On Interference Alignment for Multi-hop MIMO Networks

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***Abstract*—Interference alignment (IA) is a major advance in information theory. Despite its rapid advance in the information theory community, most results on IA remain point-to-point or single-hop and there is a lack of advance of IA in the context of *multi-hop* wireless networks. The goal of this paper is to make a concrete step toward advancing IA technique in multi-hop MIMO networks. We present an IA model consisting of a set of constraints at a transmitter and a receiver that can be used to determine a subset of interfering streams for IA. Based on this IA model, we develop an IA optimization framework for a multi-hop MIMO network. For performance evaluation, we compare the performance of a network throughput optimization problem under our proposed IA framework and the same problem when IA is not employed. Simulation results show that the use of IA can significantly decrease the DoF consumption for IC, thereby improving network throughput.**

I. Introduction

Interference alignment (IA) is widely regarded as a major advance in information theory in recent years [10]. The concept of IA refers to the construction of signals at transmitters so that these signals overlap at non-intended receivers while they remain resolvable at intended receivers. It was shown in [1] that by using IA, each user in the K-user interference channel can obtain 1/2 interference-free channel capacity regardless of the number of users. That is, the aggregate user capacity scales linearly with $K/2$. Given its potential in capacity improvement in wireless networks, IA has become a central research theme in the information theory community (see, e.g. [2], [19]).

Despite its rapid advance in the information theory community, most results on IA remain point-to-point or single-hop and there is a lack of advance of IA in the context of *multi-hop* wireless networks. This is mainly due to the complex interference pattern inherent in a multi-hop network environment (see Section IV). As a result, existing (single-hop) IA schemes cannot be easily extended into multi-hop wireless networks. In [13], Li *et al.* attempted to explore IA in a multi-hop MIMO networks. The idea of IA was described in several examples in the paper to illustrate its benefits. However, the key concept of IA (i.e., the construction of signals at transmitters such that these signals overlap at non-intended receivers while they remain resolvable at intended receivers) was not incorporated into their problem formulation and thus was absent in the final solution. In another recent effort in [7], the authors used IA in their paper title although they only considered transmitter-side zero-forcing technique.

The lack of results of IA in multi-hop networks underlines both the technical barrier in this area and the critical need to close this gap by the research community. The goal of this paper is to make a concrete step toward advancing IA technique in multi-hop MIMO networks. We consider IA as the construction of transmit data streams so that (i) they overlap at receivers where they are considered as interfering streams and (ii) they are resolvable at their intended receivers (not to be overlapped by either interfering streams or other data streams). The construction of transmit data streams is equivalent to the design of transmit vector for each data stream at each transmitter. Since the interfering streams are overlapped at a receiver, one can use fewer number of DoFs to cancel these interfering streams. As a result, the DoF resources consumed for IC will be reduced and thus more DoF resources become available for data transport. The main contributions of this paper are summarized as follows.

- We model IA for a transmitter and a receiver in a multi-hop MIMO network. Our model consists of a set of constraints at a transmitter to determine which subset of interfering streams can be used for IA and a set of constraints at a receiver to determine which subset of interfering streams.
- Based on the proposed IA model, we develop a set of constraints across multiple layers of a multi-hop MIMO network. Collectively, these constraints form an IA optimization framework for a multi-hop MIMO network. Under this framework, IA can be exploited to the fullest extent for a target network performance objective.
- For performance evaluation, we compare the performance of a network throughput optimization problem under our proposed IA framework and the same problem when IA is not employed. We show that the use of IA can reduce DoF consumption for IC at receiving nodes in the network and achieve higher throughput objective than the case when IA is not employed.

The remainder of this paper is organized as follows. Section II presents related work on IA. Section III offers some essential background on IA in MIMO networks. Section IV discusses the challenges of applying IA in multi-hop networks. In Section V, we model IA at both transmitter and receiver. In Section VI, we develop an IA optimization framework for a multi-hop MIMO network. In Section VII, we apply the IA optimization framework to evaluate the benefits of IA in a

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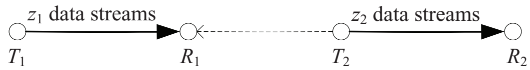

Fig. 1. SM and IC in MIMO.

multi-hop MIMO network. Section VIII concludes this paper.

II. Related Work

The concept of IA was coined in a seminar paper by Jafar and Shamai for the two-user X channel [12]. Since then, results for IA have been developed for a variety of channels and networks in increasingly sophisticated forms, such as the K-user interference channel [1], the X network with arbitrary number of users [2], the cellular network [19], ergodic capacity in fading channel [8]. A distributed IA scheme was proposed by Gomadam *et al.* in [5]. The feasibility of IA in signal vector space for K-user MIMO interference channel was studied by Yetis *et al.* in [21], and blind IA (no CSI at transmitter) was studied in [9]. A tutorial on IA from information theory perspective is [10].

In wireless communications and networking communities, efforts on IA have been mainly limited to validations on small toy networks [3], [4], [14]. In [3], El Ayach *et al.* did an experimental study of IA in MIMO-OFDM interference channels and showed that IA achieves the theoretical throughput gains. In [4], Gollakotta *et al.* demonstrated that the combination of IA and IC increases the average throughput by $1.5\times$ on the downlink and $2\times$ on the uplink in a 2×2 MIMO WLAN. In [14], Lin *et al.* proposed a distributed random access protocol (called 802.11n$^+$) based on IA and demonstrated that the system can double the average network throughput in a small network with three pairs of nodes.

III. Preliminaries: IA in MIMO

In this section, we review MIMO's DoF resources for spatial multiplexing (SM) and interference cancellation (IC). We also review how IA can help reduce the number of DoFs required for IC. Table I lists the notation used in this paper.

MIMO's DoF Resources for SM and IC. The number of DoFs of a node is typically assumed to be the same as the number of antennas at the node and represents the total available resources at the node for SM and IC [11], [18], [20]. SM refers to the use of one or multiple DoFs (both at transmitting and receiving nodes) for data transport, with each DoF corresponding to one independent data stream. IC refers to the use of one or more DoFs to cancel interference from other nodes, with each DoF being responsible for cancelling one interfering stream. IC can be done either at a transmit node (to cancel interference to another node) or a receive node (to cancel interference from another node). For example, consider two links in Fig. 1. To transmit z_1 data streams on link (T_1, R_1), both nodes T_1 and R_1 need to consume z_1 DoFs for SM. Similarly, to transmit z_2 data streams on link (T_2, R_2), both nodes T_2 and R_2 need to consume z_2 DoFs for SM. The interference from T_2 to R_1 can be cancelled by either R_1 or T_2. If R_1 cancels this interference, it needs to consume z_2 DoFs. If T_2 cancels this interference, it needs to consume z_1 DoFs.

TABLE I
NOTATION.

Symbol	Definition
$\mathcal{A}_{ij}$	The set of interfering streams from transmitter T_i to unintended receiver R_j
$\mathcal{B}_{ij}$	The subset of interfering streams in $\mathcal{A}_{ij}$ that are aligned to other interfering streams at R_j
c_i^k	An arbitrary nonzero number
$\mathbf{e}_k$	Unit vector with 1 in the k-th entry and 0 in all others
e_{ij}^k	The interfering stream from transmitter T_i to receiver R_j that corresponds to transmit vector $\mathbf{u}_i^k$
F	The number of sessions in the network
$\mathbf{H}_{ji}$	Channel matrix between transmitter i and receiver j
$\mathcal{I}_i$	The set of nodes within node i's interference range
L	The number of links in the network
$\mathcal{L}$	The set of links in the network
$\mathcal{L}_i^{\text{in}}$	The set of incoming links at node i
$\mathcal{L}_i^{\text{out}}$	The set of outgoing links at node i
M	The number of antennas at each node
N	The number of nodes in the network
$\mathcal{N}$	The set of nodes in the network
N_r	The number of receiving nodes in the network
N_t	The number of transmitting nodes in the network
$r_{\min}$	The minimum data rate among all sessions in the network
$r(f)$	The data rate of session f
$r_l(f)$	The amount of rate on link l that is attributed to session f
$\text{Rx}(l)$	The receiver of link l
R_j	The j-th receiving node in the network
T_i	The i-th transmitting node in the network
$\mathbf{u}_i^k$	The transmit vector for stream s_i^k at transmitter T_i
$x_i(t)$	A binary variable to indicate whether node i is a transmitter for some link in time slot t
$y_i(t)$	A binary variable to indicate whether node i is a receiver for some link in time slot t
$z_l(t)$	The number of data streams on link l in time slot t
$\alpha_{ij}(t)$	The cardinality of $\mathcal{A}_{ij}$ in time slot t
$\beta_{ij}(t)$	The cardinality of $\mathcal{B}_{ij}$ in time slot t
λ_i	The number of outgoing data streams at transmitter T_i
μ_j	The number of incoming data streams at receiver R_j

IA in MIMO. In the context of MIMO, IA refers to the construction of transmit data streams so that (i) they overlap at receivers where they are considered as interfering streams and (ii) they are resolvable at their intended receivers (not to be overlapped by either interfering streams or other data streams) [1], [4]. The construction of transmit data streams is equivalent to the design of transmit vector (weights) for each data stream at each transmitter. Since the interfering streams are overlapped at a receiver, one can use fewer number of DoFs to cancel these interfering streams. As a result, the DoF resources consumed for IC will be reduced and thus more DoF resources become available for data transport.

We use the following example to illustrate the benefits of IA in MIMO networks. Consider the 4-link network shown in Fig. 2. A solid line with arrow represents directed link while a dashed line with arrow represents directed interference. Assume that each node is equipped with three antennas. Suppose that there are 2 data streams on link (T_1, R_1), 2 data streams on link (T_2, R_2), and 1 data stream on link (T_3, R_3). Denote $\mathbf{u}_i^k$ as the transmit vector for the k-th data stream s_i^k

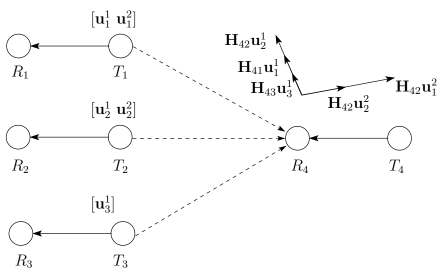

Fig. 2. An illustration of IA at node R_4.

on link (T_i, R_i) and $\mathbf{H}_{ji}$ as the channel matrix between T_i and R_j.

When IA is not employed, R_4 needs to consume 5 DoFs to cancel the interference from transmitters T_1, T_2, and T_3 [11], [18]. Since there are only 3 DoFs available at R_4, it is not possible to cancel all 5 interfering streams, let alone to receive any data stream from T_4. But when IA is used (see Fig. 2), we can align the 5 interfering data streams into 2 dimensions, which can be cancelled by R_4 with 2 DoFs. Therefore, R_4 still has 1 DoF remaining, allowing it to receive 1 data stream from T_4.

We now show one possible approach to construct the 5 transmit vectors at T_1, T_2, and T_3 so that their 5 interfering streams are aligned into 2 dimensions at receiver R_4. First, we construct the transmit vectors at T_1 independently by letting $\mathbf{u}_1^1 = \mathbf{e}_1$ and $\mathbf{u}_1^2 = \mathbf{e}_2$, where $\mathbf{e}_k$ is a unit vector with the k-th entry being 1 and other entries being 0. For the two transmit vectors $[\mathbf{u}_2^1\ \mathbf{u}_2^2]$ at T_2, we can align the interfering stream corresponding to $\mathbf{u}_2^1$ to the interfering stream corresponding to $\mathbf{u}_1^1$ at receiver R_4. This can be done by letting $\mathbf{H}_{42} \cdot \mathbf{u}_2^1 = \mathbf{H}_{41} \cdot \mathbf{u}_1^1$ and thus $\mathbf{u}_2^1 = \mathbf{H}_{42}^{-1} \cdot \mathbf{H}_{41} \cdot \mathbf{u}_1^1$. Similarly, we can align the interfering stream corresponding to $\mathbf{u}_2^2$ to the interfering stream corresponding to $\mathbf{u}_1^2$ at receiver R_4. This is done by letting $\mathbf{H}_{42} \cdot \mathbf{u}_2^2 = \mathbf{H}_{41} \cdot \mathbf{u}_1^2$ and thus $\mathbf{u}_2^2 = \mathbf{H}_{42}^{-1} \cdot \mathbf{H}_{41} \cdot \mathbf{u}_1^2$. Finally, for the transmit vector $\mathbf{u}_3^1$ at T_3, we can align its interfering stream to the interfering stream corresponding to $\mathbf{u}_1^1$ at receiver R_4. This is done by having $\mathbf{H}_{43} \cdot \mathbf{u}_3^1 = \mathbf{H}_{41} \cdot \mathbf{u}_1^1$ and thus $\mathbf{u}_3^1 = \mathbf{H}_{43}^{-1} \cdot \mathbf{H}_{41} \cdot \mathbf{u}_1^1$. As a result of IA, the 5 interfering streams are aligned into only 2 dimensions and can be cancelled by R_4 with 2 DoFs (instead of 5).

IV. Applying IA in Multi-hop Networks: Where Are the Challenges

As discussed in Section II, although there is a flourish of information theoretic research on IA at the physical layer, results on applying IA in multi-hop networks remain very limited. This is because there are a number of new challenges for applying IA in multi-hop MIMO networks, which we summarize as follows.

- How to perform IA among a large number of nodes in the network is a very hard problem. In particular, for each pair of nodes, one needs to determine which subset of interfering streams for IA and how to align them successfully at the receiver. While performing IA, one also has to ensure that the desirable data streams at each receiver remain resolvable (without being overlapped by either interfering streams or other data streams). The answers to these questions require the development of new IA constraints at both transmitter and receiver.
- In MIMO networks, IA, IC and SM are coupled together through each node's DoF resources. This makes it difficult to perform IA at each node while the node's DoF is also being used for SM and IC. The answer to this question requires the development of new DoF constraints for SM, IC, and IA at both transmitter and receiver.
- In a multi-hop environment, an IA scheme is also coupled with the upper layer scheduling and routing algorithms. The upper layer algorithms determine the set of transmitters, the set of receivers, the set of links, and the number of data streams on each link, which are different in each time slot. Thus, an IA scheme must be jointly designed with upper layer scheduling and routing algorithms, which is again a new and challenging problem.

V. Modeling IA for a Transmitter and a Receiver

In this section, we develop a set of constraints for IA in a multi-hop MIMO network. Assume that each node has M antennas. In a given time slot, suppose that we have a set of links $\mathcal{L}$. Denote $\{T_i : 1 \le i \le N_t\}$ and $\{R_j : 1 \le j \le N_r\}$ as the sets of transmitters and receivers of $\mathcal{L}$, respectively. For transmitter T_i, denote λ_i as the number of outgoing data streams and thus we have $\lambda_i = \sum_{l \in \mathcal{L}_i^{\text{out}}} z_l$, where $\mathcal{L}_i^{\text{out}}$ is the set of outgoing links from T_i and z_l as the number of data streams on link $l \in \mathcal{L}$.[1] Similarly, for receiver R_j, denote μ_j as the number of its incoming data streams and thus we have $\mu_j = \sum_{l \in \mathcal{L}_j^{\text{in}}} z_l$, where $\mathcal{L}_j^{\text{in}}$ is the set of its incoming links into R_j. At transmitter T_i, denote s_i^k as its k-th outgoing data stream and denote $\mathbf{u}_i^k$ as the transmit vector of data stream s_i^k.

Denote $\mathcal{I}_i$ as the set of nodes within node i's interference range. Consider a node pair (T_i, R_j). For the transmission of data stream s_i^k on T_i, if R_j is not the intended receiver of this data stream, then we call this data stream as an *interfering stream*, denoted as e_{ij}^k, at node R_j. Denote $\mathcal{A}_{ij}$ as the set of interfering streams from transmitter T_i to unintended receiver R_j and denote α_{ij} as the cardinality of $\mathcal{A}_{ij}$. Note that without IA, receiver R_j needs to expend α_{ij} DoFs to cancel the interference from transmitter T_i. Also, note that one data stream may be considered as an interfering stream by multiple receivers.

To reduce DoF consumption for IC at a receiver R_j, we can align a subset of its interfering streams to the other interfering streams by properly constructing their transmit vectors. Among the interfering streams in $\mathcal{A}_{ij}$, denote $\mathcal{B}_{ij}$ (with $\beta_{ij} = |\mathcal{B}_{ij}|$) as the subset of interfering streams that are aligned to the other interfering streams at receiver R_j. Then the "effective" cardinality of interfering streams at receiver R_j

[1] The activity of link l is determined by z_l. If $z_l > 0$, then link l is active. If $z_l = 0$, then link l is inactive.

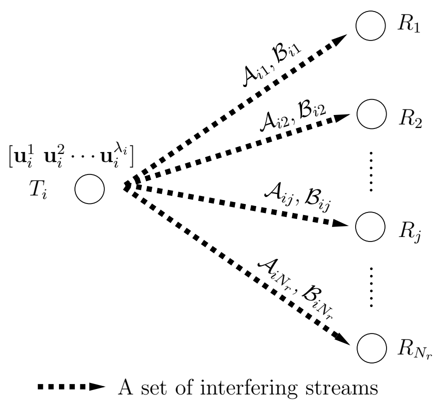

Fig. 3. IA constraints at transmitter T_i.

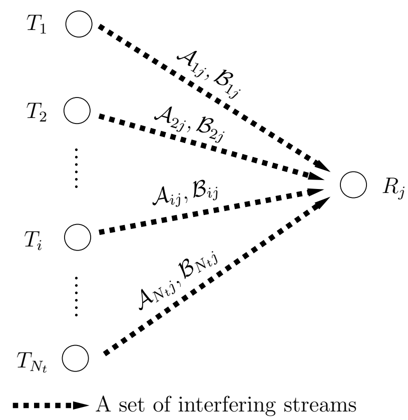

Fig. 4. IA constraints at receiver R_j.

is decreased from α_{ij} to $\alpha_{ij} - \beta_{ij}$, resulting in a saving of β_{ij} DoFs for IC.

The question to ask is then how to perform IA among the nodes in the network so that

- **(C-1):** each interfering stream in $\mathcal{B}_{ij}$'s is aligned successfully;
- **(C-2):** each data stream at its intended receiver remains resolvable (not to be overlapped by either interfering streams or other data streams).

Sections V-A and V-B answer this question by imposing constraints at a transmitter and a receiver, respectively.

A. IA Constraints at A Transmitter

Based on the definitions of β_{ij} and α_{ij}, we have the following constraints at transmitter T_i:

$$\beta_{ij} \leq \alpha_{ij}, \qquad j \in \mathcal{I}_i. \tag{1}$$

Constraint (1) gives an upper bound for each β_{ij}.

At transmitter T_i, there are λ_i transmit vectors corresponding to λ_i outgoing data streams. Each of the λ_i transmit vectors may correspond to multiple interfering streams, each for a different unintended receivers. However, one can construct each transmit vector so that only one of its corresponding interfering streams is successfully aligned to a particular direction for IA at its receiver. Given that $\lambda_i = \sum_{l \in \mathcal{L}_i^{\text{out}}} z_l$, we have the following constraints at transmitter T_i:

$$\sum_{j \in \mathcal{I}_i} \beta_{ij} \leq \sum_{l \in \mathcal{L}_i^{\text{out}}} z_l. \tag{2}$$

Constraint (2) ensures that (C-1) holds at transmitter T_i. As an example, let's consider transmitter T_i shown in Fig. 3. Transmit vector $\mathbf{u}_i^k$ corresponds to the set of interfering streams $\{e_{ij}^k : j \in \mathcal{I}_i\}$. For the set of interfering streams $\{e_{ij}^k : j \in \mathcal{I}_i\}$, only one of them can be successfully aligned to some direction for IA by constructing $\mathbf{u}_i^k$. Thus, among those interfering streams in $\cup_{j \in \mathcal{I}_i} \mathcal{A}_{ij}$ (i.e., all interfering streams from transmitter T_i), at most λ_i interfering streams can be successfully aligned to some direction for IA at their receivers. Therefore, the number of interfering streams in $\cup_{j \in \mathcal{I}_i} \mathcal{B}_{ij}$ is bounded by λ_i (i.e., $\sum_{l \in \mathcal{L}_i^{\text{out}}} z_l$).

B. IA Constraints at A Receiver

To ensure (C-1) and (C-2) at receiver R_j (see Fig. 4), we have the following three conditions on IA.

- The first condition is that each interfering stream in $\cup_{i \in \mathcal{I}_j} \mathcal{B}_{ij}$ can only be aligned to an interfering stream in $\cup_{i \in \mathcal{I}_j} (\mathcal{A}_{ij} \backslash \mathcal{B}_{ij})$.
- The second condition is that any interfering stream in $\mathcal{B}_{ij}$ cannot be aligned to an interfering stream in $\mathcal{A}_{ij}$. To show this is true, suppose that e_{ij}^k in $\mathcal{B}_{ij}$ is aligned to $e_{ij}^{k'}$ in $\mathcal{A}_{ij}$ at R_j. Then, we have $\mathbf{u}_i^k = c_i^k \mathbf{H}_{ji}^{-1} \mathbf{H}_{ji} \mathbf{u}_i^{k'} = c_i^k \mathbf{u}_i^{k'}$ (c_i^k is a nonzero number), implying that transmit vectors $\mathbf{u}_i^k$ and $\mathbf{u}_i^{k'}$ are linearly dependent. This means that data streams s_i^k and $s_i^{k'}$ are not resolvable at their intended receiver.
- The third condition is that any two interfering streams in $\mathcal{B}_{ij}$ cannot be aligned to the same (a third) interfering stream. To show this is true, suppose that both e_{ij}^k and $e_{ij}^{k'}$ in $\mathcal{B}_{ij}$ are aligned to e_{rj}^l at R_j. Then, we have $\mathbf{u}_i^k = c_i^k \mathbf{H}_{ji}^{-1} \mathbf{H}_{jr} \mathbf{u}_r^l$ and $\mathbf{u}_i^{k'} = c_i^{k'} \mathbf{H}_{ji}^{-1} \mathbf{H}_{jr} \mathbf{u}_r^l$. Based on these two equations, we have $\mathbf{u}_i^k = \frac{c_i^k}{c_i^{k'}} \mathbf{u}_i^{k'}$, indicating that transmit vectors $\mathbf{u}_i^k$ and $\mathbf{u}_i^{k'}$ are linearly dependent. This means that data streams s_i^k and $s_i^{k'}$ are not resolvable at their intended receiver.

The following lemma gives necessary and sufficient condition for the existence of IA scheme that meets the above three conditions at a receiver.

Lemma 1: There exists an IA scheme that meets the above three conditions at receiver R_j if and only if

$$\beta_{ij} \leq \sum_{k \in \mathcal{I}_j}^{k \neq i} (\alpha_{kj} - \beta_{kj}), \qquad i \in \mathcal{I}_j. \tag{3}$$

PROOF. We first show the "if" part by construction and then show the "only if" part by contradiction.

Sufficient condition: We first propose an algorithm based on (3) to obtain an IA scheme at R_j, and then show that the IA scheme obtained by the proposed algorithm satisfies the three conditions at R_j. The proposed IA algorithm is as follows: For the interfering streams in each $\mathcal{B}_{ij}$, we align them to those interfering streams in $\cup_{k\in\mathcal{I}_j}^{k\neq i}(\mathcal{A}_{kj}\backslash\mathcal{B}_{kj})$ without repetition. Since $\beta_{ij} \leq \sum_{k\in\mathcal{I}_j}^{k\neq i}(\alpha_{kj} - \beta_{kj})$ according to (3), we know that every interfering stream in $\mathcal{B}_{ij}$ can be aligned to an interfering stream in this algorithm.

We now show that the IA scheme obtained by this algorithm satisfies the three conditions at R_j. In this algorithm, every interfering stream in $\mathcal{B}_{ij}$ is aligned to an interfering stream in $\cup_{k\in\mathcal{I}_j}^{k\neq i}(\mathcal{A}_{kj}\backslash\mathcal{B}_{kj})$. Thus, we know that the first condition is satisfied. After performing this algorithm at receiver R_j, it is easy to see that any interfering stream in $\mathcal{B}_{ij}$ will not be aligned to an interfering stream in $\mathcal{A}_{ij}$ and that any two interfering streams in $\mathcal{B}_{ij}$ will not be aligned to the same (a third) interfering stream. Thus, the second and third conditions are satisfied. Therefore, the "if" part of Lemma 1 is proved.

Necessary condition: Consider any IA scheme at R_j. Suppose that $\beta_{ij} > \sum_{k\in\mathcal{I}_j}^{k\neq i}(\alpha_{kj} - \beta_{kj})$ for some $i \in \mathcal{I}_j$. Then for node pair (T_i, R_j), in order to meet the first condition, the interfering streams in $\mathcal{B}_{ij}$ must be aligned to the interfering streams in $\cup_{k\in\mathcal{I}_j}(\mathcal{A}_{kj}\backslash\mathcal{B}_{kj})$. In order to meet the second condition, the interfering streams in $\mathcal{B}_{ij}$ must be aligned to the interfering streams in $\cup_{k\in\mathcal{I}_j}^{k\neq i}(\mathcal{A}_{kj}\backslash\mathcal{B}_{kj})$. However, since the cardinality of $\mathcal{B}_{ij}$ is greater than the cardinality of $\cup_{k\in\mathcal{I}_j}^{k\neq i}(\mathcal{A}_{kj}\backslash\mathcal{B}_{kj})$ (i.e., $\beta_{ij} > \sum_{k\in\mathcal{I}_j}^{k\neq i}(\alpha_{kj} - \beta_{kj})$), there exist two interfering streams in $\mathcal{B}_{ij}$ that are aligned to the same (a third) interfering stream in $\cup_{k\in\mathcal{I}_j}^{k\neq i}(\mathcal{A}_{kj}\backslash\mathcal{B}_{kj})$. This leads to a contradiction to the third condition. This completes the proof of the "only if" part of Lemma 1. □

VI. An Optimization Framework

In this section, we develop an optimization framework for IA in multi-hop MIMO networks. Consider a multi-hop MIMO network consisting of a set of nodes $\mathcal{N}$ (with $N = |\mathcal{N}|$), each of which is equipped with M antennas. Denote $\mathcal{L}$ as the set of links in the network, with $L = |\mathcal{L}|$. Denote $\mathcal{F}$ the set of sessions in the network, with $F = |\mathcal{F}|$. Denote $r(f)$ as the data rate of session $f \in \mathcal{F}$. Denote $\mathrm{src}(f)$ and $\mathrm{dst}(f)$ as the source node and the destination node of session $f \in \mathcal{F}$, respectively. To transport data flow f from $\mathrm{src}(f)$ to $\mathrm{dst}(f)$, we allow flow splitting inside the network for better load balancing and network resource utilization. For scheduling, we assume time is slotted and a time frame consists of T time slots.

Half Duplex Constraints. We assume that a node cannot transmit and receive in the same time slot. Denote $x_i(t)$ $(1 \leq t \leq T)$ as a binary variable to indicate whether node $i \in \mathcal{N}$ is a transmitter in time slot t, i.e., $x_i(t) = 1$ if node i is a transmitter in time slot t and 0 otherwise. Similarly, denote $y_i(t)$ $(1 \leq t \leq T)$ as another binary variable to indicate whether node $i \in \mathcal{N}$ is a receiver in time slot t. Then the half duplex constraints can be written as

$$x_i(t) + y_i(t) \leq 1, \quad (1 \leq i \leq N, 1 \leq t \leq T). \tag{4}$$

Node Activity Constraints. Denote $z_l(t)$ as the number of data streams on link $l \in \mathcal{L}$ in time slot t. If node i is a transmitter, then we have $1 \leq \sum_{l\in\mathcal{L}_i^{\text{out}}} z_l(t) \leq M$. Otherwise (i.e., node i is either a receiver or inactive), then we have $\sum_{l\in\mathcal{L}_i^{\text{out}}} z_l(t) = 0$. Combining the two cases, we have the following constraints:

$$x_i(t) \leq \sum_{l\in\mathcal{L}_i^{\text{out}}} z_l(t) \leq M \cdot x_i(t), \quad (1 \leq i \leq N, 1 \leq t \leq T). \tag{5}$$

Similarly, by considering whether or not node i is a receiver, we have the following constraints:

$$y_j(t) \leq \sum_{l\in\mathcal{L}_j^{\text{in}}} z_l(t) \leq M \cdot y_j(t), \quad (1 \leq j \leq N, 1 \leq t \leq T). \tag{6}$$

General IA Constraints at a Node. In Section V, we developed IA constraints for a transmitter and a receiver. Here we generalize those constraints at a node that can be either a transmitter, receiver, or idle.

Suppose that node j is within the interference range of node i, i.e., $j \in \mathcal{I}_i$. If node j is a receiving node in time slot t (i.e., $y_j(t) = 1$), then $\alpha_{ij}(t)$ (the number of interfering streams from node i to node j in time slot t) is $\sum_{l\in\mathcal{L}_i^{\text{out}}}^{\mathrm{Rx}(l)\neq j} z_l(t)$, where $\mathrm{Rx}(l)$ is the receiver of link l. Otherwise (i.e., $y_j(t) = 0$), we have $\alpha_{ij}(t) = 0$ based on the definition of $\alpha_{ij}(t)$. In general, we have the following constraints:

$$\alpha_{ij}(t) = y_j(t) \cdot \sum_{l\in\mathcal{L}_i^{\text{out}}}^{\mathrm{Rx}(l)\neq j} z_l(t), \quad (j \in \mathcal{I}_i, 1 \leq i \leq N, 1 \leq t \leq T). \tag{7}$$

For $\beta_{ij}(t)$, if node i is a transmitter, then based on (1), we have $\beta_{ij}(t) \leq \alpha_{ij}(t)$, $j \in \mathcal{I}_i$. Otherwise (node i is either a receiver or idle), we have $\beta_{ij}(t) = 0$ and $\alpha_{ij}(t) = 0$ for each $j \in \mathcal{I}_i$ based on their definitions. Combining these two cases, we have the following constraints:

$$\beta_{ij}(t) \leq \alpha_{ij}(t), \quad (j \in \mathcal{I}_i, 1 \leq i \leq N, 1 \leq t \leq T). \tag{8}$$

If node i is a transmitter, based on (2), we have $\sum_{j\in\mathcal{I}_i} \beta_{ij}(t) \leq \sum_{l\in\mathcal{L}_i^{\text{out}}} z_l(t)$. Otherwise (node i is either a receiver or idle), we have $\sum_{j\in\mathcal{I}_i} \beta_{ij}(t) = 0$ and $\sum_{l\in\mathcal{L}_i^{\text{out}}} z_l(t) = 0$. Combining these two cases, we have the following constraints:

$$\sum_{j\in\mathcal{I}_i} \beta_{ij}(t) \leq \sum_{l\in\mathcal{L}_i^{\text{out}}} z_l(t), \quad (1 \leq i \leq N, 1 \leq t \leq T). \tag{9}$$

If node j is a receiver, based on (3), we have $\beta_{ij}(t) \leq \sum_{k\in\mathcal{I}_j}^{k\neq i}(\alpha_{kj}(t) - \beta_{kj}(t))$ for each $i \in \mathcal{I}_j$. Otherwise (node j is either a transmitter or idle), we have $\beta_{ij}(t) = 0$ and $\alpha_{ij}(t) = 0$ for each $i \in \mathcal{I}_j$ based on their definitions. Combining these two cases, we have the following constraints:

$$\beta_{ij}(t) \leq \sum_{k\in\mathcal{I}_j}^{k\neq i} \left[\alpha_{kj}(t) - \beta_{kj}(t)\right], \quad (i \in \mathcal{I}_j, 1 \leq j \leq N, 1 \leq t \leq T). \tag{10}$$

DoF Consumption Constraints. Although an interference can be cancelled at either its transmitting node or its receiving node, we only consider the case where IC is done at a receiving node in this paper.[2] Then the DoF consumption for SM and IC at a node can be summarized as follows.

- *Transmitting Node.* The number of DoFs consumed for SM at a transmitting node is equal to the number of its outgoing data streams. Furthermore, there is no DoF consumption for IC at a transmitting node, as it is not responsible for IC.
- *Receiving Node.* The DoF consumption at a receiving node consists of two parts: for SM and for IC. The number of DoFs consumed for SM at a receiving node is equal to the number of its incoming data streams, while the number of DoFs consumed for IC at a receiving node is equal to the dimension of its interference subspace (i.e., $\sum_{i\in\mathcal{I}_j}(\alpha_{ij}-\beta_{ij})$ for R_j).

Suppose that node i is a transmitter in time slot t. Then the number of DoFs it consumes is $\sum_{l\in\mathcal{L}_i^{\text{out}}} z_l(t) \le M$. Otherwise, we have $\sum_{l\in\mathcal{L}_i^{\text{out}}} z_l(t) = 0$. Combining these two cases, we have the following constraints:

$$\sum_{l\in\mathcal{L}_i^{\text{out}}} z_l(t) \le M\cdot x_i(t), \qquad (1\le i\le N, 1\le t\le T). \quad (11)$$

Suppose that node j is a receiver in time slot t. Then it consumes $\sum_{l\in\mathcal{L}_j^{\text{in}}} z_l(t)$ DoFs for SM and $\sum_{i\in\mathcal{I}_j}[\alpha_{ij}(t)-\beta_{ij}(t)]$ DoFs for IC. Since the number of DoFs consumed for SM and IC cannot exceed the total number of available DoFs at a node, then we have the following DoF constraint at node j: $\sum_{l\in\mathcal{L}_j^{\text{in}}} z_l(t)+\sum_{i\in\mathcal{I}_j}[\alpha_{ij}(t)-\beta_{ij}(t)]\le M$. Otherwise (node j is either a transmitter or idle), we have $z_l(t)=0$ for $l\in\mathcal{L}_j^{\text{in}}$ and $\alpha_{ij}(t)=\beta_{ij}(t)=0$ for $i\in\mathcal{I}_j$ based on their definitions. Combining these two cases, we have the following constraints:

$$\sum_{l\in\mathcal{L}_j^{\text{in}}} z_l(t) + \sum_{i\in\mathcal{I}_j}[\alpha_{ij}(t)-\beta_{ij}(t)] \le M\cdot y_j(t), \quad (1\le j\le N, 1\le t\le T). \quad (12)$$

Link Capacity Constraints. Denote $r_l(f)$ as the amount of data rate on link l that is attributed to session $f\in\mathcal{F}$. For simplicity, we assume that one data stream in one time slot corresponds to one unit data rate.[3] Then the average rate of link l over T time slots is $\frac{1}{T}\sum_{t=1}^{T} z_l(t)$. Since the aggregate data rates cannot exceed the average link rate, we have

$$\sum_{f=1}^{F} r_l(f) \le \frac{1}{T}\sum_{t=1}^{T} z_l(t), \qquad (1\le l\le L). \quad (13)$$

Flow Routing Constraints. At each node, flow conservation must be observed. At a source node, we have

$$\sum_{l\in\mathcal{L}_i^{\text{out}}} r_l(f) = r(f), \qquad (i=\text{src}(f), 1\le f\le F). \quad (14)$$

At an intermediate relay node, we have

$$\sum_{l\in\mathcal{L}_i^{\text{in}}} r_l(f) = \sum_{l\in\mathcal{L}_i^{\text{out}}} r_l(f), \qquad (1\le i\le N, i\ne \text{src}(f), i\ne\text{dst}(f), 1\le f\le F). \quad (15)$$

At a destination node, we have

$$\sum_{l\in\mathcal{L}_i^{\text{in}}} r_l(f) = r(f), \qquad (i=\text{dst}(f), 1\le f\le F). \quad (16)$$

It can be easily verified that if (14) and (15) are satisfied, then (16) is also satisfied. Therefore, it is sufficient to include only (14) and (15).

VII. Performance Evaluation

In this section, we apply the IA optimization framework for multi-hop MIMO networks that we developed in the previous section. In particular, we use it to study a network throughput maximization problem, and compare its performance to the case where IA is not employed.

A. A Throughput Maximization Problem

In a multi-hop MIMO network, suppose that the objective is to maximize the minimum rate among all sessions, denoted as $r_{\min}$.[4] Then we have the following constraints:

$$r_{\min} \le r(f), \qquad 1\le f\le F. \quad (17)$$

According to the constraints developed in Section VI, we have the following formulation:

Max $r_{\min}$
s.t. Half duplex constraints: (4);
Node activity constraints: (5), (6);
IA constraints: (7), (8), (9), (10);
DoF consumption constraints: (11), (12);
Link capacity constraints: (13);
Flow routing constraints: (14), (15);
Min rate constraints: (17).

Among all these constraints, only (7) is nonlinear. We linearize (7) by employing *reformulation linearization technique* (RLT) [17]. By analyzing the relationship between $\alpha_{ij}(t)$ and $\sum_{l\in\mathcal{L}_i^{\text{out}}}^{\text{Rx}(l)\ne j} z_l(t)$ in (7), we construct two new sets of constraints (18) and (19). It can be verified that the combination of (18) and (19) is equivalent to (7).

$$0 \le \sum_{l\in\mathcal{L}_i^{\text{out}}}^{\text{Rx}(l)\ne j} z_l(t) - \alpha_{ij}(t) \le (1-y_j(t))\cdot B, \quad (j\in\mathcal{I}_i, 1\le i\le N, 1\le t\le T), \quad (18)$$

and

$$0\le \alpha_{ij}(t)\le y_j(t)\cdot B, \quad (j\in\mathcal{I}_i, 1\le i\le N, 1\le t\le T), \quad (19)$$

where B is a constant integer (e.g., $B=M$).

By replacing nonlinear constraint (7) with (18) and (19), we have the following problem formulation:

OPT-IA Max $r_{\min}$
s.t. (4), (5), (6), (8), (9), (10), (11), (12), (13), (14), (15), (17), (18), (19).

[2] The case where IC can be done at both transmitting and receiving nodes will be investigated in our future work.

[3] We assume fixed modulation and coding scheme (MCS) in this paper.

[4] Note that problems with other objectives such as maximizing sum of weighted rates or a proportional increase (scaling factor) of all session rates belongs to the same category and can be solved following the same token.

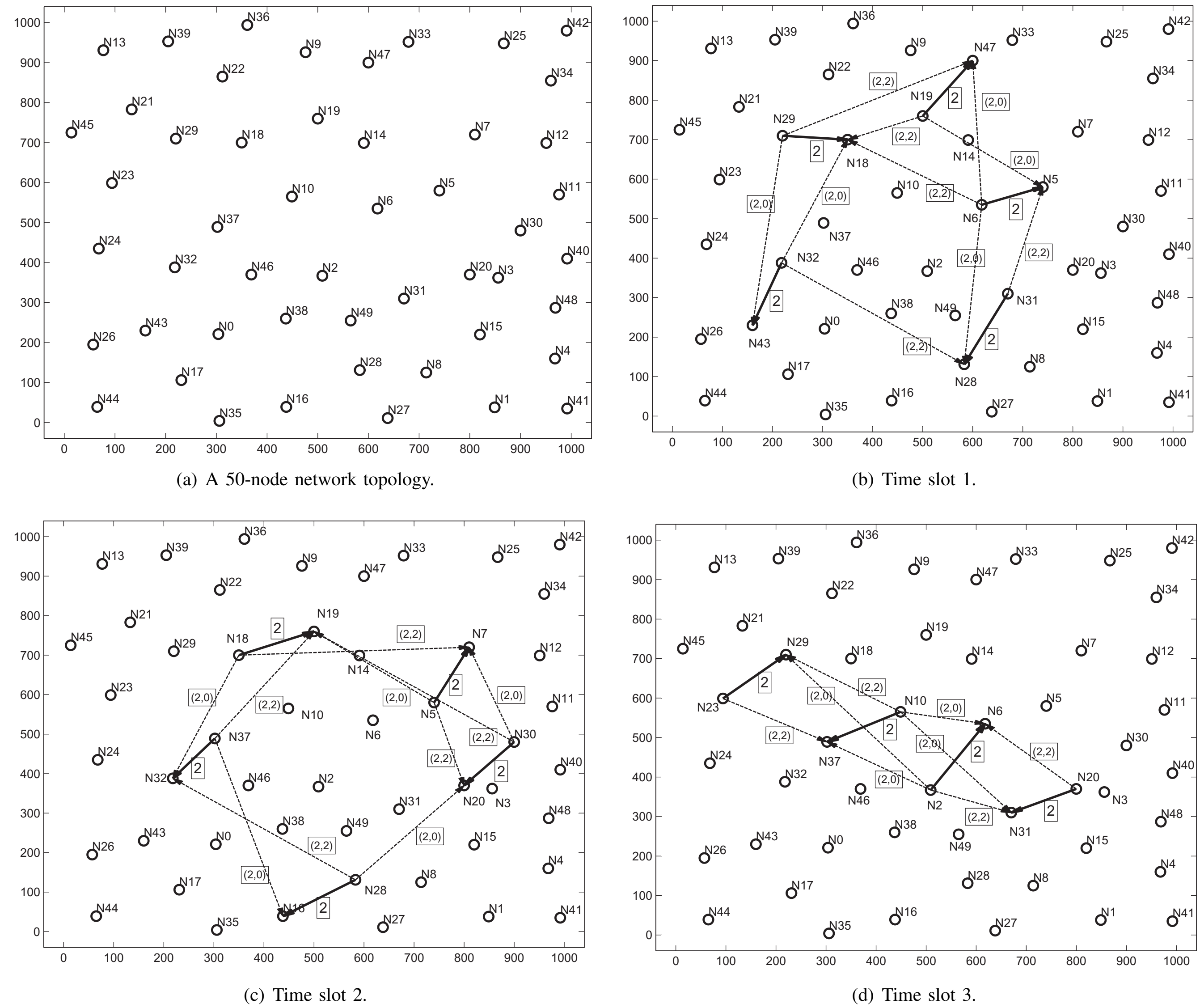

Fig. 5. Transmission/reception pattern, interference pattern, and IA scheme in each time slot. In (b)-(d), a solid arrow line represents a directed transmission link (with the number of data streams on this link shown in a box). A dashed arrow link represents an interference, with the total number of interfering streams and the number of subset interfering streams chosen for IA shown in a box, i.e., $(\alpha_{ij}, \beta_{ij})$.

where $x_i(t)$ and $y_i(t)$ are binary variables; $z_l(t)$, $\alpha_{ij}(t)$, and $\beta_{ij}(t)$ are non-negative integer variables; $r(f)$ and $r_l(f)$ are non-negative variables; M, N, L, F, T, and B are constants.

OPT-IA is a mixed integer linear programming (MILP). Although the theoretical worst-case complexity of solving a general MILP problem is exponential [15], there exist highly efficient optimal and approximation algorithms (e.g., branch-and-bound with cutting planes [16]) and heuristic algorithms (e.g., sequential fixing algorithm [6]). Another approach is to employ an off-the-shelf solver such as CPLEX [22]. Since the goal of this paper is to develop an IA optimization framework for multi-hop MIMO networks (rather than developing a solution procedure for a specific problem), we will employ CPLEX solver in this performance evaluation.

B. Simulation Setting

Without loss of generality, we normalize all units for distance, data rate, bandwidth, time and power with appropriate dimensions. We consider a randomly generated multi-hop MIMO network with 50 nodes, which are distributed in a 1000×1000 square region. Each node in the network is equipped with four antennas. We assume that all nodes have the same transmission range 250 and interference range 500.

C. A Case Study

As a case study, we investigate a network instance in Fig. 5(a) with the above setting. There are four active sessions in the network (N_{10} to N_{43}, N_{23} to N_{47}, N_{30} to N_{16}, and N_2 to N_7). For ease of illustration, we assume that there are only 3 time slots in a time frame. By solving OPT-IA, we obtain

TABLE II
A COMPARISON BETWEEN $P(N_j)$ AND $Q(N_j)$. $P(N_j)$ IS THE TOTAL NUMBER OF INTERFERING STREAMS AT NODE N_j AND $Q(N_j)$ IS THE TOTAL NUMBER OF DOFS THAT ARE CONSUMED FOR IC AT NODE N_j.

Time slot 1			Time slot 2			Time slot 3		
Rx	$P(Rx)$	$Q(Rx)$	Rx	$P(Rx)$	$Q(Rx)$	Rx	$P(Rx)$	$Q(Rx)$
N_5	4	2	N_7	4	2	N_6	4	2
N_{18}	6	2	N_{16}	2	2	N_{23}	4	2
N_{28}	4	2	N_{19}	6	2	N_{31}	4	2
N_{43}	2	2	N_{20}	4	2	N_{37}	4	2
N_{47}	4	2	N_{32}	4	2			

the optimal objective (i.e., the maximum throughput) of 0.67.

Fig. 5(b)–(d) show the transmission/reception pattern, interference pattern, and IA scheme in each time slot. Specifically, a solid arrow line represents a directed transmission link (with the number of data streams on this link shown in a box). A dashed arrow link represents an interference, with the total number of interfering streams and the number of subset interfering streams chosen for IA shown in a box, i.e., $(\alpha_{ij}, \beta_{ij})$. For example, in Fig. 5(b), on the dashed line between N_6 and N_{18}), $(2, 2)$ represents that $\alpha_{6,18} = 2$ and $\beta_{6,18} = 2$, i.e., there are two interfering streams from node N_6 to node N_{18} and both of these 2 interfering streams are selected for IA at node N_{18} in our solution.

As an example to illustrate how IA is performed in a network, let's take a look at N_{18} in time slot 1 (Fig. 5(b)). At node N_{18}, there is a total of 6 interfering streams (from transmitting nodes N_{19}, N_6, and N_{32}). In our solution, we find that for the 2 interfering streams from node N_{19}, both of them are aligned to the interfering streams from node N_{32}. Similarly, the 2 interfering streams from node N_6 have also been aligned to the interfering streams from node N_{32}. That is, among the 6 interfering streams at node N_{18}, 4 of them have been successfully aligned to the remaining 2 interfering streams. As a result, node N_{18} only needs to consume 2 DoFs for IC.

Table II summarizes the savings of DoFs in IC due to IA at each receiving node in each time slot. To abbreviate notation in the table, denote $P(N_j)$ as the total number of interfering streams at node N_j, i.e., $P(N_j) = \sum_{i \in \mathcal{I}_j} \alpha_{ij}$. Denote $Q(N_j)$ as the total number of DoFs that are consumed by node N_j for IC, i.e., $Q(N_j) = \sum_{i \in \mathcal{I}_j} (\alpha_{ij} - \beta_{ij})$. Then the difference between $P(N_j)$ and $Q(N_j)$ is the saving in DoFs at node N_j due to IA. Note that savings in DoFs directly translate into improvement in network throughput.

Comparison to OPT-base. To compare the case when our IA framework is not applied, we formulate the same network throughput optimization problem (with only MIMO's SM and IC) as OPT-base, which is given in the appendix. By solving OPT-base with CPLEX, we have that the objective is only 0.33 (comparing to 0.67 under OPT-IA).

D. Complete Results

The previous section gives results for one 50-node network instance. In this section, we perform the same drill for 50 network instances, each with 50 nodes randomly deployed in the 1000×1000 square. Again, there are four sessions in each network instance, with each session's source and destination nodes being randomly selected among the nodes. Here, a time frame has six time slots. Table III lists the objective values under OPT-IA and OPT-base. The average percentage increase in objective value (over 50 instances) is 43.4%.

TABLE III
A COMPARISON OF OBJECTIVE VALUES BETWEEN OPT-IA AND OPT-BASE.

Index	OPT-base	OPT-IA	Index	OPT-base	OPT-IA
1	0.333	0.5	26	0.333	0.5
2	0.5	0.667	27	0.333	0.5
3	0.333	0.5	28	0.5	0.667
4	0.667	0.83	29	0.333	0.5
5	0.5	0.667	30	0.333	0.5
6	0.333	0.5	31	0.333	0.667
7	0.5	0.5	32	0.333	0.5
8	0.333	0.5	33	0.5	0.833
9	0.667	0.667	34	0.5	0.667
10	0.5	0.667	35	0.333	0.5
11	0.333	0.5	36	0.333	0.5
12	0.667	0.667	37	0.5	0.5
13	0.333	0.5	38	0.333	0.5
14	0.333	0.667	39	0.333	0.5
15	0.5	0.667	40	0.333	0.5
16	0.333	0.5	41	0.333	0.5
17	0.333	0.5	42	0.5	0.667
18	0.5	0.667	43	0.333	0.5
19	0.333	0.5	44	0.667	0.667
20	0.333	0.5	45	0.333	0.5
21	0.333	0.5	46	0.333	0.5
22	0.667	0.667	47	0.667	0.833
23	0.333	0.5	48	0.333	0.5
24	0.333	0.667	49	0.333	0.5
25	0.5	0.667	50	0.333	0.5

VIII. CONCLUSIONS

The goal of this paper is to make a concrete step forward in advancing IA technique in multi-hop MIMO networks. We developed an IA model consisting of a set of constraints for each transmitter and receiver in a multi-hop MIMO network. Based on this IA model, we developed an optimization framework for IA in a multi-hop MIMO network. We anticipate that this framework (or variants of it) will be widely adopted by the networking community to study IA in a multi-hop network environment.

As an application of this optimization framework, we studied a network throughput optimization problem and compared performance objectives with our IA model and that without IA. Simulation results showed that the use of IA in a multi-hop MIMO network can significantly reduce DoF consumption for IC at the receivers, thereby improving network throughput.

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APPENDIX A
PROBLEM FORMULATION WITHOUT IA

We formulate the same network throughput optimization (with only MIMO's SM and IC). We have the same DoF consumption constraint on the transmitting node as (11) in OPT-IA. However, without IA, the DoF consumption constraint on the receiving node is different from (12) in OPT-IA.

If node j is receiver, then its DoF consumption consists of two parts: for SM and for IC. The number of its DoFs consumed for SM is $\mu_j = \sum_{l \in \mathcal{I}_j^{\text{in}}} z_l$. The number of its DoFs consumed for IC is equal to the number of its interfering streams (i.e., $\sum_{i \in \mathcal{I}_j} \alpha_{ij}$). Thus, we have the following DoF constraint at node j.

$$\sum_{l \in \mathcal{L}_j^{\text{in}}} z_l + \sum_{i \in \mathcal{I}_j} \alpha_{ij} \le M.$$

Otherwise (node j is either a transmitter or inactive), we know $z_l(t) = 0$ for $l \in \mathcal{L}_j^{\text{in}}$ and $\alpha_{ij}(t) = 0$ for $i \in \mathcal{I}_j$ based on their definitions. Combining these two cases, we have the following DoF consumption constraint on the receiving node:

$$\sum_{l \in \mathcal{L}_j^{\text{in}}} z_l(t) + \sum_{i \in \mathcal{I}_j} \alpha_{ij}(t) \le M \cdot y_j(t), \quad (1 \le j \le N, 1 \le t \le T), \tag{20}$$

where $\alpha_{ij}(t)$ is constrained by (7), which is equivalent to the combination of (18) and (19).

Now we formulate the problem as follows:

OPT-base Max $r_{\min}$
s.t. Half duplex constraints: (4);
Node activity constraints: (5), (6);
DoF consumption constraints: (11), (18–20);
Link capacity constraints: (13);
Flow routing constraints: (14–15);
Min rate constraints: (17).

where $x_i(t)$ and $y_i(t)$ are binary variables; $z_l(t)$ and $\alpha_{ij}(t)$ are non-negative integer variables; $r(f)$ and $r_l(f)$ are non-negative variables; M, N, L, F, T, and B are constants.