Some Hints on Mathematical Style

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Many years ago, just after my degree, I had the good fortune to be given some hints on mathematical writing by J.-P. Serre. Through the years I have found myself trying to repeat this very sound advice to other mathematicians who are also starting out. I have also been involved in the publishing of a proceedings volume, as well as being an editor of the Journal of Number Theory. Many of the papers coming my way are from young authors; so I have written down these hints in order to speed the process along.

This is a slightly enlarged edition of the original "hints" of the early 1990's. I have incorporated some recent (Spring 1998) suggestions and comments from E.G. Dunne of the American Mathematical Society and P. Vojta of U.C. Berkeley.

These hints are presented as a source of ideas on mathematical style. Feel free to use them in any way that you deem useful.

- Two basic rules are: 1. *Have mercy on the reader*, and, 2. *Have mercy on the editor/publisher.* We will illustrate these as we move along.
- General Flow of the Paper.
  - **Definition**: *All* basic definitions should be given if they are not a standard part of the literature. It is perhaps best to err on the side of making life easier on the reader by including a bit too much as opposed to too little (Rule 1).
    * Some redundancy should be built into the paper so that one or two misprints cannot destroy the understandability. The analogy is with "error-correcting codes" which allow transmission of information through noisy and defective channels.
  - As a very general rule, the definitions should go *before* the results that they are used in (Rule 1).
  - The "quantifiers" should always be clear (Rule 1). Some examples to avoid:
    * "We have $f(x) = g(x)$ $(x \in X)$." What does the parenthesis mean? That $f(x) = g(x)$ for *all* $x \in X$, or, for *some* $x \in X$?
    * What does "$f_{t,u}(x, y) = O(g_{t,u}(x, y))$" mean? Very often it means that t, u, y are fixed and x is allowed to vary. Quantifier problems are especially troublesome with "big O" notation.
    * The word "constant" is terribly ambiguous. It is important to make explicit *exactly* which variables the constant depends on.
  - **Theorem/Proposition/Lemma/Corollary**: Give clear and unambiguous statements of results. These are what other people are reading your paper for; so you should ensure that these, at least, can be understood by the reader (Rule 1).

    * The statement of the Theorem/Proposition/Lemma/Corollary should *not* include comments (except for an occasional brief remark in parenthesis) or examples.

- If you use or quote an important result of another person, you should give a reference. You should avoid giving the impression that such a result is obvious, a generally accepted fact, due to you, and so on.
    * A reference to a book should always give the page!
    * Try to avoid using "by the proof of" when the proof is in the paper and the statements can be rewritten to be *directly* quoted.
    * A "well-known" result that is *not* in the literature should be proved if needed (Rule 1).
- **Proof**: A proof should give enough information to make the theorem believable *and* leave the reader with the confidence (as well as the ability) to fill in details should it be necessary (Rule 1).
- Whatever format or style you choose to adopt, especially if it deviates from the publisher's style, make sure that it is consistent. This is mostly a difficulty with books (Rule 2).
    * If one proof ends with a "QED," then they all should, etc.
    * If you leave a blank page at the end of one chapter so the next one can start on an odd-numbered page, then make sure you always do so.
- **References:** The references should have a consistent (and preferably accepted) style for the entries (Rule 2).
- **TEX:** In general, advanced TEXing should be left to the experts; i.e., as a typesetter or page designer the author should tread lightly. Remember, the more one messes with the TEX-file, the less portable the manuscript will be. Your article may *not* be accepted at the first place you send it, make sure you can easily resubmit it elsewhere (Rule 2). Moreover, playing with the TEX increases the likelihood that the final output will look different on different systems. (Rule 0: have mercy on the author!)
- Writing a paper or book entails making choices of what material is important and what can be skipped. It is impossible to cover all possible results and so the material needs to be covered in a well thought out manner. A paper or book should *not* be considered an opportunity for showing off (Rule 1).

• Other comments:

- One should, of course, observe the usual conventions in terms of spelling, punctuation, and the other basic elements of style. Use complete sentences, with subject, *verb*, and complement (Rule 1).

* Words like “then,” “and,” or “or” should not be replaced by a comma. It is bad to write “If $x = 2$, $y = 3$, $z = 4$” meaning “If $x = 2$ and $y = 3$, then z is equal to 4” (or “If $x = 2$ and $y = 3$, then $z = 4$”).
  * It is better to write “... we prove *that* $\frac{\zeta_{\mathbf{Q}}(2n)}{\pi^{2n}}$ belongs to $\mathbf{Q}$” (or “is rational”) instead of “... we prove $\zeta_{\mathbf{Q}}(2n) \in \pi^{2n}\mathbf{Q}$.”
- Use the present – not the past – form.
  * As an example of bad writing, we have: “We have proved that $f(x)$ was equal to $g(x)$...”. This is corrected to: “We have proved that $f(x)$ is equal to $g(x)$...”.
- Long computations that can easily be carried out by the reader should be avoided. The ideas and results should be given with an invitation to the reader to do the calculation should it be desired (Rule 1).
  * The *exception* to this rule is when the long computation is an *essential* part of the argument. In this case, it should be given in full (Rule 1).
- Do not simply state “X is isomorphic to Y” unless it is completely obvious. Rather, it will be much easier on the reader if you state “the function $f\colon X \to Y$ is an isomorphism” where f is *explicitly* given (Rule 1).
- One should avoid giving the reader the impression that the subject matter can be mastered only with great pain. In fact, this is an *ideal* way to lose readers (or audiences!).
- One should avoid using abbreviations like “w.r.t.” (with respect to), “iff” (if and only if), and “w.l.o.g.” (without loss of generality). They simply do not look very nice (and “iff” is offensive! – Rules 1 and 2).
- You should *not* begin a sentence with a math symbol. This can confuse the printer as well as the reader (Rules 1 and 2).
  * As a example of such bad writing, we have: “... we want to prove the continuity of $f(x) = 2\cos^2 x \cdot \sin x$. $\cos x$ being continuous....”. This is corrected to: “...$f(x) = 2\cos^2 x \cdot \sin x$. Since $\cos x$ is continuous...”.
- If your paper raises a natural question, and you don’t know the answer, by all means *say so*! This may turn out to be more interesting than the theorems that you prove.
  * Conversely, refrain from making “conjectures” too hastily. Use instead the words “question” or “problem.” Remember that a good “question” should be answerable by “yes” or “no.” To ask “under what conditions does A hold” is not a question worth printing.
- It is often helpful to begin a new section of the paper with a summary of the general setting.

- After the paper is finished, it should be reread (and, perhaps, rewritten) from the reader's point of view (Rule 1).
- A good way to begin is to use a standard classic of mathematical exposition (e.g., Bourbaki-Algebra, works by Serre, Atiyah or Milnor) as a basic model.

- Some further sources to look at:
  - E. Swanson *Mathematics into Type*, American Mathematical Society, Revised edition, 1979. Reprinted with corrections, 1986.
  - P. Halmos: *How to write mathematics*, Enseign. Math. **16** (1970), 123–152.
  - W. Strunk, Jr., & E.B. White: *The Elements of Style*, Macmillan Paperbacks Edition, (1962).
  - *The Chicago Manual of Style: The Essential Guide for Writers, Editors, and Publishers* (14th Edition), Univ. of Chicago Press, 1993.
  - N.J. Higham: *Handbook of Writing for the Mathematical Sciences*, SIAM, 1993.
  - R.W. Burchfield (Editor), H. W. Fowler: *The new Fowler's modern English usage*, Oxford Univ. Press, 1996.
  - H.W. Fowler: *Fowler's Modern English Usage: Dictionary of Modern English Usage*, Oxford Univ. Press, 1983. (The classic version)
  - D. Knuth et al.: *Mathematical Writing*, MAA Notes #14 (1989).
  - S.G. Krantz: *A Primer of Mathematical Writing* (being a disquisition on having your ideas recorded, typeset, published, read and appreciated), AMS (1997).
  - Some conventions on citations and pronouns may be found in: S. Zucker: *Variation of a mixed Hodge structure II*, Invent. Math. **80** (1985), p. 545.

- Finally, I quote from a letter Serre wrote commenting on my original version: "It strikes me that mathematical writing is similar to using a language. To be understood you have to follow some grammatical rules. However, in our case, nobody has taken the trouble of writing down the grammar; we get it as a baby does from parents, by imitation of others. Some mathematicians have a good ear; some not (and some prefer the slangy expressions such as "iff"). That's life."