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On the capacity of UWB-based wireless sensor networks

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ABSTRACT

Ultra-wideband (UWB) is a promising wireless communications technology and has great potential for emerging applications such as sensor networks. This paper studies the following fundamental problems for UWB-based sensor networks. For a given network instance, what is the maximum data rate (network capacity) that can be received at the base station (i.e., sink node)? What is the network capacity bound among arbitrary network instances? We show that these problems can be cast into a cross-layer formulation with joint consideration of routing, scheduling, power control, and rate assignment. For a given network instance, we find a closed-form network capacity as well as corresponding optimal routing, scheduling, power control, and rate assignment. We also find a network capacity bound among arbitrary network instances. Our results provide fundamental results for UWB-based sensor networks.

1. Introduction

Over the last ten years, there has been a flourish of research and development efforts on UWB for a wide range of military and commercial applications. These applications include tactical handheld and network LPI/D radios, non-LOS LPI/D groundwave communications, precision geolocation systems, high-speed wireless LANs, collision avoidance sensors, and intelligent tags. There are some significant benefits of UWB for wireless communications, such as extremely simple design of radio, large processing gain in the presence of interference, extremely low power spectral density for covert operations, and fine time resolution for accurate position sensing [6,9,12].

In this paper, we consider a UWB-based sensor network for surveillance and monitoring applications. For this network application, upon an event detection, all sensing data must be relayed to a central data collection point – the base station. Although the bit rate for each UWB-based sensor node could be high, the total rate that can be collected by the single base station is limited due to the bottleneck near the base station. A fundamental question becomes: What is the maximum aggregate data rate (network capacity) that can be received at the base station? In this paper, we will systematically address this problem, both for a specific network instance and arbitrary network instances.

We start our investigation with the consideration of a single-hop UWB-based sensor network, where each sensor transmits data directly to the base station (in one-hop). We focus on scheduling, power control, and rate assignment problem (since routing is fixed as one-hop). Motivated by the work in [8], we consider how to allocate frequency sub-bands for scheduling among the nodes. The scheduling problem considers how to allocate bandwidth among the sub-bands and which sub-bands a node should use for transmission and reception. The power control problem considers how much power a node should use to transmit data in a particular sub-band. The rate assignment problem considers how much data a node should transmit so as to maximize the network capacity. Built upon the results for a single-hop network instance, we subsequently consider the network capacity problem in a multi-hop setting. In this context, we need to consider the additional routing problem so as to maximize network capacity. Finally, we explore the bound of network capacity for arbitrary network instances.

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1.1. Main contributions

The main contributions of this paper are the following:

- For a given network instance, we find a closed-form expression for network capacity as follows.
  – In the context of single-hop network, we show that the maximum network capacity is achieved if and only if each sensor node simultaneously transmits on all sub-bands with maximum power.
  – In the context of multi-hop network, we show that the maximum network capacity is achieved by those nodes that are within one hop from the base station while keeping the rest of the nodes in the network idle. Thus, we reduce the network capacity problem for a multi-hop network to one for a single-hop network. A closed-form network capacity can be obtained.
- To calculate network capacity bound among *arbitrary* network instances, we first show that for any given network instance, network capacity increases when more sensor nodes are added into the network. Subsequently, we give a closed-form network capacity bound among arbitrary network instances.

1.2. Paper organization

The remainder of the paper is organized as follows. In Section 2, we give details of the network model for our problems. Section 3 analyzes the maximum network capacity of a given network instance, for both single-hop and multi-hop networks. In Section 4, we analyze the network capacity bound among arbitrary network instances. Section 5 reviews related work and Section 6 concludes this paper.

2. Network model

We consider a UWB-based sensor network consisting of a set of $\mathcal{N}$ nodes. Within the sensor network, we assume there is a base station (or sink node) B to which all collected data from sensor nodes send. Under this network setting, we are interested in answering the following questions. Given a network instance, what is the maximum total data rate (network capacity) that can be received at the base station? For arbitrary network instances, what is the network capacity bound? In this paper, we first focus on single-hop networks, where all sensors are one-hop neighbors of the base station and they transmit data to the base station via one-hop. We then extend the result to a general multi-hop setting. Table 1 lists all notation used in this paper.

Before we discuss the nature of this problem, we give the following definition for the feasibility of a rate vector $\mathbf{r}$, where each positive element r_i corresponds to the sensing rate produced by node $i \in \mathcal{N}$.

Definition 1. A rate vector $\mathbf{r}$ is feasible if and only if there exists a solution such that for each source node $i \in \mathcal{N}$, data rate r_i can be successfully sent to the base station.

Table 1
Notation

Symbol	Definition
B	Base station
c_{iB}^m	Capacity from node i to base station B on sub-band m
C_B	Maximum network capacity at base station B
d_{iB}	Distance between node i and base station B
f_{iB}	Flow rate from node i to base station B
g_{iB}	Propagation gain from node i to base station B
g_{nom}	Propagation gain at a nominal distance
$\mathcal{I}_B$	The set of one-hop neighbors of base station B
M	Total number of sub-bands for scheduling within W
N	Total number of sensor nodes in the network
$\mathcal{N}$	The set of sensor nodes in the network
p_{iB}^m	Power spent by node i on sub-band m for sending data to base station B
$p_{\max}$	$= W\pi_{\max}/g_{\text{nom}}$ is the power limit
r_i	Bit rate generated at sensor node i
$\mathbf{r}$	The vector of $r_i, i \in \mathcal{N}$
W	Entire spectrum (7.5 GHz) for a UWB node
α	Path loss index in propagation gain
$\lambda^{(m)}$	Normalized length of sub-band m, $\sum_{m=1}^{M}\lambda^{(m)} = 1$
η	Ambient Gaussian noise spectral density
$\pi_{\max}$	Limit of power spectral density at a node

It is clear that if a given rate vector $\mathbf{r}$ is feasible, then the total receiving rate at the base station is $\sum_{i=1}^{N} r_i$, where N is the number of sensor nodes in the network. Denote C_B the network capacity, i.e., $C_B = \max\sum_{i=1}^{N} r_i$ over all feasible $\mathbf{r}$ vectors. Our problem is to determine the maximum network capacity C_B for a given network instance and an upper bound of C_B among arbitrary network instances.

To determine whether or not a solution exists to ensure that the given rate vector $\mathbf{r}$ is feasible, there are several issues that must be considered. That is, we need to determine scheduling and power control for each node such that link capacity constraint can be met satisfactorily (note that there is no routing problem in a single-hop network). Clearly, this is a problem that couples scheduling and power control. We now take a closer look at each problem.

The scheduling problem deals with how to allocate resource for access among the nodes. Motivated by Negi and Rajeswaran's work in [8], we consider how to divide and allocate frequency sub-bands, although this approach can also be applied to deal with time-slotted systems. For the total available UWB spectrum of $W = 7.5$ GHz (from 3.1 to 10.6 GHz), we divide it into M sub-bands. Given M, the scheduling problem considers how to divide the total spectrum of W into M sub-bands and which sub-bands a node should use for transmission and reception. More formally, denote $\lambda^{(m)}W$ the bandwidth of sub-band m. We have $\sum_{m=1}^{M}\lambda^{(m)} = 1$.

The power control problem considers how much power a node should use to transmit data in a particular sub-band. The total power that a node i can expend on a sub-band m must satisfy the following power limit,

$$p_{iB}^m \leqslant p_{\max}\lambda^{(m)}, \tag{1}$$

where p_{iB}^m is the power that node i expends in sub-band m for sending data to base station B. This requirement comes from the power density limitation of UWB, i.e., $\frac{g_{\text{nom}} \cdot p_{iB}^m}{W\lambda^{(m)}} \leqslant \pi_{\max}$, where $\pi_{\max}$ is the maximum allowed transmission power spectral density and g_{nom} is the gain at some fixed nominal distance [11]. Thus, we have

$$p_{\max} = \frac{W\pi_{\max}}{g_{\text{nom}}}. \tag{2}$$

A widely-used gain model is

$$g_{iB} = d_{iB}^{-\alpha}, \tag{3}$$

where d_{iB} is the distance between node i and base station B and α is the path loss index.

Denote $\mathscr{I}_B$ the set of one-hop neighbors of base station B. The capacity from node i to the base station B on sub-band m is then

$$c_{iB}^m = W\lambda^{(m)}\log_2\left(1 + \frac{g_{iB}p_{iB}^m}{\eta W\lambda^{(m)} + \sum_{k\in\mathscr{I}_B}^{k\neq i} g_{kB}p_{kB}^m}\right), \tag{4}$$

where η is the ambient Gaussian noise. Denote the flow rate from node i to base station B as f_{iB}. Then the link capacity constraint is $f_{iB} \leqslant \sum_{m=1}^{M} c_{iB}^m$.

3. Network capacity

In this section, we first consider a given single-hop network instance (Section 3.1). Our result shows that the maximum network capacity is achieved if and only if all nodes transmit data to base station with maximum power on all sub-bands. The optimal rate vector $\mathbf{r}^*$ to achieve the maximum network capacity is given in Theorem 1. We then extend this result to a multi-hop network instance in Section 3.2. We find that the maximum network capacity is achieved if and only if all nodes belong to $\mathscr{I}_B$ (where $\mathscr{I}_B$ is the set of one-hop neighbors of the base station B) transmit while the rest of the nodes in the network stay idle. This result is intuitive since any remote source sensor node will decrease the transmission rate of a relay node within $\mathscr{I}_B$ (since a node cannot transmit and receive at the same band) and thus decrease the network capacity. Again, these nodes in $\mathscr{I}_B$ should transmit data with the maximum power on all sub-bands in order to achieve maximum network capacity. We give the optimal rate vector $\hat{\mathbf{r}}^* = \{\mathbf{r}^*, \mathbf{0}\}$ in Theorem 2, where the rate vector for nodes in $\mathscr{I}_B$ is $\mathbf{r}^*$ and the rate vector for nodes not in $\mathscr{I}_B$ is $\mathbf{0}$.

3.1. Network capacity of single-hop networks

In a single-hop network, each node i is a one-hop neighbor of the base station and transmits data to the base station via one-hop. An interesting result is that the maximum network capacity is achieved when each node transmits data with the maximum power on all sub-bands. That is, to achieve the maximum network capacity, scheduling and power control is merely simultaneous transmission on all sub-bands with the maximum power. We also show that the corresponding optimal rate assignment vector is unique. We begin with the following lemma.

Lemma 1. *For any given scheduling policy (the number of allowed sub-bands M and bandwidth allocation among the M sub-bands), network capacity is maximized when every node transmits at the maximum allowed power on each sub-band.*

Proof. Under the given scheduling policy, the SNR for data from a node i in a sub-band m, where $1 \leqslant i \leqslant N, 1 \leqslant m \leqslant M$, is

$$\begin{aligned}\frac{g_{iB}p_{iB}^m}{\eta W\lambda^{(m)}} &\leqslant \frac{g_{iB}\cdot p_{\max}\lambda^{(m)}}{\eta W\lambda^{(m)}} = \frac{g_{iB}\cdot p_{\max}}{\eta W} = \frac{g_{iB}}{\eta W}\cdot\frac{W\pi_{\max}}{g_{\text{nom}}}\\ &= \frac{g_{iB}}{g_{\text{nom}}}\cdot\frac{\pi_{\max}}{\eta} \ll 1.\end{aligned} \tag{5}$$

The first inequality holds by (1). The third equality holds by (2). The last inequality holds by $\frac{\pi_{\max}}{\eta} \ll 1$ for UWB (e.g., on the order of 10^{-2} [11]) and that g_{iB} and g_{nom} are comparable. By (5), we further have

$$\frac{g_{iB}p_{iB}^m}{\eta W\lambda^{(m)} + \sum_{k\in\mathscr{N}}^{k\neq i} g_{kB}p_{kB}^m} < \frac{g_{iB}p_{iB}^m}{\eta W\lambda^{(m)}} \ll 1. \tag{6}$$

We now consider the total rate $(\sum_{i\in\mathscr{N}} c_{iB}^m)$ received at the base station in sub-band m. According to (4) and the fact that we are considering a single-hop network instance, we have

$$\begin{aligned}c_{iB}^m &= W\lambda^{(m)}\log_2\left(1 + \frac{g_{iB}p_{iB}^m}{\eta W\lambda^{(m)} + \sum_{k\in\mathscr{N}}^{k\neq i} g_{kB}p_{kB}^m}\right)\\ &\approx \frac{W\lambda^{(m)}}{\ln 2}\cdot\frac{g_{iB}p_{iB}^m}{\eta W\lambda^{(m)} + \sum_{k\in\mathscr{N}}^{k\neq i} g_{kB}p_{kB}^m}.\end{aligned}$$

This approximation is the so-called linear rate-SINR property for UWB, which is based on (6) and $\ln(1+x) \approx x$ when $0 < x \ll 1$. Thus,

$$\begin{aligned}\frac{\partial(\sum_{i\in\mathscr{N}} c_{iB}^m)}{\partial(g_{iB}p_{iB}^m)} = \frac{W\lambda^{(m)}}{\ln 2}\cdot\Bigg[&\frac{1}{\eta W\lambda^{(m)} + \sum_{k\in\mathscr{N}}^{k\neq i} g_{kB}p_{kB}^m}\\ &- \sum_{j\in\mathscr{N}}^{j\neq i}\frac{g_{jB}p_{jB}^m}{(\eta W\lambda^{(m)} + \sum_{k\in\mathscr{N}}^{k\neq i} g_{kB}p_{kB}^m)^2}\Bigg]\end{aligned}$$

To simplify notation, denote $a = \eta W\lambda^{(m)}$, $b = \sum_{k=1}^{N} g_{kB}p_{kB}^m$, and $\delta = \max_{i\in\mathscr{N}} \frac{g_{iB}p_{iB}^m}{\eta W\lambda^{(m)}}$. We have $\delta \ll 1$ by (5) and

$$\begin{aligned}\frac{\partial(\sum_{i\in\mathscr{N}} c_{iB}^m)}{\partial(g_{iB}p_{iB}^m)} &= \frac{W\lambda^{(m)}}{\ln 2}\left[\frac{1}{a+b-g_{iB}p_{iB}^m} - \sum_{j\in\mathscr{N}}^{j\neq i}\frac{g_{jB}p_{jB}^m}{(a+b-g_{jB}p_{jB}^m)^2}\right]\\ &\geqslant \frac{W\lambda^{(m)}}{\ln 2}\left[\frac{1}{a+b} - \sum_{j\in\mathscr{N}}^{j\neq i}\frac{g_{jB}p_{jB}^m}{(a+b-\delta a)^2}\right]\\ &\geqslant \frac{W\lambda^{(m)}}{\ln 2}\left[\frac{1}{a+b} - \frac{b}{(a+b-\delta a)^2}\right]\\ &= \frac{W\lambda^{(m)}}{\ln 2}\cdot\frac{(1-2\delta)(a^2+ab)+\delta^2a^2}{(a+b)(a+b-\delta a)^2} > 0.\end{aligned} \tag{7}$$

The last inequality holds because $\delta \ll 1$. Thus, $(\sum_{i\in\mathscr{N}} c_{iB}^m)$ is an increasing function of p_{iB}^m. As a result, to maximize the received data rate on a sub-band m, we must have p_{iB}^m equal the maximum allowed transmission power $p_{\max}\lambda^{(m)}$ for every node i. This completes the proof. □

We note that the above result is unique for UWB-based sensor networks, and does not hold for narrow band wireless networks.

The following theorem gives an optimal rate vector $\mathbf{r}^*$ to achieve the maximum network capacity.

Theorem 1. *For a given single-hop network instance, the rate vector* $\mathbf{r}^*$ *with* $r_i^* = W\log_2(1 + \frac{g_{iB}p_{\max}}{\eta W + \sum_{k\in\mathcal{N}}^{k\neq i} g_{kB}p_{\max}})$ *is feasible and achieves the maximum network capacity* C_B*. That is,*

$$C_B = \sum_{i\in\mathcal{N}} r_i^* = W\sum_{i\in\mathcal{N}} \log_2\left(1 + \frac{g_{iB}p_{\max}}{\eta W + \sum_{k\in\mathcal{N}}^{k\neq i} g_{kB}p_{\max}}\right).$$

Proof. First, we verify that the rate vector $\mathbf{r}^*$ stated in the theorem is feasible. That is, there exists a scheduling and power control policy such that the rate vector $\mathbf{r}^*$ can be successfully sent to the base station. As an existence proof for such a policy, let $M = 1$ and $\lambda^{(1)} = 1$, i.e., the simple case that there is only a single band. Let each node i send its data to the base station via one-hop with power $p_{iB}^1 = p_{\max}$. The link capacity c_{iB}^1 can be computed from (4). It is easy to verify that $r_i^* = c_{iB}^1$, i.e., each node i can send data r_i^* to the base station. Therefore, the rate vector $\mathbf{r}^*$ is feasible.

We now show that the rate vector $\mathbf{r}^*$ achieves the maximum network capacity. Based on Lemma 1, to achieve the maximum network capacity, each node should transmit at the maximum allowed power on all sub-bands. Thus, for a set of feasible rates to maximize the network capacity, we have

$$\begin{aligned} r_i &\leqslant \sum_{m=1}^{M} c_{iB}^m \\ &= \sum_{m=1}^{M} W\lambda^{(m)}\log_2\left(1 + \frac{g_{iB}p_{\max}\lambda^{(m)}}{\eta W\lambda^{(m)} + \sum_{k\in\mathcal{N}}^{k\neq i} g_{kB}p_{\max}\lambda^{(m)}}\right) \\ &= W\log_2\left(1 + \frac{g_{iB}p_{\max}}{\eta W + \sum_{k\in\mathcal{N}}^{k\neq i} g_{kB}p_{\max}}\right)\sum_{m=1}^{M}\lambda^{(m)} \\ &= W\log_2\left(1 + \frac{g_{iB}p_{\max}}{\eta W + \sum_{k\in\mathcal{N}}^{k\neq i} g_{kB}p_{\max}}\right) = r_i^*. \end{aligned} \tag{8}$$

That is, for any rate vector $\mathbf{r}$ to maximize the network capacity at the base station, we must have $r_i \leqslant r_i^*$. The maximum network capacity is thus

$$C_B = \sum_{i\in\mathcal{N}} r_i^* = W\sum_{i\in\mathcal{N}} \log_2\left(1 + \frac{g_{iB}p_{\max}}{\eta W + \sum_{k\in\mathcal{N}}^{k\neq i} g_{kB}p_{\max}}\right).$$

This completes the proof. □

Based on the above proof, we have the following interesting result.

Corollary 1. *The maximum network capacity is independent of sub-band division.*

Regarding the uniqueness of the optimal rate vector, we have the following corollary, which also follows from the proof of Theorem 1.

Corollary 2. *For a given single-hop network instance, the optimal rate vector to achieve the maximum network capacity* C_B *is unique.*

Proof. This result follows by the proof of Theorem 1. Among all feasible rate vectors, we have shown that $r_i \leqslant r_i^*$ in (8) for any rate vector $\mathbf{r}$ to maximize the network capacity at the base station and the maximum network capacity is $C_B = \sum_{i\in\mathcal{N}} r_i^*$. Thus, C_B is achieved only if the rate vector is $\mathbf{r}^*$. □

3.2. The case of multi-hop networks

We now consider a multi-hop network, where nodes in $\mathcal{I}_B$ can transmit data directly to the base station while other nodes must relay their data via those nodes in $\mathcal{I}_B$ to the base station. Therefore, routing is part of the problem, in addition to scheduling, power control, and rate assignment. However, we will show that to achieve the maximum network capacity, we only need to consider nodes in $\mathcal{I}_B$ and the one-hop routing for these nodes. That is, the network capacity problem for a multi-hop network instance can be reduced to the network capacity problem for a single-hop network instance, which has already been solved in Section 3.1. We first give the following lemma.

Lemma 2. *For a multi-hop network instance, for any feasible rate vector* $\mathbf{r}$ *with aggregate rate* $C = \sum_{i=1}^{N} r_i$*, there exists a feasible rate vector* $\hat{\mathbf{r}}$ *with the same aggregate rate* C *such that (1)* $\hat{r}_i = 0$ *for all* $i \notin \mathcal{I}_B$ *and (2) for each node* i *with* $\hat{r}_i > 0$*, it transmits data to the base station via one-hop.*

Proof. The proof is based on construction. That is, if the given feasible vector $\mathbf{r}$ with aggregate rate C does not meet the single-hop property in the lemma, we will construct a feasible rate vector $\hat{\mathbf{r}}$ with the same aggregate rate C that satisfies the requirements in lemma. Recall that by Definition 1, a feasible rate vector $\hat{\mathbf{r}}$ means that we can construct a solution such that for each source node $i \in \mathcal{N}$, data rate $\hat{r}_i$ can be successfully sent to the base station.

Suppose that we have a feasible solution ψ for rate vector $\mathbf{r}$ with aggregate rate C. For each node $i \in \mathcal{I}_B$, denote the flow rate from node i to the base station B in solution ψ as f_{iB}. We construct $\hat{\mathbf{r}}$ by letting $\hat{r}_i = f_{iB}$ for $i \in \mathcal{I}_B$ and $\hat{r}_i = 0$ for $i \notin \mathcal{I}_B$. Note that $\sum_{i\in\mathcal{I}_B} f_{iB}$ is the total receiving rate at the base station, we have

$$\sum_{i\in\mathcal{N}} \hat{r}_i = \sum_{i\in\mathcal{I}_B} \hat{r}_i = \sum_{i\in\mathcal{I}_B} f_{iB} = C.$$

Thus, $\hat{\mathbf{r}}$ has the same aggregate rate C and $\hat{r}_i = 0$ for all $i \notin \mathcal{I}_B$.

We now show $\hat{\mathbf{r}}$ is feasible via one-hop transmissions. In solution ψ, there may exist many transmissions not in the last hop, which can be removed to construct a new solution $\hat{\psi}$. Note that solution $\hat{\psi}$ has at least the same link capacity on each link $(i, B), i \in \mathcal{I}_B$, due to smaller interference. Since in solution ψ, each node $i \in \mathcal{I}_B$ can transmit $f_{iB} = \hat{r}_i$ directly to the base station via one-hop, then in solution $\hat{\psi}$, each node $i \in \mathcal{I}_B$ must also be able to transmit $\hat{r}_i$ to the base station via one-hop. This completes the proof. □

Based on Lemma 2, to calculate the maximum network capacity for a multi-hop network instance, it is sufficient to consider a smaller and simpler single-hop network instance, i.e., a network with all nodes in $\mathcal{I}_B$ and with one-hop routing. The maximum network capacity for a single-hop network is given in Theorem 1. Thus, we have the following theorem.

Theorem 2. *For a given multi-hop network instance, the rate vector* $\hat{\mathbf{r}}^*$ *with* $\hat{r}_i^* = W \cdot \log_2\left(1 + \frac{g_{iB}p_{\max}}{\eta W + \sum_{k \in \mathscr{I}_B}^{k \neq i} g_{kB}p_{\max}}\right)$ *for* $i \in \mathscr{I}_B$ *and* $\hat{r}_i^* = 0$ *for* $i \notin \mathscr{I}_B$ *is feasible and achieves the maximum network capacity* C_B*. That is,*

$$C_B = \sum_{i \in \mathscr{I}_B} \hat{r}_i^* = W \sum_{i \in \mathscr{I}_B} \log_2\left(1 + \frac{g_{iB}p_{\max}}{\eta W + \sum_{k \in \mathscr{I}_B}^{k \neq i} g_{kB}p_{\max}}\right).$$

In fact, the result in Theorem 2 can be further strengthened by the *uniqueness* of the rate vector that achieves C_B. This is stated in the following lemma.

Lemma 3. *For a given multi-hop network instance, the optimal rate vector that achieves the maximum network capacity* C_B *is unique.*

Proof. To show the correctness of this lemma, we need to consider all feasible rate vectors with both multi-hop and single-hop solutions. We organize our proof as follows. First, we show that for a rate vector to maximize the total rate received by the base station, it must use a single-hop solution, i.e., $\hat{r}_i > 0$ only if $i \in \mathscr{I}_B$ and these $\hat{r}_i > 0$ are sent to the base station via one-hop. Then, based on Corollary 2, we claim that this rate vector must be $\hat{\mathbf{r}}^*$.

The first result follows by the proof of Theorem 1. Since $(\sum_{i \in \mathscr{I}_B} c_{iB}^m)$ is an increasing function of p_{iB}^m by (7), to maximize the capacity on sub-band m, we must have $p_{iB}^m = p_{\max}\lambda^{(m)}$ for each node $i \in \mathscr{I}_B$ and each sub-band m. Thus, if a node $i \in \mathscr{I}_B$ is used as a relay node, then the transmission power for node i at a certain sub-band must be 0, since a node cannot send and receive within the same sub-band. As a result, the total rate received by the base station on this sub-band will be smaller, resulting in a total rate over all the sub-bands less than C_B. That is, the maximum aggregate rate C_B received by the base station is achievable only if the routing solution is one-hop, i.e., $\hat{r}_i > 0$ only if $i \in \mathscr{I}_B$ and these $\hat{r}_i > 0$ are sent to the base station via one-hop. Therefore, we already proved that the rate of node not in $\mathscr{I}_B$ must be 0 in an optimal rate vector.

By Corollary 2, we know that the rate of node in $\mathscr{I}_B$ must be $\hat{r}_i^*$ in an optimal rate vector. Thus, we conclude that the maximum network capacity C_B is achievable only if the rate vector is $\hat{\mathbf{r}}^*$. This completes the proof. □

3.3. Examples

We now present examples for computing network capacity. Given that the total UWB spectrum is W = 7.5 GHz and assume that the maximum transmission range is 10, where the distance is based on normalized length in (3). The gain model for a link (i,j) is $g_{ij} = d_{ij}^{-4}$ and the nominal gain is chosen as $g_{\text{nom}} = 0.0016$. The power density limit $\pi_{\max}$ is assumed to be 1% of the white noise η [11].

We first consider a randomly generated 20-node network instance (see Fig. 1) over a 20 × 20 area. The base station is located at the origin (center of the network). The location of each sensor in the 20-node network is shown in Table 2.

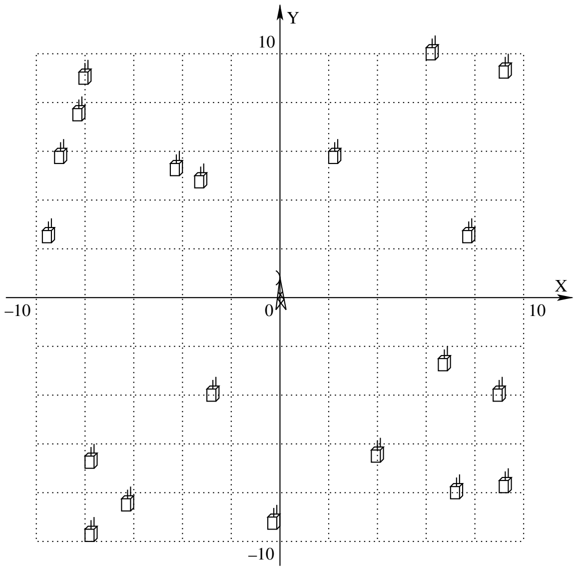

Fig. 1. A UWB-based sensor network with 20 nodes.

Table 2
Locations and optimal rates for the 20-node sensor network

i	(x_i, y_i)	r_i^*	i	(x_i, y_i)	r_i^*
1	(−8.2, 7.9)	0	11	(−0.5, −8.9)	10.40
2	(−9.4, 2.5)	7.34	12	(7.1, −8.2)	0
3	(−4.4, 5.6)	25.55	13	(2.3, 5.7)	46.09
4	(−3.4, 4.9)	52.01	14	(9.2, 9.6)	0
5	(−9.0, 5.8)	0	15	(9.6, −7.9)	0
6	(−8.0, 9.1)	0	16	(4.1, −6.8)	16.53
7	(−7.8, −9.8)	0	17	(6.1, 10.0)	0
8	(−7.7, −7.2)	0	18	(6.6, −2.8)	24.88
9	(−2.7, −4.1)	113.62	19	(7.8, 2.4)	14.81
10	(−6.2, −8.7)	0	20	(9.1, −4.0)	6.73

We now compute the maximum network capacity for the 20-node network instance. First, we need to determine the set $\mathscr{I}_B$ of one-hop neighbors of the base station B based on their locations. For example, node 2 is a member of $\mathscr{I}_B$ since the distance between node 2 and the base station B is 9.7 and is smaller than the maximum transmission range 10. Based on this distance classification, we have $\mathscr{I}_B = \{2, 3, 4, 9, 11, 13, 16, 18, 19, 20\}$. Based on Theorem 1, the maximum network capacity is achieved when the rate of node $i \notin \mathscr{I}_B$ is 0 and the rate of node $i \in \mathscr{I}_B$ is

$$\begin{aligned} r_i^* &= W \log_2\left(1 + \frac{g_{iB}p_{\max}}{\eta W + \sum_{k \in \mathscr{I}_B}^{k \neq i} g_{kB}p_{\max}}\right) \\ &= W \log_2\left(1 + \frac{g_{iB}\frac{W\pi_{\max}}{g_{\text{nom}}}}{\eta W + \sum_{k \in \mathscr{I}_B}^{k \neq i} g_{kB}\frac{W\pi_{\max}}{g_{\text{nom}}}}\right) \\ &= W \log_2\left(1 + \frac{0.01 \cdot g_{iB}}{g_{\text{nom}} + 0.01 \cdot \sum_{k \in \mathscr{I}_B}^{k \neq i} g_{kB}}\right). \end{aligned}$$

The second equality holds by (2) and the third equality holds by $\pi_{\max} = 0.01 \cdot \eta$. The optimal rate assignments

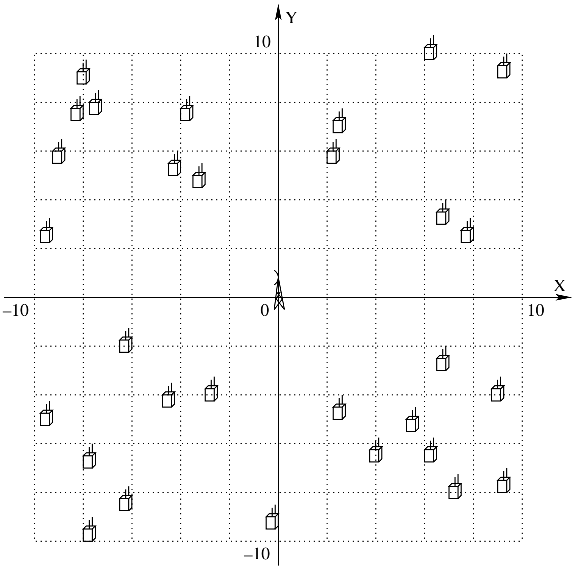

Fig. 2. A UWB-based sensor network with 30 nodes.

are shown in Table 2 and the maximum network capacity $C_B = \sum_{i=1}^{20} r_i^* = 317.96$.

Now, we randomly put 10 more nodes into the network (see Fig. 2). The location of each sensor in this 30-node network is shown in Table 3. Similarly, we can compute the optimal rates for this 30-node network instance (see Table 3) and the maximum network capacity is $C_B = \sum_{i=1}^{30} r_i^* = 541.88$. By comparing the results in Tables 2 and 3, we have the following two observations. First, due to the interference from the additional 10 nodes, for each node i, $1 \leqslant i \leqslant 20$, its rate is decreased (e.g., node 2's rate is decreased from 7.34 to 7.18). Second, although the total data rate from the first 20 nodes is decreased, the total network capacity for the entire networks (30 nodes) is increased (from 317.96 to 541.88) since the base station can receive extra data from the 10 additional nodes.

It is interesting to see in this example that the maximum network capacity is increased when we add more nodes into the network. This motivates us to explore the following question: What is the network capacity bound among arbitrary network instances? In the next section, we study this problem.

4. Capacity bound for arbitrary network

In Section 3.3, we show how to calculate the maximum network capacity for given network instances. We observe that the maximum network capacity may increase when the number of nodes increases. In Lemma 4, we will formally prove this observation. Further, in Theorem 3, we will determine the network capacity bound among arbitrary network instances.

It is clear that the more one-hop sensor nodes near the base station, the more interference at the base station. Each node's rate should decrease when the number of one-hop sensor nodes of the base station increases. However, due to the low interference property of UWB, the decrease of a node's rate is not as significant as that in narrow-band networks. We have the following lemma.

Lemma 4. *For any given network instance, the maximum network capacity increases when more sensor nodes are added as one-hop neighboring nodes of the base station.*

Proof. We first show that for any network, after we deploy one more sensor as a one-hop neighbor of the base station, the maximum network capacity increases. Once this is proved, our lemma (for multiple additional nodes) is also proved.

We will show a contradiction if the result (for one additional node) is not true. Suppose that there is a network with k one-hop neighbors and the maximum network capacity is C_B^k. Denote $\{r_1^*, r_2^*, \ldots, r_k^*\}$ the optimal rate vector for nodes in $\mathscr{I}_B$. We now deploy $(k+1)$th sensor as a one-hop neighbor of the base station. Denote C_B^{k+1} the new maximum network capacity. Since $\{r_1^*, r_2^*, \ldots, r_k^*, 0\}$ is a feasible rate vector, where the rate for the $(k+1)$th sensor is $0, C_B^{k+1}$ is at least $\sum_{i=1}^{k} r_i^* = C_B^k$. If the maximum network capacity does not increase, then we must have $C_B^{k+1} = C_B^k$ and $\{r_1^*, r_2^*, \ldots, r_k^*, 0\}$ is an optimal rate vector. However, in the optimal solution shown in Theorem 2, $(k+1)$th sensor has a positive data rate. Thus, we have two different optimal rate vectors. But based on Lemma 3, this leads to a contradiction. Therefore, for any network, after we deploy more sensor nodes as a one-hop neighbor of the base station, the maximum network capacity always increases. □

Based on Lemma 4, after we deploy one more sensor as a one-hop neighbor of the base station, the maximum network capacity also increases. However, the maximum net-

Table 3
Locations and optimal rates for the 30-node sensor network

i	(x_i, y_i)	r_i^*	i	(x_i, y_i)	r_i^*	i	(x_i, y_i)	r_i^*
1	(−8.2,7.9)	0	11	(−0.5,−8.9)	10.18	21	(−7.5,7.8)	0
2	(−9.4,2.5)	7.18	12	(7.1,−8.2)	0	22	(−2.3,7.0)	21.82
3	(−4.4,5.6)	25.00	13	(2.3,5.7)	45.11	23	(5.7,−5.4)	16.92
4	(−3.4,4.9)	50.90	14	(9.2,9.6)	0	24	(−6.1,2.1)	37.15
5	(−9.0,5.8)	0	15	(9.6,−7.9)	0	25	(−9.8,5.2)	0
6	(−8.0,9.1)	0	16	(4.1,−6.8)	16.17	26	(−3.8,7.6)	12.33
7	(−7.8,−9.8)	0	17	(6.1,10.0)	0	27	(6.1,−6.3)	10.87
8	(−7.7,−7.2)	0	18	(6.6,−2.8)	24.35	28	(6.7,3.0)	22.15
9	(−2.7,−4.1)	111.19	19	(7.8,2.4)	14.49	29	(2.3,−5.0)	70.25
10	(−6.2,−8.7)	0	20	(9.1,−4.0)	6.58	30	(−4.5,−4.5)	39.24

work capacity cannot increase indefinitely, even if $|\mathcal{I}_B| \to \infty$, i.e., the number of one-hop sensor nodes of the base station increases to infinity. The following theorem gives an upper bound on network capacity among arbitrary network instances.

Theorem 3. *Among arbitrary network instances, the maximum network capacity is upper bounded by* $\frac{W}{\ln 2}$.

Proof. From Theorem 2, the maximum network capacity is

$$C_B = W \sum_{i \in \mathcal{I}_B} \log_2 \left(1 + \frac{g_{iB} p_{\max}}{\eta W + \sum_{k \in \mathcal{I}_B}^{k \neq i} g_{kB} p_{\max}}\right).$$

When $|\mathcal{I}_B| \to \infty$, we have

$$\begin{aligned} C_B &= W \sum_{i \in \mathcal{I}_B} \log_2 \left(1 + \frac{g_{iB} p_{\max}}{\eta W + \sum_{k \in \mathcal{I}_B}^{k \neq i} g_{kB} p_{\max}}\right) \\ &\approx \frac{W}{\ln 2} \sum_{i \in \mathcal{I}_B} \frac{g_{iB} p_{\max}}{\eta W + \sum_{k \in \mathcal{I}_B}^{k \neq i} g_{kB} p_{\max}} \\ &\approx \frac{W}{\ln 2} \sum_{i \in \mathcal{I}_B} \frac{g_{iB} p_{\max}}{\sum_{k \in \mathcal{I}_B}^{k \neq i} g_{kB} p_{\max}} \\ &\approx \frac{W}{\ln 2} \sum_{i \in \mathcal{I}_B} \frac{g_{iB}}{\sum_{k \in \mathcal{I}_B} g_{kB}} = \frac{W}{\ln 2}. \end{aligned}$$

The second approximation holds because $\frac{g_{iB} p_{\max}}{\eta W + \sum_{k \in \mathcal{I}_B}^{k \neq i} g_{kB} p_{\max}} \ll 1$. The third approximation holds because $\sum_{k \in \mathcal{I}_B}^{k \neq i} g_{kB} p_{\max} \gg \eta W$ when $|\mathcal{I}_B| \to \infty$. The fourth approximation holds because $\sum_{k \in \mathcal{I}_B}^{k \neq i} g_{kB} \approx \sum_{k \in \mathcal{I}_B} g_{kB}$ when $|\mathcal{I}_B| \to \infty$. Thus, the network capacity bound among arbitrary network instances is $\frac{W}{\ln 2}$. □

5. Related work

Capacity problems for wireless networks have been an active research area over the past few years. However, most of these investigation has been focusing on narrow-band wireless networks (e.g., [1,4,5,13]) and these results cannot be carried over to UWB-based networks.

A recent overview paper on UWB is given [9]. Physical layer issues associated with UWB-based multiple access communications can be found in [2,3,14] and references therein. In this section, we focus on related work addressing cross-layer optimization problems with UWB.

In [8], Negi and Rajeswaran studied how to maximize proportional rate allocation in a single-hop UWB ad hoc network. The problem was formulated as a cross-layer optimization problem with similar scheduling and power control constraints as in this paper. The impact of routing, however, was not addressed in this research. In [10], Radunovic and Le Boudec studied how to maximize the total log-utility of flow rates in multi-hop ad hoc networks. As the optimization problem is NP-hard, the authors then studied a simple ring network as well as a small-sized network with pre-defined scheduling and routing policies.

The most relevant effects to this work are [7,15], which are for ad hoc networks. These asymptotic results show that, in contrast to previously published results, the throughput of UWB-based ad hoc networks increases with node density. This important result demonstrates the significance of physical layer properties on network capacity. In this paper, our focus is on UWB-based sensor networks where there is only one sink node in the network. We find that the maximum network capacity only depends on the one-hop neighbors of the base station and this capacity cannot increase to infinity (different from the results for ad hoc networks given in [7,15]).

6. Conclusion

In this paper, we studied important network capacity problems for UWB-based sensor networks. We follow a cross-layer approach with joint consideration of routing, scheduling, power control, and rate assignment. Our contributions are two-fold. First, for a given network instance, we find a closed-form network capacity as well as corresponding optimal routing, scheduling, power control, and rate assignment for both single-hop and multi-hop networks. Second, we find a network capacity bound for arbitrary network instances. These results provide fundamental understanding for UWB-based sensor networks.

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