This full text paper was peer reviewed at the direction of IEEE Communications Society subject matter experts for publication in the Sarnoff 2016 proceedings.

Modeling and Optimization for Programmable Unified Control Plane in Heterogeneous Wireless Networks

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Abstract—Tactical networks are wireless in nature to support mobility and rapid deployment in contested environments. These networks also consist of various heterogeneous networking technologies. Since the tactical networks are specialized purpose built networks and their main motivation is to solve a special purpose functions, the interoperability between different technologies was not initially viewed as essential. As the deployment of wireless networks become ubiquitous both in private and government sectors, the lack of interoperability has become a disadvantage in creating a unified control plane. A unified control plane can abstract the complexity of heterogeneous wireless networks and can provide a centralized control over the network resources. In this paper, we develop necessary mathematical model to realize the unified programmable control plane for heterogeneous wireless networks. We develop a cross-layer optimization framework, which characterizes the interaction between physical, link, and network layer for the unified programmable control plane in a heterogeneous wireless network. By applying the framework on a throughput maximization problem, we will show an application of the model to solve practical issues in a tactical network and gain some theoretical insight on the optimal behavior of the unified programmable control plane for a heterogeneous wireless network.

I. INTRODUCTION

The recent advances in the development of software defined networking (SDN) make the programmable control plane in the wired network a reality by separating the control and data plane[1], [2], [3], [4]. In SDN architecture, the control plane is completely removed from the hardware and externally centralized on a server called controller. Fig. 1 illustrates the differences in the control plane for the traditional and programmable control plane.

Software defined wireless network (SDWN) recently has been proposed as a natural extension of SDN from wired to wireless network[2], [5], [6]. SDWN envisioned to provide flexible resource management, end-user/application aware control plane , and vendor-independent wireless network hardwares. In addition to packet forwarding problem, SDWN has to also focus on wireless access and interference management with respect to physical layer design in complex radio environment[6]. The current research efforts on SDWN[5], [6], [7] mainly focuses on proposing network architecture. To the best of our knowledge, the previous work does not show how



Fig. 1. Difference in Control Plane implementation in traditional and programmable networks

to clearly model and solve the complex functionality of the control plane.

In Fig. 2, we demonstrate the logical blocks of the unified programmable control plane for heterogeneous wireless network. The network objective corresponds to the dynamic objective which sought to be achieved by the network. The network layer block ensures the flow balance constraints hold in every node between source and destination for each session. This block is further responsible to find the optimal routing path for each active session. The MAC/link layer has two main functions: i) resolve the medium access contention by link scheduling over different time slots. ii) control the amount of flow on each wireless link to ensure it does not exceed the capacity of the link. The physical layer block manages the signal processing, power management, modulation and coding. This block is also responsible for interference management. The programmable nature of the unified control plane allows the modification of the network objective, constraints within each individual block or even introducing new blocks. The interoperability between different technologies for wireless heterogeneous network can be captured by having necessary constraints in these building blocks.

We claim that the functionality of the unified programmable control plane can be modeled as a centralized network opti-





Fig. 2. Wireless Programmable Control Plane Building Blocks

Fig. 3. Programmable Network Architecture

mization problem, in which the optimal network decisions are to be identified and the optimal wireless network resources are to be allocated to achieve the optimal objective for the network at a give time. Fig. 3 demonstrates a realization of a network architecture for the unified programmable control plane. The centralized network control center performs the control plane tasks and the data will be carried through the core and wireless network.

In this paper, we develop necessary mathematical model to realize the unified programmable control plane in wireless heterogeneous network. We develop a cross-layer optimization framework, which characterizes the interaction between physical, link, and network layer. By applying the framework on a throughput maximization problem, we will show an application of the model to solve practical issues in a tactical network and gain some theoretical insight on the optimal behavior of the unified programmable control plane for a heterogeneous wireless network.

The remainder of this work is organized as following. In Section II, we propose necessary mathematical model to realize the unified programmable control plane for heterogeneous wireless networks. In III, we develop a cross-layer optimization framework, which characterizes the interaction between physical, link, and network layer for the unified programmable control plane in a heterogeneous wireless network. In Section IV, we will show an application of the model to solve practical issues in a tactical network and gain some theoretical insight on the optimal behavior of the unified programmable control plane for a heterogeneous wireless network. Section V concludes this paper.

II. MODELING UNIFIED CONTROL PLANE FOR WIRELESS HETEROGENEOUS NETWORK

In this section, we will present the functionality of each individual module in the unified control plane for wireless heterogeneous network in mathematical programming.

Consider a multi-hop MIMO network consisting of a set of nodes \mathcal{N} which has N elements. Each node is assumed to have M antennas. Suppose that there are L possible links in the network. Denote Tx(l) and Rx(l) as the transmit and receive nodes of link $l, 1 \leq l \leq L$. We consider a timeslotted scheduling, where a time frame consists of T time slots. Depending on link scheduling, a subset of links will be active in time slot $t, 1 \leq t \leq T$.

A. Mac and Link Layer Constraints

Half-Duplex Constraint. Although there has been significant advance on full duplex for single antenna node, there remain significant challenges to have a practical design for full duplex on a MIMO node. Therefore, we assume half duplex on a MIMO node in this paper. Denote $x_i[t]$ as a binary variable to indicate whether node $i \in \mathcal{N}$ is transmitting in time slot t, i.e., $x_i[t] = 1$ if node i is a transmitter in time slot t and 0 otherwise. Similarly, denote $y_i[t]$ as a binary variable to indicate whether node $i \in \mathcal{N}$ is receiving in time slot t, i.e., $y_i[t] = 1$ if node i is a receiver in time slot t and 0 otherwise. For half-duplex, we have the following constraint:

$$x_i[t] + y_i[t] \le 1,$$
 $(1 \le i \le N, 1 \le t \le T).$ (1)

Node's SM Constraints. If node *i* is not a transmitter, then we have $\sum_{l \in \mathcal{L}_i^{\text{out}}} z_l[t] = 0$. Otherwise, the total number of outgoing streams should be positive and lesser than the number of antennas, i.e., $1 \leq \sum_{l \in \mathcal{L}_i^{\text{out}}} z_l[t] \leq M$. These two cases can be expressed in a compact form as follows:

$$x_i[t] \le \sum_{l \in \mathcal{L}_i^{\text{out}}} z_l[t] \le M x_i[t], \quad (1 \le i \le N, 1 \le t \le T) .$$
 (2)

Similarly, depending on whether node i is an active receiver, we have the following constraint:

$$y_i[t] \le \sum_{l \in \mathcal{L}_i^{\text{in}}} z_l[t] \le M y_i[t], \quad (1 \le i \le N, 1 \le t \le T) .$$
(3)

B. Interference Management

Strong Interference vs Weak Interference. We first introduce a concept of "strong" interference range. For a receive node j, it may be interfered by all the unintended transmit nodes in the network. We distinguish an interference as either a "strong" interference or a "weak" interference through strong interference range. Specifically, if the distance from a transmit node to its unintended receive node is less than or equal to interference range, we consider this interference as strong interference; otherwise, we consider it as week interference. In our model, only strong interference will be considered for IC, while weak interference will not be considered for IC. Instead, weak interference will be treated as noise at the receive node when calculating its achievable data rate. Denote \mathcal{I}_i as the set of nodes that are located within the strong interference range of transmitter *i*.

Ordering Constraint. In a multi-hop MIMO network, to avoid duplication in IC while ensuring feasibility of DoF allocation, Shi *et al.* [8] introduced a novel IC scheme among the nodes based on a node ordering concept. Under this scheme, all nodes in the network are put into a logical list with the position of the node in the list representing its order. Specifically, denote $\pi[t]$ as an ordered list of nodes in the network in time slot t and denote $\pi_i[t]$ as the position of node $i \in \mathcal{N}$ in $\pi[t]$. Then we have:

$$1 \le \pi_i[t] \le N,$$
 $(1 \le i \le N, 1 \le t \le T).$ (4)

To model the relative ordering between any two nodes i and j in $\pi[t]$, we define an indicator variable $\theta_{ji}[t]$ as follows:

$$\theta_{ji}[t] = \begin{cases} 1 & \text{if node } j \text{ is before node } i \text{ in } \pi[t], \\ 0 & \text{otherwise.} \end{cases}$$

Denote \mathcal{I}_i as the set of nodes that are located within the interference range of transmitter *i*. Then the ordering relationship between any two nodes in the network can be represented by the following mathematical programming constraints[8]:

$$\pi_{i}[t] - N \cdot \theta_{ji}[t] + 1 \le \pi_{j}[t] \le \pi_{i}[t] - N \cdot \theta_{ji}[t] + N - 1, (1 \le i \le N, j \in \mathcal{I}_{i}, 1 \le t \le T) .$$
(5)

Based on $\pi[t]$, each node in this list has the following responsibility in IC:

- *Transmit node*. If this node is a transmit node, then it only needs to cancel its interference to those receive nodes that are before itself in the ordered node list. It does not need to consume DoFs to cancel its interference to those receive nodes that are after itself in the ordered node list. Interference from this transmit node to receive nodes after itself will be canceled by those receive nodes later. The number of DoFs consumed at this transmit node for IC is equal to the total number of desired data streams received by those receive nodes.
- *Receive node.* If this node is a receive node, then it only needs to cancel interference from those transmit nodes that are before itself in the ordered node list. It does not need to cancel interference from those transmit nodes that are after itself in the ordered node list. Interference from transmit nodes after this node will be canceled by those transmit nodes later. The number of DoFs consumed at this receive node for IC is equal to the total number of data streams transmitted by those transmit nodes.

The above IC rules can also be cast into mathematical programming constraints. Then the DoF constraint at a transmit node and a receive node for $(1 \le i \le N, 1 \le t \le T)$ can be written as follows [8]:

$$\sum_{l \in \mathcal{L}_i^{\text{out}}} z_{(l)}[t] + \sum_{j \in \mathcal{I}_i} \theta_{ji}[t] \sum_{k \in \mathcal{L}_j^{\text{in}}} z_{(k)}[t] \le M x_i[t] + (1 - x_i[t]) B_i, \quad (6)$$

$$\operatorname{Bx}(l) \neq i$$

$$\sum_{k \in \mathcal{L}_i^{\mathrm{in}}} z_{(k)}[t] + \sum_{j \in \mathcal{I}_i} \theta_{ji}[t] \sum_{l \in \mathcal{L}_j^{\mathrm{out}}} z_{(l)}[t] \le M \cdot y_i[t] + (1 - y_i[t])B_i, \quad (7)$$

where B_i is a large constant and is no small than $\sum_{j \in \mathcal{I}_i} \theta_{ji}[t] \sum_{k \in \mathcal{L}_j^{\text{in}}}^{\mathrm{Tx}(k) \neq i} z_{(k)}[t]$ and $\sum_{j \in \mathcal{I}_i} \theta_{ji}[t] \sum_{l \in \mathcal{L}_j^{\text{out}}}^{\mathrm{Rx}(l) \neq i} z_{(l)}[t]$. For example, we can set $B_i = M \cdot |\mathcal{I}_i|$.

C. Physical Layer Constraints

Power Allocation Constraints. For power control, we assume that the transmission power allocated to qth outgoing data stream at transmit node of link l can be tuned to a finite number of levels between 0 and P_{\max} . We further assume that the total transmit power of a node is P_s . To model this discrete data stream power control, we introduce an integer parameter Q that represents the total number of power levels to which a transmitter can be adjusted, i.e., transmission power can be $0, \frac{1}{Q}P_{\max}, \frac{2}{Q}P_{\max}, \ldots, P_{\max}$. Denote $p_l^q \in \{0, 1, \ldots, Q\}$ the integer levels for transmission power allocated to qth outgoing data stream at transmit node of link l. If node i is an active transmitter (i.e., $x_i(t) = 1$), then its total transmit power over all data streams is P_{\max} . Otherwise (i.e., $x_i(t) = 0$), its transmit power for each data stream is zero. Then we have

$$\sum_{l \in \mathcal{L}_i^{\text{out}}} \sum_{q=1}^{z_l[t]} p_l^q[t] = Q \cdot x_i[t], (1 \le i \le N, 1 \le t \le T) .$$
(8)

SINR Constraints. Denote $c_l^q[t]$ as the achievable rate of qth data stream of link l in time slot t. Denote $\gamma_l^q[t]$ as the effective SINR at the receive node of link l for receiving the kth data streams of link l. Then the data stream capacity constraints for $(1 \le l \le L, 1 \le q \le z_l[t], 1 \le t \le T)$ can be written as follows:

$$c_l^q[t] = W \cdot \log_2(1 + \gamma_l^q[t]) , \qquad (9)$$

where W is the system bandwidth.

We know calculate the effective SINR $\gamma_l^q[t]$. The DoF IC cancels the strong interfering signals and weak interference signals is treated as noise. Therefore, the effective SINR at the receive node of link l for receiving the qth data streams of link l for $(1 \le l \le L, 1 \le q \le z_l[t], 1 \le t \le T)$ can be written as

$$\gamma_l^q[t] = \frac{G_{\mathrm{Tx}(l)\mathrm{Rx}(l)} \frac{p_l^{\gamma}[t]}{Q} P_{\mathrm{max}}}{A_l^q[t] \Big[\sum_{i \in \mathcal{N} \setminus \mathcal{I}_{\mathrm{Rx}(l)}} G_{i\mathrm{Rx}(l)} P_{\mathrm{max}} x_i[t] + P_n \Big]}, \qquad (10)$$

a . . .

where G_{ij} is the path loss from the transmit node *i* to the receive node *j*; P_n is the noise power at the receiver; $A_l^q[t]$ is a power scaling coefficient of the interference and the noise for the *q*th data stream of link *l*, which is determined by the channel matrices.

Link Capacity Constraints. Denote $c_l[t]$ as the aggregate data rate at link l over its $z_l[t]$ data streams in time slot t. Then we have

$$c_l[t] = \sum_{q=1}^{z_l[t]} c_l^q[t], \qquad (1 \le l \le L, 1 \le t \le T) . \quad (11)$$

Denote $r_l(f)$ as the amount of data rate on link l that is attributed to session $f \in \mathcal{F}$. Since the aggregate data rate on link l cannot exceed the link's average rate, we have

$$\sum_{f \in \mathcal{F}} r_l(f) \le \frac{1}{T} \sum_{t=1}^T c_l[t], \quad (1 \le l \le L)$$
(12)

where the right-hand-side represents the average throughput on link l over a frame (T time slots).

D. Network Layer Constraints

Flow Routing Constraints. Suppose there is a set of active sessions \mathcal{F} . Denote r(f) as the rate of session $f \in \mathcal{F}$ and r_{\min} as the minimum session rate, i.e., $r_{\min} = \min_{f \in \mathcal{F}} r(f)$. Denote s(f) and d(f) as the source and destination nodes of session $f \in \mathcal{F}$, respectively. Then at source node, s(f), $f \in \mathcal{F}$, we have the following flow balance:

$$\sum_{l \in \mathcal{L}_i^{\text{out}}} r_l(f) = r(f), \quad (i = s(f), f \in \mathcal{F}) .$$
(13)

At any intermediate relay node, we have

$$\sum_{l \in \mathcal{L}_i^{\text{in}}} r_l(f) = \sum_{l \in \mathcal{L}_i^{\text{out}}} r_l(f), \quad (1 \le i \le N, i \ne s(f), i \ne d(f), f \in \mathcal{F}) .$$
(14)

At a destination node, we have

$$\sum_{l \in \mathcal{L}_i^{\text{in}}} r_l(f) = r(f), \quad (i = d(f), f \in \mathcal{F}) .$$
(15)

III. PROBLEM FORMULATION AND SOLUTION

In this section, we show how the unified heterogeneous wireless network model developed in the previous section can be used to study tactical networking problems. Let's consider a typical throughput maximization problem in a multi-hop MIMO network. It can be easily verified that if (13) and (14) are satisfied, then (15) is also satisfied. Therefore, it is sufficient to have (13) and (14).

Putting all the constraints together, we have the following formulation for the throughput maximization problem:

TMP max
$$r_{\min}$$

s.t. $r_{\min} \leq r(f)$ $(f \in \mathcal{F})$;
Ordering Constraints.:(4),(5);
Half Duplex Constraints.(1);
SM Constraints.: (2),(3);
DoF IC Constraints.: (6),(7);
Power Allocation Constraints.: (8);
SINR Constraints.: (9)–(10);
Link Capacity Constraints.: (11),(12);
Flow Balance Constraints.: (13), (14);

In TMP, constraints (6),(7), (8), and (9)–(11) are nonlinear. Therefore, TMP is a mixed-integer nonlinear program (MINLP), which in general is NP-hard [9]. MINLP problems are known to be difficult due to the combinatorial nature of mixed-integer programs and the difficulty in solving nonlinear programs. Note that there exist some techniques to address *general* MINLP problems (e.g., outer-approximation methods [10], branch-and-bound [11], extended cutting plane methods [12], and generalized Benders' decomposition [13]). However, these techniques do not exploit our problem-specific structures and properties, and hence can only handle small-sized problems.

Through reformulation on (6),(7), (8), and (9)–(11) as well as linearization of the logarithmic function in (9), **TMP** can be reformulated into a mixed integer linear program (MILP). Although the theoretical worst-case complexity to a general MILP problem is exponential [9], [14], there exist highly efficient heuristics (e.g., sequential fixing algorithm [15, Chapter 10]) to solve it. Another approach is to apply an offthe-shelf solver (CPLEX [16]), which we found can handle up to a moderate-sized network successfully. Since the main goal of this paper is to explore DoF IC and FD jointly, it is sufficient to demonstrate our results with moderate-sized networks. Therefore, we will use CPLEX to solve MILP. The solution to this optimization problem gives us the optimal decision on when and how to perform scheduling, SM, and IC in different time-slots.

IV. NUMERICAL RESULTS

In this section, we present some numerical results to study the performance of the unified control plane for wireless heterogeneous network described in Section II. The goal of this effort is twofold. First, we want to show how a solution to the **TMP** formulation looks like for an example network. By studying the details of our solution for an example network, we will develop some quantitative understanding on how the intelligent network decision made in the unified control plane can significantly improve the network throughput. Second, we will show how the intelligent centralized interference management scheme implemented in the unified control plane for the wireless heterogeneous network can cancel the interference and improves the network throughput compared to the case without such an intelligent interference management in the unified control plane.

A. Simulation Settings

We consider a randomly generated multi-hop wireless network with 30 nodes that are distributed in a 100×100 area. For generality, we normalize all units for distance, data rate, bandwidth, and power with appropriate dimensions. At the network layer, minimum-hop routing is employed. There are 2 active sessions in the network with each session's source node and destination node given in Table I. Each node is equipped with M = 4 antennas. We assume the bandwidth W = 1. The transmit power for each node is set to 100. The path loss parameter $G_{\text{Tx}(k)\text{Rx}(k)} = [20 \log_{10}(d) + 38.25]$ (in dB) [17], where d is the distance between Tx(k) and Rx(k). The number of time slots in a frame is T = 4. The This full text paper was peer reviewed at the direction of IEEE Communications Society subject matter experts for publication in the Sarnoff 2016 proceedings.

worst case upper bound value for $A_l^q[t]$ is 7.3753. We set the maximum acceptable performance gap between the optimal objective values of

 TABLE I

 Source node and Destination node in the 25-Node Network

Session	Source Node	Dest. Node
f	s(f)	d(f)
1	7	22
2	11	2

Figure 4 shows the set of active links and the number of data streams per link in each time slot in the solution. Figure 5 shows the combined results for all time slots. Table II shows the details of DoF allocation for SM, aggregated capacity over data streams, and session's rate for all time slots for MIMO half duplex. Table III shows the details of IC for time slot T = 3. The IC within the network follows the node ordering, which is shown in the third column of Table III. The number of DoFs used for IC to/from other nodes is shown in the column of Table III. As shown in Table II, the objective value r_{\min} (minimum session rate) for this network scenario is 0.483653.



Fig. 5. Shows the combined results for all time slots (with the number of data streams for each time slot on the link shown in a box).

V. CONCLUSION

The separation of the data and control plane is a key element for designing a highly programmable wireless network. The unfied programmable control plane for wireless heterogeneous network can abstract the complexity of heterogeneous wireless network and provides a centralized control over the network resources. The abstraction of the complexity enables the interoperability between different technologies in tactical wireless networks. Moreover such a unified programmable control plane enables advanced routing, MAC/Link/Phy layer functionalities, and advanced interference management scheme. It is also providing a dynamic structure to re-configure the network to achieve various network objectives over time.

TABLE II DOF ALLOCATION FOR SM, AGGREGATED CAPACITY OVER DATA STREAMS, AND SESSION'S RATE FOR ALL TIME SLOTS.

Session	Link	Time Slot	DoF for SM	Capacity	Session's Rate
	$N_7 \rightarrow N_{13}$	1	1	11.1414	0.441784
		2	0	0	
		3	0	0	
1		4	0	0	
1	$N_{13} \rightarrow N_{22}$	1	0	0	
		2	2	1.15608	
		3	0	0	
		4	1	1.01216	
	$N_{11} \rightarrow N_{24}$	1	0	0	0.441784
		2	0	0	
		3	3	10.6387	
		4	0	0	
	$N_{24} \rightarrow N_{21}$	1	0	0	
2		2	0	0	
2		3	0	0	
		4	4	1.76713	
	$N_{21} \rightarrow N_2$	1	0	0	
		2	3	1.1577	
		3	1	0.845915	
		4	0	0	

TABLE III DOF ALLOCATION FOR SM/IC FOR TIME SLOT T = 3.

Node i	TX/RX	$\pi_{i}[1]$	DoF for SM	DoF for IC to/from
N_{11}	TX	2	3	0
N_{24}	RX	1	3	0
N_{21}	TX	7	1	3 to N ₂₄
N_2	RX	3	1	3 to N_1

We developed the necessary mathematical model to realize a unified programmable control plane for wireless heterogeneous network. Our proposed network optimization framework characterizes the interaction between physical, link, and network layer. The unified programmable control plane can dynamically solve the optimization problem to decide the optimal values for network decision variables. By applying the framework on a throughput maximization problem, we evaluate our model and show the proposed unified programmable control plane for wireless heterogeneous network can solve complex problems in tactical wireless networks.

ACKNOWLEDGMENT

The authors express their gratitude to U.S Army Research Laboratory for supporting this work. The work of B. Jalaian is supported in part by an appointment to the Student Research Participation Program at the U.S. Army Research Laboratory administered by the the Oak Ridge Institute for Science and Education through an interagency agreement between the U.S. Department of Energy and USARL. The work of Y.T. Hou was supported in part by the US National Science Foundation (NSF).

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Fig. 4. Scheduled links, DoFs allocation on each link, and interference pattern in time slots 1 to 4, respectively.

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