Squeezing the Most Out of Interference: An Optimization Framework for Joint Interference Exploitation and Avoidance

Canming Jiang† Yi Shi† Y. Thomas Hou† Wenjing Lou† Sastry Kompella‡ Scott F. Midkiff†
† Virginia Polytechnic Institute and State University, USA
‡ U.S. Naval Research Laboratory, Washington, DC, USA

***Abstract*—There is a growing interest on exploiting interference (rather than avoiding it) to increase network throughput. In particular, the so-called *successive interference cancellation* (SIC) scheme appears very promising, due to its ability to enable concurrent receptions from multiple transmitters and interference rejection. Although SIC has been extensively studied as a physical layer technology, its research and advances in the context of multi-hop wireless network remain limited. In this paper, we try to answer the following fundamental questions. (i) What are the limitations of SIC? How to overcome such limitations? (ii) How to optimize the interaction between SIC and interference avoidance? How to incorporate multiple layers (physical, link, and network) in an optimization framework? We find that SIC alone is not adequate to handle interference in a multi-hop wireless network, and advocate the use of joint SIC and interference avoidance. To optimize the joint scheme, we propose a cross-layer optimization framework that incorporates variables at physical, link, and network layers. We use numerical results to affirm the validity of our optimization framework and give insights on how SIC and interference avoidance can complement each other in an optimal manner.**

I. Introduction

Interference is widely regarded as the fundamental impediment to throughput performance in wireless networks. In networking community, a natural and main stream approach to handle interference is to employ certain *interference avoidance* scheme, which can be done either through deterministic resource allocation (e.g., TDMA, FDMA, or CDMA) or random access based schemes (e.g., CSMA, CSMA/CA). The essence of an interference avoidance scheme is to remove any overlap among the transmitting signals (the root of interference). Although easy to understand and simple to implement, an interference avoidance scheme, in general, cannot offer a performance close to network information theoretical limit [25].

Recently, there is a growing interest on exploiting interference (rather than avoiding it) to increase network throughput (see Section II for related work). In essence, such an *interference exploitation* approach allows overlap among transmitting signals and relies on some advanced decoding schemes to remove interference. In particular, the so-called *successive interference cancellation* (SIC) scheme appears very promising [1], [3], [7], [9], [15], [30] and has already attracted development efforts from industry (e.g., QUALCOMM's CSM6850 chipset for cellular base station [18]). Under SIC, a receiver attempts to decode the concurrent signals from multiple transmitters successively, starting from the strongest signal. If the strongest signal can be decoded, it will be subtracted from the aggregate signal so that the SINR (signal-to-interference-and-noise-ratio) for the remaining signals can be improved. Then the SIC receiver continues to decode the second strongest signals and so forth, until all signals are decoded, or terminates if the signal is no longer decodable (see Section III for more details).

Although SIC has been extensively studied as a physical layer technology, its limitation and optimal application in the context of multi-hop wireless network remain limited. In this paper, we will try to answer the following fundamental questions.

- What are the limitations of SIC? How to overcome such limitations?
- How to develop an optimization framework for optimal interaction between SIC and interference avoidance? How to incorporate variables from multiple layers (physical, link, and network) into such an optimization framework?

We take a formal optimization approach to address these fundamental questions. We find that the limitations of SIC come from its stringent constraints when decoding multiple signals. Specifically, in order to decode aggregate signals successively, an SIC receiver must meet a series of SINR constraints on its received signal powers. Further, due to these constraints, there exists a decoding limit on SIC for concurrent receptions or interference rejection. As a result, SIC alone is inadequate to handle all concurrent interference in a multi-user wireless network.

However, the limitations of SIC can be overcome precisely by the traditional interference avoidance scheme. Therefore, we advocate a joint interference exploitation and avoidance approach, which combines the best of both worlds while remove each other's pitfalls. We believe such an approach is most appropriate to handle interference in a multi-hop wireless network.

Although the need of such a joint approach is easy to understand, there are a number of new technical challenges in the context of a multi-hop network. This is particularly true when the optimization space encompasses physical layer SIC, link layer scheduling, and network layer flow routing. We address these new challenges by developing a formal optimization framework, with cross-layer formulation of physical, link, and network layers. This new optimization framework offers a holistic design space to squeeze the most out of interference and lay a mathematical foundation for the modeling and analysis of a joint interference exploitation and avoidance scheme in a multi-hop wireless network. To the best of our knowledge, this is the first effort toward this direction. To demonstrate the practical utility of our optimization framework, we conduct a case study on maximizing network throughput. Our numerical results affirm the efficacy of this framework and give us insights on how SIC should optimally interact with an interference avoidance scheme.

The rest of this paper is organized as follows. Section II presents related work on interference exploitation. Section III offers a primer on SIC and illustrates its benefits. In Section IV, we discuss some inherent limitations of SIC. In Section V, we advocate a joint interference exploitation and avoidance approach to overcome these limitations. In Section VI, we develop mathematical models for constraints under such a scheme. In Section VII, we develop a formal optimization framework for the joint interference exploitation-avoidance scheme. In Section VIII, we apply our optimization framework on a case study and present some numerical results. Section IX concludes this paper. Table I lists the relevant notation used in this paper.

TABLE I
NOTATION.

Symbol	Definition
A_j	The maximum number of signals an SIC receiver j can decode
B	The channel bandwidth in our network
C_{ij}	The maximum achievable rate on link $i \rightarrow j$
d_{ij}	Distance between nodes i and j
D_{ijm}	A lower bound of $P_{m,j} - \sum_{k \neq i}^{P_{k,j} \leq P_{m,j}} \beta P_{kj} \cdot \lambda_k[t]$ $-\beta\sigma^2$
$d(f)$	Destination node of session $f \in \mathcal{F}$
$\mathcal{F}$	The set of user sessions in the network
g_{ij}	Channel gain from node i to node j
H_{ij}	A lower bound of $P_{i,j} - \sum_{k \neq i}^{P_{k,j} \leq P_{i,j}} \beta P_{k,j} \cdot \lambda_k[t]$ $-\beta\sigma^2$
$\mathcal{I}_i$	The set of neighboring nodes of node i
$\mathcal{L}$	The set of links in the network
$\mathcal{N}$	The set of nodes in the network
$\mathcal{N}_j$	The set of nodes which are transmitting when j is receiving
M_{ij}	A lower bound of $P_{i,j} - \sum_{k \neq i} \beta P_{k,j} \lambda_k[t] - \beta\sigma^2$
P	The transmission power of each node
$P_j^{\max}$	$= \max_{i \in \mathcal{N}_j} P_{ij}$, the maximum power of all signals received at node j
P_{ij}	The received power at node j from node i
$r(f)$	Data rate of session $f \in \mathcal{F}$
$r_{ij}(f)$	Data rate that is attributed to session f on link l
$s(f)$	Source nodes of session f
T	The total number of time slots in each time frame
$x_{ij}[t]$	The indicator of weather the transmission on link $i \rightarrow j$ is successful or not on time slot t
$y_{(i,j)(m)}[t]$	A bridge binary variable
$w(f)$	The weight associated with session f
R	The data rate of a successful transmission
β	The SINR threshold for successful decoding
γ	Path loss index
$\lambda_i[t]$	The indicator of weather node i is transmitting or not in time slot t
σ^2	The power level of ambient noise

II. RELATED WORK

At the physical layer, a classic reference on interference exploitation (cancellation) is the book by Verdu [27] and references therein. For more details and new advances of some important interference cancellation techniques, we refer readers to see SIC [5], [28], parallel interference cancellation [8], [26], iterative interference cancellation (turbo multiuser user detection) [12], [29], which all aim to enable a receiver to decode multiple signals at the same time, and reject interference from other unintended transmitters. A recent review on how to apply interference cancellation for cellular systems was given in [1], which positioned SIC as one of the most promising techniques to mitigate interference due to its simplicity and effectiveness.

Note that the SIC considered in this paper differs from some new interference cancellation schemes such as analog network coding [13] and ZigZag decoding [17]. Both were proposed to resolve packet collisions, and they require that some bits in one of the collision packets be known in advance. SIC also differs from smart antenna-based interference cancellation schemes, such as Zero-Forcing Beam Forming (ZFBF) [2], [23], [31] in MIMO[1] and directional antennas [14], [19], [24].

Very recently, there is a growing interest to exploit SIC at the physical layer to improve performance of upper layers in a wireless network [3], [7], [9], [15], [16], [30]. In [9], Halperin *et al.* built a ZigBee prototype of SIC based on [27, Ch. 7] using software radios and used experimental results to validate that SIC is an effective way to improve system throughput. In [15], Lv *et al.* studied a scheduling problem in an ad hoc network with SIC. To simplify network-layer problem, the authors considered fixed routes in the network (e.g., based on shortest path), and subsequently developed a greedy heuristic scheduling algorithm based on conflict set graph. Link scheduling problem for wireless networks with SIC was also studied in [16], but the aggregate interference effect of the practical SINR model was not considered. In [7], Gelal *et al.* proposed a topology control framework for exploiting SIC. They studied how to divide a network topology into a minimum number of sub-topologies where the set of links in each sub-topology can be active at the same time. In [30], Weber *et al.* studied the asymptotic transmission capacity of one-hop ad hoc networks with SIC under a simplified model where all signals from transmitters within a specific radius can all be successfully decoded. More realistic SIC model for asymptotic transmission capacity was later explored by Blomer and Jindal in [3]. We also notice a recent paper [21] claiming that the potential gain by SIC is very marginal. This is in contrast to the state-of-the-art [3], [7], [9], [15], [16], [30] as well as our findings in this paper (see Footnote 7). A closer look at [21] shows that the claim was made based on

[1]Note that MIMO requires multiple antennas for interference cancellation, while SIC does not have such requirement. This paper considers SIC with a single antenna on each node.

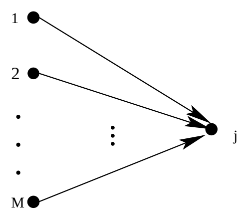

Fig. 1. A receiver with M concurrent transmitters.

unfair comparison. In [21], the authors considered a simple network with two links. They compared the time needed to complete one packet on both links with SIC and without SIC. Without SIC, the two links transmits data sequentially and the completion time is the sum of the time used on both links. With SIC, the two links can transmit data simultaneously and the completion time is defined as the maximum time used by these two links. We argue that their comparison method is unfair. Since the link that finishes the transmission first can transmit other packets afterwards instead of being idle.

To date, results on how to apply SIC in a *multi-hop* network remain very limited, particularly those results based on a formal optimization framework that explores the interaction of interference exploitation and cancellation.

III. SIC and Its Benefits

Under the classical information reception model in a wireless network, a receiver j treats all the interfering signals from other concurrent (non-intended) transmissions as noise. For the signal from the intended transmitting node i, if its SINR at node j is greater than or equal to a threshold β, then the transmission is said to be successful (i.e., the signal from node i to j can be decoded successfully). [2] Denote P_{ij} the power level of the signal from node i that is received at node j. Denote $\mathcal{N}_j$ the set of other concurrent transmitting nodes that can be heard by node j. Then, under the classical model, a successful transmission from transmitting node i to node j occurs if

$$\frac{P_{ij}}{\sum_{k\in\mathcal{N}_j}^{k\neq i} P_{kj} + \sigma^2} \geq \beta ,$$

where constant σ^2 is the power level of the ambient noise.

In contrast to the above classical paradigm, a receiver with SIC capability can decode a number of concurrent signals (including some interfering signals) rather than treating them blindly as noise [9], [27, Ch. 7], [30]. This is done by decoding concurrent signals in a *sequential* order and subtracting each successfully decoded signal before proceeding to decode the next one. Figure 1 illustrates a communication scenario where

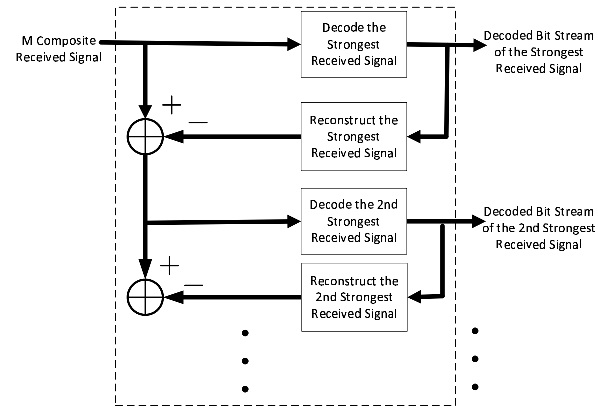

Fig. 2. The process of SIC.

a node j is receiving from M concurrent transmitters. Under SIC, receiver j first attempts to decode the strongest signal. If the strongest signal can be decoded successfully (i.e., the SINR for this signal is no less than the threshold β), then this signal will be subtracted from the aggregate signal (see Fig. 2). Then the receiving node j tries to decode the second strongest signal and so forth. The process continues until all the signals are successfully decoded or at some stage the SINR criterion for the underlying signal is no longer satisfied.

Without loss of generality, referring to Fig. 1, suppose that the power levels of the signals from the M transmitters received at node j are in nondecreasing order as $P_{1j} \leq P_{2j} \leq \cdots \leq P_{Mj}$. Receiving node j tries to decode the signals from transmitting nodes in the order of $M, M-1, \cdots 1$. Then, the signal with received power P_{ij} can be decoded successfully if and only if

$$\begin{array}{lc} \text{Step 1} & \dfrac{P_{Mj}}{\sum_{k=1}^{M-1} P_{kj} + \sigma^2} \geq \beta , \\ \text{Step 2} & \dfrac{P_{M-1,j}}{\sum_{k=1}^{M-2} P_{kj} + \sigma^2} \geq \beta , \\ \vdots & \vdots \\ \text{Step } (M-i+1) & \dfrac{P_{ij}}{\sum_{k=1}^{i-1} P_{kj} + \sigma^2} \geq \beta . \end{array} \tag{1}$$

As shown in (1), in order to decode the signal with received power P_{ij}, it is necessary to decode all the stronger signals first. Note that we assume perfect cancellation of a successfully decoded signal in the iterative process. Similar to [7], [15], [16], we do not consider link rate adaptation in our model and assume that the data rate on each successful transmission is $R = B\log_2(1+\beta)$, where B is the channel bandwidth.[3]

There are two key benefits associated with SIC, namely, enabling *concurrent receptions from multiple transmitters* and *interference rejection*. In the rest of this section, we elaborate these two benefits.

[2]In communication theory, the SINR of a received signal determines the receiver's ability to recover the signal. Suppose the transmitting node is sending data at a rate R under certain encoding scheme. If the SINR of this transmission is no less than the threshold β, then the error probability is considered to be within a certain bound and the receiver can successfully decode the signal and recover the same data rate R. Otherwise (i.e., SINR less than β), the error probability is considered too high for the receiver to recover the data rate R and retransmission may be necessary.

[3]We leave the more complex case with link rate adaption as our future work.

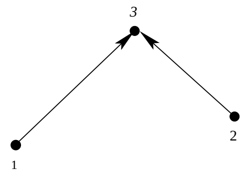

Fig. 3. An example of concurrent receptions from multiple transmitters.

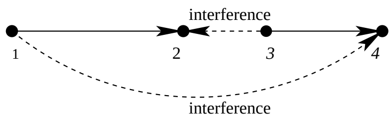

Fig. 4. An example for interference rejection.

Concurrent Receptions from Multiple Transmitters. Note that under the classical reception model, only one intended transmitter is allowed to transmit; concurrent transmissions to the same receiver will lead to a collision and are considered wasteful of resource. In contrast, a SIC receiver is capable of receiving from multiple transmitters at the same time (if the criteria in (1) are met) and thus can substantially increase throughput in the network. As a simple example, consider Fig. 3, where both nodes 1 and 2 wish to transmit to receiving node 3. Assume $P_{13} = 1$, $P_{23} = 1.2$, $\sigma^2 = 0.1$, and $\beta = 1$, where all units are normalized with appropriate dimensions. Under the traditional interference avoidance model, nodes 1 and 2 cannot transmit to node 3 at the same time due to interference. Under SIC, receiver 3 can first decode the stronger signal from node 2 by treating the signal from 1 as interference. We have $\frac{1.2}{1+0.1} = 1.1 > \beta$. Next, receiver 3 subtracts the decoded signal from the aggregate signal. The SINR from node 1 is $\frac{P_{13}}{P_{23}-P_{23}+\sigma^2} = \frac{1}{1.2-1.2+0.1} = 10 > \beta$, which shows transmission from node 1 is also successful.

Interference Rejection. The ability to decode multiple received signals can also help the receiving node to selectively reject interference from other unintended transmitters. As a simple example, consider the two-transmitter two-receiver case in Fig. 4. Node 1 wishes to send data to node 2 while node 3 wishes to send data to node 4. Due to the broadcast nature of a wireless channel, the signal from node 3 will interfere with the reception at node 2 and likewise the signal from node 1 will interfere with the reception at node 4. Assume $P_{12} = 1$, $P_{14} = 0.5$, $P_{32} = 2$, $P_{34} = 1.5$, $\sigma^2 = 0.1$, and $\beta = 1$. Under the traditional model, links $1 \to 2$ and $3 \to 4$ cannot be active at the same time. Under SIC, receiver 2 can first try to decode the strongest received signal, which is the signal from node 3. Since $\frac{P_{32}}{P_{12}+\sigma^2} = \frac{2}{1+0.1} = 1.82 > \beta$, such decoding is successful. Then, node 2 subtracts this decoded signal from the aggregate signal, and tries to decode the second strongest signal, which is from node 1. We have $\frac{P_{12}}{P_{32}-P_{32}+\sigma^2} = \frac{1}{2-2+0.1} = 10 > \beta$. So this decoding is again successful. Likewise, on node 4, it tries to decode the strongest received signal first, which is from node 3. Since $\frac{P_{34}}{P_{14}+\sigma^2} = \frac{1.5}{0.5+0.1} = 2.5 > \beta$, this decoding is successful.

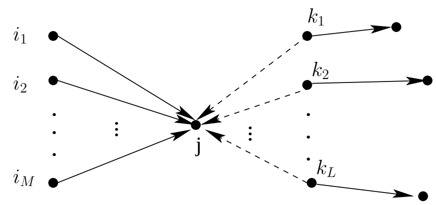

Fig. 5. The general case of concurrent reception and interference rejection at a receiving node j. A solid arrow represents intended transmission and a dashed arrow represents interference.

Summary. Our discussion for the above two benefits, i.e., concurrent reception from multiple transmitters (Fig. 3) and interference rejection (Fig. 4) can be generalized by Fig. 5. In this figure, a receiving node j tries to decode all the signals it receives, among which it tries to retain the desired bit streams from the M intended transmitters and cancel the interfering bit streams from the L unintended transmitters.

IV. Limitations of SIC

Although the potential benefits of SIC are not hard to recognize, we now show that these benefits may not always be readily available. In other words, there are some stringent constraints and hard limits for SIC to satisfy before reaping any potential benefit.

Sequential SINR Constraint. As we have shown in Section III, at any stage when a receiver tries to decode the desired signal from the aggregate signal, the SINR must satisfy (1). Otherwise, the current signal cannot be decoded successfully, neither will all the remaining weaker signals.

Again let's use the two-transmitter one-receiver example in Fig. 3. Assuming $P_{13} < P_{23}$, to decode both the signals from nodes 1 and 2 successfully, P_{13} and P_{23} must satisfy

$$\frac{P_{23}}{P_{13}+\sigma^2} \geq \beta \text{ and } \frac{P_{13}}{\sigma^2} \geq \beta \ .$$

But suppose $P_{23} = 1.2, P_{13} = 1$, $\sigma^2 = 0.5$ and $\beta = 1$. Then we have $\frac{P_{23}}{P_{13}+\sigma^2} = \frac{1.2}{1+0.5} = 0.8 < \beta$. This means that even the strongest signal from node 2 cannot be successfully decoded. Therefore, the weaker signal from node 1 cannot be decoded either. In this case, SIC will not work.

Sequential Decoding Limit. Another limitation of SIC is that it can only decode a limited number of signals (either intended or unintended). Such limit is determined by (1) and sets up a cap on the number of decodable signals. Before we calculate this limit, we present the following property.

Property 1: **(Geometric Power Property)** *Denote P_{1j}, P_{2j}, $\cdots$, P_{Mj} the received powers of the signals that can be successfully decoded at node j via SIC. Without loss of generality, suppose $P_{1j} \leq P_{2j} \leq \ldots \leq P_{Mj}$. Then, we have*

$$P_{ij} \geq \beta(1+\beta)^{i-1}\sigma^2, \textit{ for } i = 1, \ldots, M \ .$$

Proof: Our proof is based on induction. First consider $i = 1$. Since all previous stronger interference are removed from the composite interference when decoding the weakest signal, the SINR for P_{1j} is $\frac{P_{1j}}{\sigma^2}$, which must be no less than β. Then, we have $\frac{P_{1j}}{\sigma^2} \geq \beta$, which is $P_{1j} \geq \beta\sigma^2$.

Next, suppose that

$$P_{ij} \geq \beta(1+\beta)^{i-1}\sigma^2, \ i = 1, \ldots, l \ . \tag{2}$$

We will prove that $P_{l+1,j} \geq \beta(1+\beta)^l\sigma^2$. We know that we still have all the interference from the weaker signals when we decode the signal from $P_{l+1,j}$. Then, we have $P_{l+1,j}/(\sum_{i=1}^{l} P_{ij} + \sigma^2) \geq \beta$, which gives us

$$\begin{aligned} P_{l+1,j} &\geq \beta \left(\sum_{i=1}^{l} P_{ij} + \sigma^2 \right) \geq \beta \left[\sum_{i=1}^{l} \beta(1+\beta)^{i-1}\sigma^2 + \sigma^2 \right] \\ &= \beta \left[1 + \beta \sum_{i=1}^{l} (1+\beta)^{i-1} \right] \sigma^2 = \beta(1+\beta)^l\sigma^2 \ , \end{aligned}$$

where the second inequality holds due to (2). ■

Now we are ready to calculate the limit on the number of signals that can be decoded. More formally, denote A_j an upper bound of the number of signals that receiver j can decode. Then we have the following lemma.

Lemma 1: Denote $P_j^{\max}$ the strongest received power at receiver j, i.e., $P_j^{\max} = \max_{i \in \mathcal{N}_j} P_{ij}$, where $\mathcal{N}_j$ is the set of all active concurrent transmitters. Then the number of successfully decoded signals at receiver j is no more than $A_j = 1 + \log_{\beta+1}(\frac{P_j^{\max}}{\beta\sigma^2})$.

Proof: Let $P_{1j} \leq P_{2j} \leq \ldots \leq P_{Mj}$ be a set of powers of the signals successfully decoded at receiver j, we have $P_j^{\max} = P_{Mj}$. Combining $P_j^{\max} = P_{Mj}$ with Property 1 gives us $P_j^{\max} = P_{Mj} \geq \beta(1+\beta)^{M-1}\sigma^2$, which gives us

$$M \leq 1 + \log_{\beta+1} \left(\frac{P_j^{\max}}{\beta\sigma^2} \right) \ .$$

The above inequality says that the number of successfully decoded signals at receiver j is upper bounded by $A_j = 1 + \log_{\beta+1}(\frac{P_j^{\max}}{\beta\sigma^2})$. ■

As an example of the sequential decoding limit, we assume that $P_j^{\max} = 5$, $\sigma^2 = 0.1$, and $\beta = 1$. Based on Lemma 1, we have $A_j = 1 + \log_{1+1}(\frac{5}{1 \cdot 0.1}) = 6.6$. That is, only up to 6 signals can be successfully decoded at receiver j.

Remark 1: Note that A_j given in Lemma 1 is only an upper bound. The actual number of decodable signals may be much lower than this bound. This is because that the powers of decodable signals must also satisfy the sequential SINR constraints in (1). ■

V. An Optimal Approach for Interference Exploitation

A. Approach

Based on the discussion in Section III, an interference exploitation scheme such as SIC has clear advantage over a pure interference avoidance scheme. On the other hand, due to the intrinsic limitations associated with SIC, it is evident that SIC alone is inadequate in a multi-hop network. As a result, it is necessary to incorporate interference avoidance (e.g., scheduling) to complement such limitations. This is true for both the sequential SINR constraints and sequential decoding limit. In particular, when the sequential SINR constraints are no longer satisfied at certain stage, one has to resort to scheduling (e.g., time slot assignment) to avoid interference so that different transmissions can be carried out successfully. Likewise, once the number of interfering transmissions exceeds the sequential decoding limit, one again has to employ scheduling to allocate theses transmissions into different time slots such that the number of interfering transmissions in each time slot is within the decoding limit. In other words, we should take the best of both worlds (interference exploitation and interference avoidance) while avoid each other's pitfalls.

B. New Challenges

There are several new challenges when developing a joint interference exploitation and interference avoidance scheme, particularly in a multi-hop network.

- At the physical layer, under the classical SINR model, a receiving node treats all the other concurrent (unintended) interfering transmissions as noise when deciding whether or not the underlying intended transmission is successful. This itself is not a trivial problem as the set of interfering transmissions is usually coupled with upper layer scheduling and routing algorithms. In the context of SIC, not only one needs to deal with such coupling with upper layer algorithms, one also has to deal with multiple transmissions at the same time, in the sense that one has to decode those stronger signals before decoding its own signal (in a sequential order). This sequential decoding imposes significant difficulty to develop a tractable model for mathematical programming.
- At the link layer, a scheduling algorithm (i.e., interference avoidance scheme) is needed to address the limitations of SIC at the physical layer. Note that such scheduling algorithm is also coupled with routing in a multi-hop network environment. How to design an optimal scheduling algorithm to fulfill certain network performance objective in this context is a new and non-trivial problem.
- As discussed in Section III, SIC allows more concurrent transmissions in the network than traditional interference avoidance model. This offers many more available links for choosing a path at the network layer. Consequently, the design space at the network layer is much larger, leading to a more complex optimization problem.

To address these new challenges, it is necessary to develop a tractable cross-layer model that is suitable for a formal optimization framework.

VI. Modeling of Cross-layer Constraints

As a first step toward a formal optimization framework, we examine constraints across the three lower layers for a multi-hop network. Consider a single antenna multi-hop wireless network, with a set of nodes $\mathcal{N}$ operating within the same channel and SIC-capable. For interference avoidance, we consider TDMA in the time domain.[4] Under TDMA, we assume a frame is divided into T time slots, each of equal length. For simplicity, we do not consider individual power control of each node and assume each node transmit at the same power P. Denote g_{ij} the channel gain from node i to node j. Then, when node i is transmitting, the received power at node j is $P_{ij} = P \cdot g_{ij}$.

Scheduling Constraints. We first define a binary scheduling variable $x_{ij}[t]$ for link $i \to j$ in time slot t ($1 \le t \le T$).

$$x_{ij}[t] = \begin{cases} 1 & \text{if node } i \text{ transmits data to node } j \\ & \text{successfully in time slot } t \\ 0 & \text{otherwise.} \end{cases}$$

By "successfully," we mean that the intended transmission from node i can be decoded at node j via SIC, i.e., the sequential SINR constraints in (1) are satisfied for this signal. In the case of an "unsuccessful" transmission (i.e., the sequential SINR constraints in (1) are not satisfied for this signal), it is desirable to turn off the transmitter rather than having it transmit undecodable signals. Therefore, when $x_{ij}[t] = 0$, we will not have any transmission from node i to node j.

Denote $\mathcal{I}_i$ the set of all neighboring nodes of node $i \in \mathcal{N}$. For unicast communication in the network, a node transmits data to only one node in a time slot, i.e.,

$$\sum_{j \in \mathcal{I}_i} x_{ij}[t] \le 1 \; (i \in \mathcal{N}, 1 \le t \le T) \, . \tag{3}$$

For reception at a node, it becomes more interesting, as a node can receive data from multiple nodes in a time slot due to SIC. That is, for a receiver j, we may have $\sum_{i \in I_j} x_{ij}[t] > 1$.

Based on the state-of-the-art in the literature, there is no evidence that SIC can achieve full-duplex with single antenna. Therefore, half-duplex will still be necessary at each node. To model half-duplex at a node i, we have

$$x_{ki}[t] + x_{ij}[t] \le 1 \quad (i \in \mathcal{N}, k, j \in \mathcal{I}_i, 1 \le t \le T) \, . \tag{4}$$

That is, node i cannot transmit and receive at the same time.

Denote C_{ij} the achievable link rate on link $i \to j$. Then, we have $C_{ij} = \frac{1}{T} \sum_{t=1}^{T} R \cdot x_{ij}[t]$.

Joint PHY-Link Constraints. We first give a definition for *residual SINR*, which characterize the SINR value in a sequential fashion under SIC. For a signal from node i to node j in time slot t (from either intended or unintended transmission), we define the residual SINR (or r-SINR) of this signal, r-SINR$_{(i,j)}[t]$, as

$$\begin{aligned} &\text{r-SINR}_{(i,j)}[t] \\ =&\frac{P_{ij}}{\sum_{k \ne i} \sum_{l \in \mathcal{I}_k} P_{kj} x_{kl}[t] - \sum_{k \ne i}^{P_{kj} > P_{ij}} \sum_{l \in \mathcal{I}_k} P_{kj} x_{kl}[t] + \sigma^2} \\ =&\frac{P_{ij}}{\sum_{k \ne i}^{P_{kj} \le P_{ij}} \sum_{l \in \mathcal{I}_k} P_{kj} \cdot x_{kl}[t] + \sigma^2} \, . \end{aligned} \tag{5}$$

[4]Interference avoidance in the frequency domain via FDMA can also be done in the same manner.

Note that $\sum_{k \ne i}^{P_{kj} \le P_{ij}} \sum_{l \in \mathcal{I}_k} P_{kj} \cdot x_{kl}[t]$ is the residual interference when node j attempts to decode the signal from node i after subtracting all the stronger received signals from other concurrent transmissions.

To see the coupling of r-SINR with scheduling, note that when $x_{ij}[t] = 1$, we have a successful decoding for the signal from node i to node j under SIC. This implies that

- The r-SINR's of all stronger received signals at node j from other concurrent transmissions are no less than the SINR threshold β.
- The r-SINR of the signal from node i to j is no less than the SINR threshold β.

More formally, we have following coupling constraints for PHY-Link layers.

$$\begin{aligned} &\text{If } x_{ij}[t] = 1, \text{ then r-SINR}_{(m,j)}[t] \ge \beta \; (j \in \mathcal{N}, i \in \mathcal{I}_j, \\ &\quad m \ne i, n \in \mathcal{I}_m, P_{mj} > P_{ij}, x_{mn}[t] = 1, 1 \le t \le T) \end{aligned} \tag{6}$$

$$\text{If } x_{ij}[t] = 1, \text{ then r-SINR}_{(i,j)}[t] \ge \beta \; (j \in \mathcal{N}, i \in \mathcal{I}_j, 1 \le t \le T). \tag{7}$$

Flow Routing Constraints. For a set of unicast communication sessions $\mathcal{F}$, denote $r(f)$ the data rate of session $f \in \mathcal{F}$, $s(f)$ and $d(f)$ the source and the destination nodes of session $f \in \mathcal{F}$, respectively. Denote $r_{ij}(f)$ the amount of rate on link $i \to j$ that is attributed to session $f \in \mathcal{F}$. Then we have the following flow balance. If node i is the source node of session f, i.e., $i = s(f)$, then

$$\sum_{j \in \mathcal{I}_i} r_{ij}(f) = r(f) \quad (f \in \mathcal{F}, i = s(f)) \, . \tag{8}$$

If node i is an intermediate relay node for session f, i.e., $i \ne s(f)$ and $i \ne d(f)$, then

$$\sum_{j \in \mathcal{I}_i}^{j \ne s(f)} r_{ij}(f) = \sum_{k \in \mathcal{I}_i}^{k \ne d(f)} r_{ki}(f) \quad (f \in \mathcal{F}, i \ne s(f), d(f)) \, . \tag{9}$$

If node i is the destination node of session f, i.e., $i = d(f)$, then

$$\sum_{k \in \mathcal{I}_i} r_{ki}(f) = r(f) \quad (f \in \mathcal{F}, i = d(f)) \, . \tag{10}$$

Note that in the above flow balance equations, we allow flow splitting/merging inside the network, which is more general than single-path flow routing. Further, it can be easily verified that if (8) and (9) are satisfied, then (10) is also satisfied. As a result, it is sufficient to list only (8) and (9) in the optimization framework.

Since the aggregate flow rate on any link $i \to j$ cannot exceed the achievable link rate C_{ij}, we have

$$\sum_{f \in \mathcal{F}}^{s(f) \ne j, d(f) \ne i} r_{ij}(f) \le C_{ij} = \sum_{t=1}^{T} \frac{R}{T} \cdot x_{ij}[t] \quad (j \in \mathcal{N}, i \in \mathcal{I}_j). \tag{11}$$

VII. A Formal Optimization Framework

A. Motivation

Note that the two sets of constraints in (6) and (7) are stated in the form of sufficient conditions rather than in the form of mathematical programming suitable for problem solving.[5] Therefore, a reformulation of (6) and (7) is needed.

As the first step to reformulate (6), we move $x_{mn}[t] = 1$ out of the range in (6). By treating $x_{mn}[t] = 1$ as part of the sufficient condition, (6) can be re-stated as follows:

$$\text{If } (x_{ij}[t] = 1 \text{ and } x_{mn}[t] = 1), \text{ then r-SINR}_{(m,j)}[t] \geq \beta \\ (j \in \mathcal{N}, i \in \mathcal{I}_j, m \neq i, n \in \mathcal{I}_m, P_{mj} > P_{ij}, 1 \leq t \leq T) \,. \tag{12}$$

To combine $x_{ij}[t] = 1$ and $x_{mn}[t] = 1$ into one condition, we can introduce a binary variable, $y_{(i,j)(m,n)}[t]$, as follows.

$$y_{(i,j)(m,n)}[t] = 1 \text{ if and only if } (x_{ij}[t] = 1 \text{ and } x_{mn}[t] = 1) \\ (j \in \mathcal{N}, i \in \mathcal{I}_j, m \neq i, n \in \mathcal{I}_m, P_{mj} > P_{ij}, 1 \leq t \leq T).$$

For time slot t, we note that binary variable y has subscripts for four node dimensions, i, j, m, n, which means the number of such y variables could be a very large number. However, we find that we can remove the last node dimension n and reduce the number of y variables based on the following lemma.

Lemma 2: Statement (12) is equivalent to the following statement:

$$\textit{If } \left(x_{ij}[t] = 1 \textit{ and } \textstyle\sum_{n \in \mathcal{I}_m} x_{mn}[t] = 1\right)\textit{, then} \\ \textit{r-SINR}_{(m,j)}[t] \geq \beta \; (j \in \mathcal{N}, i \in \mathcal{I}_j, m \neq i, \\ P_{mj} > P_{ij}, 1 \leq t \leq T). \tag{13}$$

Note that the differences between (12) and (13) are that $x_{mn}[t] = 1$ in (12) is replaced by $\sum_{n \in \mathcal{I}_m} x_{mn}[t] = 1$ in (13) and that $n \in \mathcal{I}_m$ in the range of (12) disappears in that of (13).

Proof: We first show that if (12) holds, then (13) also holds. If $x_{ij}[t] = 1$ and $\sum_{n \in \mathcal{I}_m} x_{mn}[t] = 1$, then there must exists one node $\hat{n} \in \mathcal{I}_m$ such that

$$x_{m\hat{n}}[t] = 1 \,.$$

Combining $x_{ij}[t] = 1$ and $x_{m\hat{n}}[t] = 1$, based on (12), we have r-SINR$_{(m,j)}[t] \geq \beta$.

Next, we show that if (13) holds, then (12) also holds. If $x_{ij}[t] = 1$ and $x_{mn}[t] = 1$, we have

$$\sum_{n \in \mathcal{I}_m} x_{mn}[t] = 1$$

based on (3). Combining $x_{ij}[t] = 1$ and $\sum_{n \in \mathcal{I}_m} x_{mn}[t] = 1$, based on (13), we have r-SINR$_{(m,j)}[t] \geq \beta$. ■

To simplify (13), we introduce a new binary variable $\lambda_m[t]$ and define it as follows:

$$\lambda_m[t] = \sum_{n \in \mathcal{I}_m} x_{mn} \quad (m \in \mathcal{N}, 1 \leq t \leq T) \,. \tag{14}$$

[5] By "the form of mathematical programming," we mean that a constraint should be written in the form: $h(\mathbf{x}) \leq 0$ or $h(\mathbf{x}) = 0$, where $\mathbf{x}$ is the set of variables in the constraint and h is a function mapping $\mathbf{x}$ into real space.

Intuitively, $\lambda_m[t]$ can be regarded as a variable representing whether or not node m is transmitting in time slot t, regardless of to whom it is transmitting. Then, (13) becomes

$$\text{If } (x_{ij}[t] = 1 \text{ and } \lambda_m[t] = 1), \text{ then r-SINR}_{(m,j)}[t] \geq \beta \\ (j \in \mathcal{N}, i \in \mathcal{I}_j, m \neq i, P_{mj} > P_{ij}, 1 \leq t \leq T). \tag{15}$$

To combine both conditions $x_{ij}[t] = 1$ and $\lambda_m[t] = 1$ into just one condition, we introduce a binary variable $y_{(i,j)(m)}[t]$ as follows:

$$y_{(i,j)(m)}[t] = 1 \text{ if and only if } (x_{ij}[t] = 1 \text{ and } \lambda_m[t] = 1) \\ (j \in \mathcal{N}, i \in \mathcal{I}_j, m \neq i, n \in \mathcal{I}_m, P_{mj} > P_{ij}, 1 \leq t \leq T) \,. \tag{16}$$

Note that variable y only has three node dimensions, i, j, m, which shows that the number of variables in the optimization framework has been decreased. Combining (16) and (15), we have

$$\text{If } y_{(i,j)(m)}[t] = 1, \text{ then r-SINR}_{(m,j)}[t] \geq \beta \; (j \in \mathcal{N}, \\ i \in \mathcal{I}_j, m \neq i, P_{mj} > P_{ij}, 1 \leq t \leq T) \,. \tag{17}$$

Now, (6) is replaced by (14), (16) and (17). Although (16) and (17) are still not in the form of mathematical programming, they are ready to be reformulated into such form. In the rest of this section, we show how to reformulate (16), (17) and (7).

B. Revised PHY-Link Constraints

Based on the definition of new variable $\lambda_m[t]$, we can refine the earlier definition of residual SINR in (5) as follows.

Definition 1: (r-SINR). For a signal from node i to node j in time slot t (from either intended or unintended transmission), the residual SINR (or r-SINR) of this signal is

$$\text{r-SINR}_{(i,j)}[t] = \frac{P_{ij}}{\sum_{k \neq i}^{P_{kj} \leq P_{ij}} P_{kj} \cdot \lambda_k[t] + \sigma^2} \,. \tag{18}$$

(i) Reformulation of (16)

Statement (16) is equivalent to the following two statements:

$$\text{If } (x_{ij}[t] = 1 \text{ and } \lambda_m[t] = 1), \text{ then } y_{(i,j)(m)}[t] = 1 \\ (j \in \mathcal{N}, i \in \mathcal{I}_j, m \neq i, P_{mj} > P_{ij}, 1 \leq t \leq T) \,. \tag{19}$$

$$\text{If } y_{(i,j)(m)}[t] = 1, \text{ then } (x_{ij}[t] = 1 \text{ and } \lambda_m[t] = 1) \\ (j \in \mathcal{N}, i \in \mathcal{I}_j, m \neq i, P_{mj} > P_{ij}, 1 \leq t \leq T) \,. \tag{20}$$

Statement (19) can be written as

$$y_{(i,j)(m)}[t] \geq x_{ij}[t] + \lambda_m[t] - 1 \; (j \in \mathcal{N}, i \in \mathcal{I}_j, \\ m \neq i, P_{mj} > P_{ij}, 1 \leq t \leq T) \,, \tag{21}$$

which means that when $x_{ij}[t] = 1$ and $\lambda_m[t] = 1$, we have $y_{(i,j)(m)}[t] = 1$. Statement (20) can be written as

$$x_{ij}[t] \geq y_{(i,j)(m)}[t] \quad (j \in \mathcal{N}, i \in \mathcal{I}_j, m \neq i, \\ P_{mj} > P_{ij}, 1 \leq t \leq T) \tag{22}$$

$$\lambda_m[t] \geq y_{(i,j)(m)}[t] \quad (j \in \mathcal{N}, i \in \mathcal{I}_j, m \neq i, \\ P_{mj} > P_{ij}, 1 \leq t \leq T) \,. \tag{23}$$

$$
\begin{aligned}
&\textbf{Scheduling:} && \\
&\lambda_m[t] = \sum_{n\in\mathcal{I}_m} x_{mn} && (m\in\mathcal{N}, 1\le t\le T) \\
&\frac{1}{\min\{A_j, |\mathcal{I}_j|\}} \sum_{i\in\mathcal{I}_j} x_{ij}[t] + \lambda_j[t] \le 1 && (j\in\mathcal{N}, 1\le t\le T) \\
&\textbf{PHY-Link:} && \\
&y_{(i,j)(m)}[t] \ge x_{ij}[t] + \lambda_m[t] - 1 && (j\in\mathcal{N}, i\in\mathcal{I}_j, m\ne i, P_{mj} > P_{ij}, 1\le t\le T) \\
&x_{ij}[t] \ge y_{(i,j)(m)}[t] && (j\in\mathcal{N}, i\in\mathcal{I}_j, m\ne i, P_{mj} > P_{ij}, 1\le t\le T) \\
&\lambda_m[t] \ge y_{(i,j)(m)}[t] && (j\in\mathcal{N}, i\in\mathcal{I}_j, m\ne i, P_{mj} > P_{ij}, 1\le t\le T) \\
&P_{mj} - \sum_{k\ne m}^{P_{kj}\le P_{mj}} \beta P_{kj}\lambda_k[t] - \beta\sigma^2 \ge \left(1 - y_{(i,j)(m)}[t]\right) D_{ijm} && (j\in\mathcal{N}, i\in\mathcal{I}_j, m\ne i, P_{mj} > P_{ij}, 1\le t\le T) \\
&P_{ij} - \sum_{k\ne i}^{P_{kj}\le P_{ij}} \beta P_{kj}\cdot\lambda_k[t] - \beta\sigma^2 \ge (1 - x_{ij}[t])H_{ij} && (j\in\mathcal{N}, i\in\mathcal{I}_j, 1\le t\le T) \\
&\textbf{Flow routing:} && \\
&\sum_{j\in\mathcal{I}_i} r_{ij}(f) = r(f) && (f\in\mathcal{F}, i = s(f)) \\
&\sum_{j\in\mathcal{I}_i}^{j\ne s(f)} r_{ij}(f) = \sum_{k\in\mathcal{I}_i}^{k\ne d(f)} r_{ki}(f) && (f\in\mathcal{F}, i\ne s(f), d(f)) \\
&\sum_{f\in\mathcal{F}}^{s(f)\ne j, d(f)\ne i} r_{ij}(f) \le \sum_{t=1}^{T} \frac{R}{T}\cdot x_{ij}[t] && (j\in\mathcal{N}, i\in\mathcal{I}_j)
\end{aligned}
$$

Fig. 6. An optimization framework for joint SIC and interference avoidance.

Inequalities (22) and (23) ensure that when $y_{(i,j)(m)}[t] = 1$, we have $x_{ij}[t] = 1$ and $\lambda_m[t] = 1$.

Now statement (16) is reformulated as (21), (22), and (23), which are in the form of mathematical programming.

(ii) Reformulation of (17)

By substituting (18) to (17), (17) becomes

$$\text{If } y_{(i,j)(m)}[t] = 1, \text{ then } \frac{P_{mj}}{\sum_{k\ne m}^{P_{kj}\le P_{mj}} P_{kj}\cdot\lambda_k[t] + \sigma^2} \ge \beta$$
$$(j\in\mathcal{N}, i\in\mathcal{I}_j, m\ne i, P_{mj} > P_{ij}, 1\le t\le T)\ ,$$

which is equivalent to

$$P_{mj} - \sum_{k\ne m}^{P_{kj}\le P_{mj}} \beta P_{kj}\lambda_k[t] - \beta\sigma^2 \ge \left(1 - y_{(i,j)(m)}[t]\right) D_{ijm}$$
$$(j\in\mathcal{N}, i\in\mathcal{I}_j, m\ne i, P_{mj} > P_{ij}, 1\le t\le T), \quad (24)$$

where D_{ijm} is a lower bound of $P_{mj} - \sum_{k\ne m}^{P_{kj}\le P_{mj}} \beta P_{kj}$ $\lambda_k[t] - \beta\sigma^2$ (e.g., we can set $D_{ijm} = P_{mj} - \sum_{k\ne m}^{P_{kj}\le P_{mj}} \beta P_{kj} - \beta\sigma^2$). We can verify that when $y_{(i,j)(m)}[t] = 1$, (24) becomes $P_{mj} - \sum_{k\ne m}^{P_{kj}\le P_{mj}} \beta P_{kj}\cdot\lambda_k[t] - \beta\sigma^2 \ge 0$, which is r-SINR$_{(m,j)}[t] \ge \beta$; when $y_{(i,j)(m)}[t] = 0$, (24) becomes $P_{mj} - \sum_{k\ne m}^{P_{kj}\le P_{mj}} \beta P_{kj}\cdot\lambda_k[t] - \beta\sigma^2 \ge D_{ijm}$, which holds by the definition of D_{ijm}.

(iii) Reformulation of (7)

Following the same token in reformulating (17) into (24), we can rewrite (7) as

$$P_{ij} - \sum_{k\ne i}^{P_{kj}\le P_{ij}} \beta P_{kj}\cdot\lambda_k[t] - \beta\sigma^2 \ge (1 - x_{ij}[t])H_{ij}$$
$$(j\in\mathcal{N}, i\in\mathcal{I}_j, 1\le t\le T)\ , \quad (25)$$

where H_{ij} is a lower bound of $P_{ij} - \sum_{k\ne i}^{P_{kj}\le P_{ij}} \beta P_{kj}\cdot\lambda_k[t] - \beta\sigma^2$ (e.g., we can set $H_{ij} = P_{ij} - \sum_{k\ne i}^{P_{kj}\le P_{ij}} \beta P_{kj} - \beta\sigma^2$).

C. Revised Scheduling Constraints

Inspired by the λ-variable's ability to reduce the dimension of y-variable from four to three, we would like to use λ-variable to formulate the half-duplex constraints. We have

$$\frac{1}{\min\{A_j, |\mathcal{I}_j|\}} \sum_{i\in\mathcal{I}_j} x_{ij}[t] + \lambda_j[t] \le 1 \quad (j\in\mathcal{N}, 1\le t\le T)\ , \quad (26)$$

where A_j is an upper bound of the number of signals node j can decode (see Lemma 1) and $|\mathcal{I}_j|$ is the number of neighboring nodes of node j. If node j is receiving from some node, the first term of the Left-Hand-Side in (26) is greater than 0. Then, $\lambda_j[t]$ must be 0. If node j is transmitting to some node (i.e., $\lambda_j[t] = 1$), then we must have $\frac{1}{|\mathcal{I}_j|}\sum_{i\in\mathcal{I}_j} x_{ij}[t] = 0$, which means that node j is not receiving from any node. Comparing the new half-duplex constraints (26) (formulated by using λ-variable) to the half-duplex constraints (formulated previous in (4)), we find the number of constraints in (26) is much smaller.

Moreover, due to the definition of variable λ in (14) and the fact that λ is binary, constraints (3) are redundant and can be removed from the framework.

D. Summary

Now we have all the constraints needed in an optimization framework for a multi-hop network, which include scheduling constraints (14), (26), joint PHY-Link constraints (21), (22), (23), (24), (25), and flow routing constraints (8), (9), (11). We summarize them in Fig. 6.

VIII. A Case Study

The goal of this effort is twofold. First, we want to validate the efficacy of our optimization framework in solving a practical network optimization problem. Second, we would like to have a closer look at how an interference exploitation scheme such as SIC can optimally interact with an interference avoidance scheme in a multi-hop wireless network.

TABLE II
LOCATION OF EACH NODE IN THE 20-NODE NETWORK.

Node	Location	Node	Location
1	$(54, 45)$	11	$(4, 29)$
2	$(70, 49)$	12	$(75, 75)$
3	$(40, 33)$	13	$(19, 11)$
4	$(7, 21)$	14	$(21, 58)$
5	$(79, 5)$	15	$(53, 12)$
6	$(29, 7)$	16	$(68, 96)$
7	$(3, 9)$	17	$(82, 36)$
8	$(49, 93)$	18	$(42, 51)$
9	$(88, 72)$	19	$(13, 78)$
10	$(59, 73)$	20	$(17, 45)$

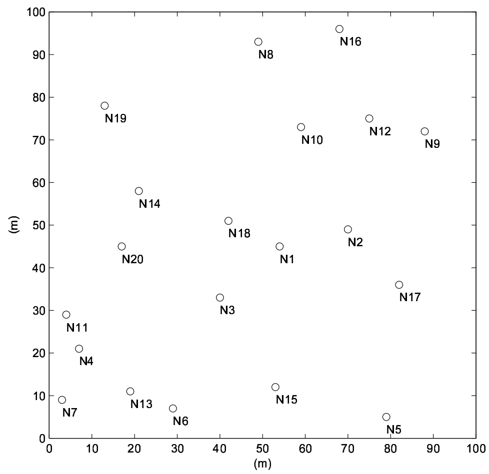

Fig. 7. The topology of a 20-node network.

A. A Throughput Maximization Problem

In a multi-hop wireless network, suppose we are interested in maximizing the weighted sum rate of active user sessions.[6] We assume each session $f \in \mathcal{F}$ is associated with a weight $w(f)$. Then, our objective is to maximize $\sum_{f\in\mathcal{F}} w(f) \cdot r(f)$. Listing all the constraints summarized in Fig. 6, we have the following network throughput maximization problem (TMP).

$$\textbf{TMP:} \quad \max \sum_{f\in\mathcal{F}} w(f) \cdot r(f)$$
$$\text{s.t. All constraints in Fig. 6.}$$

TMP is a mixed integer linear program (MILP). Although the theoretical worst-case complexity to a general MILP problem is exponential [6], [20], there exist highly efficient optimality/approximation algorithms (e.g., branch-and-bound with cutting planes [22]) and heuristics (e.g., sequential fixing algorithm [10], [11]) to solve it. Another approach is to apply an off-the-shelf solver (CPLEX [4]), which can successfully handle a moderate-sized network. We will adopt this approach as it is sufficient to serve our purpose in this section.

[6]Note that problems with objectives such as maximizing the minimum session rate among all sessions or maximizing a scaling factor of all session rates belong to the same category and can be solved similarly.

TABLE III
SOURCE NODE, DESTINATION NODE, AND WEIGHT OF EACH SESSION IN THE 20-NODE NETWORK.

Session f	Source Node $s(f)$	Dest. Node $d(f)$	Weight $w(f)$
1	2	11	5.0
2	8	3	6.0
3	19	9	7.0

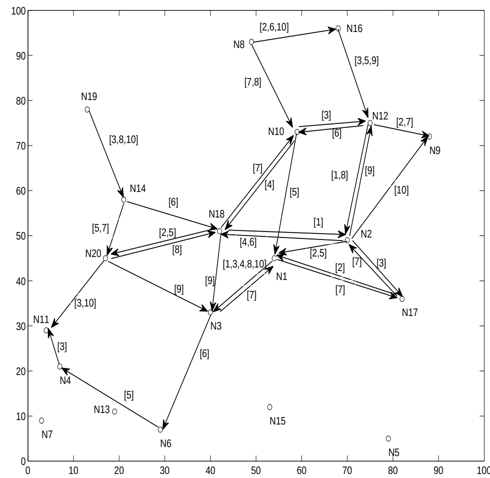

Fig. 8. Optimal routing and scheduling solution to TMP problem for the 20-node network.

B. Simulation Setting

We consider a randomly generated multi-hop wireless network with 20 nodes, which are distributed in a square region of 100×100. For generality, we normalize all units for distance, data rate, and power with appropriate dimensions. The topology of the network is shown in Fig. 7 and the location of each node is shown in Table II. There are three active sessions in the network, with each session's source node, destination node, and weight given in Table III.

The transmission power of each node is set to $P = 1$. For simplicity, we assume that channel gain g_{ij} only includes the path loss between nodes i and j and is given by $g_{ij} = d_{ij}^{\gamma}$, where d_{ij} is the distance between nodes i and j, and $\gamma = 3$ is the path loss index. The power of ambient noise is $\sigma^2 = 10^{-6}$. There are $T = 10$ time slots in each time frame. The SINR threshold for a successful transmission is $\beta = 1$. When a node i transmits to node j successfully in time slot t (i.e. $x_{ij}[t] = 1$), the achieved data rate is $R = 1$.

C. Joint Interference Exploitation and Avoidance

For the 20-node network, we apply CPLEX solver for the TMP formulation. The optimal objective value (maximum weighted sum throughput) is 6.6, with respective data rates for sessions 1, 2 and 3 being 0.3, 0.5 and 0.3. Fig. 8 shows the optimal routing and scheduling in the solution, where the numbers in the brackets next to a link show the time slots in a frame when the link is active. For example, $[3, 8, 10]$ next

TABLE IV
ACTIVE LINKS IN EACH TIME SLOT IN THE OPTIMAL SOLUTION FOR THE 20-NODE NETWORK.

Time slot	Active links
1	$1 \to 3$, $12 \to 2$, $18 \to 2$
2	$2 \to 1$, $8 \to 16$, $12 \to 9$, $17 \to 1$, $18 \to 20$
3	$1 \to 3$, $2 \to 17$, $4 \to 11$, $10 \to 12$, $16 \to 12$, $19 \to 14$, $20 \to 11$
4	$1 \to 3$, $2 \to 18$, $10 \to 18$
5	$2 \to 1$, $6 \to 4$, $10 \to 1$, $14 \to 20$, $16 \to 12$, $18 \to 20$
6	$1 \to 17$, $2 \to 18$, $3 \to 6$, $8 \to 16$, $12 \to 10$, $14 \to 18$
7	$3 \to 1$, $8 \to 10$, $12 \to 9$, $14 \to 20$, $17 \to 2$, $18 \to 10$
8	$1 \to 3$, $8 \to 10$, $12 \to 2$, $19 \to 14$, $20 \to 18$
9	$2 \to 12$, $10 \to 1$, $16 \to 12$, $18 \to 3$, $20 \to 3$
10	$1 \to 3$, $2 \to 9$, $8 \to 16$, $19 \to 14$, $20 \to 11$

to link $19 \to 14$ means that this link is active in time slots 3, 8, and 10.

Table IV shows the set of active links in each time slot. Our solution divides different links which are used to support the end-to-end sessions into different time slots so that the set of links in each time slot can successfully coexist (i.e., all links in each time slot satisfy the sequential SINR constraints in (1)). We use interference avoidance (scheduling) to overcome the limitations of SIC (clearly, the links in Table IV cannot be active in one single time slot). By exploiting the interference through SIC, we are able to activate as many links as possible in a time slot to maximize the network throughput. For example, in time slot 2, both nodes 2 and 17 transmit to node 1 simultaneously.

D. Some Details

From Table IV, we can validate the behavior of SIC quantitatively as follows. SIC allows a node to receive signals from multiple transmitters and reject the interference from other nodes in the same time slot. As an example, we look at the active links in time slot 1 in Table IV. In this time slot, links $1 \to 3$, $12 \to 2$ and $18 \to 2$ are active simultaneously. For receiver 2, the signal from node 1 (transmitting to node 3) is an interference to receiver 2, while the signals from nodes 12 and 18 are intended signals. In this example, we will show that receiver 2 rejects the interference from node 1 and receives concurrent transmissions from nodes 12 and 18.

The received signal powers from nodes 1, 12 and 18 at node 2 are $P_{1,2} = 22.29 \times 10^{-5}$, $P_{12,2} = 5.39 \times 10^{-5}$ and $P_{18,2} = 4.52 \times 10^{-5}$, respectively. Receiver 2 first tries to decode the strongest signal, which is from node 1. Note that this is an interference signal. The r-SINR for decoding this signal is

$$\frac{P_{1,2}}{P_{12,2} + P_{18,2} + \sigma^2} = \frac{22.29 \times 10^{-5}}{(5.39 + 4.52 + 0.1) \times 10^{-5}} = 2.23 > \beta = 1 \ ,$$

which shows that the interference signal from node 1 can be successfully decoded at receiver 2. After subtracting the interference from node 1 from the composite signal (i.e., interference rejection), receiver 2 moves on to decode the second strongest signal, which is from node 12. For this intended signal, its r-SINR is

$$\frac{P_{12,2}}{P_{1,2} + P_{18,2} - P_{1,2} + \sigma^2} = \frac{5.39 \times 10^{-5}}{(4.52 + 0.1) \times 10^{-5}} = 1.17 > \beta = 1 \ .$$

Thus, the signal from node 12 can be decoded successfully at receiver 2. Receiver 2 subtracts this signal from node 12 from the remaining composite signal and continues to decode the intended signal from node 18. The r-SINR for for decoding this signal is

$$\frac{P_{18,2}}{\sigma^2} = \frac{4.52 \times 10^{-5}}{10^{-6}} = 45.2 > \beta = 1 \ ,$$

which shows a successful decoding and reception.

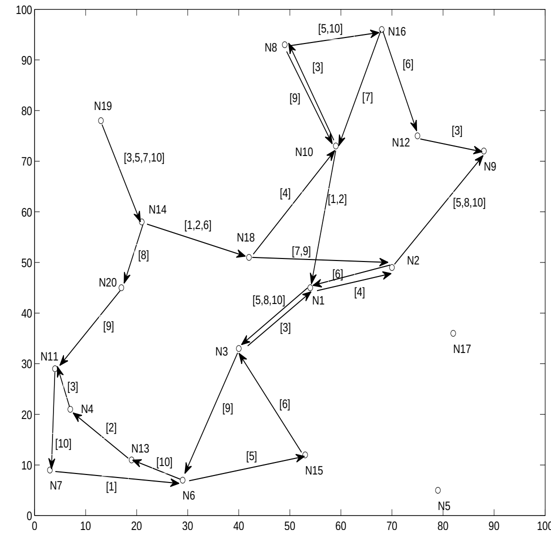

Fig. 9. The routing and scheduling results under pure interference avoidance model for the 20-node network.

E. Comparison to Pure Interference Avoidance Model

As a final part of our numerical results, we compare our optimal result to the TMP problem to the optimal result under pure interference avoidance model (i.e., SIC is not employed and all interference in the network is handled by scheduling). The problem formulation under pure interference avoidance model (called TMP-Pure) is given in the appendix, which is also a MILP problem. Again, we use CPLEX to solve TMP-Pure for the same 20-node network.

The optimal objective value (maximum weighted sum of throughput) is now 4.5 (vs. 6.6 for TMP), with the data rates for the three sessions being 0.1, 0.2 and 0.4, respectively. In other words, comparing to the pure interference avoidance model, our joint interference exploitation-avoidance can increase throughput by $\frac{6.6-4.5}{4.5} = 47\%$.[7] This increase affirms that traditional interference avoidance schemes are far from approaching network information theoretical limit.

[7]Note that this result is in contrast to that described in [21], which claimed that the improvement by SIC is marginal.

TABLE V
THE ACTIVE LINKS IN EACH TIME SLOT UNDER PURE INTERFERENCE AVOIDANCE MODEL FOR THE 20-NODE NETWORK.

Time slot	Active links
1	$7 \to 6$, $10 \to 1$, $14 \to 18$
2	$10 \to 1$,$13 \to 4$, $14 \to 18$
3	$3 \to 1$,$4 \to 11$,$10 \to 8$,$12 \to 9$,$19 \to 14$
4	$1 \to 2$, $18 \to 10$
5	$1 \to 3$, $2 \to 9$, $6 \to 15$, $8 \to 16$, $19 \to 14$
6	$2 \to 1$, $14 \to 18$, $15 \to 3$, $16 \to 12$
7	$16 \to 10$, $18 \to 2$, $19 \to 14$
8	$1 \to 3$, $2 \to 9$, $14 \to 20$
9	$3 \to 6$, $8 \to 10$, $18 \to 2$,$20 \to 11$
10	$1 \to 3$,$2 \to 9$, $6 \to 13$,$8 \to 16$

The optimal routing and scheduling results are shown in Fig. 9. The active links in each time slot are given in Table V. We now compare Fig. 9 and Table V to Fig. 8 and Table IV, respectively. It is clear that without SIC, fewer number of links are active in a pure interference avoidance solution.

IX. CONCLUSION

In this paper, we advocated a joint interference exploitation and avoidance approach, which combines the best of both worlds while avoids each's pitfalls. We discussed new challenges of such a approach in a multi-hop wireless network and proposed a formal optimization framework, with cross-layer formulation of physical, link, and network layers. This framework offered a rather complete design space for SIC, with the goal to squeeze the most out of interference. We claim that such an optimization framework is suitable for studying a broad class of network throughput optimization problems. As a case study, we demonstrated how to apply such framework for a network throughout optimization problem. Our numerical results affirmed the efficacy of this framework and gave insights on the optimal interaction between interference exploitation and interference avoidance.

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Appendix – Problem Formulation under Pure Interference Avoidance Model

Under the pure interference avoidance model, only scheduling is employed (no SIC). The joint PHY-Link constraints will change. When decoding a signal from i to node j, we treat all the signals from other transmitting nodes as noise. Then, for a successful transmission from node i to node j in time slot t (i.e., $x_{ij}[t] = 1$), we need the following statement:

$$\text{If } x_{ij}[t] = 1, \text{ then } \frac{P_{ij}}{\sum_{k \neq i} P_{kj}\lambda_k[t] + \sigma^2} \geq \beta$$
$$(j \in \mathcal{N}, i \in \mathcal{I}_j, 1 \leq t \leq T) \ .$$

The above statement can be written as

$$P_{ij} - \sum_{k \neq i} \beta P_{kj}\lambda_k[t] - \beta\sigma^2 \geq (1 - x_{ij}[t])M_{ij}$$
$$(j \in \mathcal{N}, i \in \mathcal{I}_j, 1 \leq t \leq T) \ , \qquad (27)$$

where M_{ij} is a lower bound of $P_{ij} - \beta\sum_{k \neq i} P_{kj}\lambda_k[t] - \beta\sigma^2$. Under this model, TMP-Pure has the same scheduling and flow routing constraints as that of problem TMP. Then, the formulation of TMP-Pure is as follows.

$$\max \sum_{f \in \mathcal{F}} w(f) \cdot r(f)$$
$$\text{s.t. Constraints } (8), (9), (11), (14), (26), (27)$$
$$x_{ij}[t], \lambda_i[t] \in \{0, 1\} \ (i \in \mathcal{N}, j \in \mathcal{I}_i, 1 \leq t \leq T)$$
$$r(f), r_{ij}(f) \geq 0 \quad (f \in \mathcal{F}, i \in \mathcal{N}, j \in \mathcal{I}_i)$$

The formulated problem TMP-Pure is also a mixed integer linear program (MILP).