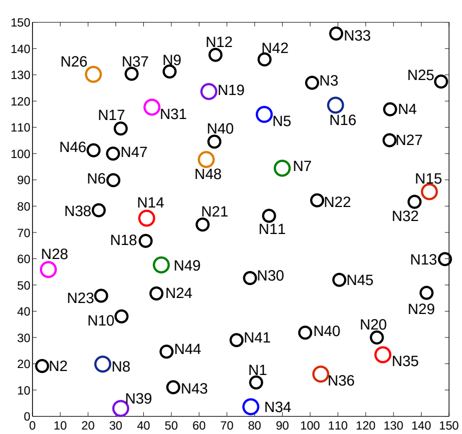



(a) Topology for a 40-node network.

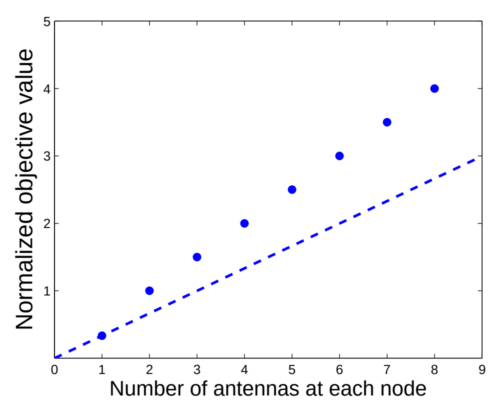



(b) Normalized objective value under different antennas for a 40-node network.

Fig. 9. Comparison of $f_{\text{CRN}^{\text{MIMO}}}$ vs $A_{\text{MIMO}} \times f_{\text{CRN}}$ for a $40$-node network.




(a) Topology for a 50-node network.




(b) Normalized objective value under different antennas for a 50-node network.

Fig. 10. Comparison of $f_{\text{CRN}^{\text{MIMO}}}$ vs $A_{\text{MIMO}} \times f_{\text{CRN}}$ for a $50$-node network.

TABLE 8
Source and destination nodes of each session in a $20$-node network

| Session $q$ | Source Node $s(q)$ | Destination Node $d(q)$ |
|---|---|---|
| 1 | N4 | N8 |
| 2 | N7 | N5 |
| 3 | N12 | N2 |
| 4 | N18 | N20 |

TABLE 10
Source and destination nodes of each session in a $40$-node network

| Session $q$ | Source Node $s(q)$ | Destination Node $d(q)$ |
|---|---|---|
| 1 | N3 | N4 |
| 2 | N29 | N33 |
| 3 | N38 | N20 |
| 4 | N5 | N2 |
| 5 | N7 | N37 |
| 6 | N28 | N26 |

last section. Figure 7 shows our results. Also shown in this figure is a dashed line $y = A_{\text{MIMO}} \times f_{\text{CRN}}$ so that we can compare $f_{\text{CRN}^{\text{MIMO}}}$ with $A_{\text{MIMO}} \times f_{\text{CRN}}$. Note that the equality in (16) only coincides on the first point, i.e., single antenna (no MIMO). When the number of antennas at each node is greater than $1$, we have an inequality, i.e., $f_{\text{CRN}^{\text{MIMO}}} > A_{\text{MIMO}} \times f_{\text{CRN}}$. That is, with joint CR and MIMO optimization, we have more than $A_{\text{MIMO}}$-fold increase in the optimal solution. This confirms that joint optimization of CR at channel level and MIMO at co-channel level is highly desirable.

In Figs. 8(b), 9(b), and 10(b), we further compare $f_{\text{CRN}^{\text{MIMO}}}$ vs. $A_{\text{MIMO}} \times f_{\text{CRN}}$ for 20-, 40-, 50-node networks under varying number of antennas, respectively. The location and available bands at each node and source/destination node of each session are given in Tables 7 to 12 for the three networks. Again, we confirm our findings that joint optimization of CR at channel level and MIMO at co-channel level offers more than $A_{\text{MIMO}}$-fold increase in throughput.

## 5 Conclusion

In this paper, we explored joint optimization of CR and MIMO in a multi-hop ad hoc network. By exploiting CR's flexibility at channel level and MIMO's capability within a channel, we showed that we can have much larger optimization space to mitigate interference in the network. We developed a tractable mathematical model for a multi-hop ad hoc network that captures the essence of channel assignment (for CR) and DoF allocation (for MIMO). Based on this mathematical model, we used numerical results to show how channel assignment in CRN and DoF allocation in MIMO can be jointly optimized to maximize throughput. More important, for a $\text{CRN}^{\text{MIMO}}$ with $A_{\text{MIMO}}$ antennas at each node, we showed that the joint optimization of both techniques offers more than $A_{\text{MIMO}}$-fold increase in throughput than a CRN (without MIMO).

TABLE 12
Source and destination nodes of each session in a $50$-node network

| Session $q$ | Source Node $s(q)$ | Destination Node $d(q)$ |
|---|---|---|
| 1 | N35 | N14 |
| 2 | N8 | N16 |
| 3 | N34 | N5 |
| 4 | N39 | N19 |
| 5 | N31 | N28 |
| 6 | N36 | N15 |
| 7 | N49 | N7 |
| 8 | N26 | N48 |

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## References

[1] R. Bhatia and L. Li, "Throughput optimization of wireless mesh networks with MIMO links," *in Proc. IEEE INFOCOM,* pp. 2326–2330, Anchorage, AK, May 2007.

[2] E. Biglieri, R. Calderbank, A. Constantinides, A. Goldsmith, A. Paulraj, and H.V. Poor, *MIMO Wireless Communications,* Cambridge University Press, Jan. 2007.

[3] G. Caire and S. Shamai, "On the achievable throughput of a multiantenna gaussian broadcast channel," *IEEE Trans. on Information Theory,* vol. 49, no. 7, pp. 1691–1706, July 2003.

[4] S. Chu and X. Wang, "Opportunistic and cooperative spatial multiplexing in MIMO ad hoc networks," *in Proc. ACM MobiHoc,* pp. 63–72, Hong Kong, China, May 2008.

[5] G.J. Foschini, "Layered space-time architecture for wireless communication in a fading envorinment when using multi-element antennas," *Bell Labs Tech. J.,* vol. 1, no. 2, pp. 41–59, 1996.

[6] D. Gesbert, M. Shafi, D.S. Shiu, P.J. Smith, and A. Naguib, "From theory to practice: An overview of MIMO space-time coded wireless systems," *IEEE Journal on Selected Areas in Communications,* vol. 21, no. 3, pp. 281–302, April 2003.

[7] B. Hamdaoui and K.G. Shin, "Characterization and analysis of multi-hop wireless MIMO network throughput," *in Proc. ACM MobiHoc,* pp. 120–129, Montréal, Québec, Canada, Sept. 2007.

[8] S. Haykin, D.J. Thomson, and J.H. Reed, "Spectrum sensing for cognitive radio," *Proc. IEEE,* vol. 97, no. 5, pp. 849–877, May 2009.

[9] S. Haykin, "Cognitive radio: Brain-empowered wireless communications," *IEEE Journal on Selected Areas in Communications,* vol. 23, no. 2, pp. 201–220, Feb. 2005.

[10] Y.T. Hou, Y. Shi, and H.D. Sherali, "Spectrum sharing for multihop networking with cognitive radio," *IEEE Journal on Selected Areas in Communications,* vol. 26, no. 1, pp. 146–155, Jan. 2008.

[11] Joint Tactical Radio System, http://www.globalsecurity.org/military/systems/ground/jtrs.htm/.

[12] *IEEE Journal on Selected Areas in Communications – Special Issue on MIMO Systems and Applications: Field Experience, Practical Aspects, Limitations and Challenges,* Guest Editors: M. Shafi, H. Huang, A. Hottinen, P.J. Smith, and R.A. Valenzuela, vol. 26, no. 6, Aug. 2008.

[13] *IEEE Journal on Selected Areas in Communications – Special Issue on Optimization of MIMO Transceivers for Realistic Communication Networks: Challenges and Opportunities,* Guest Editors: D. Palomar, T. Davidson, S. Barbarossa, A. Goldsmith, and G. Giannakis, vol. 25, no. 7, Sept. 2007.

[14] Y.-T. Kim, H. Lee, S. Park, and I. Lee, "Optimal precoding for orthogonalized spatial multiplexing in closed-loop MIMO systems," *IEEE Journal on Selected Areas in Communications,* vol. 26, no. 8, pp. 1556–1566, Oct. 2008.

[15] S-J. Kim, X. Wang, and M. Madihian, "Cross-layer design of wireless multihop backhaul networks with multiantenna beamforming," *IEEE Trans. on Mobile Computing,* vol. 6, no. 11, pp. 1259–1269, Nov. 2007.

[16] J. Ma, G.Y. Li, and B.H. Juang, "Signal processing in cognitive radio," *Proc. IEEE,* vol. 97, no. 5, pp. 805–823, May 2009.

[17] J. Mitola, "Cognitive radio architecture evolution," *Proc. IEEE,* vol. 97, no. 4, pp. 626–641, April 2009.

[18] J.M. Peha, "Sharing spectrum through spectrum policy reform and cognitive radio," *Proc. IEEE,* vol. 97, no. 4, pp. 708–719, April 2009.

[19] A.S.Y. Poon, R.W. Brodersen, and D.N.C. Tse, "Degrees of freedom in multiple-antenna channels: A signal space approach," *IEEE Trans. on Information Theory,* vol. 51, no. 2, pp. 523–536, Feb. 2005.

[20] SAFECOM, http://www.safecomprogram.gov/SAFECOM/.

[21] P. Setoodeh and S. Haykin, "Robust transmit power control for cognitive radio," *Proc. IEEE,* vol. 97, no. 5, pp. 915–939, May 2009.

[22] M. Shenouda and T. Davidson, "A design framework for limited feedback MIMO systems with zero-forcing DFE," *IEEE Journal on Selected Areas in Communications,* vol. 26, no. 8, pp. 1578–1587, Oct. 2008.

[23] Y. Shi, Y.T. Hou, and H. Zhou, "Per-node based optimal power control for multi-hop cognitive radio networks," *IEEE Transactions on Wireless Communications,* vol. 8, no. 10, pp. 5290–5299, October 2009.

[24] Y. Shi, J. Liu, C. Jiang, C. Gao, and Y.T. Hou, "An optimal MIMO link model for multi-hop wireless networks," *in Proc. IEEE INFOCOM,* Shanghai, China, April 10–15, 2011.

[25] Y. Shi, J. Liu, C. Jiang, C. Gao, and Y.T. Hou, "An optimal MIMO link model for multi-hop wireless networks," Technical Report, the Bradley Department of Electrical and Computer Engineering, Virginia Polytechnic Institute and State University, Blacksburg, VA, July 2010. Available at http://filebox.vt.edu/users/yshi/papers/model.pdf.

[26] K. Sundaresan, R. Sivakumar, M.A. Ingram, and T-Y Chang, "A fair medium access control protocol for ad-hoc networks with MIMO links," *in Proc. IEEE INFOCOM,* pp. 2559–2570, Hong Kong, China, March 2004.

[27] I.E. Telatar, "Capacity of multi-antenna gaussian channels," *European Trans. Telecomm.,* vol. 10, no. 6, pp. 585–596, Nov. 1999.

[28] M. Trivellato, F. Boccardi, and H. Huang, "On transceiver design and channel quantization for downlink multiuser MIMO systems with limited feedback," *IEEE Journal on Selected Areas in Communications,* vol. 26, no. 8, pp. 1494–1504, Oct. 2008.

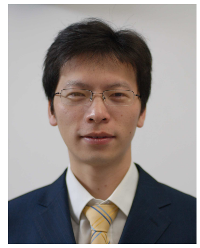
[29] D. Tse and P. Viswanath, *Fundamentals of Wireless Communication,* Cambridge University Press, June 2005.

[30] Vanu Inc., http://www.vanu.com/.

[31] H. Viswanathan, S. Venkatesan, and H. Huang, "Downlink capacity evaluation of cellular networks with known-interference cancellation," *IEEE Journal on Selected Areas in Communications,* vol. 21, no. 5, pp. 802–811, June 2003.

[32] A. Wyglinski, M. Nekovee, and Y.T. Hou, Editors, *Cognitive Radio Communications and Networks: Principles and Practice,* Academic Press/Elsevier, 2010.

[33] L. Zheng and D.N.C. Tse, "Diversity and multiplexing: A fundamental tradeoff in multiple-antenna channels," *IEEE Trans. on Information Theory,* vol. 49, no. 5, pp. 1073–1096, May 2003.

**Cunhao Gao** (S'08) received his B.S. degree in Computer Science from the Department of Special Class for the Gifted Young, University of Science and Technology of China, Hefei, China, in 2004. He received his M.E. degree in Computer Science from University of Science and Technology of China in 2007. In 2009, he earned his M.S. degree in Computer Engineering from the Bradley Department of Electrical and Computer Engineering, Virginia Polytechnic Institute and State University (Virginia Tech), Blacksburg, VA. He is currently working toward his Ph.D. degree in the Department of Electrical and Computer Engineering at Stony Brook University, Stony Brook, NY. His current research focuses on cross-layer design and optimizations for wireless networks, ad hoc networks, cognitive radio networks, and MIMO.

**Yi Shi** (S'02–M'08) received his Ph.D. degree in computer engineering from Virginia Polytechnic Institute and State University (Virginia Tech), in 2007. He is currently a Research Scientist in the Department of Electrical and Computer Engineering at Virginia Tech. His research focuses on algorithms and optimization for cognitive radio networks, MIMO and cooperative communication networks, sensor networks, and ad hoc networks.

Dr. Shi was a recipient of IEEE INFOCOM 2008 Best Paper Award. He was a recipient of Chinese Government Award for Outstanding Ph.D. Students Abroad in 2006. While an undergraduate, he was a recipient of Meritorious Award in International Mathematical Contest in Modeling in 1997 and 1998, respectively. He served as a TPC member for major international conferences (including ACM MobiHoc 2009 and IEEE INFOCOM 2009–2011).

**Y. Thomas Hou** (S'91–M'98–SM'04) received his Ph.D. degree in Electrical Engineering from Polytechnic Institute of New York University in 1998. From 1997 to 2002, Dr. Hou was a Researcher at Fujitsu Laboratories of America, Sunnyvale, CA. Since 2002, he has been with Virginia Polytechnic Institute and State University ("Virginia Tech"), the Bradley Department of Electrical and Computer Engineering, Blacksburg, VA, where he is now an Associate Professor.

Prof. Hou's research interests are cross-layer design and optimization for cognitive radio wireless networks, cooperative communications, MIMO-based ad hoc networks, and new interference management schemes for wireless networks. He was a recipient of an Office of Naval Research (ONR) Young Investigator Award (2003) and a National Science Foundation (NSF) CAREER Award (2004) for his research on optimizations and algorithm design for wireless ad hoc and sensor networks.

Prof. Hou is currently serving as an Area Editor of *IEEE Transactions on Wireless Communications*, and Editor for *IEEE Transactions on Mobile Computing*, *IEEE Wireless Communications*, *ACM/Springer Wireless Networks (WINET)*, and *Elsevier Ad Hoc Networks.* He was a past Associate Editor of *IEEE Transactions on Vehicular Technology.* He was Technical Program Co-Chair of IEEE INFOCOM 2009.

Prof. Hou recently co-edited a textbook titled *Cognitive Radio Communications and Networks: Principles and Practices,* which was published by Academic Press/Elsevier, 2010.

**Sastry Kompella** (S'04–M'07) received the Ph.D. degree in electrical and computer engineering from Virginia Polytechnic Institute and State University, Blacksburg, in 2006. Currently, he is a Senior Research Scientist in the Information Technology Division at the U.S. Naval Research Laboratory, Washington, DC. His research focuses on scheduling, cross-layer optimization and cooperation in wireless and cognitive radio networks.