A DoF-based Link Layer Model for Multi-hop MIMO Networks

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Abstract—The rapid advances of MIMO to date have mainly stayed at the physical layer. Such fruits have not fully benefited MIMO research at the network layer mainly due to the computational complexity associated with the matrix-based model that MIMO involves. Recently, there have been some efforts to simplify link layer model for MIMO so as to facilitate research at the upper layers. These models only require simple numeric computations on MIMO's degrees-of-freedom (DoFs) to characterize spatial multiplexing (SM) and interference cancellation (IC). Thus, these models are much simpler than the original matrix-based model from the communications world. However, achievable DoF regions of these DoF-based models are not analyzed. In this paper, we re-visit this important problem of MIMO modeling. Based on accounting of how DoFs are consumed for SM and IC, we develop a tractable link layer model for multi-hop MIMO networks. We show that under common assumptions of DoF-based models and additional assumption of no dependency cycle, this model includes all the feasible solutions by the matrix-based model under SM and IC for any network topology. This work offers an important building block for theoretical research on multi-hop MIMO networks.

Index Terms—MIMO, link layer, spatial multiplexing, interference cancellation, degree-of-freedom (DoF), achievable DoF region, ad hoc network

1 Introduction

MIMO is a powerful physical layer technology to increase link capacity [2], [6], [7], [26], [27]. However, most of the technological advances of MIMO to date have stayed at the physical layer [2], [11], [13], [14], [15], [27]. Although there are some early efforts on exploiting MIMO's benefits for MAC and routing schemes [10], [25], fundamental understanding and optimal results (even in certain limited settings) on translating MIMO capability to upper layers remain very limited. The major technical barrier in this stagnation is the lack of a tractable and accurate MIMO model that is amenable for cross-layer optimization. Existing models for MIMO based on physical layer channel gain matrices, although accurate, are cumbersome to handle, due to the computational complexity associated with matrix manipulations. As a result, networking research based on these models has resulted in very limited success [5], [16].

Recognizing the difficulties in dealing with MIMO channel gain matrices, some researchers attempted to simplify MIMO models for upper-layer networking research (see [1], [9], [17], [21], [24]). Instead of modeling the maximum achievable rate region (or the capacity region) under all possible MIMO schemes, these models focus on a particular MIMO scheme of spatial multiplexing (SM) and interference cancellation (IC) [4], [18], [23] and its degrees-of-freedom (DoFs) representation [27, Chapter 7]. Under this approach, a node can exploit its DoFs for either SM or IC[1] so that more data streams can be achieved. Instead of carrying complex manipulations on matrices, DoF-based MIMO models only require simple numeric computations to identify a feasible DoF region, with each DoF corresponds to one data stream. Such models have since been applied to study throughput optimization problems [1], [9], [17] and to design MAC protocols [21], [24].

1.1 Limitation of Existing DoF-based Models

Although DoF-based MIMO modeling offers significant advantages over traditional matrix-based representation, such an approach has its own limitations. Most existing DoF models are based on zero forcing scheme and do not consider array and diversity gain. As a result, even if we find the best DoF-based model, it is still sub-optimal from capacity perspective. This is the price to pay for using the DoF-based approach.

Moreover, the achievable DoF regions by the existing DoF-based models have not been carefully analyzed

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1. Since interference among neighboring links can be cancelled by using MIMO's DoFs, several links can be active simultaneously in the same vicinity. This is also known as *spatial reuse* [9], [18].

and may be much smaller than that by the matrix-based model under SM and IC. Instead, most existing DoF-based models [1], [9], [17], [21], [24] focused on identifying *sufficient conditions* for feasible data streams under SM and IC. In particular, Bhatia and Li [1] stated that IC could be done by both transmitters and receivers. In [21], Park *et al.* stated that to have a newly active transmission join other ongoing transmissions, one could let the newly active transmitter and receiver to cancel interference. Sundaresan *et al.* [24] stated that IC could be done by receivers only. Hamdaoui and Shin [9] stated that for each interference between two links, one of the two nodes (the transmitter of one link and the receiver of the other link) is sufficient to cancel this interference (see CiM in [9]). However, there is an unfortunate error in their CiM modeling that have both transmitter and receiver use their DoFs for IC. Correct CiM equations are not available. Therefore, none of DoF-based models in [1], [9], [21], [24] is optimal (i.e., achieve the same DoF region as that by the matrix-based model under SM and IC). The DoF model in [3] was said to be optimal but there was no proof on either feasibility or optimality. In [17], we proposed a node-level ordering scheme to identify which node should perform IC. In particular, a transmitter should cancel its interference to all non-intended receivers that are before itself in the node order, while a receiver should cancel interference from all non-intended transmitters before itself in the node order. A model based on a node-level ordering can guarantee feasibility. Such a model was shown to achieve a larger feasible DoF region than that by previous DoF-based models. However, there was no theoretical result on its performance.

1.2 Main Contributions

The goal of this paper is to develop a DoF-based link layer model for multi-hop MIMO networks, which can provide some performance guarantee on its achievable DoF region for any network topology under certain assumptions. Specifically, recognizing that the maximum rate region by considering all possible MIMO schemes (e.g., multi-user detection, dirty paper coding) is still an open problem, our investigation will be limited in the scope of SM and IC, i.e., our model is based on zero forcing scheme and does not consider array and diversity gain. Further, our model does not include solutions with dependency cycles. In Section 4.3, we will present more details on this limitation and the reason to introduce this assumption. Under the above assumptions for MIMO, our main contributions can be summarized as follows.

- We start from the matrix-based model under SM and IC to formally derive how DoFs are consumed in a multi-hop MIMO network. We find that DoF consumption in IC only needs to involve either the transmit weight vector or the receive weight vector. To ensure feasibility, the vectors involved in IC can be determined by a vector-level ordering.
- We prove that for the purpose of achieving the same DoF region, it is sufficient to work with a "node-level" ordering instead of a vector-level ordering.
- Based on the above analysis, we propose a DoF-based link layer model for any multi-hop MIMO network topology. This model encompasses the ordering of transmitting and receiving nodes and the number of DoFs consumed for both SM and IC. Same as previous DoF-based models, our model only requires simple numeric computations to characterize a feasible DoF region for a multi-hop MIMO network. But unlike previous DoF models, our model is proved to include any feasible solution by the matrix-based model under SM and IC when there is no dependency cycle.
- To show the application of our DoF model, we apply it to a cross-layer optimization problem for a multi-hop MIMO network as a case study. We show that the resulting problem has a similar mathematical structure as that for a single-antenna network. We use CPLEX solver to offer some numerical results in the case study and leave it as future research to develop efficient solutions.

1.3 Paper Organization

The remainder of this paper is organized as follows. In Section 2, we review existing DoF-based models. In Section 3, we offer necessary background on MIMO and the matrix-based model under SM and IC. Section 4 analyzes DoF consumption in the matrix-based model under SM and IC. In Section 5, we develop our new DoF-based link layer model for multi-hop MIMO networks. We also compare our DoF model to the matrix-based model and a previous model in terms of DoF region and complexity. In Section 6, we apply our model to study a cross-layer optimization problem for a multi-hop network. Section 7 concludes this paper.

2 Related Work

Existing DoF-based MIMO models assume that the number of DoFs at a node is equal to the number of antennas at this node. In [1], Bhatia and Li stated that if the number of DoFs at a receiver is no fewer than the total number of its incoming data streams plus all interfering data streams, then this receiver can receive its incoming data streams and cancel interference from all other transmitters. Similarly, if the number of DoFs at a transmitter is no fewer than the total number of its transmitting data streams plus all data streams being interfered by its transmission, then this transmitter can transmit its data streams and cancel its interference to all other receivers. Thus, a DoF-based model can be built based on the above two sufficient conditions. The benefit of this model is that we do not need to find transmission and receive weight vectors (physical layer issues), which involves complex manipulations on matrices. However, the problem with this model is that IC is done by both transmitter and receiver, which is wasteful of DoF resources.

Park *et al.* [21] stated that to have a newly active transmission join other ongoing transmissions, one can let the newly active transmitter and receiver to cancel interference. SM was not considered in [21]. Thus, if the number of DoFs at the newly active receiver is no fewer than one plus the number of all interfering transmissions, then this receiver can receive its data stream and cancel interference from all other transmitters. Similarly, if the number of DoFs at the newly active transmitter is no fewer than one plus the number of all interfered transmissions, then this transmitter can transmit its data stream and cancel its interference to all other receivers. Although this model could achieve a larger feasible DoF region than that in [1], potential benefits from adjusting the DoFs at other active transmitters and receivers were not explored. In [24], Sundaresan *et al.* stated that it is sufficient to have the receivers meet the requirement as in [1] (i.e., the number of DoFs at each receiver is no less than the total number of its receiving data streams plus all interfering data streams); the requirement on transmitters can however be relaxed by merely having each transmitter's DoFs be no fewer than the total number of its transmitting data streams (without adding the number of data streams being interfered by its transmission). This corresponds to a scheme where IC is done by receivers only. This model could also achieve a larger feasible DoF region than that in [1]. However, since IC was only done by the receivers, potential benefits of DoFs at the transmitters were not explored.

To fully explore DoFs at all nodes, Hamdaoui and Shin [9] stated that for each interference between two links, one of the two nodes (the transmitter of one link and the receiver of the other link) is sufficient to cancel this interference (see CiM in [9]). However, there was an unfortunate error in the CiM equations where both transmitter and receiver use their DoFs for IC. Since correct CiM modeling is not available, the only available model in [9] is NiM, which is the same as the approach in [1]. In [3], Blough *et al.* built a DoF model based on the assumption that one can arbitrarily select a transmitter or a receiver to perform IC. However, no proof was given on either feasibility or optimality in [3].

In [17], we proposed a node-level ordering scheme to identify which node should perform IC. In particular, we required that a transmitter cancels its interference to all non-intended receivers before itself in the node order and a receiver cancels interference from all non-intended transmitters before itself in the node order. Although a model based on node-level ordering was shown to achieve a larger feasible DoF region than previous DoF-based models, there was no theoretical result on its performance.

3 Link Layer Model for MIMO Networks: A Primer

In this section, we formalize a matrix-based model for linear MIMO transceiver [4], [18], [23] under the scope of SM and IC and discuss its limitation. This model will also serve as a starting point in our new DoF-based modeling in Sections 4 and 5.

Consider a multi-hop MIMO network with N nodes. Suppose that there are L possible links in this network. Denote Tx(l) and Rx(l) the transmitter and receiver of link l, $1 \le l \le L$, respectively. The number of antennas at nodes Tx(l) and Rx(l) are denoted as $A_{\text{Tx}(l)}$ and $A_{\text{Rx}(l)}$, respectively. Due to potential interference, these links may not be active at the same time. We consider a time slot based scheduling. That is, we consider a time frame with T equal-length time slots and within a time slot t, $1 \le t \le T$, only a subset of these L links can be active. Since a MIMO link can support multiple data streams by SM, we denote $z_l[t]$ the number of data streams on link l in time slot t. Then the average DoF of each link l over T time slots is

$$c_l = \frac{1}{T}\sum_{t=1}^{T} z_l[t] \quad (1 \le l \le L)\,. \tag{1}$$

We now describe SM and IC and their constraints.

SM. Spatial multiplexing refers that a transmitter multiplexes several data streams in spatial domain when sending to its receiver. For a link l in time slot t, denote $s_{lj}[t]$ the signal of data stream j, $1 \le j \le z_l[t]$. To transmit $z_l[t]$ data streams, transmitter Tx(l) chooses an $A_{\text{Tx}(l)} \times 1$ transmit weight vector $\mathbf{u}_{lj}[t]$ for each data stream j and sends the combined signal vector $\sum_{j=1}^{z_l[t]} \mathbf{u}_{lj}[t] s_{lj}[t]$ through its $A_{\text{Tx}(l)}$ antennas. Denote $\mathbf{H}_{(l,l)}$ the $A_{\text{Tx}(l)} \times A_{\text{Rx}(l)}$ channel gain matrix between nodes Tx(l) and Rx(l), which is assumed full rank.[2] The signal vector at receiver Rx(l)'s $A_{\text{Rx}(l)}$ antennas is $(\sum_{j=1}^{z_l[t]} \mathbf{u}_{lj}[t] s_{lj}[t])^T \mathbf{H}_{(l,l)}$. Receiver Rx($l$) uses an $A_{\text{Rx}(l)} \times 1$ receive weight vector $\mathbf{v}_{li}[t]$ to receive data stream i, $1 \le i \le z_l[t]$. The received signal $r_{li}[t]$ for data stream i is

$$\begin{aligned} r_{li}[t] &= \left(\sum_{j=1}^{z_l[t]} \mathbf{u}_{lj}[t] s_{lj}[t]\right)^T \mathbf{H}_{(l,l)} \mathbf{v}_{li}[t] \\ &= ((\mathbf{u}_{li}[t])^T \mathbf{H}_{(l,l)} \mathbf{v}_{li}[t]) \cdot s_{li}[t] \\ &\quad + \sum_{1 \le j \le z_l[t]}^{j \ne i} ((\mathbf{u}_{lj}[t])^T \mathbf{H}_{(l,l)} \mathbf{v}_{li}[t]) \cdot s_{lj}[t]\,. \end{aligned}$$

By choosing appropriate $\mathbf{u}$ and $\mathbf{v}$ vectors, the received signal $r_{li}[t]$ can achieve a unit gain (i.e., $(\mathbf{u}_{li}[t])^T \mathbf{H}_{(l,l)} \mathbf{v}_{li}[t] = 1$) and zero interference (i.e., $(\mathbf{u}_{lj}[t])^T \mathbf{H}_{(l,l)} \mathbf{v}_{li}[t] = 0, \forall i \ne j$) such that the data

2. This holds when the scattering in the environment is sufficiently rich, e.g., an urban environment. Note that all channel matrices are assumed full rank in this paper, which is a common assumption for DoF-based models (see, e.g., [9], [21]).

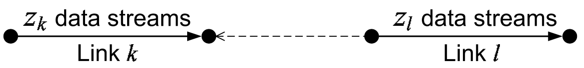


Fig. 1. Interference cancellation between two MIMO links.

stream i can be successfully received. Thus, we have the following SM constraints for each link l, $1 \le l \le L$.

$$(\mathbf{u}_{li}[t])^T \mathbf{H}_{(l,l)} \mathbf{v}_{li}[t] = 1 \qquad (1 \le i \le z_l[t]) \tag{2}$$

$$(\mathbf{u}_{lj}[t])^T \mathbf{H}_{(l,l)} \mathbf{v}_{li}[t] = 0 \quad (1 \le i, j \le z_l[t], j \ne i)\ . \tag{3}$$

IC. In addition to SM, MIMO nodes can cancel interference so that several links can be active simultaneously in the same vicinity.[3] This is also known as *spatial reuse* [9], [18]. We now consider two links l and k in a time slot t (see Fig. 1), where the receiver on link k is interfered by the transmitter on link l. As discussed, transmitter Tx(l) sends the combined signal vector $\sum_{i=1}^{z_l[t]} \mathbf{u}_{li}[t] s_{li}[t]$ through its $A_{\mathrm{Tx}(l)}$ antennas. Denote $\mathbf{H}_{(l,k)}$ the full-rank $A_{\mathrm{Tx}(l)} \times A_{\mathrm{Rx}(k)}$ channel gain matrix between nodes Tx(l) and Rx(k). The interference at receiver Rx(k)'s $A_{\mathrm{Rx}(k)}$ antennas is $(\sum_{i=1}^{z_l[t]} \mathbf{u}_{li}[t] s_{li}[t])^T \mathbf{H}_{(l,k)}$. Receiver Rx($k$) uses an $A_{\mathrm{Rx}(k)} \times 1$ receive weight vector $\mathbf{v}_{kj}[t]$ to receive data stream j from transmitter Tx(k), $1 \le j \le z_k[t]$. The interference to data stream j is

$$\left(\sum_{i=1}^{z_l[t]} \mathbf{u}_{li}[t] s_{li}[t]\right)^T \mathbf{H}_{(l,k)} \mathbf{v}_{kj}[t] = \sum_{i=1}^{z_l[t]} ((\mathbf{u}_{li}[t])^T \mathbf{H}_{(l,k)} \mathbf{v}_{kj}[t]) \cdot s_{li}[t].$$

In order to cancel the interference on each data stream j, the following IC constraints must be satisfied:

$$(\mathbf{u}_{li}[t])^T \mathbf{H}_{(l,k)} \mathbf{v}_{kj}[t] = 0 \ (1 \le i \le z_l[t], 1 \le j \le z_k[t])\ . \tag{4}$$

Based on the above discussion, a set of values for $(c_1, c_2, \cdots, c_L)$ is feasible if and only if we can find a feasible solution for all the transmit weight vectors and receive weight vectors in each time slot such that (1), (2), (3), and (4) hold. Note that although this matrix-based MIMO model is optimal in terms of identifying all feasible sets of values for $(c_1, c_2, \cdots, c_L)$ under SM and IC, its practical utility as an analytic tool is extremely limited. There are two troubling issues with this model. First, to obtain the DoF region by (1), one needs to verify the feasibility of each set of values for $(z_1[t], z_2[t], \cdots, z_L[t])$ by (2), (3), and (4). Note that each set of values for $(z_1[t], z_2[t], \cdots, z_L[t])$ yields a different set of constraints and variables in (2), (3), and (4). Since one has to solve a different problem for each set of values for $(z_1[t], z_2[t], \cdots, z_L[t])$, the number of problems that need to be solved is exponential with L. Second, verifying the feasibility of a given set of values for $(z_1[t], z_2[t], \cdots, z_L[t])$ requires to solve a problem with a large number of bilinear equations (2), (3), and (4). Unlike linear equation systems, a general solution to bilinear equation systems remains unknown [12]. In Section 5.4, we will show the high complexity of the matrix-based model even for small-sized networks.

3. IC discussed in this paper is different from successive interference cancellation (SIC) in [28, Chapter 7]. SIC needs to decode interference before performing cancellation while IC does not require that interference be decoded first. On the other hand, IC requires multiple antennas at each node while SIC does not.

4 Understanding DoF Consumption in the Matrix-based Model

Before we construct a DoF-based link layer model for multi-hop MIMO networks, we must have a deep understanding of DoF consumption in the matrix-based model and have accurate accounting of the number of DoFs consumed for SM and IC, respectively.

4.1 Basic Idea

First, let's determine the total available DoFs for a transmit (or receive) weight vector at a node, which is associated for each data stream transmitted (or received) at this node. Initially, there is no constraint at a vector. Then each of its elements is undetermined and can be set arbitrarily. There is a feasible region (a space) that includes all possible values by such an unconstrained vector. The DoFs of this feasible region is equal to the number of elements in the unconstrained vector (or the number of antennas at the node). A vector's total available DoFs is defined as this initial DoFs.[4]

We now show how DoFs are consumed when constraints are imposed on a vector. This can be illustrated by a substraction approach, i.e., first finding the vector's total available DoFs, then finding the vector's remaining DoFs (after constraints are imposed), and finally subtracting this remaining DoFs from the total available DoFs. Here, a vector's remaining DoFs is the DoFs of its feasible region after constraints are imposed, which is equal to the number of its linearly independent elements.

Alternatively, the number of consumed DoFs can be determined directly by analyzing the given constraints. Note that constraints considered in this paper are all linear constraints. A linear constraint can either set some element in a vector to a value (e.g., $x_1 = 1$) or set some linear relationship among multiple elements (e.g., $x_2 + x_3 - x_4 = 0$). In either case, the number of vector's DoFs is decreased by 1. When there is no linear dependency among the given constraints, the number of consumed DoFs is equal to the number of constraints. When there is linear dependency among the given constraints, we should consider a subset of linearly independent constraints, and the number of consumed DoFs is equal to the number of linearly independent constraints. As an example, consider a vector $[x_1, x_2, x_3, x_4, x_5]^T$ and the following three constraints.

$$2x_1 = 1 \tag{5}$$

$$2x_1 + x_3 + 5x_4 = 2 \tag{6}$$

$$6x_1 + x_3 + 5x_4 = 4 \tag{7}$$

4. Note that all vectors at a node have the same number of initial DoFs, which is equal to the number of antennas at this node. This number is called the node's DoFs in [1], [9], [21], [24].

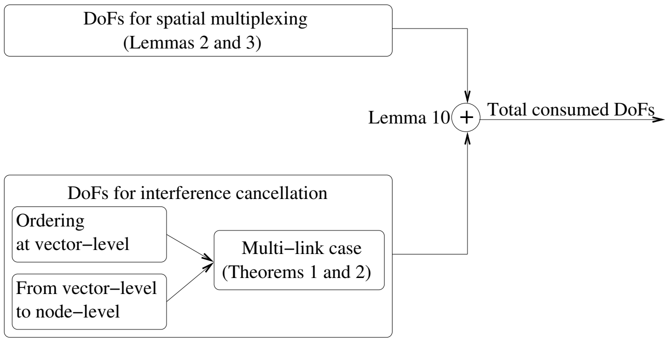


Fig. 2. Our roadmap for DoF analysis.

Since (7) is a linear combination of (5) and (6), we have only two independent constraints. Thus, the number of DoFs consumed by these constraints is 2. We summarize our discussion in the following lemma.

Lemma 1: The number of consumed DoFs of a vector due to a set of linear constraints among its elements is equal to the number of linearly independent constraints in this set.

With Lemma 1, we now analyze DoF consumption in the matrix-based model. Our roadmap for DoF analysis is shown in Fig. 2. Since the constraints for a transmit/receive vector are due to SM and IC, we will analyze each case in Sections 4.2 and 4.3, respectively. In Section 4.4, we show that the total consumed DoFs is the sum of DoFs consumed by SM and IC.

4.2 DoF Consumption by SM

We now analyze DoF consumption by SM in transmit and receive weight vectors in the matrix-based model (see the top block in Fig. 2). For a time slot t, we first consider a transmit weight vector $\mathbf{u}_{li}$ at transmitter Tx(l). For simplicity, we omit time slot $[t]$ in this section, e.g., using $\mathbf{u}_{li}$ instead of $\mathbf{u}_{li}[t]$. Under SM constraints (2) and (3), $\mathbf{u}_{li}$ must satisfy the following constraints.

$$\mathbf{u}_{li}^T(\mathbf{H}_{(l,l)}\mathbf{v}_{li}) = 1 \tag{8}$$

$$\mathbf{u}_{li}^T(\mathbf{H}_{(l,l)}\mathbf{v}_{lj}) = 0 \quad (1 \le j \le z_l, j \ne i) \tag{9}$$

All the constraints in (8) and (9) are linear constraints. By Lemma 1, we need to analyze linear dependency among these constraints. We find that these constraints are all linearly independent. Thus, we have the following lemma.

Lemma 2: Denote the number of data streams on link l as z_l. Then the number of DoFs consumed by SM in each transmit weight vector $\mathbf{u}_{li}$ at transmitter Tx(l) is z_l.

Proof: By Lemma 1, we need to prove that all SM constraints in (8) and (9) are linearly independent. We prove this by contradiction.

Suppose that we can represent one constraint in (8) or (9) as a linear combination of other constraints. Note that the right-hand-sides (RHS) of all constraints in (9) are zero and thus the RHS of their linear combination must be zero. Then we cannot represent (8) as a linear combination of constraints in (9). Now suppose that we can represent a constraint in (9), say $\mathbf{u}_{li}^T(\mathbf{H}_{(l,l)}\mathbf{v}_{lj}) = 0$, as a linear combination of other constraints. Since any linear combination that includes (8) has a non-zero RHS, the linear combination for $\mathbf{u}_{li}^T(\mathbf{H}_{(l,l)}\mathbf{v}_{lj}) = 0$ cannot include (8). Thus, we have $\mathbf{H}_{(l,l)}\mathbf{v}_{lj} = \sum_{1\le m\le z_l}^{m\ne i,j} w_m \cdot \mathbf{H}_{(l,l)}\mathbf{v}_{lm}$ under some weights w_m. To find a contradiction, multiplying $\mathbf{u}_{lj}^T$ on both sides, we have

$$\begin{aligned}
\mathbf{u}_{lj}^T\mathbf{H}_{(l,l)}\mathbf{v}_{lj} &= \mathbf{u}_{lj}^T \sum_{1\le m\le z_l}^{m\ne i,j} w_m \cdot \mathbf{H}_{(l,l)}\mathbf{v}_{lm} \\
&= \sum_{1\le m\le z_l}^{m\ne i,j} w_m \cdot \mathbf{u}_{lj}^T\mathbf{H}_{(l,l)}\mathbf{v}_{lm} \\
&= \sum_{1\le m\le z_l}^{m\ne i,j} w_m \cdot 0 = 0 \, ,
\end{aligned}$$

where the third equality holds by (3). However, this is a contradiction since we have $\mathbf{u}_{lj}^T\mathbf{H}_{(l,l)}\mathbf{v}_{lj} = 1$ by (2). This proves that all SM constraints in (8) and (9) are linearly independent. □

Lemma 2 can be explained intuitively as follows. The constraints in (8) and (9) ensure multiple orthogonal channels in the spatial domain. Since each data stream should be transmitted in its own channel, the total number of channels required (corresponding to the number of consumed DoFs) is equal to the number of data streams z_l.

Now we consider a receive weight vector. Following the same token as for a transmit weight vector, we can prove the following lemma.

Lemma 3: For a link l with z_l data streams, the number of DoFs consumed by SM in each receive weight vector $\mathbf{v}_{lj}$ at receiver Rx(l) is z_l.

Lemma 3 can also be intuitively explained by that z_l data streams will need z_l channels in the spatial domain and thus the number of consumed DoFs in each receive weight vector is z_l.

4.3 DoF Consumption by IC

We now analyze DoF consumption by IC in transmit and receive weight vectors in the matrix-based model (see the bottom block in Fig. 2). It turns out that unlike SM, DoFs consumed by IC only involve either a transmit weight vector or a receive weight vector, but not both. Now a new problem is: Which vector (transmit or receive weight vector) should consume its DoFs for IC? We show that one cannot arbitrarily select a vector to consume its DoFs for IC. Otherwise, one may end up with some infeasible solution. To ensure feasibility, we consider an order-based approach, which follows some *order* to determine all the vectors. That is, when we determine a vector, this vector is determined by considering related IC constraints and previously determined vectors. Under this order-based approach, a vector determined later should consume its DoFs for IC. Formal results for these findings are given in the rest of this section.

Based on the ordering concept, we can build a mathematical model by calculating DoF consumption for all vectors. However, such "vector-level" model will involve many variables and constraints and is cumbersome to work with. A good question to ask is: Can we simplify this vector-level model without any loss of DoF region? We find that it is sufficient to consider a "node-level" ordering instead of a vector-level ordering. Such a node-level operation can significantly decrease the number of variables and constraints. Further, we prove that a model based on node-level ordering can achieve the same DoF region as that by a model based on vector-level ordering.

We organize this section as follows. We start with the simple $1 \to 1$ case, where one link interferes the other link. The ordering concept and the transition from vector-level ordering to node-level ordering are introduced here. We then present result for the general $M \to 1$ and $1 \to M$ cases, where multiple links interfere one link or one link interferes multiple links, respectively.

4.3.1 $1 \to 1$ *Case*

Let's consider the $1 \to 1$ case in Fig. 1, where the receiver Rx(k) of link k is interfered by the transmitter Tx(l) of a single link l and the transmitter Tx(l) of link l interferes the receiver Rx(k) of a single link k. Under the matrix-based model, some DoFs of transmit weight vectors at node Tx(l) or receive weight vectors at node Rx(k) will be consumed by IC constraints (4).

The Concept of Sequential Ordering. One IC constraint only consumes one DoF of a vector at either the transmitter or the receiver, but not both. Intuitively, this means that an IC constraint can be satisfied by (i) transmitter Tx(l) does not transmit on a channel (and thus receiver Rx(k) can receive on that channel) or (ii) receiver Rx(k) does not receive on a channel (and thus transmitter Tx(l) can transmit on that channel). As a result, either vector $\mathbf{u}_{li}$ or vector $\mathbf{v}_{kj}$ consumes one DoF.

We now determine which vector (transmit or receive weight vector) should consume its DoFs for each IC constraint. This can be done by analyzing dependency relationships among the vectors. When we consider all vectors, there may or may not exist dependency cycles among them. Ideally, we should include solutions regardless of dependency cycles in our model. However, there is no proof to guarantee that we can always find a feasible weight assignment for arbitrarily specified dependency cycles among the vectors. Since it is not clear how to specify dependency cycles to guarantee a feasible weight assignment, we decide not to model solutions with dependency cycles. That is, we focus on an order-based approach to determine all the vectors.

Denote such a sequential order of vectors as Π. For a transmit weight vector $\mathbf{u}_{li}$, denote $\Pi_{\mathbf{u}_{li}}$ the position of this vector in Π. Similarly, denote $\Pi_{\mathbf{v}_{kj}}$ the position of the receive weight vector $\mathbf{v}_{kj}$ in Π. Given that we have z_l transmit weight vectors and z_k receive weight vectors, the number of vectors in Π is $z_l + z_k$. To select a transmit weight vector $\mathbf{u}_{li}$, a transmitter only considers IC constraints for those receive weight vectors placed before $\mathbf{u}_{li}$ in Π. As a result, $\mathbf{u}_{li}$ consumes its DoFs for these IC constraints. On the other hand, the selection of $\mathbf{u}_{li}$ does not consider IC constraints for those receive weight vectors placed after $\mathbf{u}_{li}$ in Π. As a result, $\mathbf{u}_{li}$ does not consume its DoFs for these IC constraints. Instead, these IC constraints will be satisfied by selecting corresponding receive weight vectors after $\mathbf{u}_{li}$ is chosen. Similarly, a receive weight vector $\mathbf{v}_{kj}$ consumes its DoFs only for transmit weight vectors before itself in Π.

DoF Consumption Under A Sequential Order. We now analyze DoF consumption for a transmit weight vector and a receive weight vector, respectively.

Case A: Transmit weight vector $\mathbf{u}_{li}$. By (4), vector $\mathbf{u}_{li}$ must satisfy $\mathbf{u}_{li}^T(\mathbf{H}_{(l,k)}\mathbf{v}_{kj}) = 0$ for $1 \le j \le z_k$. Let's begin by considering one constraint $\mathbf{u}_{li}^T(\mathbf{H}_{(l,k)}\mathbf{v}_{kj}) = 0$ for a given j.

- If $\Pi_{\mathbf{v}_{kj}} < \Pi_{\mathbf{u}_{li}}$, then by the time we consider $\mathbf{u}_{li}$, vector $\mathbf{v}_{kj}$ has already been determined and we now have a linear constraint on $\mathbf{u}_{li}$, which decreases $\mathbf{u}_{li}$'s DoFs by one.
- On the other hand, if $\Pi_{\mathbf{u}_{li}} < \Pi_{\mathbf{v}_{kj}}$, i.e., $\mathbf{u}_{li}$ is before $\mathbf{v}_{kj}$ in Π, it is not possible to impose any constraint on $\mathbf{u}_{li}$ since $\mathbf{v}_{kj}$ is yet to be determined. Constraint $\mathbf{u}_{li}^T(\mathbf{H}_{(l,k)}\mathbf{v}_{kj}) = 0$ will be satisfied when we consider $\mathbf{v}_{kj}$ in the future. As a result, $\mathbf{u}_{li}$ does not need to concern itself with this constraint and will thus not consume any DoF.

Thus, to analyze $\mathbf{u}_{li}$'s DoF consumption, we only need to consider the following constraints: $\mathbf{u}_{li}^T(\mathbf{H}_{(l,k)}\mathbf{v}_{kj}) = 0$ for $1 \le j \le z_k$ and $\Pi_{\mathbf{v}_{kj}} < \Pi_{\mathbf{u}_{li}}$. The number of these constraints is equal to the number of receive weight vectors that are placed before $\mathbf{u}_{li}$ in Π. Further, we verify that these constraints are all linearly independent (see Proof of Lemma 4). Therefore, the number of DoFs consumed by IC in a transmit weight vector $\mathbf{u}_{li}$ is equal to the number of receive weight vectors that are placed before $\mathbf{u}_{li}$ in Π.

Case B: Receive weight vector $\mathbf{v}_{kj}$. Following the same token, we have that the number of DoFs consumed by IC at a receive weight vector $\mathbf{v}_{kj}$ is equal to the number of transmit weight vectors that are placed before $\mathbf{v}_{kj}$ in Π.

The following lemma summarizes our discussion.

Lemma 4: Consider the interference from transmitter Tx(l)'s z_l data streams to receiver Rx(k)'s z_k data streams and an order Π for vectors. Based on IC constraint (4) in the matrix-based model, we have (i) for a transmit weight vector $\mathbf{u}_{li}$, the number of DoFs consumed by IC in $\mathbf{u}_{li}$ is equal to the number of receive weight vectors at Rx(k) that are placed before $\mathbf{u}_{li}$ in Π; (ii) for a receive weight vector $\mathbf{v}_{kj}$, the number of DoFs consumed by IC in $\mathbf{v}_{kj}$ is equal to the number of transmit weight vectors at Tx(l) that are placed before $\mathbf{v}_{kj}$ in Π.

Proof: This proof is based on mathematical induction. Without loss of generality, we assume that the last vector in Π is a transmit weight vector. We further assume that the last $x_1 \geq 1$ vectors are all transmit weight vectors. Denote the set of these vectors as $\mathcal{X}_1$. Suppose that the next $y_1 \geq 1$ vectors are all receive weight vectors (denote as $\mathcal{Y}_1$), the next $x_2 \geq 1$ vectors are all transmit weight vectors (denote as $\mathcal{X}_2$), the next $y_2 \geq 1$ vectors are all receive weight vectors (denote as $\mathcal{Y}_2$), etc. We will first prove that Lemma 4 holds for vectors in $\mathcal{X}_1$. We then prove that if Lemma 4 holds for vectors in $\mathcal{X}_j$, $1 \leq j \leq i+1$, and $\mathcal{Y}_j$, $1 \leq j \leq i$, then Lemma 4 holds for vectors in $\mathcal{Y}_{i+1}$. Moreover, if Lemma 4 holds for vectors in $\mathcal{X}_j$ and $\mathcal{Y}_j$, $1 \leq j \leq i$, then Lemma 4 holds for vectors in $\mathcal{X}_{i+1}$. Once all these results are proved, we have Lemma 4 holds for all transmit/receive weight vectors.

We first prove that Lemma 4 holds for vectors in $\mathcal{X}_1$. When we determine these transmit weight vectors, all receive weight vectors are already determined. Thus, each $\mathbf{u}_{li} \in \mathcal{X}_1$ needs to satisfy the following z_k constraints.

$$\mathbf{u}_{li}^T(\mathbf{H}_{(l,k)}\mathbf{v}_{kj}) = 0 \quad (1 \leq j \leq z_k) \ .$$

We assume a rich scattering environment, which is a common assumption made by existing DoF models. Then $\mathbf{H}_{(l,k)}$ is of full rank and the above constraints are linearly independent under given receive weight vectors $\mathbf{v}_{kj}$, $1 \leq j \leq z_k$. Thus by Lemma 1, each $\mathbf{u}_{li} \in \mathcal{X}_1$ consumes z_k DoFs for IC, where z_k is the number of receive weight vectors that are placed before $\mathbf{u}_{li}$ in Π.

We now prove that Lemma 4 holds for vectors in $\mathcal{Y}_1$, given that Lemma 4 holds for vectors in $\mathcal{X}_1$. When we determine these receive weight vectors in $\mathcal{Y}_1$, the number of transmit weight vectors that are already determined is $z_l - x_1$. Thus, each of $\mathbf{v}_{kj} \in \mathcal{Y}_1$ needs to satisfy the following $z_l - x_1$ constraints.

$$(\mathbf{u}_{li}^T\mathbf{H}_{(l,k)})\mathbf{v}_{kj} = 0 \quad (1 \leq i \leq z_l, \mathbf{u}_{li} \notin \mathcal{X}_1) \ .$$

For full-rank $\mathbf{H}_{(l,k)}$ and given transmit weight vectors $\mathbf{u}_{li}$, $1 \leq i \leq z_l, \mathbf{u}_{li} \notin \mathcal{X}_1$, these constraints are linearly independent. Thus, each $\mathbf{v}_{kj} \in \mathcal{Y}_1$ consumes $z_l - x_1$ DoFs for IC, where $z_l - x_1$ is the number of transmit weight vectors that are placed before $\mathbf{v}_{kj}$ in Π.

Following the same token, we can prove that if Lemma 4 holds for vectors in $\mathcal{X}_j$ and $\mathcal{Y}_j$, $1 \leq j \leq i$, then Lemma 4 holds for vectors in $\mathcal{X}_{i+1}$; and if Lemma 4 holds for vectors in $\mathcal{X}_j$, $1 \leq j \leq i+1$, and $\mathcal{Y}_j$, $1 \leq j \leq i$, then Lemma 4 holds for vectors in $\mathcal{Y}_{i+1}$. These proofs are omitted to conserve space.

Based on all these results, Lemma 4 is proved. □

Since different sequential order Π will yield different DoF consumption in a transmit/receive weight vector, such order Π should be subject to optimization in a particular problem.

From Vector-Level to Node-Level. The sequential order in Lemma 4 is on vector level. A model based on such a vector-level ordering would have too many variables and constraints, which is cumbersome to work with. To simplify the model, we now consider a special vector-level ordering, under which we visit each node following some sequential order π, and once we are at a node, we can determine all the vectors at this node following an arbitrary order. That is, now we have an order π among nodes and when at a node, some order for vectors at this node. For this special vector-level order, it is easy to verify that, by Lemma 4, all vectors at the same node will have the same DoF consumption and the order among the vectors at the same node does not affect DoF consumption.

As a result of this finding, we may consider a "node-level" ordering π among the nodes in the network. There is no need to consider the ordering among the vectors at the same node. For a transmitter Tx(l), denote $\pi_{\text{Tx}(l)}$ the position of this node in π. Similarly, denote $\pi_{\text{Rx}(k)}$ the position of a receiver Rx(k) in π. We have the following lemma.

Lemma 5: Consider the interference from transmitter Tx(l)'s z_l data streams to receiver Rx(k)'s z_k data streams and an order π for nodes. Based on IC constraint (4) in the matrix-based model, we have (i) if $\pi_{\text{Tx}(l)} > \pi_{\text{Rx}(k)}$, then the number of DoFs consumed by IC are z_k and 0 at Tx(l) and Rx(k), respectively; (ii) if $\pi_{\text{Tx}(l)} < \pi_{\text{Rx}(k)}$, then the number of DoFs consumed by IC are 0 and z_l at Tx(l) and Rx(k), respectively.

Proof: We first consider (i), i.e., the case when $\pi_{\text{Tx}(l)} > \pi_{\text{Rx}(k)}$. We employ such a vector-level ordering Π by assigning an arbitrary order to all the vectors at the same node. For any transmit weight vector $\mathbf{u}_{li}$ and any receive weight vector $\mathbf{v}_{kj}$, we always have $\Pi_{\mathbf{v}_{kj}} < \Pi_{\mathbf{u}_{li}}$ (due to $\pi_{\text{Tx}(l)} > \pi_{\text{Rx}(k)}$ in the node-level order π). Thus, by Lemma 4, the number of consumed DoFs at a transmit weight vector $\mathbf{u}_{li}$ for IC is equal to the number of all receive weight vectors z_k while the number of consumed DoFs at a receive weight vector $\mathbf{v}_{kj}$ for IC is 0. In other words, the number of consumed DoFs at transmitter Tx(l) for IC is z_k while the number of consumed DoFs at receiver Rx(k) for IC is 0. Case (i) is therefore proved.

The proof for (ii) follows the same token as that for (i) and is omitted to conserve space. □

For the DoF region achieved by a node-level ordering, we have the following lemma.

Lemma 6: For the $1 \to 1$ case, the achievable DoF region by the matrix-based model with a node-level ordering is the same as that under the matrix-based model with a vector-level ordering.

To prove this lemma, it is sufficient to show that for any feasible (z_l, z_k) values that can be achieved by the matrix-based model with a vector-level ordering Π, we can construct a node-level ordering π to achieve the same (z_l, z_k) values. Let's see the following example.

Example 1: Consider two links l and k with $z_l = 2$ and $z_k = 2$ data streams, respectively. Assume that there are 4 antennas on each of the transmitting and receiving nodes. Then the total available DoFs in each transmit or receive weight vector is 4. Suppose that the vector-level ordering Π is $\mathbf{u}_{l2}, \mathbf{v}_{k1}, \mathbf{u}_{l1}, \mathbf{v}_{k2}$ (see the first line in Fig. 3). We now show how to construct a node-level ordering π

A vector–level ordering: $\boldsymbol{u}_{l2}$ $\boldsymbol{v}_{k1}$ $\boldsymbol{u}_{l1}$ $\boldsymbol{v}_{k2}$

The constructed vector–level ordering: $\boldsymbol{u}_{l2}$ $\boldsymbol{u}_{l1}$ $\boldsymbol{v}_{k1}$ $\boldsymbol{v}_{k2}$

The constructed node–level ordering: Tx_l Rx_k

Fig. 3. An example showing the transition from vector-level ordering to node-level ordering.

based on Π such that $z_l = 2$ and $z_k = 2$ remain feasible.

Denote $DoF_l(\mathbf{v}_{kj})$ the DoFs consumed by IC in receive weight vector $\mathbf{v}_{kj}$. Based on Lemma 4, we have $DoF_l(\mathbf{v}_{k1}) = 1$ and $DoF_l(\mathbf{v}_{k2}) = 2$. Since $DoF_l(\mathbf{v}_{k1}) < DoF_l(\mathbf{v}_{k2})$, $\mathbf{v}_{k2}$ is the bottleneck receive weight vector at receiver Rx(k), which has the fewest remaining DoFs ($4-2=2$) for SM. Similarly, denote $DoF_k(\mathbf{u}_{li})$ the DoFs consumed for IC at transmit weight vector $\mathbf{u}_{li}$. Based on Lemma 4, we have $DoF_k(\mathbf{u}_{l2}) = 0$, and $DoF_k(\mathbf{u}_{l1}) = 1$. We can see that $\mathbf{u}_{l1}$ is the bottleneck transmit weight vector at transmitter Tx(l), which has the fewest remaining DoFs ($4-1=3$) for SM.

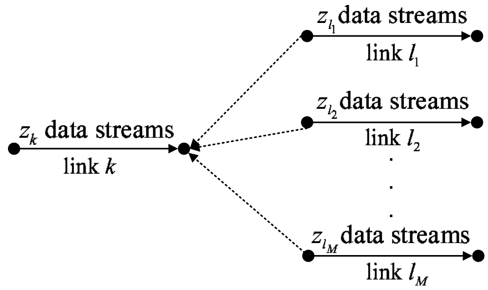
To construct a node-level ordering π, we first re-order the vectors as $\mathbf{u}_{l2}, \mathbf{u}_{l1}, \mathbf{v}_{k1}, \mathbf{v}_{k2}$, where the first two vectors are the transmit weight vectors and the remaining two vectors are the receive weight vectors (see the second line in Fig. 3). Based on Lemma 4, we have $DoF_l(\mathbf{v}_{k1}) = DoF_l(\mathbf{v}_{k2}) = 2$ under this new order. We find that although $DoF_l(\mathbf{v}_{k1})$ is increased from 1 to 2, $\mathbf{v}_{k2}$ remains a bottleneck receive weight vector. Thus, $z_k = 2$ is still feasible under this new ordering. Similarly, based on Lemma 4, we have $DoF_k(\mathbf{u}_{l2}) = DoF_k(\mathbf{u}_{l1}) = 0$ under this new order. Since $DoF_k(\mathbf{u}_{l1})$ is decreased from 1 to 0, this vector has more DoFs remaining than in Π. Thus, $z_l = 2$ is still feasible under this new order. In summary, (z_l, z_k) values remain feasible under the new ordering.

For this new vector-level order, if we group the first two transmit weight vectors at transmitter Tx(l) and the other two receive weight vectors at receiver Rx(k), then we have a node-level order (see the third row in Fig. 3). Note that for this node-level order, the DoF consumption in each vector at the same node is identical (i.e., $DoF_l(\mathbf{v}_{k1}) = DoF_l(\mathbf{v}_{k2}) = 2$ and $DoF_k(\mathbf{u}_{l2}) = DoF_k(\mathbf{u}_{l1}) = 0$). We call this node-level order π, which can achieve the same (z_l, z_k) values as in Π. □

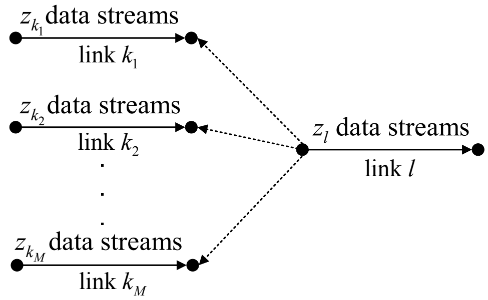
A formal proof of Lemma 6 based on the idea in Example 1 is omitted to conserve space.

4.3.2 $M \to 1$ and $1 \to M$ Cases

In general, we need to analyze (i) DoF consumption at a receiver that is being interfered by multiple other links (see the $M \to 1$ case in Fig. 4(a)), and (ii) DoF consumption at a transmitter that interferes multiple other links (see the $1 \to M$ case in Fig. 4(b)). When we determine vectors at different nodes, there may or may not exist dependency cycles among the vectors. Ideally, our model should include solutions regardless of dependency cycles. However, there is no proof to guarantee that we can always find a feasible weight assignment for arbitrarily specified dependency cycles in a multi-hop MIMO network. Since it is not clear how to specify dependency cycles to guarantee a feasible weight assignment, we decide not to model solutions with dependency cycles. That is, we focus on an order-based approach to determine all the vectors.



(a) One link (k) is interfered by multiple links ($l_1, l_2, \cdots, l_M$).



(b) One link (l) interferes multiple links ($k_1, k_2, \cdots, k_M$).

Fig. 4. The $M \to 1$ and $1 \to M$ cases.

We first analyze (i), which is illustrated in Fig. 4(a). There are z_k data streams on link k and M interfering links $l_1, l_2, \cdots, l_M$ with $z_{l_1}, z_{l_2}, \cdots, z_{l_M}$ data streams, respectively, where transmitter Tx(l_m) of each link l_m interferes receiver Rx(k) of link k. Under an order-based approach, only when $\Pi_{\mathbf{u}_{l_m,i}} < \Pi_{\mathbf{v}_{kj}}$, receive weight vector $\mathbf{v}_{kj}$ at receiver Rx(k) needs to consume its DoFs to cancel interference from the i-th data stream at transmitter Tx(l_m). Thus, among all the IC constraints (4) in the matrix-based model, vector $\mathbf{v}_{kj}$ only needs to satisfy the following constraints.

$$(\mathbf{u}_{l_m,i}^T \mathbf{H}_{(l_m,k)})\mathbf{v}_{kj} = 0 \quad (1 \le m \le M, 1 \le i \le z_{l_m}, \Pi_{\mathbf{u}_{l_m,i}} < \Pi_{\mathbf{v}_{kj}}) \quad (10)$$

Under the order-based approach, transmitters Tx(l_m) with $\Pi_{\mathbf{u}_{l_m,i}} < \Pi_{\mathbf{v}_{kj}}$ (for some i and j) cannot collaborate to determine their transmit weight vectors $\mathbf{u}_{l_m,i}$. Then it is unlikely that some IC constraints in (10) are linearly dependent. Under a given order Π, the total DoFs consumed for IC at $\mathbf{v}_{kj}$ is $\sum_{m=1}^{M}\sum_{i=1}^{z_{l_m}} 1^+\{\Pi_{\mathbf{u}_{l_m,i}} < \Pi_{\mathbf{v}_{kj}}\}$, where $1^+\{\Pi_{\mathbf{u}_{l_m,i}} < \Pi_{\mathbf{v}_{kj}}\}$ is an indicator function and is defined to be 1 when $\Pi_{\mathbf{u}_{l_m,i}} < \Pi_{\mathbf{v}_{kj}}$ and 0 otherwise. We state this result in the following lemma.

Lemma 7: For the scenario in Fig. 4(a), the number of DoFs consumed by IC in a receive weight vector $\mathbf{v}_{kj}$ at receiver Rx(k) is $\sum_{m=1}^{M}\sum_{i=1}^{z_{l_m}} 1^+\{\Pi_{\mathbf{u}_{l_m,i}} < \Pi_{\mathbf{v}_{kj}}\}$ under an order Π

for vectors.

We now analyze (ii), which is illustrated in Fig. 4(b). Similarly, under an order-based approach, a transmit weight vector $\mathbf{u}_{li}$ at Tx(l) only needs to satisfy the following constraints.

$$\mathbf{u}_{li}^T(\mathbf{H}_{(l,k_m)}\mathbf{v}_{k_m,j}) = 0 \quad (1 \le m \le M, 1 \le j \le z_{k_m}, \Pi_{\mathbf{v}_{k_m,j}} < \Pi_{\mathbf{u}_{li}}) \quad (11)$$

These receivers Rx(k_m) with $\Pi_{\mathbf{v}_{k_m,j}} < \Pi_{\mathbf{u}_{li}}$ for some i and j may or may not collaborate to determine their receive weight vectors $\mathbf{u}_{k_m,j}$. If they do not collaborate, then we have the following lemma.

Lemma 8: For the scenario in Fig. 4(b), the number of DoFs consumed by IC in a transmit weight vector $\mathbf{u}_{li}$ at transmitter Tx(l) is $\sum_{m=1}^{M}\sum_{j=1}^{z_{k_m}} 1^+\{\Pi_{\mathbf{v}_{k_m,j}} < \Pi_{\mathbf{u}_{li}}\}$ under an order Π for vectors.

Remark 1: Note that our order-based IC scheme does not include potential feasible solutions with dependency cycles among the vectors. As a consequence, our model via Lemmas 7 and 8 cannot achieve the same DoF region as that under SM and IC. However, our model includes all feasible solutions under SM and IC where dependency cycles are not allowed.

From Vector-Level to Node-Level. Similar to that for the $1 \rightarrow 1$ case, we may consider a node-level ordering π, instead of a vector-level ordering Π. Then for (i), constraints in (10) under vector-level ordering are changed to the following constraints

$$(\mathbf{u}_{l_m,i}^T\mathbf{H}_{(l_m,k)})\mathbf{v}_{kj} = 0 \quad (1 \le m \le M, 1 \le i \le z_{l_m}, \pi_{\text{Tx}(l_m)} < \pi_{\text{Rx}(k)}) \quad (12)$$

and receiver Rx(k) only needs to determine each of its receive weight vectors to satisfy the above constraints. Thus, we have the following theorem.

Theorem 1: For the scenario in Fig. 4(a), the number of DoFs consumed by IC at receiver Rx(k) is $\sum_{m=1}^{M} z_{l_m} \cdot 1^+\{\pi_{\text{Tx}(l_m)} < \pi_{\text{Rx}(k)}\}$ under an order π for nodes.

For (ii), constraints in (11) under vector-level ordering are changed to the following constraints

$$\mathbf{u}_{li}^T(\mathbf{H}_{(l,k_m)}\mathbf{v}_{k_m,j}) = 0 \quad (1 \le m \le M, 1 \le j \le z_{k_m}, \pi_{\text{Rx}(k_m)} < \pi_{\text{Tx}(l)}) \quad (13)$$

and transmitter Tx(l) only needs to determine each of its transmit weight vectors to satisfy the above constraints. Thus, we have the following theorem.

Theorem 2: For the scenario in Fig. 4(b), the number of DoFs consumed by IC at transmitter Tx(l) is $\sum_{m=1}^{M} z_{k_m} \cdot 1^+\{\pi_{\text{Rx}(k_m)} < \pi_{\text{Tx}(l)}\}$ under an order π for nodes.

For the achievable DoF region by a node-level ordering, we can prove the following lemma by using a similar construction in the proof of Lemma 6.

Lemma 9: For the general case of multiple links, the DoF region by a node-level ordering is the same as that under the matrix-based model with a vector-level ordering.

4.4 Total Consumed DoFs

We have analyzed DoF consumptions by SM and IC in Sections 4.2 and 4.3, respectively. The remaining question becomes: Is the total number of consumed DoFs a simple sum of those by SM and IC? The analysis for this question is the last step shown in Fig. 2.

To answer this question, we need to find out whether there is any linear dependency between the set of SM constraints and the set of IC constraints. For example, for a transmit weight vector, we need to check whether any SM constraint in (8) and (9) or IC constraint in (13) can be represented as a linear combination of other SM and IC constraints. We will prove that there is no linear dependency between these two sets of constraints. Thus, the answer to our question is positive and we have the following lemma.

Lemma 10: The total consumed DoFs in the matrix-based model is the sum of DoFs consumed by SM and IC.

Proof: Suppose that we are considering DoF consumption for a transmit weight vector $\mathbf{u}_{li}$. (The proof for a reception vector $\mathbf{v}_{kj}$ is similar and is omitted.) Based on the proof of Lemma 2, all the SM constraints in (8) and (9) are linearly independent. Based on Theorem 2, all the IC constraints in (13) are linearly independent. By Lemma 1, we need to prove that the union of linearly independent SM constraints in (8) and (9) and linearly independent IC constraints in (13) is a linearly independent set. That is, we need to prove that a constraint in (8), (9), or (13) cannot be represented as a linear combination of some constraints in (8) or (9) and some constraints in (13). Since (8) is the only constraint with non-zero RHS, it cannot be represented as a linear combination of other constraints. Moreover, since any linear combination that includes (8) has a non-zero RHS, a constraint in (9) and (13) cannot be represented by a linear combination that includes (8). Thus, we only need to prove that a constraint in (9) or (13) cannot be represented as a linear combination of some constraints in (9) and some constraints in (13).

We now prove, by contradiction, that a constraint in (9) cannot be represented as a linear combination of some constraints in (9) and some constraints in (13). Suppose that we can represent an SM constraint in (9), say $\mathbf{u}_{li}^T(\mathbf{H}_{(l,l)}\mathbf{v}_{lj}) = 0$, as a linear combination of other SM constraints in (9) and some IC constraints in (13). Then we have

$$\begin{aligned}\mathbf{H}_{(l,l)}\mathbf{v}_{lj} \;=\; & \sum_{1 \le i \le z_l}^{i \ne j} w_i \mathbf{H}_{(l,l)}\mathbf{v}_{li} \\ & + \sum_{1 \le m \le M}^{\pi_{\text{Rx}(k_m)} < \pi_{\text{Tx}(l)}} \sum_{1 \le i \le z_{k_m}} w_{mi}\mathbf{H}_{(l,k_m)}\mathbf{v}_{k_m,i}\end{aligned}$$

under some weights w_i and w_{mi}. To find a contradiction, multiplying $\mathbf{u}_{lj}^T$ on both sides of the above equality, we have

$$\begin{aligned}
& \mathbf{u}_{lj}^T \mathbf{H}_{(l,l)} \mathbf{v}_{lj} \\
= & \sum_{1\le i\le z_l}^{i\neq j} \mathbf{u}_{lj}^T w_i \mathbf{H}_{(l,l)} \mathbf{v}_{li} \\
& + \sum_{1\le m\le M}^{\pi_{\mathrm{Rx}(k_m)}<\pi_{\mathrm{Tx}(l)}} \sum_{1\le i\le z_{k_m}} \mathbf{u}_{lj}^T w_{mi} \mathbf{H}_{(l,k_m)} \mathbf{v}_{k_m,i} \\
= & \sum_{1\le i\le z_l}^{i\neq j} w_i \cdot \mathbf{u}_{lj}^T \mathbf{H}_{(l,l)} \mathbf{v}_{li} \\
& + \sum_{1\le m\le M}^{\pi_{\mathrm{Rx}(k_m)}<\pi_{\mathrm{Tx}(l)}} \sum_{1\le i\le z_{k_m}} w_{mi} \cdot \mathbf{u}_{lj}^T \mathbf{H}_{(l,k_m)} \mathbf{v}_{k_m,i} \\
= & \ 0
\end{aligned}$$

where the last equality holds by (3) and (4). On the other hand, we have $\mathbf{u}_{lj}^T \mathbf{H}_{(l,l)} \mathbf{v}_{lj} = 1$ by (2). This is a contradiction.

Similarly, we can prove that a constraint in (13) cannot be represented as a linear combination of some constraints in (9) and some constraints in (13). The details are omitted to conserve space. □

5 A DoF-based Model

5.1 Mathematical Modeling

Based on the results in the previous section, we are now ready to develop a DoF-based link layer model for a multi-hop MIMO network. We have the following four sets of constraints.

Half-Duplex Constraint. Due to the half-duplex property, a node cannot be the transmitter of one link and the receiver of another link in the same time slot. We use a binary variable $x_i[t]$, $1 \le i \le N$ and $1 \le t \le T$, to indicate whether node i is a transmitter for some link in time slot t. That is, if node i is a transmitter in time slot t, then $x_i[t] = 1$, otherwise $x_i[t] = 0$. We use another binary variable $y_i[t]$, $1 \le i \le N$ and $1 \le t \le T$, to indicate whether node i is a receiver for some link in time slot t. Then the half-duplex property can be modeled as

$$x_i[t] + y_i[t] \le 1 \quad (1 \le i \le N, 1 \le t \le T)\ . \tag{14}$$

Constraints for Node Activity. Denote $\mathcal{L}_i^{\mathrm{in}}$ and $\mathcal{L}_i^{\mathrm{out}}$ the set of possible incoming and outgoing links at node i, respectively. Note that a node can be the transmitter of multiple links (i.e., multi-packet transmission) or the receiver of multiple links (i.e., multi-packet reception) in the same time slot by IC.

If node i is not an active transmitter, then we have $\sum_{l\in\mathcal{L}_i^{\mathrm{out}}} z_l[t] = 0$. Otherwise, by the fact that the total available DoFs of a vector at node i is equal to A_i (the number of antennas at node i), we have $1 \le \sum_{l\in\mathcal{L}_i^{\mathrm{out}}} z_l[t] \le A_i$ by Lemmas 2. These two cases can be formulated by the following constraint.

$$x_i[t] \le \sum_{l\in\mathcal{L}_i^{\mathrm{out}}} z_l[t] \le A_i \cdot x_i[t] \quad (1 \le i \le N, 1 \le t \le T). \tag{15}$$

Similarly, considering whether or not node i is an active receiver, we have

$$y_i[t] \le \sum_{l\in\mathcal{L}_i^{\mathrm{in}}} z_l[t] \le A_i \cdot y_i[t] \quad (1 \le i \le N, 1 \le t \le T). \tag{16}$$

Ordering Constraints. For any order $\pi[t]$, we have

$$1 \le \pi_i[t] \le N \quad (1 \le i \le N, 1 \le t \le T)\ . \tag{17}$$

To model the "relative" ordering between any two nodes i and j in $\pi[t]$, we use a binary variable $\theta_{ji}[t]$ and define it as follows: $\theta_{ji}[t] = 1$ if node i is after node j in $\pi[t]$ (not necessarily consecutive) and 0 otherwise. It is easy to verify that the following relationships hold among $\pi_i[t]$, $\pi_j[t]$, and $\theta_{ji}[t]$.

$$\pi_i[t] - N\cdot\theta_{ji}[t] + 1 \le \pi_j[t] \le \pi_i[t] - N\cdot\theta_{ji}[t] + N - 1$$
$$(1 \le i \le N, j \in \mathcal{I}_i, 1 \le t \le T)\ , \tag{18}$$

where $\mathcal{I}_i$ is the set of nodes within node i's interference range.

DoF Consumption Constraints. It is clear that the total consumed DoFs at a node cannot exceed its total available DoFs (or the number of antennas at this node). Thus, if node i is a transmitter, then we have $\sum_{l\in\mathcal{L}_i^{\mathrm{out}}} z_l[t] + \sum_{j\in\mathcal{I}_i} \theta_{ji}[t] \sum_{k\in\mathcal{L}_j^{\mathrm{in}}}^{\mathrm{Tx}(k)\neq i} z_k[t] \le A_i$ by Lemma 2 and Theorem 2. Otherwise, if node i is not a transmitter, then $\sum_{l\in\mathcal{L}_i^{\mathrm{out}}} z_l[t] = 0$ and there is no constraint on $\sum_{j\in\mathcal{I}_i} \theta_{ji}[t] \sum_{k\in\mathcal{L}_j^{\mathrm{in}}}^{\mathrm{Tx}(k)\neq i} z_k[t]$. To develop one constraint for both cases, we introduce a large constant B_i (e.g., setting $B_i = \sum_{j\in\mathcal{I}_i} A_j$) to ensure that B_i is an upper bound of $\sum_{j\in\mathcal{I}_i} \theta_{ji}[t] \sum_{k\in\mathcal{L}_j^{\mathrm{in}}}^{\mathrm{Tx}(k)\neq i} z_k[t]$. Then we have

$$\sum_{l\in\mathcal{L}_i^{\mathrm{out}}} z_l[t] + \sum_{j\in\mathcal{I}_i} \theta_{ji}[t] \sum_{k\in\mathcal{L}_j^{\mathrm{in}}}^{\mathrm{Tx}(k)\neq i} z_k[t] \le A_i x_i[t] + (1 - x_i[t]) B_i\ ,$$
$$(1 \le i \le N, 1 \le t \le T). \tag{19}$$

Now we consider the case of whether or not node i is a receiver. Following the same token, we have

$$\sum_{k\in\mathcal{L}_i^{\mathrm{in}}} z_k[t] + \sum_{j\in\mathcal{I}_i} \theta_{ji}[t] \sum_{l\in\mathcal{L}_j^{\mathrm{out}}}^{\mathrm{Rx}(l)\neq i} z_l[t] \le A_i y_i[t] + (1 - y_i[t]) B_i\ ,$$
$$(1 \le i \le N, 1 \le t \le T). \tag{20}$$

Note that $z_l[t]$ is the number of data streams on link l in time slot t. By (1), we can calculate the achievable DoF c_l, which is the average of $z_l[t]$ over all T time slots. Thus, a model for the set of $(c_1, c_2, \cdots, c_L)$ values includes constraints (1), (14)–(20). Note that there is no matrix representation involved in this DoF-based model.

5.2 Linearization

One thing we can improve upon the above model is to remove the non-linearity in (19) and (20). The non-linear terms in (19) and (20) are products of one binary variable ($\theta_{ji}[t]$) and another linear term ($\sum_{k\in\mathcal{L}_j^{\text{in}}}^{\text{Tx}(k)\neq i} z_k[t]$ or $\sum_{l\in\mathcal{L}_j^{\text{out}}}^{\text{Rx}(l)\neq i} z_l[t]$). Such non-linear terms can be removed by introducing new variables and adding new linear constraints. To do this for (19), we define new variables $\lambda_{ji}[t] = \theta_{ji}[t]\sum_{k\in\mathcal{L}_j^{\text{in}}}^{\text{Tx}(k)\neq i} z_k[t]$. Then we can re-write (19) as

$$\sum_{l\in\mathcal{L}_i^{\text{out}}} z_l[t] + \sum_{j\in\mathcal{I}_i} \lambda_{ji}[t] \leq A_i x_i[t] + (1 - x_i[t])B_i\ , \quad (1 \leq i \leq N, 1 \leq t \leq T). \tag{21}$$

Since $\theta_{ji}[t] \in \{0, 1\}$ and $0 \leq \sum_{k\in\mathcal{L}_j^{\text{in}}}^{\text{Tx}(k)\neq i} z_k[t] \leq A_j$, the new constraints among $\lambda_{ji}[t], \theta_{ji}[t]$, and $\sum_{k\in\mathcal{L}_j^{\text{in}}}^{\text{Tx}(k)\neq i} z_k[t]$ that we need to add are

$$\lambda_{ji}[t] \leq \sum_{k\in\mathcal{L}_j^{\text{in}}}^{\text{Tx}(k)\neq i} z_k[t] \quad (1 \leq i \leq N, j \in \mathcal{I}_i, 1 \leq t \leq T), \tag{22}$$

$$\lambda_{ji}[t] \leq A_j \cdot \theta_{ji}[t] \quad (1 \leq i \leq N, j \in \mathcal{I}_i, 1 \leq t \leq T), \tag{23}$$

$$\lambda_{ji}[t] \geq A_j \cdot \theta_{ji}[t] + \sum_{k\in\mathcal{L}_j^{\text{in}}}^{\text{Tx}(k)\neq i} z_k[t] - A_j \quad (1 \leq i \leq N, j \in \mathcal{I}_i, 1 \leq t \leq T). \tag{24}$$

Similarly, by letting $\mu_{ji}[t] = \theta_{ji}[t]\sum_{l\in\mathcal{L}_j^{\text{out}}}^{\text{Rx}(l)\neq i} z_l[t]$, we can replace (20) by

$$\sum_{k\in\mathcal{L}_i^{\text{in}}} z_k[t] + \sum_{j\in\mathcal{I}_i} \mu_{ji}[t] \leq A_i y_i[t] + (1 - y_i[t])B_i \quad (1 \leq i \leq N, 1 \leq t \leq T), \tag{25}$$

$$\mu_{ji}[t] \leq \sum_{l\in\mathcal{L}_j^{\text{out}}}^{\text{Rx}(l)\neq i} z_l[t] \quad (1 \leq i \leq N, j \in \mathcal{I}_i, 1 \leq t \leq T), \tag{26}$$

$$\mu_{ji}[t] \leq A_j \cdot \theta_{ji}[t] \quad (1 \leq i \leq N, j \in \mathcal{I}_i, 1 \leq t \leq T), \tag{27}$$

$$\mu_{ji}[t] \geq A_j \cdot \theta_{ji}[t] + \sum_{l\in\mathcal{L}_j^{\text{out}}}^{\text{Rx}(l)\neq i} z_l[t] - A_j \quad (1 \leq i \leq N, j \in \mathcal{I}_i, 1 \leq t \leq T). \tag{28}$$

5.3 Complexity and Performance Comparison

After removing non-linear constraints (19) and (20), we now have a model for the set of $(c_1, c_2, \cdots, c_L)$ values with linear constraints (1), (14)–(18), (21)–(28).

Complexity Comparison. We now show that, comparing to the matrix-based model (1)–(4), our model is much simpler. First, note that for the matrix-based model, the set of constraints and variables in (2), (3), and (4) depend on the set of $(z_1[t], z_2[t], \cdots, z_L[t])$ values. Since we have to solve one problem for each set of values of $(z_1[t], z_2[t], \cdots, z_L[t])$, the number of problems that need to be solved is exponential with L. Second, in the matrix-based model, verifying the feasibility of a given set of $(z_1[t], z_2[t], \cdots, z_L[t])$ requires to solve a large number of bilinear equations (2), (3), and (4), which are very difficult.

In our DoF-based model, the set of constraints and variables does not depend on the set of $(z_1[t], z_2[t], \cdots, z_L[t])$ values. Thus, we only need to solve one problem. Moreover, all constraints in this problem are linear.

Achievable DoF Region. Since our scheme is solely based on SM and IC, it does not include those solutions beyond SM and IC (e.g., multi-user detection, dirty paper coding). Moreover, during our analysis on DoF consumption for IC, we made the assumption that all transmit and receive weight vectors involved in IC are determined by following some order. As we discussed in Remark 1, our model cannot include all feasible solutions under SM and IC. Instead, our model includes all feasible solution by SM and IC when dependency cycles among the vectors are not allowed. Thus, we have the following theorem.

Theorem 3: Our DoF-based link layer model includes all feasible solutions for a multi-hop MIMO network under the matrix-based model for SM and IC when there is no dependency cycle among the vectors.

Our order-based approach converts a (zero-forcing) matrix formulation into a simpler, DoF-based formulation that provably produces a feasible solution. Theorem 3 shows that this approach considers a restricted MIMO model (no dependency cycle), so the DoF region computed by our approach may be smaller than that achieved by a more general MIMO model with consideration of dependency cycles. This is the price we pay for a DoF-based formulation which, on the other hand, is useful to simplify the cumbersome task of computing MIMO weights.

Our model includes all possible orders while previous DoF-based models in [1], [9], [21], [24] only consider some pre-determined order. For example, the model in [24] considers the case that all receivers placed after all transmitters in an order. As a result, these models only consider some special cases within our model, i.e., they all have a smaller DoF region than that by our model.

5.4 Numerical Examples

We now compare the DoF region and complexity between our DoF-based model and the matrix-based model for a three-link network. We will show that they achieve the same DoF region while our DoF-based model incurs significantly less complexity. We will also show that the DoF region under a previous DoF-based model, NiM [9], is smaller than that under our model. As discussed in

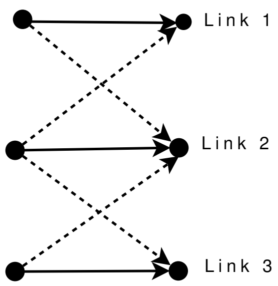


Fig. 5. A three-link network.

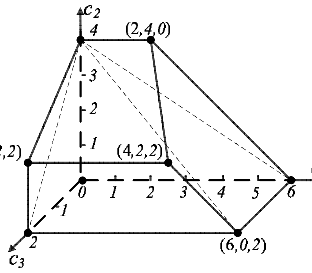


Fig. 6. The DoF region by our DoF-based model coincides with that by the matrix-based model for a three-link network. Also shown within the dashed lines is the DoF region by NiM.

Section 2, the NiM model is the same as the approach in [1].

Fig. 5 shows the topology of three active links, where links 1 and 2 are interfering with each other and links 2 and 3 are interfering with each other. Suppose that the number of antennas at both the transmitter and the receiver of link 1 are six, the number of antennas at both the transmitter and the receiver of link 2 are four, and the number of antennas at both the transmitter and the receiver of link 3 are two.

We now show how to obtain the DoF region for this three-link network under the matrix-based model and our DoF-based model.

- Under the matrix-based model, to compute (c_1, c_2, c_3) by (1), we need to verify the feasibility of each set of values for $(z_1[t], z_2[t], z_3[t])$. We use $(z_1[t], z_2[t], z_3[t]) = (3, 1, 1)$ as an example. We need to check whether we can find three 6×1 transmit weight vectors $\mathbf{u}_{11}[t], \mathbf{u}_{12}[t]$, and $\mathbf{u}_{13}[t]$ at Tx(1), three 6×1 receive weight vectors $\mathbf{v}_{11}[t], \mathbf{v}_{12}[t]$, and $\mathbf{v}_{13}[t]$ at Rx(1), one 4×1 transmit weight vectors $\mathbf{u}_{21}[t]$ at Tx(2), one 4×1 receive weight vectors $\mathbf{v}_{21}[t]$ at Rx(2), one 2×1 transmit weight vectors $\mathbf{u}_{31}[t]$ at Tx(3), and one 2×1 receive weight vectors $\mathbf{v}_{31}[t]$ at Rx(3) such that bilinear constraints (2), (3), and (4) hold. There are $(z_1[t])^2 + (z_2[t])^2 + (z_3[t])^2 + 2z_1[t]z_2[t] + 2z_2[t]z_3[t] = 19$ constraints and $12z_1[t] + 8z_2[t] + 4z_3[t] = 48$ variables (note that a 6×1 vector has six variables) in these bilinear constraints. Since a general solution to bilinear equation systems remains unknown [12], it can only be solved via exhaustive search. We finally find vectors to satisfy all the constraints and thus $(3, 1, 1)$ is feasible. This verification process for a single set of $(z_1[t], z_2[t], z_3[t]) = (3, 1, 1)$ is already very complex.
  Now suppose we want to check the feasibility of $(z_1[t], z_2[t], z_3[t]) = (1, 2, 1)$. We have a problem of 14 bilinear constraints with 32 variables. Thus, the problem for different set of $(z_1[t], z_2[t], z_3[t])$ is different.
  Since $z_1[t] \in \{0, 1, 2, 3, 4, 5, 6\}$, $z_2[t] \in \{0, 1, 2, 3, 4\}$, and $z_3[t] \in \{0, 1, 2\}$, we need to solve $7 \times 5 \times 3 = 105$ bilinear problems to determine feasibility of each set of $(z_1[t], z_2[t], z_3[t])$. Then we obtain the DoF region in Fig. 6.
- Now we compute the DoF region under our DoF-based model. Instead of verifying each set of values for $(z_1[t], z_2[t], z_3[t])$ by solving 105 different problems as in the matrix-based model, our DoF-based model only needs to solve one linear problem and obtain all possible sets of $(z_1[t], z_2[t], z_3[t])$ values. The problem we are solving now is to find $z_l[t]$ on each link l and an order $\pi[t]$ such that all constraints in Section 5.1 hold. After solving this linear program, we have $z_3[t] \leq 2$, $z_1[t] + z_2[t] \leq 6$, $z_2[t] + z_3[t] \leq 4$. This DoF region is shown in Fig. 6, which is the same as that by the matrix-based model.

TABLE 1
Complexity comparison between the matrix-based model and our DoF-based model for a three-link network.

	Matrix-based model	DoF-based model
Number of Problems	105	1
Type of Problems	Bilinear problems	Linear program

Table 1 summarizes the above discussion. Although the DoF regions by both models are identical, our DoF-based model can be solved with a much lower complexity because (i) it only requires to solve one problem, instead of many problems under the matrix-based model, and (ii) the problem under our DoF-based model is a linear problem while the problems under the matrix-based model are bilinear problems.

We also show the DoF region under CiM in Fig. 6, which is the inside tetrahedron. The ratio between the DoF regions under CiM and our model is $\frac{3}{10}$.

6 An Application of Our Model

As an application of our model, we show how to apply it to formulate a cross-layer throughput maximization problem for a multi-hop MIMO network. We study how to maximize, say, the sum of weighted rates for a set of sessions $\mathcal{F}$ in a multi-hop MIMO network. For each session $f \in \mathcal{F}$, denote $r(f)$ the rate of session f and $w(f)$ the weight of session f. Denote $r_l(f)$ the amount of rate on link l attributed to session f. We assume that one data stream corresponds to one unit data rate. The transmission and interference ranges are 300 and 500, respectively. At the network layer, minimum-hop routing is employed. Since the total data rate on any link cannot exceed its achievable rate, we have

$$\sum_{f \in \mathcal{F}} r_l(f) \leq c_l \quad (1 \leq l \leq L) \; . \tag{29}$$

The problem formulation is then to maximize $\sum_{f \in \mathcal{F}} w(f) \cdot r(f)$, subject to constraints (1), (14)–(18), (21)–(29).

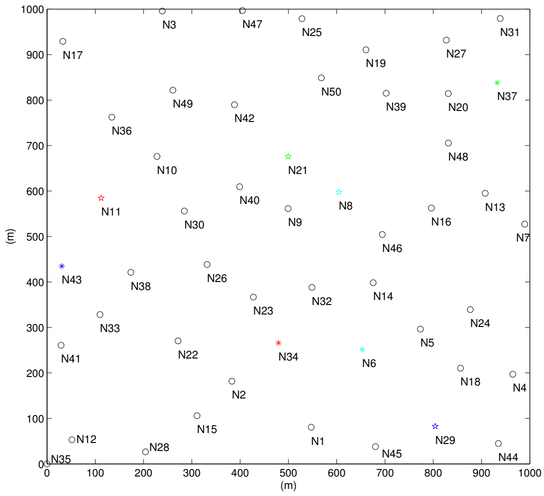


Fig. 7. A 50-node multi-hop MIMO network.

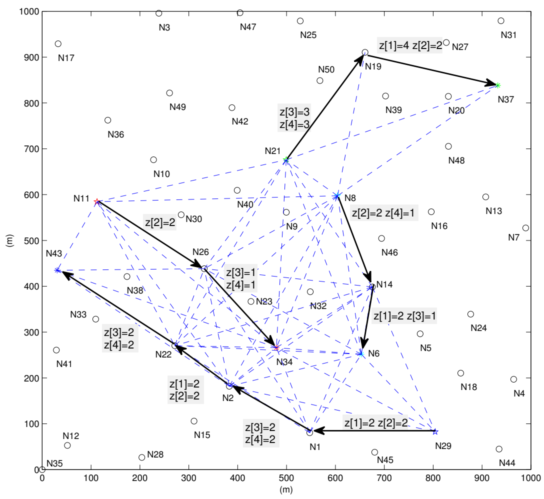


Fig. 8. Scheduling result on each link for the 50-node network.

TABLE 2
Node ordering results in each time slot of a frame for the 50-node network.

i	$\pi_i[1]$	$\pi_i[2]$	$\pi_i[3]$	$\pi_i[4]$
1	7	13	8	11
2	11	8	10	8
6	12	7	7	3
8	2	1	3	9
11	9	12	5	12
14	8	9	11	6
19	3	14	14	10
21	6	6	4	4
22	13	11	9	7
26	10	5	12	13
29	14	10	2	2
34	5	3	6	5
37	1	4	1	1
43	4	2	13	14

As a case study, consider a multi-hop MIMO network consisting of 50 nodes in Fig. 7. Each node in the network is equipped with four antennas. There are four sessions in the network: N11 to N34, N21 to N37, N29 to N43, N8 to N6 with weights $0.7, 0.4, 0.8$ and 0.9, respectively. Suppose that there are $T = 4$ time slots in each time frame. This cross-layer optimization problem may be solved by CPLEX when the number of variables is not large. We have the objective value 2.425, the scheduling solutions in Fig. 8, and the node ordering in Table 2, where only those active nodes (involved in multi-hop routing) are shown in Table 2. As an example, the shaded box next to link N8 $\rightarrow$ N14 contains $z[2] = 2, z[4] = 1$, which means that there are two data streams on this link in time slot 2 and one data stream on this link in time slot 4. In other time slots (time slots 1 and 3), this link is not active.

7 Conclusions

Matrix-based MIMO model is too complex for network level analysis and cross-layer optimization. Simple models based on DoF abstraction only require numeric computations on DoFs for SM and IC and thus offer significant advantages over the matrix-based model. However, existing DoF-based models are based on sufficient conditions on DoFs and data streams and may have a much smaller DoF region than that under the matrix-based model. In this paper, we developed a DoF-based model for a multi-hop MIMO network. It retains the same simplicity as previous DoF models while includes the same set of feasible solutions under SM and IC when there is no dependency cycle among the vectors. Our DoF model is well suited as a reference model for studying multi-hop MIMO networks.

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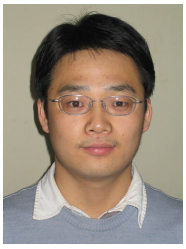

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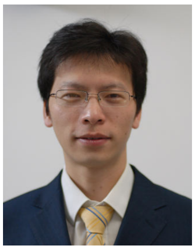

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