Bridging the Gap between Protocol and Physical Models for Wireless Networks

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Abstract—This paper tries to reconcile the tension between the physical model and the protocol model that have been used to characterize interference relationship in a multihop wireless network. The physical model (a.k.a. signal-to-interference-and-noise ratio model) is widely considered as a reference model for physical layer behavior but its application in multihop wireless networks is limited by its complexity. On the other hand, the protocol model (a.k.a. disk graph model) is simple but there have been doubts on its validity. This paper explores the following fundamental question: How to correctly use the protocol interference model? We show that, in general, solutions obtained under the protocol model may be *infeasible* and, thus, results based on blind use of protocol model can be misleading. We propose a new concept called "reality check" and present a method of using a protocol model with reality check for wireless networks. Subsequently, we show that by appropriate setting of the interference range in the protocol model, it is possible to narrow the solution gap between the two models. Our simulation results confirm that this gap is indeed small (or even negligible). Thus, our methodology of joint reality check and interference range setting retains the protocol model as a viable approach to analyze multihop wireless networks.

Index Terms—Interference modeling, protocol model, physical model, multihop wireless network, cross-layer optimization

1 Introduction

There are two widely used models to characterize interference relationship in a wireless network, namely, the physical model and the protocol model. The physical model, also known as the signal-to-interference-and-noise ratio (SINR) model, is based on practical transceiver designs of communication systems that treat interference as noise. Under this model, a transmission is successful if and only if SINR at the intended receiver exceeds a threshold so that the transmitted signal can be decoded with an acceptable bit error rate (BER). Further, achievable rate calculation is based on SINR (via Shannon's formula), which takes into account interference due to simultaneous transmissions by other nodes. In wireless communications, such interference model is considered as a reference model since there exist practical coding schemes to approach its solution in real systems. As a result, physical model is widely regarded as an accurate representation of physical layer behavior.

However, the difficulty associated with the physical model is its computational complexity in obtaining a solution, particularly when it involves cross-layer optimization in a multihop network environment. This is because, SINR calculation is a nonconvex function with respect to the transmission powers. As a result, a solution to cross-layer optimization using the physical model is difficult to develop and its computational complexity is very high for large-sized networks. Consequently, most of the current approaches to cross-layer optimization employing the physical layer model follow a simplified layer-by-layer (or "layer-decoupled") approach and thus yield suboptimal solutions (e.g., [4], [8], [10]) or instead, focus on providing asymptotic lower and upper bounds (e.g., [13], [14], [18]).

To circumvent the complexity issue associated with the physical model, the so-called protocol model [13], also known as disk graph model, has been widely used by researchers in wireless networking community as a way to simplify the mathematical characterization of the physical layer. Under the protocol model, a successful transmission occurs when the intended receiving node falls inside the transmission range of its transmitting node and falls outside the interference ranges of other nonintended transmitters. The setting of transmission range is based on a signal-to-noise ratio threshold. The setting of interference range is rather heuristic and remains an open problem. Under the protocol model, the impact of interference from a transmitting node is binary and is solely determined by whether or not a receiver falls within the interference range of this transmitting node. That is, if a receiving node falls in the interference range of a nonintended transmitter, then this node is considered to be interfered and thus cannot receive correctly from its intended transmitter; otherwise, the interference is assumed to be negligible. Due to such simplification, the protocol model has been widely used in developing algorithms and protocols in wireless networks (e.g., [1], [3], [16], [17], [20], [23], [25], [27]) and can be easily applied to analyze large-sized wireless networks.

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The controversy surrounding (or arguments against) the protocol model is that a binary decision of whether interference exists (based on interference range) does not accurately capture physical layer characteristics. For the case when a node falls in the interference range of a nonintended transmitter, the protocol model assumes that this node cannot receive correctly from its intended transmitter (due to interference). But this is overly conservative, as based on capacity formula, there could still be some capacity even with interference. On the other hand, for the case when a node falls outside the interference range of each nonintended transmitter, the protocol model assumes that there is no interference. But this is somewhat optimistic as small interference from different transmitters can aggregate and may not be negligible in achievable rate calculation. As a result, there have been some serious doubts in the research community on the correctness of the protocol interference model for wireless networks.

The goal of this paper is to reconcile the tension between physical model and protocol model by answering the following fundamental question: How to correctly use the protocol interference model? The answer to this question is important for current and future investigations on multihop wireless networks.

It is worth pointing out that in the physical model, interference is treated as noise. Information-theoretic study has shown that if the interference information is exploited wisely (e.g., successive decoding [7], [24], superposition coding [2], [6], dirty paper coding [5]), a larger achievable rate region can be achieved. However, practical implementations of these techniques for multihop wireless networks remain to be developed due to the following issues: 1) These techniques, although theoretically attractive, are hard to implement for real systems due to extremely high hardware/software requirements and computational complexity. 2) In a multihop ad hoc network, there is no centralized infrastructure. As a result, exploiting interference information in such setting is extremely difficult. Thus, these advanced physical layer techniques will not be considered in this paper.

1.1 Main Contributions

The main contributions of this paper are the following:

- We show that, in general, solutions obtained under the protocol model may not be feasible in practice. Thus, solutions based on blind use of the protocol model may offer incorrect results as there is no feasibility checking mechanism in place after a solution is obtained. Due to this oversight, the doubt on blind use of the protocol model is legitimate.
- To obtain a feasible solution for the protocol model, we propose a new concept called "reality check" and a new methodology on how to use it with the protocol model to obtain a feasible solution.
- We further show that by combining reality check with appropriate setting of the interference range, it is possible to have the protocol model offer comparable results as those under the physical model. This offers us the correct approach of using the protocol model in practice.

1.2 Paper Organization

The rest of this paper is organized as follows: Section 2 presents a general cross-layer optimization problem for wireless networks. We briefly discuss the approaches and complexities to solve this problem under both physical and protocol models. Section 3 identifies potential infeasibility issue associated with a protocol model solution. We introduce a reality check mechanism and show how it can be used to obtain a revised solution that is feasible. In Section 4, we show the impact of interference range setting. Section 5 shows that by appropriate setting of the interference range in the protocol model, it is possible to obtain comparable results under both models. Section 6 discusses how to apply the protocol model in practice. Section 7 concludes this paper.

2 Mathematical Models and Problem Formulation

For the sake of generality in this investigation, we consider a multihop cognitive radio network (CRN), which not only encompasses all the features in existing multichannel multiradio [1], [9], [17], [18], [20], [21] (including 802.11-based radio platform) but also is positioned to be the primary radio platform in the coming decades [26]. Thus, algorithmic and optimization results for CRNs are not only important for future wireless networks, but are also generalizations of traditional wireless networks.

2.1 Models at Multiple Layers

We consider a CRN consisting of a set of $\mathcal{N}$ nodes. In a CRN, the available frequency bands at each node depend on its location and may not be the same. Denote $\mathcal{M}_i$ the set of available frequency bands at node i and assume the bandwidth of each frequency band is W. Denote $\mathcal{M}$ the set of all frequency bands present in the network, i.e., $\mathcal{M} = \bigcup_{i \in \mathcal{N}} \mathcal{M}_i$. Denote $\mathcal{M}_{ij} = \mathcal{M}_i \bigcap \mathcal{M}_j$, which is the set of common available bands on nodes i and j and thus can be used for transmission between these two nodes.

2.1.1 Scheduling and Power Control for Both Physical and Protocol Models

Scheduling can be done solely in frequency domain if the available spectrum is divided into a sufficiently large number of small bands. Alternatively, scheduling can be done solely in the time domain if the time frame is divided into sufficiently large number of small time slots. In this study, we consider scheduling in the frequency domain. Denote

$$x_{ij}^m = \begin{cases} 1 & \text{If node } i \text{ transmits to node } j \text{ on band } m, \\ 0 & \text{otherwise.} \end{cases} \quad (1)$$

Then, for a band $m \in \mathcal{M}_i$, node i cannot use it for transmission to multiple nodes or for reception from multiple nodes. Further, due to self-interference, node i cannot use it for both transmission and reception. Putting these constraints together, we have

$$\sum_{i \in \mathcal{T}_k^m} x_{ki}^m + \sum_{j \in \mathcal{T}_i^m} x_{ij}^m \leq 1 \quad (i \in \mathcal{N}, m \in \mathcal{M}_i), \quad (2)$$

where $\mathcal{T}_i^m$ is the set of nodes that are within the maximum transmission range from node i (under transmission power $P_{\max}$) on band m.

Denote p_{ij}^m as the transmission power at node i when node i transmits data to node j on band m. Clearly, when node i does not transmit data to node j on band m, p_{ij}^m should be 0. Under the maximum allowed transmission power limit $P_{\max}$ on one band, we have

$$p_{ij}^m \le P_{\max} x_{ij}^m \quad \left(i \in \mathcal{N}, m \in \mathcal{M}_i, j \in \mathcal{T}_i^m\right). \tag{3}$$

Denote P_i the maximum total transmission power at node i on all bands. We have $P_i \ge P_{\max}$ and

$$\sum_{m \in \mathcal{M}_i} \sum_{j \in \mathcal{T}_i^m} p_{ij}^m \le P_i \quad (i \in \mathcal{N}). \tag{4}$$

2.1.2 Scheduling Feasibility Constraints under the Physical Model

Under the physical model, a transmission is successful if and only if the SINR at the receiving node exceeds a certain threshold, say α. We now formulate this constraint. For a transmission from nodes i to j on band m, the SINR at node j is

$$s_{ij}^m = \frac{g_{ij} p_{ij}^m}{\eta W + \sum_{k \in \mathcal{N}}^{k \ne i,j} \sum_{h \in \mathcal{T}_k^m}^{h \ne i,j} g_{kj} p_{kh}^m},$$

where η is the ambient Gaussian noise density, g_{ij} is the propagation gain from nodes i to j, and $\mathcal{T}_k^m$ is the set of nodes to which node k can transmit on band m.

Since there is a transmission from nodes i to j on band m, neither i nor j can receive from other nodes on band m, i.e., $p_{ki}^m = 0$ and $p_{kj}^m = 0$. We have $\sum_{h \in \mathcal{T}_k^m} g_{kj} p_{kh}^m = \sum_{h \in \mathcal{T}_k^m}^{h \ne i,j} g_{kj} p_{kh}^m$. Denote

$$t_k^m = \sum_{h \in \mathcal{T}_k^m} p_{kh}^m = \sum_{h \in \mathcal{T}_k^m}^{h \ne i,j} p_{kh}^m \quad (k \in \mathcal{N}, m \in \mathcal{M}_k). \tag{5}$$

We have $s_{ij}^m = \frac{g_{ij} p_{ij}^m}{\eta W + \sum_{k \in \mathcal{N}}^{k \ne i,j} g_{kj} t_k^m}$, i.e.,

$$\eta W s_{ij}^m + \sum_{k \in \mathcal{N}}^{k \ne i,j} g_{kj} t_k^m s_{ij}^m - !g_{ij} p_{ij}^m = 0 \quad \left(i \in \mathcal{N}, min\mathcal{M}_i, j \in \mathcal{T}_i^m\right). \tag{6}$$

Note that this SINR computation also holds when $p_{ij}^m = 0$, i.e., when there is no transmission from nodes i to j on band m.

Recall that under the physical model, a transmission from nodes i to j on band m is successful if and only if SINR at node j exceeds a threshold α, i.e., $s_{ij}^m \ge \alpha$. Then, by (1), we have

$$s_{ij}^m \ge \alpha x_{ij}^m \quad \left(i \in \mathcal{N}, m \in \mathcal{M}_i, j \in \mathcal{T}_i^m\right), \tag{7}$$

which is the necessary and sufficient condition for successful transmission under the physical model.

For a successful transmission (i.e., if the above constraints are satisfied), the achievable rate by this s_{ij}^m is at most

$$c_{ij}^m = W \log_2\left(1 + s_{ij}^m\right) \quad \left(i \in \mathcal{N}, m \in \mathcal{M}_i, j \in \mathcal{T}_i^m\right). \tag{8}$$

Of course, the actual data rate depends on a number of other parameters, such as modulation, coding schemes, BER constraints, detector schemes, and so on, and will be lower than that obtained by the Shannon capacity formula.

2.1.3 Scheduling Feasibility Constraints under the Protocol Model

Under the protocol model, a transmission is successful if and only if the receiving node is within the transmission range of the intended transmitting node and is outside the interference range of each nonintended transmitting node. When power control is employed at each transmitting node, the transmission range and interference range can be varied and may be different from the others. As a result, the interference relationship among nodes becomes more complicated. In [22], Shi and Hou showed that the conditions for successful transmission from nodes i to j with an interfering transmission from nodes k to h can be formulated as follows:

$$p_{ij}^m \in \left[\left(\frac{d_{ij}}{R_T^{\max}}\right)^n P_{\max} x_{ij}^m,\ P_{\max} x_{ij}^m\right] \quad (i \in \mathcal{N}, m \in \mathcal{M}_i, j \in \mathcal{T}_i^m),$$

$$p_{kh}^m \le P_{\max} - \left[1 - \left(\frac{d_{kj}}{R_I^{\max}}\right)^n\right] P_{\max} x_{ij}^m \quad \left(i \in \mathcal{N}, m \in \mathcal{M}_i, j \in \mathcal{T}_i^m, k \in \mathcal{I}_j^m, k \ne i, h \in \mathcal{T}_k^m\right),$$

where d_{ij} is the physical distance between nodes i and j, $R_T^{\max}$ and $R_I^{\max}$ are the maximum transmission and interference ranges (under transmission power $P_{\max}$), respectively, and $\mathcal{I}_j^m$ is the set of nodes that may contribute towards nonnegligible interference at node j. These constraints are based on the uniform propagation gain $g_{ij} = d_{ij}^{-n}$, where n is the path loss index.

To better understand the physical meaning of these two constraints, we consider a general propagation gain function $g_{ij} = g(d_{ij})$. Further, denote

$$P_{ij}^T = \frac{g(R_T^{\max})}{g(d_{ij})} P_{\max} \tag{9}$$

and $P_{kj}^I = \frac{g(R_I^{\max})}{g(d_{kj})} P_{\max}$, which are the minimum required power for transmission from nodes i to j and the maximum allowed transmission power at node k when node j is receiving, respectively. We have the following constraints for successful transmission from nodes i to j (with a concurrent transmission from nodes k to h) under the protocol model:

$$p_{ij}^m \in [P_{ij}^T x_{ij}^m,\ P_{\max} x_{ij}^m] \quad (i \in \mathcal{N}, m \in \mathcal{M}_i, j \in \mathcal{T}_i^m), \tag{10}$$

$$p_{kh}^m \le P_{\max} - (P_{\max} - P_{kj}^I) x_{ij}^m \quad (i \in \mathcal{N}, m \in \mathcal{M}_i, j \in \mathcal{T}_i^m, k \in \mathcal{I}_j^m, k \ne i, h \in \mathcal{T}_k^m). \tag{11}$$

For a successful transmission (i.e., the above two constraints are satisfied), the interference from any other transmitter is considered "negligible" under the protocol model and the achieved rate is

$$c_{ij}^m = W \log_2\left(1 + \frac{g_{ij} p_{ij}^m}{\eta W}\right) \quad (i \in \mathcal{N}, m \in \mathcal{M}_i, j \in \mathcal{T}_i^m). \tag{12}$$

Note that this achievable rate computation also holds for $p_{ij}^m = 0$, i.e., when there is no transmission from nodes i to j on band m.

In our problem, power control can be performed at each node. When the transmission power $P_{\max}$ is used at a node i, this node has the maximum transmission range $R_T^{\max}$, which can be computed based on minimum required receiving power ($g(R_T^{\max}) \cdot P_{\max}$) at a receiving node j. When the transmission power p is less than $P_{\max}$, the same minimum required receiving power should be met. If node i can transmit to node j, then we have $g(d_{ij}) \cdot p \geq g(R_T^{\max}) \cdot P_{\max}$. Thus, the transmission range is

$$R_T(p) = g^{-1}\left(\frac{g(R_T^{\max}) \cdot P_{\max}}{p}\right), \tag{13}$$

which is a function of transmission power p. Similarly, the interference range is

$$R_I(p) = g^{-1}\left(\frac{g(R_I^{\max}) \cdot P_{\max}}{p}\right). \tag{14}$$

2.1.4 Routing for Both Physical and Protocol Models

Among the set of $\mathcal{N}$ nodes in the ad hoc network, we assume there is a set of $\mathcal{L}$ active user communication (unicast) sessions. Denote $s(l)$ and $d(l)$ the source and destination nodes of session $l \in \mathcal{L}$ and $r(l)$ the minimum rate requirement (in b/s) of session l. Suppose that our objective is to maximize a scaling factor K for all sessions' requirements. That is, for each session $l \in \mathcal{L}$, $Kr(l)$ amount of data rate is to be transmitted from $s(l)$ to $d(l)$. To route each of these flows from its respective source node to destination node, it is necessary to employ multihop. Further, to maximize the scaling factor, it is necessary to employ multipath (i.e., allow flow splitting). This is because, a single path is overly restrictive and may not yield optimal solution.

Mathematically, this can be modeled as follows: Denote $f_{ij}(l)$ the data rate from nodes i to j that is attributed to session l, where $i \in \mathcal{N}, j \in \mathcal{T}_i = \bigcup_{m\in\mathcal{M}_i} \mathcal{T}_i^m$. If node i is the source of session l, i.e., $i = s(l)$, then

$$\sum_{j\in\mathcal{T}_i} f_{ij}(l) = Kr(l) \quad (l \in \mathcal{L}, i = s(l)). \tag{15}$$

If node i is an intermediate relay node for session l, i.e., $i \neq s(l)$ and $i \neq d(l)$, then

$$\sum_{j\in\mathcal{T}_i}^{j\neq s(l)} f_{ij}(l) = \sum_{k\in\mathcal{T}_i}^{k\neq d(l)} f_{ki}(l) \quad (l \in \mathcal{L}, i \in \mathcal{N}, i \neq s(l), d(l)). \tag{16}$$

If node i is the destination of session l, i.e., $i = d(l)$, then

$$\sum_{k\in\mathcal{T}_i} f_{ki}(l) = Kr(l) \quad (l \in \mathcal{L}, i = d(l)). \tag{17}$$

It can be easily verified that once (15) and (16) are satisfied, (17) must also be satisfied. As a result, it is sufficient to have (15) and (16) in the formulation.

In addition to the above flow balance equations at each node $i \in \mathcal{N}$ for session $l \in \mathcal{L}$, the aggregated flow rates on each radio link cannot exceed this link's capacity. Therefore, for a link $i \to j$, we have

$$\sum_{l\in\mathcal{L}}^{s(l)\neq j, d(l)\neq i} f_{ij}(l) \leq \sum_{m\in\mathcal{M}_{ij}} c_{ij}^m \quad (i \in \mathcal{N}, j \in \mathcal{T}_i), \tag{18}$$

where c_{ij}^m is computed by (8) under the physical model or by (12) under the protocol model.

2.2 Problem Formulation and Solution Approach

2.2.1 Objective Function

In our problem formulation, we are interested in maximizing a rate related objective function. Specifically, we choose to maximize the scaling factor K for all sessions' rate requirements. There are many other objectives that can also be used in this investigation, e.g., the sum of all sessions' rates, the sum of log utility of session rates, and so on. In general, we could consider an objective function in the form of the total utility of session rates, with the utility of a session being a concave function of its rate. We emphasize that the same methodology that we will develop regarding how to correctly use the protocol model is applicable to all these objective settings.

2.2.2 Problem Formulation

Under the physical model, putting together all the constraints for scheduling, power control, and flow routing, we have

$$\begin{aligned}
&\text{Max} && K \\
&\text{s.t.} && (2), (3), (4), (5), (6), (7), (8), (15), (16), (18) \\
& && K, f_{ij}(l) \geq 0 \ (l \in \mathcal{L}, i \in \mathcal{N}, i \neq d(l), j \in \mathcal{T}_i, j \neq s(l)) \\
& && x_{ij}^m \in \{0,1\}, 0 \leq p_{ij}^m \leq P_{\max}, t_i^m, s_{ij}^m \geq 0 \\
& && \qquad \big(i \in \mathcal{N}, m \in \mathcal{M}_i, j \in \mathcal{T}_i^m\big).
\end{aligned}$$

While under the protocol model, we have

$$\begin{aligned}
&\text{Max} && K \\
&\text{s.t.} && (2), (4), (10), (11), (12), (15), (16), (18) \\
& && K, f_{ij}(l) \geq 0 \ (l \in \mathcal{L}, i \in \mathcal{N}, i \neq d(l), j \in \mathcal{T}_i, j \neq s(l)) \\
& && x_{ij}^m \in \{0,1\}, 0 \leq p_{ij}^m \leq P_{\max}\big(i \in \mathcal{N}, m \in \mathcal{M}_i, j \in \mathcal{T}_i^m\big).
\end{aligned}$$

2.2.3 Solution Approach

Both cross-layer optimization problems are in the form of *mixed-integer nonlinear programming* (MINLP) problem, which is NP-hard in general [11]. A solution procedure based on the branch-and-bound (similar to [22]) can be developed to solve these MINLP optimization problems.

An important step in branch-and-bound approach is constructing and solving a linear relaxation for the original optimization problem at each iteration. To have a linear relaxation for a nonlinear term, we introduce a new variable to replace this nonlinear term and add some linear constraints for this new variable (see [22]). Under the physical model, the linear relaxation has $O(N^3M)$ variables, while under the protocol model, the linear relaxation has $O(N^2M)$ variables. Since the number of variables directly impacts complexity, the complexity of solving a protocol model problem is much lower than a physical model problem.

We point out that to compare the capability (optimal performance) between the protocol model and the physical model, we have to develop optimal solutions for both models, despite that the complexities may be very high. In

Reality Check for A Given Protocol Model Solution.
1. Compute actual link capacity by (8) under the scheduling and power control in the given protocol model solution.
2. With calculated link capacities, obtain the maximal K and routing via an LP with constraints (15), (16), and (18).
3. The reality check result is the maximal K.

Fig. 1. Pseudocode for reality check.

Section 6, we will discuss how our findings on the protocol model can be applied in the development of efficient solutions.

3 A Reality Check Mechanism for Protocol Model Solution

In this section, we identify the potential infeasibility issue associated with a protocol model solution. Then, we introduce a reality check mechanism and show how it can be used to obtain a revised solution that is feasible.

Under the protocol model, the impact of interference from neighboring nodes is binary and is solely determined by whether or not the node falls within the interference range of non-intended transmitters. However, as nonzero interferences are neglected, achievable rate calculated under the protocol model may be larger than the actual achievable rate. As a result, solutions obtained under this model may not be feasible in practice. Due to such potential infeasibility in a protocol model solution, results based on blind use of the protocol model may be incorrect.

To find out the actually achievable result under a protocol model solution, it is necessary to go through a validation process. In this section, we introduce the notion of a "reality check" mechanism for a protocol model solution. The goal of reality check is to find the achievable result under a given protocol model solution. Reality check result can also be viewed as a corrected/revised result based on the given protocol model solution.

The reality check procedure is shown in Fig. 1. Specifically, for a given protocol model solution, we have the knowledge of scheduling and power control for each node in the network. Under reality check, instead of using the achievable rate computed by (12) (which neglects the impact of interference), we use (8) to *recompute* actual achievable rate between the nodes under the scheduling and power control in the given protocol model solution. Using this accurate rate calculation among the nodes, we can recompute the achievable result (i.e., objective function) via an LP to obtain a feasible routing solution. This new routing, along with the original scheduling and power control, offer a feasible solution. We call this achievable objective the reality check result, which is formally defined as follows:

Definition 1. *Reality check result is defined as the achievable objective for a given protocol model solution.*

In reality check, we only need to recompute capacities and adjust flow rates (see Fig. 1). Although we use the same achievable rate formula as that in the physical model, this formula is only used for simple calculations instead of being part of a complex optimization problem as under the physical model. Therefore, the complexity of this reality check mechanism is very small.

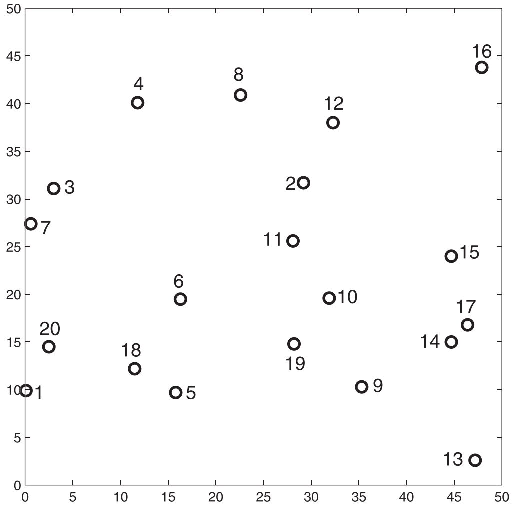


Fig. 2. A 20-node 5-session network topology.

Example 1. Consider a 20-node 5-session network in Fig. 2. The location and available bands at each node are shown in Table 1. The source node, destination node, and minimum rate requirement of each session are shown in Table 2. The setting of parameters are $W = 50$, $\alpha = 3$, $R_T^{\max} = 20$, $P_{\max} = 4.8 \cdot 10^5 \eta W$, $P_i = 10 P_{\max}$, and $g(d_{ij}) = d_{ij}^{-4}$. All units are normalized appropriately. When the maximum interference range is $R_I^{\max} = 35$, we have the following power control and scheduling solution: $x_{8,11}^2 = 1, p_{8,11}^2 = 0.6 \cdot P_{\max}; x_{8,11}^2 = 1, p_{8,11}^2 = 0.6 \cdot P_{\max}$; $x_{9,17}^7 = 1$, $p_{9,17}^7 = 0.2 \cdot P_{\max}$; $x_{11,10}^9 = 1$, $p_{11,10}^9 = 0.1 \cdot P_{\max}$; $x_{12,8}^7 = 1$, $p_{12,8}^7 = 0.1 \cdot P_{\max}$; $x_{12,11}^8 = 1$, $p_{12,11}^8 = 0.4 \cdot P_{\max}$; $x_{13,9}^2 = 1$, $p_{13,9}^2 = 0.4 \cdot P_{\max}$; $x_{15,14}^5 = 1$, $p_{15,14}^5 = 0.1 \cdot P_{\max}$; $x_{16,12}^6 = 1$, $p_{16,12}^6 = P_{\max}$, $x_{16,12}^{10} = 1$, $p_{16,12}^{10} = 0.5 \cdot P_{\max}$; $x_{18,1}^5 = 1$, $p_{18,1}^5 = 0.2 \cdot P_{\max}$. The objective function (i.e., the maximum scaling factor) under the protocol model solution is 20.47 (without reality check).

We now examine a transmission from nodes 8 to 11 on band 2. In the protocol model solution, the computed rate for link $8 \rightarrow 11$ is 117.84. There is another transmission from nodes 13 to 9 also on band 2 (see the location of each node in Table 1). Under the protocol model, it is easy to verify that the interference range at node 13 is smaller than the distance between nodes 13 and 11. Thus, the interference from node 13 is assumed to be negligible at node 11. However, this interference is $g_{13,11} \cdot p_{13,9}^2 = 0.2403 \cdot \eta W$, which is not zero. By (8), the actual rate from nodes 8 to 11 is 105.61. But in the protocol model solution, the flow rate on this link is 117.84, which is larger than this link's achievable rate. Therefore, the solution obtained under the protocol model is infeasible in practice.

To obtain the achievable objective function, we recompute achievable rate for all links following the same token. We list the actual rate and the rate computed in the protocol model in Table 3. Using revised capacities (from reality check), we can recompute a feasible solution using a linear program. The new achieved objective value is 18.34 (versus 20.47 in the blind use of the protocol model solution).

TABLE 1
Location and Available Frequency Bands at Each Node for a 20-Node 5-Session Network

Node	Location	Available Bands	Node	Location	Available Bands	Node	Location	Available Bands
1	$(0.1, 9.9)$	2, 5, 6, 7, 9, 10	8	$(22.6, 40.9)$	2, 5, 6, 7, 8, 9	15	$(44.7, 24)$	2, 5, 6, 8
2	$(29.2, 31.7)$	1, 3, 4	9	$(35.3, 10.3)$	2, 3, 4, 5, 6, 7, 8, 9, 10	16	$(47.9, 43.8)$	2, 5, 6, 8, 9, 10
3	$(3, 31.1)$	2, 6, 7, 8, 9, 10	10	$(31.9, 19.6)$	2, 6, 7, 8, 9, 10	17	$(46.4, 16.8)$	2, 5, 6, 7, 8
4	$(11.8, 40.1)$	2, 8	11	$(28.1, 25.6)$	2, 5, 6, 8, 9, 10	18	$(11.5, 12.2)$	1, 2, 5, 6, 7, 8, 9, 10
5	$(15.8, 9.7)$	7, 8, 9, 10	12	$(32.3, 38)$	2, 5, 6, 7, 8, 10	19	$(28.2, 14.8)$	3, 4
6	$(16.3, 19.5)$	2, 6, 10	13	$(47.2, 2.6)$	2, 6	20	$(2.5, 14.5)$	5, 6, 10
7	$(0.6, 27.4)$	1, 3	14	$(44.7, 15)$	5, 6, 7, 8, 9, 10			

The following statement summarizes our discussion on the reality check result:

Principle 1. *The reality check result offers a correct measure of achievable result by a given protocol model solution.*

The above principle offers a meaningful performance measurement criteria for the protocol model. The efficacy of the protocol model depends on the performance gap between its optimal reality check result and the optimal result obtained under the physical model. If this performance gap is small, then the protocol model is a good approximation and can be used as an effective tool for analyzing wireless networks. On the other hand, if this performance gap is large, then the protocol model may not be very useful.

Note that the reality check result also employs the accurate link capacity computation (8) and thus is a feasible solution under the physical model. Thus, reality check result cannot exceed the optimal result under the physical model. We state this relationship in the following fact:

Fact 1. *Reality check result is upper bounded by the optimal result under the physical model.*

4 Impact of the Interference Range Setting in the Protocol Model

In this section, we will show that interference range is an important parameter for the protocol model and study its impact on the protocol model solution.

4.1 Maximum Transmission Range ${R_T}^{\max}$

To perform a meaningful comparison, both the physical and protocol models should use the same underlying physical layer mechanism. Thus, the parameters for the two models should be set appropriately based on the same physical layer behavior. One parameter for the protocol model is the maximum transmission range $R_T^{\max}$. Since the underlying physical layer mechanism is the same, this parameter should be consistent with the α parameter in the physical model.

Under an ideal scenario, when there is no concurrent transmission in the same band, two nodes with distance $R_T^{\max}$ should be able to communicate with each other under the maximum transmission power $P_{\max}$ and the SINR should be α (the same as that under the physical model). Thus, we have $\frac{g(R_T^{\max})\cdot P_{\max}}{\eta W} = \alpha$. As a result, we should set transmission range as

$$R_T^{\max} = g^{-1}\left(\frac{P_{\max}}{\alpha\eta W}\right). \tag{19}$$

We assume a uniform propagation gain $g(d_{ij}) = d_{ij}^{-4}$ for numerical results.

4.2 Bounds for Maximum Interference Range ${R_I}^{\max}$

Another parameter for the protocol model is the maximum interference range $R_I^{\max}$. This parameter is introduced by the protocol model, and there is no corresponding parameter in the underlying physical layer mechanism. This is the only tunable parameter in the protocol model. The requirement on $R_I^{\max}$ is $R_I^{\max} > R_T^{\max}$, i.e., a lower bound for $R_I^{\max}$ is $R_T^{\max}$.

To find an upper bound for $R_I^{\max}$, we can determine a constant $(R_I)^U$ such that if $R_I^{\max} \geq (R_I)^U$, then when any link is active, all other links on the same band in the network cannot be active. Consider a link $k \to h$ on a band m. Using (9) and (14), its interference range is at least

$$R_I(P_{kh}^T) = g^{-1}\left(\frac{g(R_I^{\max})\cdot P_{\max}}{P_{kh}^T}\right) = g^{-1}\left(\frac{g(R_I^{\max})\cdot g(d_{kh})}{g(R_T^{\max})}\right).$$

TABLE 2
Source Node, Destination Node, and Minimum Rate Requirement of Each Session in the 20-Node 5-Session Network

Session l	Source Node $s(l)$	Dest. Node $d(l)$	Min Rate Req. $r(l)$
1	16	10	9
2	18	1	1
3	12	11	4
4	13	17	3
5	15	14	2

TABLE 3
Actual Achievable Rate Computed by Reality Check versus Rate Computed in the Protocol Model

Link	Actual Achievable Rate	Rate Computed in the Protocol Model
$8 \to 11$	105.61	117.84
$9 \to 17$	102.67	108.67
$11 \to 10$	328.78	328.78
$12 \to 8$	119.47	123.98
$12 \to 11$	145.70	145.70
$13 \to 9$	113.75	126.28
$15 \to 14$	148.07	152.81
$16 \to 12$	243.25	243.25
$18 \to 1$	131.58	132.18

For another node j with $d_{jk} \leq R_I(P_{kh}^T)$ (or equivalently, $R_I^{\max} \geq g^{-1}(\frac{g(R_T^{\max}) \cdot g(d_{jk})}{g(d_{kh})})$), it cannot receive on band m when node i is transmitting to node j on band m. Thus, we can set

$$\begin{aligned}(R_I)^U &= \max\left\{ g^{-1}\left(\frac{g(R_T^{\max}) \cdot g(d_{jk})}{g(d_{kh})}\right) \right. \\ &\quad \left. : j \in \mathcal{N}, m \in \mathcal{M}_j, k \in \mathcal{I}_j^m, h \in \mathcal{T}_k^m \right\} \\ &= \max\left\{ g^{-1}\left(\frac{g(R_T^{\max}) \cdot g\big(\max\{d_{jk} : k \in \mathcal{I}_j^m\}\big)}{\min\{g(d_{kh}) : h \in \mathcal{T}_k^m\}}\right) \right. \\ &\quad \left. : k \in \mathcal{N}, m \in \mathcal{M}_j \right\}.\end{aligned}$$

Any $R_I^{\max} \geq (R_I)^U$ will lead to the same interference relationship in the network, which in turn yields the same protocol model solution and the same reality check result. Thus, without loss of generality, the range for $R_I^{\max}$ is within $[R_T^{\max}, (R_I)^U]$.

4.3 Varying Maximum Interference Range $R_I{}^{\max}$

As one would expect, the setting of $R_I^{\max}$ directly affects the performance gap between the two models. We now investigate the impact of the maximum interference range ($R_I^{\max}$) setting. In this study, we consider two groups of networks. The networks in the first group are all in a fixed area while the networks in the second group have a fixed node density.

4.3.1 Networks in a Fixed Area

We consider randomly generated networks with different number of nodes and sessions. The number of nodes in the network is within $[20, 50]$ with each node randomly located in a 50×50 area. For the ease of exposition, we normalize all units for distance, bandwidth, rate, and power based on (6) and (8) with appropriate dimensions. At each node, there are up to 10 available frequency bands, and each band has a bandwidth of $W = 50$. The set of available bands at different nodes can be different. The number of sessions is within $[5, 10]$. The source node and destination node of each session are randomly selected. The minimum rate requirement for each session is randomly generated within $[1, 10]$.

We assume that the SINR threshold α for the physical model is $\alpha = 3$ [12], which reflects the sensitivity of the radio receiver's signal detection and decoding capability. For the protocol model, the maximum transmission range $R_T^{\max}$ and maximum interference range $R_I^{\max}$ under transmission power $P_{\max}$ are two constant parameters. Note that the transmission range $R_T(p)$ and interference range $R_I(p)$ are variables that depend on transmission power p (see (13) and (14)). We assume the maximum transmission power $P_{\max} = 4.8 \cdot 10^5 \eta W$, with the corresponding maximum transmission range $R_T^{\max} = 20$ (by (19)). We assume the maximum total transmission power $P_i = 10P_{\max}$ for each node i. We will set different values for the maximum interference range $R_I^{\max}$ (under transmission power $P_{\max}$) in this study and investigate the impact of this parameter setting in our study. We apply the reality check mechanism for each protocol model solution.

The first set of results is for the 20-node 5-session network discussed in Example 1. The solution under the physical model has an objective value of 18.89. The results under the protocol model are shown in Fig. 3. We can see that the reality check result for a protocol model solution is different under different maximum interference range setting. The largest objective value among these reality check results is 18.34 (with $R_I^{\max} = 35$), which is very close to the physical model solution.

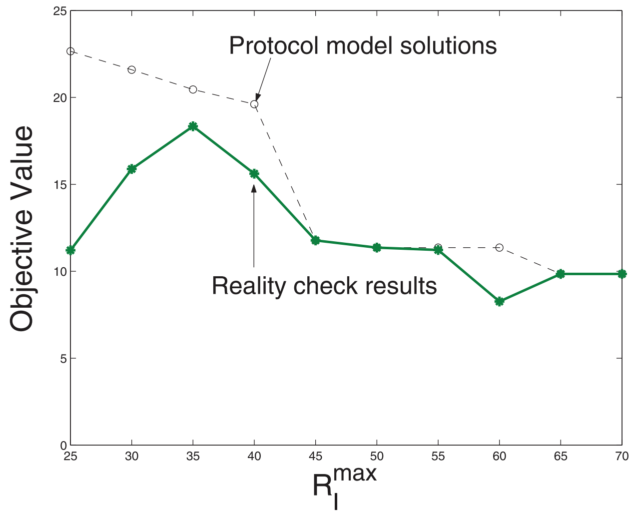


Fig. 3. Protocol model solutions and corresponding reality check results for the 20-node 5-session network.

By Principle 1, the reality check result offers a measure of achievable result by a protocol model solution. Thus, for this network, the best maximum interference range value should be 35, and the ratio between $R_I^{\max}$ and $R_T^{\max}$ is $\frac{35}{20} = 1.75$. Our results on different networks will show that such ratio is within $[1.5, 2]$.

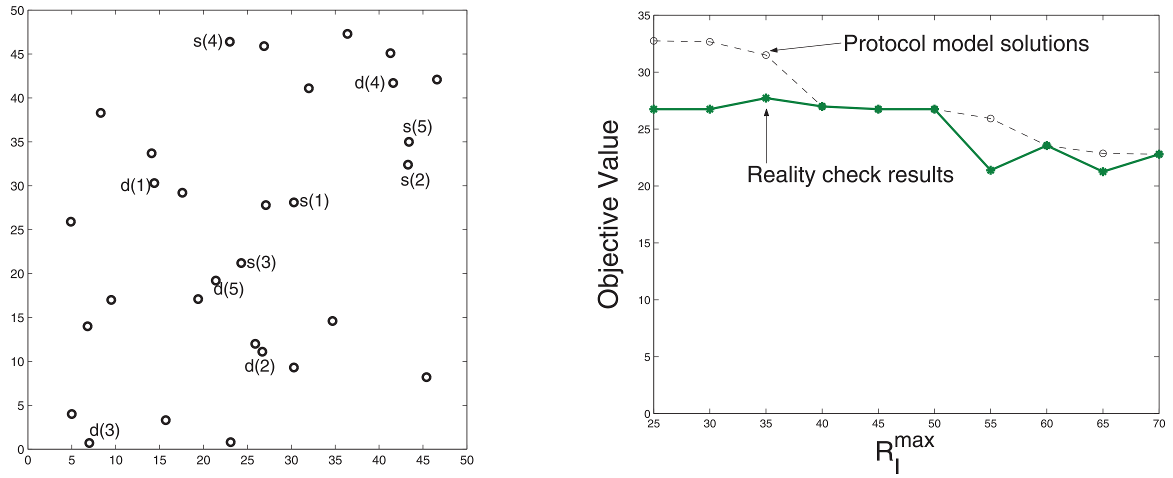
In general, we have the following rule on how to set the maximum interference range for an algorithm developed under the protocol model:

Rule 1. *For an algorithm designed under the protocol model, the maximum interference range should be set to the value corresponding to the maximum reality check result.*

We emphasize that, to set this range optimally, it is *not* necessary to solve the problem under the physical model, which involves much higher complexity.

The network topology of a 30-node 5-session network is shown in Fig. 4a. The location and available bands at each node are shown in Table 4. The source node, destination node, and minimum rate requirement of each session are shown in Table 5. The solution under the physical model has an objective value of 31.18. The results under the protocol model are shown in Fig. 4b. We find that the best reality check result has an objective value of 27.72, which is within 11 percent of the optimum (i.e., 31.18) and the maximum interference range should be set to 35 by Rule 1. The ratio between two ranges is again 1.75.

The network topology of a 40-node 10-session network is shown in Fig. 5a. The location and available bands at each node are shown in Table 6. The source node, destination node, and minimum rate requirement of each session are shown in Table 7. The solution under the physical model has a maximum objective value of 16.43. The results under the protocol model are shown in Fig. 5b



(a) Network topology.

(b) Protocol model solutions and corresponding reality check results.

Fig. 4. A 30-node 5-session network.

TABLE 4
Location and Available Frequency Bands at Each Node for a 30-Node 5-Session Network

Node	Location	Available Bands	Node	Location	Available Bands
1	$(7, 0.7)$	1, 2, 6, 7, 16, 17, 19, 20	16	$(30.3, 28.1)$	7, 8, 11, 16, 17, 19, 20
2	$(5, 4)$	3, 5, 9, 12, 14, 15	17	$(32, 41.1)$	7, 11, 16, 17, 19, 20
3	$(6.8, 14)$	1, 2, 6, 7, 8, 11, 16, 17, 19, 20	18	$(14.1, 33.7)$	3, 4, 5
4	$(15.7, 3.3)$	1, 2, 7, 16, 20	19	$(23, 46.4)$	3, 12, 15
5	$(9.5, 17)$	3, 4, 5, 9, 12	20	$(30.3, 9.3)$	5, 9
6	$(19.4, 17.1)$	1, 2, 6, 7, 8, 16, 19, 20	21	$(17.6, 29.2)$	1, 2, 6, 7, 8, 11, 16, 17, 19, 20
7	$(34.7, 14.6)$	3, 4, 5, 9, 12, 14	22	$(27.1, 27.8)$	9, 12, 14, 15
8	$(4.9, 25.9)$	3, 4, 12	23	$(26.9, 45.9)$	3, 4, 5, 9, 10, 12, 13, 14, 15, 17
9	$(46.6, 42.1)$	10, 18	24	$(43.3, 32.4)$	1, 2, 11, 16, 17, 20
10	$(8.3, 38.3)$	3, 4, 5, 9, 14	25	$(45.4, 8.2)$	3, 4, 5, 9, 12, 14
11	$(26.7, 11.1)$	1, 6, 7, 8, 11, 16, 17, 19, 20	26	$(43.4, 35)$	3, 5, 9, 15
12	$(36.4, 47.3)$	10, 13, 18	27	$(41.3, 45.1)$	1, 16, 20
13	$(24.3, 21.2)$	1, 2, 6, 8, 11, 19	28	$(14.4, 30.3)$	1, 2, 6, 7, 8, 11, 16, 17, 20
14	$(23.1, 0.8)$	3, 5, 9, 14	29	$(41.6, 41.7)$	3, 4, 5, 9, 10, 12, 14, 15, 18
15	$(21.4, 19.2)$	4, 9, 12, 14	30	$(25.9, 12)$	1, 2, 6, 7, 8, 11, 16, 17, 19, 20

with the same maximum reality check result being 16.43. By Rule 1, we find that the maximum interference range in the protocol model should be set to 30 and the ratio between two ranges is 1.5.

For a 50-node 5-session network in Fig. 6a, the location and available bands at each node are shown in Table 8. The source node, destination node, and minimum rate requirement of each session are shown in Table 9. The solution under the physical model has an objective value of 25.27. The results under the protocol model are shown in Fig. 6b. By Rule 1, we find that the maximum interference range in the protocol model should be set to 35 or 40, and the ratio between two ranges can be set to either 1.75 or 2. Both values can achieve the same objective value of 25.02, which is very close to the optimum 25.27.

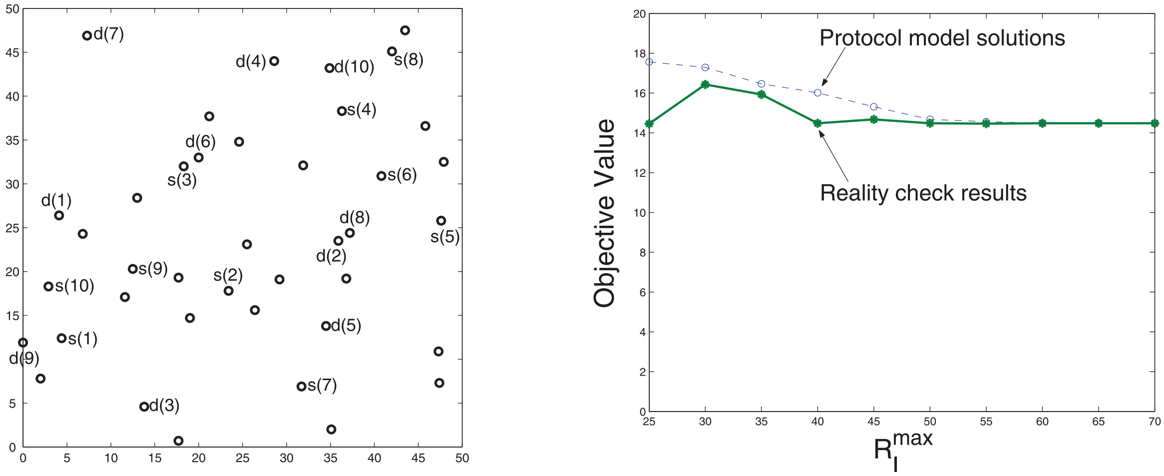
Finally, we have the results for a 50-node 10-session network in Fig. 7. The location and available bands at each node are shown in Table 10. The source node, destination node, and minimum rate requirement of each session are shown in Table 11. The solution under the physical model has an objective value of 13.36. By Rule 1, we find that the maximum interference range in the protocol model should be set to 40 and the ratio between two ranges is 2. The corresponding objective value is the same as that under the physical model, i.e., 13.36.

TABLE 5
Source Node, Destination Node, and Minimum Rate Requirement of Each Session in the 30-Node 5-Session Network

Session l	Source Node $s(l)$	Dest. Node $d(l)$	Min Rate Req. $r(l)$
1	16	28	4
2	24	11	7
3	13	1	1
4	19	29	8
5	26	15	1

4.3.2 Networks with Fixed Node Density

We consider randomly generated networks with different number of nodes, sessions, and network sizes. The number of nodes in the network is within $[16, 49]$ with each node randomly located in a square with length in $[50, 70]$. The node density is 0.01. The numbers of active user communication sessions is within $[4, 8]$. We again set the bandwidth of each band as $W = 50$, the minimum rate requirement for each session as a random number in $[1, 10]$. Due to space



(a) Network topology.

(b) Protocol model solutions and corresponding reality check results.

Fig. 5. A 40-node 10-session network.

TABLE 6
Location and Available Frequency Bands at Each Node for a 40-Node 10-Session Network

Node	Location	Available Bands	Node	Location	Available Bands
1	$(0, 11.9)$	1, 11, 23	21	$(12.5, 20.3)$	1, 9, 11
2	$(2, 7.8)$	5, 6, 8, 12, 28	22	$(13, 28.4)$	1, 9, 11, 16, 21, 23, 24, 26
3	$(2.9, 18.3)$	1, 3, 9, 11, 20, 21, 23, 24, 26	23	$(47.9, 32.5)$	2, 13, 14, 15, 18, 22
4	$(6.8, 24.3)$	3, 20, 21, 23, 24, 26	24	$(21.2, 37.7)$	1, 9, 11, 16, 20, 21, 23, 24, 26
5	$(7.3, 46.9)$	1, 11, 26	25	$(19, 14.7)$	4, 5, 6, 8, 12, 28, 29
6	$(4.4, 12.4)$	4, 5, 6, 8, 19, 29	26	$(31.7, 6.9)$	1, 3, 9, 11, 16, 26
7	$(4.1, 26.4)$	6, 8, 27, 29	27	$(24.6, 34.8)$	4, 5, 6, 8, 12, 17, 19, 27, 28, 29
8	$(13.8, 4.6)$	4, 12	28	$(29.2, 19.1)$	1, 11, 16, 20, 21, 24, 26
9	$(26.4, 15.6)$	2, 13, 14, 15, 18, 22, 25	29	$(11.6, 17.1)$	5, 6, 8, 17, 29
10	$(23.4, 17.8)$	19, 29	30	$(45.8, 36.6)$	2, 7, 10, 13, 14, 15, 18, 22, 25
11	$(20, 33)$	1, 2, 7, 10, 13, 14, 15, 18, 22, 25	31	$(31.9, 32.1)$	1, 21, 23
12	$(17.7, 0.7)$	3, 11, 20, 23, 24, 26	32	$(47.6, 25.8)$	7, 18
13	$(28.6, 44)$	2, 7, 10, 14, 15, 22, 25	33	$(25.5, 23.1)$	2, 7, 10, 14, 22, 25
14	$(34.5, 13.8)$	7, 10, 13, 14, 15, 25	34	$(34.9, 43.2)$	1, 3, 9, 11, 16, 20, 21, 23, 24, 26
15	$(18.3, 32)$	4, 5, 6, 8, 13, 17, 28, 29	35	$(47.4, 7.3)$	3, 9, 16, 20, 21
16	$(40.8, 30.9)$	2, 7, 13, 15, 18, 22, 25	36	$(36.8, 19.2)$	1, 3, 9, 11, 16, 20, 21, 24
17	$(36.3, 38.3)$	2, 10, 13, 14, 18, 25	37	$(37.2, 24.4)$	1, 9, 16, 20, 21, 23, 24, 26
18	$(35.1, 2)$	4, 17	38	$(42, 45.1)$	1, 9, 11, 20, 21
19	$(47.3, 10.9)$	7, 10, 13, 14, 15, 25	39	$(17.7, 19.3)$	4, 12, 17, 19, 27, 28
20	$(35.9, 23.5)$	4, 5, 6, 8, 12, 17, 19, 27, 28, 29	40	$(43.5, 47.5)$	7, 10

limitation, we omit the location and available bands at each node, the source node, destination node, and minimum rate requirement of each session.

TABLE 7
Source Node, Destination Node, and Minimum Rate Requirement of Each Session in the 40-Node 10-Session Network

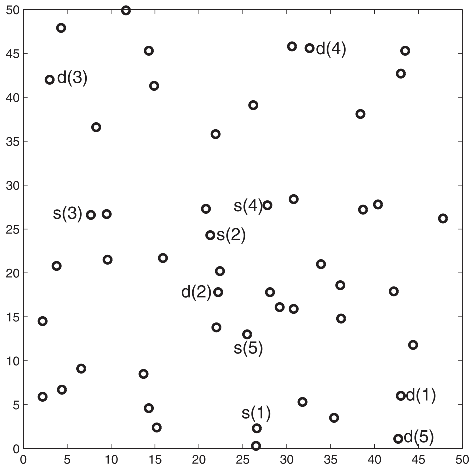
Session l	Source Node $s(l)$	Dest. Node $d(l)$	Min Rate Req. $r(l)$
1	6	7	8
2	10	20	9
3	15	8	9
4	17	13	8
5	32	14	9
6	16	11	4
7	26	5	7
8	38	37	2
9	21	1	3
10	3	34	3

For these networks, we also observed the impact of maximum interference range setting. The detailed reality check results under different interference range setting are omitted due to space limitation. Instead, we list the optimal ratio between maximum interference range $R_I^{\max}$ and maximum transmission range $R_T^{\max}$, the achieved objective value by the protocol model solution (with reality check), and the optimal objective value achieved by the physical model solution in Table 13.

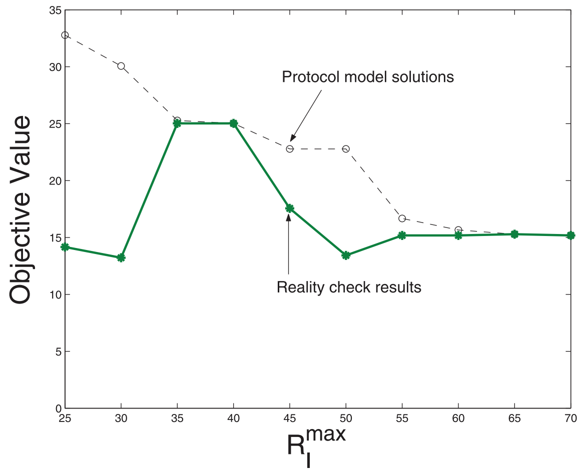
In summary, the setting of maximum interference range in the protocol model has a direct impact on the performance result for wireless networks. Now, the question is how to set interference range, which will be addressed in the next section.

5 Closing the Gap: Optimal Setting of ${R_I}^{\max}$

In this section, we discuss how to set $R_I^{\max}$ such that the performance gap between the protocol model solution (after reality check) and the optimal solution by the



(a) Network topology.



(b) Protocol model solutions and corresponding reality check results.

Fig. 6. A 50-node 5-session network.

TABLE 8
Location and Available Frequency Bands at Each Node for a 50-Node 5-Session Network

Node	Location	Available Bands	Node	Location	Available Bands
1	(6.6, 9.1)	1, 2, 7, 19	26	(26.2, 39.1)	1, 2, 7, 16, 17
2	(2.2, 5.9)	3, 4, 9, 10, 12, 15	27	(42.7, 1.1)	3, 4, 9, 10, 15, 18
3	(3.8, 20.8)	1, 7, 17, 19, 20	28	(36.2, 14.8)	3, 4, 9, 10, 15, 18
4	(2.2, 14.5)	3, 4, 9, 10, 12, 15	29	(22, 13.8)	5, 6, 8, 11, 13, 14
5	(9.5, 26.7)	4, 10	30	(47.8, 26.2)	3, 4, 9, 10, 12, 15, 18
6	(7.7, 26.6)	1, 2, 7, 19, 20	31	(9.6, 21.5)	5, 11, 13, 14
7	(43, 42.7)	2, 7, 16, 19	32	(36.1, 18.6)	5, 6, 11, 13, 14
8	(13.7, 8.5)	1, 2, 7, 17, 19, 20	33	(43, 6)	1, 2, 7, 11, 16, 17, 19, 20
9	(4.3, 47.9)	3, 15, 18	34	(38.4, 38.1)	3, 4, 9, 10, 12, 14, 15, 18
10	(15.2, 2.4)	3, 12, 18	35	(42.2, 17.9)	1, 2, 7, 17, 19, 20
11	(22.4, 20.2)	3, 9, 15, 18	36	(28.1, 17.8)	5, 11, 13, 14
12	(21.9, 35.8)	3, 4, 12, 15, 18	37	(40.4, 27.8)	1, 2, 7, 16, 17, 19, 20
13	(22.2, 17.8)	2, 7, 17, 19, 20	38	(38.7, 27.2)	6, 8, 14
14	(14.9, 41.3)	3, 4, 9, 18	39	(4.4, 6.7)	6, 14
15	(31.8, 5.3)	4, 10, 12, 15, 18	40	(43.5, 45.3)	1, 7, 16, 17, 20
16	(21.3, 24.3)	1, 7, 19, 20	41	(8.3, 36.6)	1, 2, 7, 16, 19, 20
17	(14.3, 4.6)	6, 8, 11	42	(44.4, 11.8)	1, 2, 7, 16, 19, 20
18	(33.9, 21)	3, 4, 9, 10, 12, 15, 18	43	(20.8, 27.3)	1, 2, 7, 16, 17, 19
19	(30.8, 28.4)	3, 4, 9, 10, 12, 15	44	(15.9, 21.7)	3, 4, 9, 12, 15, 18
20	(14.3, 45.3)	2, 7, 16, 17, 19, 20	45	(11.7, 49.9)	3, 10, 12, 15, 18
21	(26.6, 2.3)	1, 2, 7, 17, 19, 20	46	(30.6, 45.8)	8, 11
22	(30.8, 15.9)	1, 2, 7, 16, 17	47	(25.5, 13)	4, 10
23	(27.8, 27.7)	1, 2, 16, 17, 19, 20	48	(26.5, 0.3)	5, 13
24	(29.2, 16.1)	6, 8, 11, 13, 14	49	(3, 42)	1, 2, 7, 16, 19, 20
25	(32.6, 45.6)	4, 9, 12, 15, 18	50	(35.4, 3.5)	3, 4, 9, 15

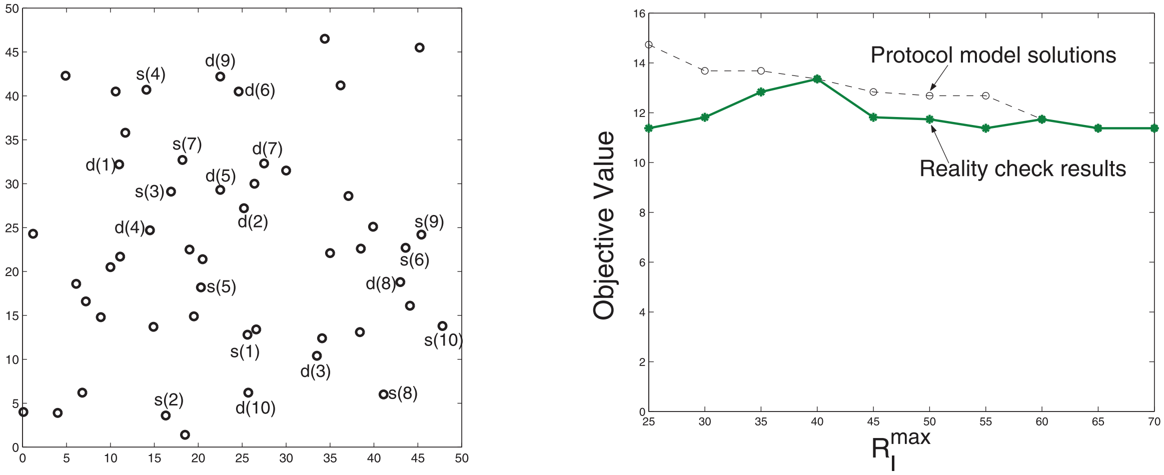
physical model is small. The protocol model is heuristic in nature and serves as the basis for the reality check results. An attempt to develop a theoretical result (on performance guarantee) for the reality check results for a general network topology seems difficult. Thus, our study is mainly simulation based. Nevertheless, such an approach is still valuable as we are indeed able to gain many insights through this effort.

Under protocol model solutions, we can see that (in Figs. 3, 4b, 5b, 6b, and 7b) the actual objective value in reality check result is no more than that in the protocol model solution.

When the maximum interference range $R_I^{\max}$ is set too small (e.g., 25), the protocol model provides an incorrect (overly optimistic) solution, which is even larger than that under the physical model. After we perform reality check on the protocol model solution, the achieved result could be much lower than that under the physical model. This is because, under a very small interference range, the

TABLE 9
Source Node, Destination Node, and Minimum Rate Requirement of Each Session in the 50-Node 5-Session Network

Session l	Source Node $s(l)$	Dest. Node $d(l)$	Min Rate Req. $r(l)$
1	21	33	8
2	16	13	9
3	6	49	5
4	19	25	2
5	47	27	4



(a) Network topology.

(b) Protocol model solutions and corresponding reality check results.

Fig. 7. A 50-node 10-session network.

TABLE 10
Location and Available Frequency Bands at Each Node for a 50-Node 10-Session Network

Node	Location	Available Bands	Node	Location	Available Bands
1	(11.1, 21.7)	2, 3, 4, 8, 25	26	(25.2, 27.2)	10, 14, 20, 24, 26
2	(0.1, 4)	6, 7, 10, 13, 14, 20, 23, 24, 26, 28	27	(22.5, 42.2)	5, 9, 12, 16, 18, 27, 29, 30
3	(7.2, 16.6)	6, 10, 14, 20, 23, 24, 26	28	(30, 31.5)	6, 13, 24, 26, 28
4	(11, 32.2)	6, 7, 10, 13, 14, 20, 23, 24, 26, 28	29	(35, 22.1)	6, 10
5	(16.3, 3.6)	10, 13, 14, 20, 23	30	(25.7, 6.2)	5, 9, 12, 17, 18, 22, 27, 29, 30
6	(14.5, 24.7)	8, 11, 25	31	(34.1, 12.4)	9, 12, 16, 17, 30
7	(10.6, 40.5)	4, 15, 19, 21, 25	32	(26.4, 30)	5, 9, 12, 16, 17, 18, 22, 27, 29, 30
8	(19.5, 14.9)	7, 24, 28	33	(14.1, 40.7)	1, 2, 25
9	(26.6, 13.4)	1, 19, 21, 25	34	(34.4, 46.5)	9, 17, 18, 30
10	(22.5, 29.3)	1, 3, 4, 8, 11, 15, 19	35	(19, 22.5)	1, 6, 7, 10, 13, 14, 20, 23, 24, 28
11	(24.6, 40.5)	3, 8, 25	36	(39.9, 25.1)	6, 13, 14, 20, 23, 24, 26, 28
12	(38.4, 13.1)	2, 8, 11, 15	37	(20.3, 18.2)	1, 2, 3, 4, 8, 11, 15, 19, 21, 27
13	(4, 3.9)	9, 12, 16, 22, 27, 29, 30	38	(10, 20.5)	6, 7, 10, 13, 14, 20, 23, 24, 26, 28
14	(6.1, 18.6)	9, 12, 16, 17, 18, 22, 27, 30	39	(20.5, 21.4)	1, 2, 3, 4, 8, 11, 15, 19, 21, 25
15	(38.5, 22.6)	2, 4, 11, 15, 19, 21, 25	40	(37.1, 28.6)	7, 10, 13, 14, 20, 23, 24, 26
16	(1.2, 24.3)	5, 9, 12, 17, 22, 29, 30	41	(44.1, 16.1)	1, 15, 21
17	(4.9, 42.3)	5, 27	42	(41.1, 6)	9, 29
18	(18.5, 1.4)	5, 9, 12, 17, 18, 27, 30	43	(43, 18.8)	5, 9, 12, 16, 18, 22
19	(16.9, 29.1)	3, 4, 10, 11, 12, 15	44	(45.4, 24.2)	9, 12, 16, 17, 18, 30
20	(33.5, 10.4)	7, 13, 14, 20, 23, 24, 26, 28	45	(36.2, 41.2)	5, 9, 17, 27, 29, 30
21	(25.6, 12.8)	6, 7, 20, 23, 24, 28	46	(27.5, 32.3)	12, 16, 17, 18, 29, 30
22	(45.2, 45.5)	2, 8, 15, 19	47	(47.8, 13.8)	22, 27, 29, 30
23	(43.6, 22.7)	1, 2, 3, 4, 11, 15, 19, 21	48	(8.9, 14.8)	5, 30
24	(14.9, 13.7)	5, 9, 12, 16, 17, 18, 22, 27, 29, 30	49	(6.8, 6.2)	5, 9, 12, 16, 17, 27, 30
25	(18.2, 32.7)	9, 12, 18, 22, 27	50	(11.7, 35.8)	1, 2, 3, 4, 8, 11, 15, 19, 21, 25

"negligible" interference from neighboring nodes could be large (nonnegligible). As a result, the accurate rate calculated via (8) could be much smaller than that computed in the protocol model by (12).

On the other hand, when the maximum interference range is set too large, the gap between a protocol model solution and its reality check result can be small. This is because, with a very large interference range, spectrum may not be reused at different nodes. As a result, there is no interference from other nodes. In this case, the link capacity computation in (12) is the same as that in (8), and thus, there is no performance degradation after reality check. However, as we have seen in all the results, setting the maximum interference range too large will lead to conservative results, i.e., much smaller than those under the physical model.

By Rule 1, we should set $R_I^{\max}$ appropriately to obtain a good protocol model solution. Since distance is normalized, instead of studying the value for interference range, we focus on determining a suitable ratio for $R_I^{\max}/R_T^{\max}$. Based on the results in Section 4.3, we list the optimal ratio between interference range $R_I^{\max}$ and transmission range $R_T^{\max}$, the achievable objective value by the protocol model solution (with reality check), and the optimal objective value achieved by the physical model solution in Table 12. We can see that for each network instance, the protocol model can provide a solution very close to the physical model solution under the optimal ratio.

We also performed simulations on networks with a fixed node density while the area of network was changed. Results for this set of simulations are summarized in Table 13. We

TABLE 11
Source Node, Destination Node, and Minimum Rate Requirement of Each Session in the 50-Node 10-Session Network

Session l	Source Node $s(l)$	Dest. Node $d(l)$	Min Rate Req. $r(l)$
1	21	4	4
2	5	26	7
3	19	20	6
4	33	6	10
5	37	10	9
6	23	11	2
7	25	46	3
8	42	43	9
9	44	27	8
10	47	30	1

TABLE 12
Results for Networks within the Same Network Coverage Area

Network Instance	Optimal Ratio	Achieved Objective Value	Optimal Objective Value
20-node 5-session network	1.75	18.34	18.89
30-node 5-session network	1.75	27.72	31.18
40-node 10-session network	1.5	16.43	16.43
50-node 5-session network	1.75	25.02	25.27
50-node 10-session network	2	13.36	13.36

TABLE 13
Results for Networks with the Same Node Density

Network Instance	Optimal Ratio	Achieved Objective Value	Optimal Objective Value
16-node 4-session network	1.75	38.54	38.54
25-node 5-session network	2	13.61	14.14
36-node 6-session network	2	38.03	38.03
49-node 8-session network	1.75	25.72	26.90

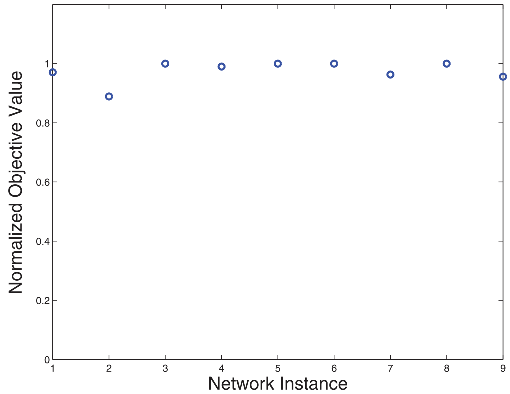


Fig. 8. Normalized objective value under optimal setting of $R_I^{\max}$ for all

can see that the results in Table 13 are consistent with that in Table 12.

To visually show the performance gap, we normalize the achieved objective value by the optimal objective value and show this normalized objective value for all networks in Fig. 8. We can see that for most networks, the normalized objective value is very close to one (average 0.97). Based on these results, we draw the following observation:

Observation 1. Under the optimal setting of the maximum interference range, the reality check result for the protocol model solution is close to (or the same as) the physical model solution.

The significance of the above observation is that it enables us to use the protocol model (with reality check) as a good simplification for the physical model as long as maximum interference range $R_I^{\max}$ is set correctly. Based on all results in Tables 12 and 13, the optimal ratio between $R_I^{\max}$ and $R_T^{\max}$ should be within $[1.5, 2]$.

6 Applying the Protocol Model in Practice

It is important to realize that to measure the capability of the protocol model against the physical model, we must develop optimal solutions for both models, despite that the complexity of such optimal solutions is very high. This is what we did in Section 2. Subsequently, in Sections 3 to 5, we have shown that the protocol model, once used correctly, can offer similar performance as the physical model and thus can be a viable approach to analyze wireless networks.

In practice, it is necessary to develop highly efficient algorithms (with polynomial-time complexity) while to have optimal or near-optimal (e.g., approximation) solutions. How to design such an algorithm is problem specific as it depends on the desired objective and the underlying problem setting. So a general discussion on such an efficient and high-performance algorithm is not possible.

Nevertheless, we now show how the findings in this paper can be applied after an efficient protocol model algorithm has been developed for some problem under investigation. To ensure that the obtained protocol model solution is feasible, we need to apply reality check to this solution so as to obtain a modified (feasible) solution. The complexity of this step is very small. Then, we can tune the interference range $R_I^{\max}$ such that the reality check result can be optimized. To do this, we should simulate the network with different interference ranges and pick the value that optimizes the desired objective. Note that the reality check mechanism should be used in evaluating the achievable objective under each interference range setting. This tuning of interference range does not require us to solve the problem under the physical model, which involves much higher complexity.

7 Conclusions

This paper aimed to reconcile the tension between the physical and protocol models. We showed that, in general, solutions obtained directly under the protocol model are likely to be infeasible in practice and thus blind use of the protocol model is likely to offer incorrect results. To address this problem, we proposed a new mechanism called "reality check" and showed how it can be used to obtain a feasible protocol model solution. Subsequently, we showed that by appropriate setting of the interference range in the protocol model, it is possible to narrow the solution gap between protocol model and physical model. Our simulation results confirmed that this gap is indeed small, thereby suggesting that our method of joint reality check and optimal interference range setting can make the protocol model a viable approach to analyze multihop wireless networks.

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