Cross-Layer Optimization for Routing Data Traffic in UWB-based Sensor Networks

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ABSTRACT

Ultra-wideband (UWB) has great potential for wireless communications in emerging applications such as sensor networks. This paper considers UWB-based sensor networks and studies the following problem: given a set of source sensor nodes in the network each generating a certain data rate, is it possible to relay all these rates successfully to the base-station? We follow a cross-layer optimization approach, with joint consideration of link layer scheduling, power control, and network layer routing. The optimization problem is formulated as a non-linear programming problem. For small-sized networks, we develop a powerful approximation solution procedure to this problem based on the branch-and-bound approach and the novel Reformulation-Linearization Technique (RLT). For large-sized networks, we propose an efficient heuristic algorithm by partitioning the sensor network into a core centered around the base-station and an edge that is outside the core. We also provide a closed-form analysis for the maximum rate that a base-station can receive. Simulation results exhibit the efficacy of our proposed optimization solution procedure and demonstrate the importance of the cross-layer approach to UWB-based sensor networks.

Categories and Subject Descriptors

C.2.1 [**Network Architecture and Design**]: Wireless communication; G.1.6 [**Optimization**]: Nonlinear programming

General Terms

Algorithms, Performance

Keywords

Sensor networks, ultra-wide band (UWB), scheduling, power control, routing, cross-layer optimization, branch-and-bound, Reformulation-Linearization Technique (RLT)

1. INTRODUCTION

The U.S. Federal Communications Commission (FCC) Notice of Inquiry in 1998 inspired a flourish of research and development efforts on UWB for a wide range of military and commercial applications. These applications include tactical handheld & network LPI/D radios, non-LOS LPI/D groundwave communications, precision geolocation systems, high-speed wireless LANs, collision avoidance sensors, and intelligent tags, among others. There are some significant benefits of UWB for wireless communications, such as extremely simple design (and thus cost) of radio, large processing gain in the presence of interference, extremely low power spectral density for covert operations, and fine time resolution for accurate position sensing [11].

In this paper, we consider a UWB-based sensor network for surveillance and monitoring applications. For this network application, upon an event detection, all sensing data must be relayed to a central data collection point, which is here a base-station. The multi-hop nature of a sensor network introduces some unique challenges. Specifically, due to interference from neighboring links, a change of power level on one link will produce a change in capacity in all neighboring links. As a result, the capacity-based routing problem at the network layer is deeply coupled with link layer problems such as scheduling and power control. An optimal solution to a network level problem thus must be pursued via a cross-layer approach for such networks [4].

In this paper, we study the data collection problem associated with UWB-based sensor networks. For such networks, although the bit rate for each UWB-based sensor node could be high, the total rate that can be collected by the single base-station is limited due to the network resource bottleneck near the base-station as well as interference among the incoming data traffic. Therefore, a fundamental question is the following: *Given a set of source sensor nodes in the network with each node generating a certain data rate, is it possible to relay all these rates successfully to the base-station?*

A naive approach to this problem is to calculate the maximum bit rate that the base-station can receive and then perform a simple comparison between this limit with the sum of bit rates produced by the set of source sensor nodes. Indeed, if this limit is exceeded, it is impossible to relay all these rates successfully to the base-station. But even if the sum of bit rates generated by the set of source sensor nodes is less than this limit, it may still be infeasible to relay all these rates successfully to the base-station. This is because, due to interference and the fact that a node cannot send and receive at the same time, the actual sum of bit rates that can be relayed to the base-station can be substantially smaller than the bit rate limit that a base-station can receive. Further, such an admissibility is highly dependent upon the network topology, locations of source sensor nodes, bit rate produced by a source sensor node, and other network parameters. As a result, testing for this feasibility is not trivial and it is important to devise a methodology and solution procedure to address this problem.

In this paper, we study this admissibility (or feasibility) problem through a cross-layer optimization approach, with joint consideration of link layer scheduling, power control, and network layer routing. The link layer scheduling problem deals with how to allocate resources for access among the nodes. Motivated by the work in [8], we consider how to allocate frequency sub-bands, although this approach can also be applied to a time-slot based system. For a total available UWB spectrum of W, we divide it into M sub-bands. For a given M, the scheduling problem considers how to allocate bandwidth to each sub-band and in which sub-bands a node should transmit or receive data. Note that a node cannot transmit and receive within the same sub-band. The power control problem considers how much power a node should use to transmit data in a particular sub-band. Finally, the routing problem at the network level considers which path a flow should take from the source sensor node toward the base-station. For optimality, we allow a flow from a source node to be split into sub-flows that take different paths to the base-station.

We formulate this feasibility problem as an optimization problem, which turns out to be a mixed-integer non-polynomial programming problem. To reduce the problem complexity, we modify the integrality and the non-polynomial components in the constraints by exploiting a reformulation technique and the approximately linear property between the rate and SNR, which is unique to UWB. The resulting new optimization problem is then cast into the form of a *non-linear program* (NLP). Since a NLP problem is NP-hard in general, our specific NLP problem is likely to be NP-hard, although its formal proof is not given in this paper. *The first contribution of this paper is the development of a novel approximation solution procedure to this feasibility problem based on a branch-and-bound approach and the powerful Reformulation-Linearization Technique (RLT).* This solution procedure performs efficiently for a network on the order of 100 nodes on an ordinary desktop PC platform.

For large-sized networks, due to storage and computational requirements, it is necessary to develop a more scalable solution procedure. *Our second contribution in this paper is the development of a fast heuristic algorithm that is effective for large-sized networks.* Our approach is to partition the network into two parts: a network core that is centered around the base-station and a network edge that is outside the core. The size of the network core is determined by the computational capability of the proposed solution procedure for small-sized networks (e.g., 100 nodes). The solution procedure consists of formulating a new optimization problem with the objective of maximizing the total incoming rates to the network core (from nodes outside the core), subject to the constraint that the bit rates generated by source sensor nodes inside the core can be delivered to the base-station, among other constraints. During the iterations of this optimization problem (with a similar solution procedure developed for small-sized networks), we examine whether it is possible to "re-connect" source sensor nodes that are *outside* the core with a feasible solution.

The third contribution of this paper is an analysis of the maximum rate that can be received by the base-station, which can be used as a first test (upper bound) for feasibility. We give a closed-form solution for this maximum rate and show that the source rate vector that achieves this performance limit is also unique.

The remainder of the paper is organized as follows. In Section 2, we give details of the network model for our problem and discuss the inherent cross-layer nature of this problem. Section 3 presents the mathematical formulation of the cross-layer optimization problem and a solution procedure based on the branch-and-bound and RLT procedures. In Section 4, we present an efficient heuristic algorithm for large-sized networks. Section 5 analyzes the maximum rate that can be received by the base-station. In Section 6, we present simulation results to demonstrate the efficacy of our proposed solution procedures and give insights on the impact of the different optimization components. Section 7 reviews related work and Section 8 concludes this paper.

2. NETWORK MODEL

We consider a UWB-based sensor network. Although the size of the network (in terms of the number of sensor nodes N) is potentially large, we expect the number of simultaneous source sensor nodes that produce sensing data to be limited, assuming that the number of simultaneous events that need to be reported in different parts of the network is not large. Nevertheless, the number of nodes involved in relaying (routing) may still be significant due to the limited range of a UWB-based sensor node and the range of the network.

Within such a sensor network, we assume there is a base-station (or sink node) to which all collected data from source sensor nodes must be relayed (see Fig. 5). For simplicity, we denote the base-station as node 0 in the network.

Under this sensor network setting, we are interested in answering the following questions.

- Suppose we have a small group of nodes in $\mathcal{N}$ that have detected certain events and each of these nodes is generating data. Can we determine whether the bit rates from these source sensor nodes can be successfully sent to the base-station?
- If the determination is "yes", then how should we relay the data from each source sensor node to the base-station?

Before we further explore this problem, we give the following definition for the feasibility of a rate vector $\mathbf{r}$, where each element, r_i, of the vector corresponds to the sensing rate produced by node $i \in \mathcal{N}$.

DEFINITION 1. *For a given rate vector $\mathbf{r}$ having $r_i > 0$ for $i \in \mathcal{N}$, we say that this rate vector is* feasible *if and only if there exists a solution such that all r_i, $i \in \mathcal{N}$, can be relayed to the base-station.*

To determine whether or not a given rate vector $\mathbf{r}$ is feasible, there are several issues from different layers that must be considered. At the network level, we need to find a multi-hop route from the source to the sink node. At the link level, we need to find a scheduling policy and power control for each node such that constraints associated with link bit rate, flow balance at each node, and that a node cannot send and receive within the same sub-band can all be met satisfactorily. Clearly, this is a cross-layer problem that couples scheduling, power control, and routing. We now take a closer look at each problem. Table 1 lists all notation used in this paper.

Scheduling. At the link level, the scheduling problem deals with how to allocate link resources for access among the nodes. Motivated by Negi and Rajeswaran's work in [8], we consider how to allocate frequency sub-bands, although this approach can be also applied to time-slot based systems. For the total available UWB spectrum of $W = 7.5$ GHz (from 3.1 GHz to 10.5 GHz), we divide it into M sub-bands. Since the minimum bandwidth of a UWB sub-band is 500 MHz, we have $1 \leq M \leq 15$. For a given number of total sub-bands M, the scheduling problem considers how to allocate the total spectrum of W into M sub-bands and in which sub-bands a node should transmit or receive data. More formally, we consider a sub-band m with normalized bandwidth $\lambda^{(m)}$. We have

$$\sum_{m=1}^{M} \lambda^{(m)} = 1$$

and

$$\lambda_{min} \leq \lambda^{(m)} \leq \lambda_{max} \qquad \text{for } 1 \leq m \leq M \ ,$$

where $\lambda_{min} = 1/15$ and $\lambda_{max} = 1 - (M-1) \cdot \lambda_{min}$.

Power Control. The power control problem considers how much power a node should use in a particular sub-band to transmit data. Denote p_{ij}^m as the power that node i spends in sub-band m for sending data to node j. Since a node cannot send and receive data within the same sub-band, we have the following: if $p_{ik}^m > 0$ for any node k, then p_{ji}^m should be 0 for all node j.

The power density limit for each node i must satisfy

$$\frac{g_{nom} \cdot \sum_{j \in \mathcal{S}_i} p_{ij}^m}{W \cdot \lambda^{(m)}} \leq \pi_{max} \ ,$$

where g_{nom} is the gain at some fixed nominal distance and $\mathcal{S}_i$ is the set of nodes that node i can send data to in one hop. A popular model for gain is

$$g_{ij} = \min(d_{ij}^{-n}, 1) \ , \tag{1}$$

where d_{ij} is the distance between nodes i and j and n is the path loss index. Denote

$$p_{max} = \frac{W \cdot \pi_{max}}{g_{nom}} \ . \tag{2}$$

Then the total power that a node i can use at sub-band m must satisfy the following power limit,

$$\sum_{j \in \mathcal{S}_i} p_{ij}^m \leq p_{max} \lambda^{(m)} \ . \tag{3}$$

Denote $\mathcal{I}_i$ as the set of nodes that can make interference at node i. The achievable rate from node i to node j within sub-band m is then

$$b_{ij}^m = W\lambda^{(m)} \cdot \log_2 \left(1 + \frac{g_{ij} \cdot p_{ij}^m}{\eta W \lambda^{(m)} + \sum_{k \in \mathcal{I}_j, l \in \mathcal{S}_k}^{(k,l) \neq (i,j)} g_{kj} p_{kl}^m} \right) \ , \tag{4}$$

where η is the ambient Gaussian noise. Denoting b_{ij} as the total achievable rate from node i to node j among all M sub-bands, we have

$$b_{ij} = \sum_{m=1}^{M} b_{ij}^m \ . \tag{5}$$

Table 1: Notation

Symbol	Definition
W	$= 7.5$ GHz is the entire spectrum for UWB networks.
N	Total number of sensor nodes in the network
M	Total number of sub-bands for scheduling in W
Node 0	Denotes the base-station
C_0	Maximum rate that the base-station can receive
η	Power spectral density of ambient Gaussian noise
π_{max}	Limit of power spectral density at a node
g_{ij}	Propagation gain from node i to node j
g_{jj}	Self-interference parameter at node j
g_{nom}	Propagation gain at a nominal distance
p_{max}	$= W\pi_{max}/g_{nom}$ is the power limit.
$\mathcal{I}_i$	The set of nodes that can produce interference on node i
$\mathcal{R}_i$	The set of nodes that can send data directly to node i
$\mathcal{S}_i$	The set of nodes to which node i can send data in one hop
$\mathcal{H}_0^1$	The set of one-hop neighboring nodes from the base-station
$\mathcal{H}_0^c$	The set of nodes in network core
$\mathcal{H}_0^d$	The set of nodes in $\mathcal{H}_0^c$ that can receive data from nodes outside $\mathcal{H}_0^c$
r_i	Bit rate generated at source sensor node i
K	The feasibility factor used in optimization problem formulation
$\lambda^{(m)}$	Normalized length of sub-band m, $\sum_{m=1}^{M} \lambda^{(m)} = 1$.
$\mathbf{\Lambda}$	The vector of $\lambda^{(m)}$.
λ_{min}	The minimum value of $\lambda^{(m)}$
λ_{max}	The maximum value of $\lambda^{(m)}$
p_{ij}^m	Power spent by node i in sub-band m for sending data to node j
$\mathbf{p}$	The vector of p_{ij}^m, $1 \leq i \leq N, 0 \leq j \leq N, j \neq i$, $1 \leq m \leq M$
q_j^m	Total power (signal and noise) received by node j in sub-band m (see Eq. (9))
$\mathbf{q}$	The vector for q_j^m, $1 \leq j \leq N, 1 \leq m \leq M$
b_{ij}^m	Achievable rate from node i to node j in sub-band m for p_{ij}^m (see Eq. (4))
$\mathbf{b}$	The vector of b_{ij}^m, $1 \leq i \leq N, 0 \leq j \leq N, j \neq i$, $1 \leq m \leq M$
b_{ij}	Total achievable rate from node i to node j in all sub-bands
f_{ij}	Flow rate from node i to node j
f_i^{in}	Total rate of incoming flows to node i ($i \in \mathcal{H}_0^d$) from nodes outside network core
L	The problem list in the branch-and-bound procedure
LB_z	The lower bound of problem z in the branch-and-bound procedure
LB	The best (maximum) lower bound among all problems in the branch-and-bound procedure
UB_z	The upper bound of problem z in the branch-and-bound procedure
UB	The worst (maximum) upper bound among all problems in the branch-and-bound procedure

Routing. The routing problem at the network level considers the path that a flow takes from the source node toward the base-station. For optimality, we allow a flow from a source node to be split into sub-flows and take different paths to the base-station. Denoting the flow rate from node i to node j as f_{ij}, we must have

$$f_{ij} \leq b_{ij}$$
$$\sum_{j \in \mathcal{S}_i} f_{ij} - \sum_{j \in \mathcal{R}_i} f_{ji} = r_i \ ,$$

where $\mathcal{R}_i$ is the set of nodes that can send data directly to node i. The first constraint says that a flow's bit rate is upper bounded by the link capacity and the second constraint is for flow balance at node i.

3. FEASIBILITY AND SOLUTION FOR SMALL-SIZED NETWORKS

In Section 5, we analyze the maximum rate (denoted as C_0) that the base-station can receive. For a given source rate vector $\mathbf{r}$, where $r_i > 0$ denotes that node i is a source sensor node that produces sensing data at rate r_i, if $\sum_{i=1}^{N} r_i > C_0$, then the rate vector must be infeasible. But $\sum_{i=1}^{N} r_i \leq C_0$ does not guarantee the feasibility of the rate vector $\mathbf{r}$ and further determination is needed. Moreover, if we indeed find that a given rate vector $\mathbf{r}$ is feasible, we also would like to obtain a complete solution that implements $\mathbf{r}$ over the network, i.e., a solution showing the scheduling, power control, and routing for each node.

3.1 Rate Feasibility Problem Formulation

Our approach to this feasibility determination problem is to solve an optimization (maximization) problem for the scaled rate vector $K \cdot \mathbf{r}$, under the optimization space of scheduling, power control, and routing. K is an optimization variable which we call the *feasibility factor.* If the optimization problem yields $K \geq 1$, we claim that the rate vector $\mathbf{r}$ is feasible; otherwise (i.e., $K < 1$), we say that the rate vector $\mathbf{r}$ is infeasible.

Since a node is not allowed to send and receive within the same sub-band, we have that if $p_{jl}^m > 0$ for any $l \in \mathcal{S}_j$ then p_{ij}^m should be 0 for all $i \in \mathcal{R}_j$. Mathematically, this property can be formulated as follows. Denote x_j^m ($1 \leq j \leq N$ and $1 \leq m \leq M$) as a binary variable with the following definition: if sub-band m is used for receiving data at node j then $x_j^m = 1$; otherwise, $x_j^m = 0$. Since $\sum_{i \in \mathcal{R}_j} p_{ij}^m \leq |\mathcal{R}_j| p_{max} \lambda^{(m)}$ and $\sum_{l \in \mathcal{S}_j} p_{jl}^m \leq p_{max} \lambda^{(m)}$, we have the following constraints, which capture both the constraint that a node j cannot send and receive within the same sub-band m and the constraint on the power level.

$$\begin{aligned}
\sum_{i \in \mathcal{R}_j} p_{ij}^m &\leq |\mathcal{R}_j| \cdot p_{max} \lambda^{(m)} \cdot x_j^m \ , \\
\sum_{l \in \mathcal{S}_j} p_{jl}^m &\leq p_{max} \lambda^{(m)} \cdot (1 - x_j^m) \ .
\end{aligned}$$

The rate feasibility problem (RFP) can now be formulated as follows.

Rate Feasibility Problem (RFP):

Maximize $\quad K$

subject to

$$\sum_{m=1}^{M} \lambda^{(m)} = 1$$

$$\sum_{j \in \mathcal{S}_i} p_{ij}^m - p_{max} \lambda^{(m)} \leq 0 \quad (1 \leq i \leq N, 1 \leq m \leq M)$$

$$b_{ij}^m = W \lambda^{(m)} \log_2 \left(1 + \frac{g_{ij} p_{ij}^m}{\eta W \lambda^{(m)} + \sum_{k \in \mathcal{I}_j, l \in \mathcal{S}_k}^{(k,l) \neq (i,j)} g_{kj} p_{kl}^m} \right) \quad (1 \leq i \leq N, j \in \mathcal{S}_i, 1 \leq m \leq M)$$

$$\sum_{i \in \mathcal{R}_j} p_{ij}^m \leq |\mathcal{R}_j| p_{max} \lambda^{(m)} x_j^m \qquad (1 \leq j \leq N, 1 \leq m \leq M) \quad (6)$$

$$\sum_{l \in \mathcal{S}_j} p_{jl}^m \leq p_{max} \lambda^{(m)} (1 - x_j^m) \qquad (1 \leq j \leq N, 1 \leq m \leq M) \quad (7)$$

$$\begin{aligned}
\sum_{m=1}^{M} b_{ij}^m - f_{ij} &\geq 0 && (1 \leq i \leq N, j \in \mathcal{S}_i) \\
\sum_{j \in \mathcal{S}_i} f_{ij} - \sum_{j \in \mathcal{R}_i} f_{ji} - r_i K &= 0 && (1 \leq i \leq N) \\
\lambda_{min} \leq \lambda^{(m)} &\leq \lambda_{max} && (1 \leq m \leq M) \\
x_j^m &= 0 \text{ or } 1 && (1 \leq j \leq N, 1 \leq m \leq M) \\
K, p_{ij}^m, b_{ij}^m, f_{ij} &\geq 0 && (1 \leq i \leq N, j \in \mathcal{S}_i, 1 \leq m \leq M) \ .
\end{aligned}$$

The formulation for problem RFP is a *mixed-integer non-polynomial programming* problem. Since even a special case of the mixed-integer non-polynomial programming problem such as a mixed-integer programming problem or a non-polynomial programming problem, is NP-hard in general [3], the current formulation of the RFP problem is also NP-hard. We conjecture that the RFP problem is also NP-hard, although its formal proof is not given in this paper. Our approach to this problem is as follows. As a first step, we show how to remove the integer (binary) variables and the non-polynomial terms in the RFP problem formulation and reformulate the RFP problem as a non-linear programming problem (NLP). Since an NLP problem remains NP-hard in general, in Section 3.2, we devise a solution by exploring a branch-and-bound procedure and the novel *Reformulation-Linearization Technique* (RLT) [14].

Reformulation of Integer and Non-Polynomial Constraints. The purpose of integer (binary) variables x_j^m is to capture the fact that a node cannot send and receive within the same sub-band, i.e., if a node j sends data to any node l in a sub-band m, then the data rate that can be received by node j within this sub-band must be 0. Instead of using integer (binary) variables, we use the following approach to achieve the same purpose. We introduce a notion called *self-interference parameter* g_{jj}, with the following property:

$$g_{jj} \cdot p_{jl}^m \gg \eta W \lambda^{(m)} \ .$$

We incorporate this into the bit rate calculation in Eq. (4), i.e.,

$$b_{ij}^m = W \lambda^{(m)} \cdot \log_2 \left(1 + \frac{g_{ij} p_{ij}^m}{\eta W \lambda^{(m)} + \sum_{k \in \mathcal{I}_j, l \in \mathcal{S}_k}^{(k,l) \neq (i,j)} g_{kj} p_{kl}^m + \sum_{l \in \mathcal{S}_j} g_{jj} p_{jl}^m} \right) . \quad (8)$$

Thus, when $p_{jl}^m > 0$, i.e., node j is transmitting to any node l, then in Eq. (8), we have $b_{ij}^m \approx 0$ even if $p_{ij}^m > 0$. In other words, when node j is transmitting to any node l, the link capacity on node i to j is *effectively* shut down to 0.

With this new notion of g_{jj}, we can capture the same tramission/receiving behavior of a node without the need of using integer (binary) variables x_j^m as in the RFP formulation. As a result, we can remove constraints (6) and (7).

To write Eq. (8) in a more compact form, we re-define $\mathcal{I}_j$ to include node j as long as j is not the base-station node (i.e., node 0). Thus, Eq. (8) is now in the same form as Eq. (4). Denote

$$q_j^m = \sum_{k \in \mathcal{I}_j, l \in \mathcal{S}_k} g_{kj} p_{kl}^m \ . \tag{9}$$

Then we have

$$\begin{aligned} b_{ij}^m &= W\lambda^{(m)} \log_2 \left(1 + \frac{g_{ij} p_{ij}^m}{\eta W \lambda^{(m)} + \sum_{k \in \mathcal{I}_j, l \in \mathcal{S}_k}^{(k,l) \neq (i,j)} g_{kj} p_{kl}^m} \right) \\ &= W\lambda^{(m)} \log_2 \left(1 + \frac{g_{ij} p_{ij}^m}{\eta W \lambda^{(m)} + q_j^m - g_{ij} p_{ij}^m} \right) \ . \end{aligned}$$

To remove the non-polynomial terms, we apply the low SNR property that is unique to UWB and the linearity approximation of the log function, i.e., $\ln(1+x) \approx x$ for $x > 0$ and $x \ll 1$. We have

$$b_{ij}^m \approx \frac{W\lambda^{(m)}}{\ln 2} \cdot \frac{g_{ij} p_{ij}^m}{\eta W \lambda^{(m)} + q_j^m - g_{ij} p_{ij}^m} \ ,$$

which is equivalent to

$$\eta W \lambda^{(m)} b_{ij}^m + q_j^m b_{ij}^m - g_{ij} p_{ij}^m b_{ij}^m - \frac{W}{\ln 2} g_{ij} \lambda^{(m)} p_{ij}^m = 0 \ .$$

Finally, without loss of generality, we let $\lambda^{(m)}$ conform the following property.

$$\lambda^{(1)} \leq \lambda^{(2)} \leq \cdots \leq \lambda^{(m)} \ .$$

Although this additional constraint does not affect the optimal result, it will help speed up the computational time in our algorithm.

With the above re-formulations, we can now re-write the RFP problem as follows.

RFP-2: Maximize K

subject to

$$\sum_{m=1}^{M} \lambda^{(m)} = 1$$

$$\lambda^{(m)} - \lambda^{(m-1)} \geq 0 \qquad (2 \leq m \leq M)$$

$$\sum_{j \in \mathcal{S}_i} p_{ij}^m - p_{max} \lambda^{(m)} \leq 0 \quad (1 \leq i \leq N, 1 \leq m \leq M)$$

$$\sum_{k \in \mathcal{I}_j, l \in \mathcal{S}_k} g_{kj} p_{kl}^m - q_j^m = 0 \quad (0 \leq j \leq N, 1 \leq m \leq M)$$

$$\eta W \lambda^{(m)} b_{ij}^m + q_j^m b_{ij}^m - g_{ij} p_{ij}^m b_{ij}^m - \frac{W}{\ln 2} g_{ij} \lambda^{(m)} p_{ij}^m = 0 \quad (1 \leq i \leq N, j \in \mathcal{S}_i, 1 \leq m \leq M) \tag{10}$$

$$\sum_{m=1}^{M} b_{ij}^m - f_{ij} \geq 0 \qquad (1 \leq i \leq N, j \in \mathcal{S}_i)$$

$$\sum_{j \in \mathcal{S}_i} f_{ij} - \sum_{j \in \mathcal{R}_i} f_{ji} - r_i K = 0 \qquad (1 \leq i \leq N)$$

$$K, p_{ij}^m, b_{ij}^m, q_j^m, f_{ij} \geq 0 \quad (1 \leq i \leq N, j \in \mathcal{S}_i, 1 \leq m \leq M)$$

$$\lambda^{(1)} \geq \lambda_{min}, \ \ \lambda^{(M)} \leq \lambda_{max} \ .$$

Although problem RFP-2 is simpler than the original RFP problem, it is still a *non-linear programming* problem (NLP), which remains NP-hard in general [3]. However, it is more amenable to apply certain optimization techniques. In the next section, we develop a solution procedure based on the branch-and-bound approach [9] and the novel *Reformulation-Linearization Technique* (RLT) [14, 15] to solve this NLP optimization problem.

3.2 A Solution Procedure

Branch-and-Bound. Using a branch-and-bound approach, we aim to provide an ε-optimal solution, where ε is a small pre-defined constant reflecting our tolerance for approximation in the final solution. Initially, we determine suitable intervals for each variable that appears in nonlinear terms. By using a *relaxation technique*, we then obtain an upper bound UB on the objective function value. Although the solution to such a relaxation usually yields infeasibility to the original NLP problem, we can apply a *local search algorithm* starting from this solution to find a feasible solution to the original NLP problem. This feasible solution now provides a lower bound LB on the objective function value.

If the distance between the above two bounds is small enough, i.e., $LB \geq (1-\varepsilon)UB$, we are done with the ε-optimal solution obtained by the local search. Otherwise, we will use the branch-and-bound procedure to find an ε-optimal solution. The branch-and-bound procedure is based on the divide-and-conquer idea. That is, although the original problem is hard to solve, it may be easier to solve a problem with a smaller solution space, e.g., if we can further limit $\lambda_1 \leq 0.05$. So, we divide the original problem into sub-problems, each with a smaller solution space. We then solve the original problem by solving all these sub-problems. The branch-and-bound procedure can remove certain sub-problems before solving them entirely and thus, can provide a solution much faster than a general divide-and-conquer approach.

During the branch-and-bound procedure, we put all these sub-problems into a problem list L. Initial, there is only Problem 1 in L, which is the original problem. For each problem in the list, we can obtain an upper bound and a lower bound with a feasible solution, just as we did initially. Then, the upper bound for the original problem is $UB = \max_{z \in L}\{UB_z\}$ and the lower bound for the original problem is $LB = \max_{z \in L}\{LB_z\}$. We choose Problem z having the current worst (maximum) upper bound $UB_z = UB$ and then partition this problem into two new Problems z_1 and z_2 that replace Problem z. This partitioning is done by choosing a variable and partitioning the interval of this variable into two new intervals, e.g., from $0 \leq \lambda_1 \leq 0.1$ to $0 \leq \lambda_1 \leq 0.05$ and $0.05 \leq \lambda_1 \leq 0.1$. For each new problem created, we obtain an upper bound and a lower bound with a feasible solution. The procedure then updates the lower bound LB and the upper bound UB for the original problem.

When $LB \geq (1-\varepsilon)UB$, we can claim that the current feasible solution is ε-optimal and we are done. This is the termination criterion. Otherwise, for any Problem z', if we have $(1-\varepsilon)UB_{z'} < LB$, where UB_z' is the upper bound obtained for Problem z', then Problem z' cannot offer an ε-optimal solution to the original problem and can be removed from the problem list L. The method then proceeds to the next iteration.

Note that since we are interested in determining whether or not K is greater than or equal to 1 (as for the feasibility determination problem), we can terminate the branch-and-bound procedure if any of the following two cases holds: (1) if the upper bound of K is smaller than 1, then the RFP-2 problem is infeasible; or (2) if we find any feasible solution with $K \geq 1$, then the RFP-2 problem is feasible.

Relaxation with RLT Technique. Throughout the branch-and-bound procedure (both initially and during each iteration), we need a relaxation technique to obtain an upper bound of the objective function. For this purpose, we apply a novel method based on *Reformulation-Linearization Technique* (RLT) [14, 15], which can provide a linear relaxation for a polynomial NLP problem. Specifically, in Eq. (10), RLT introduces new variables to replace the inherent polynomial terms and adds linear constraints for these new variables. These new RLT constraints are derived from the intervals of the original variables.

In particular, since $\lambda^{(m)} b_{ij}^m$ in Eq. (10) can be viewed as a single term, we introduce a new variable y_{ij}^m and let $y_{ij}^m = \lambda^{(m)} b_{ij}^m$. Since $\lambda^{(m)}$ and b_{ij}^m are each bounded by $(\lambda^{(m)})_L \le \lambda^{(m)} \le (\lambda^{(m)})_U$ and $(b_{ij}^m)_L \le b_{ij}^m \le (b_{ij}^m)_U$, respectively, we have $[\lambda^{(m)} - (\lambda^{(m)})_L] \cdot [b_{ij}^m - (b_{ij}^m)_L] \ge 0$, $[\lambda^{(m)} - (\lambda^{(m)})_L] \cdot [(b_{ij}^m)_U - b_{ij}^m] \ge 0$, $[(\lambda^{(m)})_U - \lambda^{(m)}] \cdot [b_{ij}^m - (b_{ij}^m)_L] \ge 0$, and $[(\lambda^{(m)})_U - \lambda^{(m)}] \cdot [(b_{ij}^m)_U - b_{ij}^m] \ge 0$. From the above relationships and substituting $y_{ij}^m = \lambda^{(m)} b_{ij}^m$, we have the following RLT constraints for y_{ij}^m.

$$
\begin{aligned}
(\lambda^{(m)})_L \cdot b_{ij}^m + (b_{ij}^m)_L \cdot \lambda^{(m)} - y_{ij}^m &\le (\lambda^{(m)})_L \cdot (b_{ij}^m)_L \\
(\lambda^{(m)})_U \cdot b_{ij}^m + (b_{ij}^m)_L \cdot \lambda^{(m)} - y_{ij}^m &\ge (\lambda^{(m)})_U \cdot (b_{ij}^m)_L \\
(\lambda^{(m)})_L \cdot b_{ij}^m + (b_{ij}^m)_U \cdot \lambda^{(m)} - y_{ij}^m &\ge (\lambda^{(m)})_L \cdot (b_{ij}^m)_U \\
(\lambda^{(m)})_U \cdot b_{ij}^m + (b_{ij}^m)_U \cdot \lambda^{(m)} - y_{ij}^m &\le (\lambda^{(m)})_U \cdot (b_{ij}^m)_U \ .
\end{aligned}
$$

We therefore replace $\lambda^{(m)} b_{ij}^m$ with y_{ij}^m in Eq. (10) and add the above RLT constraints for y_{ij}^m into the RFP-2 problem formulation. Similarly, we let $u_{ij}^m = q_j^m b_{ij}^m$, $v_{ij}^m = p_{ij}^m b_{ij}^m$, and $w_{ij}^m = \lambda^{(m)} p_{ij}^m$. From $(p_{ij}^m)_L \le p_{ij}^m \le (p_{ij}^m)_U$ and $(q_j^m)_L \le q_j^m \le (q_j^m)_U$, we can obtain the RLT constraints for u_{ij}^m, v_{ij}^m, and w_{ij}^m as well.

Denote $\mathbf{\Lambda}$, $\mathbf{p}$, $\mathbf{b}$, and $\mathbf{q}$ as vectors for $\lambda^{(m)}$, p_{ij}^m, b_{ij}^m, and q_j^m, respectively. After we replace all non-linear terms as above and add the corresponding RLT constraints into the RFP-2 problem formulation, we obtain the following LP.

$$
\begin{aligned}
&\text{Maximize} && K \\
&\text{subject to} && \sum_{m=1}^{M} \lambda^{(m)} = 1 \\
& && \lambda^{(m)} - \lambda^{(m-1)} \ge 0 \qquad (2 \le m \le M) \\
& && \sum_{j \in \mathcal{S}_i} p_{ij}^m - p_{max} \lambda^{(m)} \le 0 \quad (1 \le i \le N, 1 \le m \le M) \\
& && \sum_{k \in \mathcal{I}_j, l \in \mathcal{S}_k} g_{kj} p_{kl}^m - q_j^m = 0 \quad (0 \le j \le N, 1 \le m \le M) \\
& && \eta W y_{ij}^m + u_{ij}^m - g_{ij} v_{ij}^m - \frac{W}{\ln 2} g_{ij} w_{ij}^m = 0 \\
& && \qquad (1 \le i \le N, j \in \mathcal{S}_i, 1 \le m \le M) \\
& && \text{RLT constraints for } y_{ij}^m, u_{ij}^m, v_{ij}^m, \text{ and } w_{ij}^m \\
& && \qquad (1 \le i \le N, j \in \mathcal{S}_i, 1 \le m \le M) \\
& && \sum_{m=1}^{M} b_{ij}^m - f_{ij} \ge 0 \qquad (1 \le i \le N, j \in \mathcal{S}_i) \\
& && \sum_{j \in \mathcal{S}_i} f_{ij} - \sum_{j \in \mathcal{R}_i} f_{ji} - r_i K = 0 \qquad (1 \le i \le N) \\
& && K, f_{ij}, y_{ij}^m, u_{ij}^m, v_{ij}^m, w_{ij}^m \ge 0 \quad (1 \le i \le N, j \in \mathcal{S}_i, 1 \le m \le M) \\
& && (\mathbf{\Lambda}, \mathbf{p}, \mathbf{b}, \mathbf{q}) \in \Omega \ .
\end{aligned}
$$

where $\Omega = \{(\mathbf{\Lambda}, \mathbf{p}, \mathbf{b}, \mathbf{q}) : (\lambda^{(m)})_L \le \lambda^{(m)} \le (\lambda^{(m)})_U, (p_{ij}^m)_L \le p_{ij}^m \le (p_{ij}^m)_U, (b_{ij}^m)_L \le b_{ij}^m \le (b_{ij}^m)_U, (q_j^m)_L \le q_j^m \le (q_j^m)_U\}$.

0. **Feasibility Check Algorithm**
1. Initialization:
2. Let the initial best solution $\psi^* = \emptyset$ and the initial best lower bound
3. $LB = -\infty$.
4. Let the initial problem list L include only the original problem, denoted
5. as Problem 1.
6. Relaxation:
7. Solve the RLT relaxation for Problem 1 based on $\Omega_1 =$
8. $\{(\mathbf{\Lambda}, \mathbf{p}, \mathbf{b}, \mathbf{q}) : \lambda_{min} \le \lambda^{(m)} \le \lambda_{max}, 0 \le p_{ij}^m \le p_{max} \cdot$
9. $\lambda_{max}, 0 \le b_{ij}^m \le \frac{g_{ij}}{\eta \ln 2} p_{max} \lambda_{max}, 0 \le q_j^m \le p_{max} \cdot$
10. $\lambda_{max} \sum_{k \in \mathcal{I}_j} g_{kj}\}$.
11. Denote the obtained relaxation solution as $(\hat{\mathbf{\Lambda}}, \hat{\mathbf{p}}, \hat{\mathbf{b}}, \hat{\mathbf{q}})$ and the
12. objective value as the upper bound UB_1.
13. Let the initial worst upper bound $UB = UB_1$.
14. Iteration:
15. Select Problem z that has the maximum UB_z among all problems in
16. the problem list L.
17. Local search:
18. A feasible solution ψ can be obtained from its $\hat{\mathbf{\Lambda}}$ and $\hat{\mathbf{p}}$ via a local
19. search algorithm.
20. Denote the received data at the base-station as LB_z.
21. If $(LB_z > LB)$ {
22. Update $\psi^* = \psi$ and $LB = LB_z$.
23. If $(LB \ge (1 - \varepsilon) UB)$, we stop with the ε-optimal solution ψ^*.
24. Otherwise, remove all Problems z' with $(1 - \varepsilon) UB_{z'} \le LB$
25. from the problem list L. }
26. Partition:
27. First we find the maximum relaxation error among $|\hat{\lambda}^{(m)} \hat{b}_{ij}^m -$
28. $\hat{y}_{ij}^m|, |\hat{q}_j^m \hat{b}_{ij}^m - \hat{u}_{ij}^m|, |\hat{p}_{ij}^m \hat{b}_{ij}^m - \hat{v}_{ij}^m|$, and $|\hat{\lambda}^{(m)} \hat{p}_{ij}^m - \hat{w}_{ij}^m|$,
29. for $1 \le i \le N, j \in \mathcal{S}_i, 1 \le m \le M$.
30. In the case that the maximum relaxation error is $|\hat{\lambda}^{(m)} \hat{b}_{ij}^m - \hat{y}_{ij}^m|$,
31. if $((\lambda^{(m)})_U - (\lambda^{(m)})_L) \cdot \min\{\hat{\lambda}^{(m)} - (\lambda^{(m)})_L,$
32. $(\lambda^{(m)})_U - \hat{\lambda}^{(m)}\} \ge ((b_{ij}^m)_U - (b_{ij}^m)_L) \cdot$
33. $\min\{\hat{b}_{ij}^m - (b_{ij}^m)_L, (b_{ij}^m)_U - \hat{b}_{ij}^m\}$, we partition Ω_z into
34. two new regions Ω_{z1} and Ω_{z2} by dividing $[(\lambda^{(m)})_L,$
35. $(\lambda^{(m)})_U]$ into $[(\lambda^{(m)})_L, \hat{\lambda}^{(m)}]$ and $[\hat{\lambda}^{(m)}, (\lambda^{(m)})_U]$;
36. otherwise, we partition Ω_z into two new regions by dividing
37. $[(b_{ij}^m)_L, (b_{ij}^m)_U]$ into $[(b_{ij}^m)_L, \hat{b}_{ij}^m]$ and $[\hat{b}_{ij}^m, (b_{ij}^m)_U]$.
38. Similarly, we can perform a corresponding partition if the
39. maximum relaxation error is $|\hat{q}_j^m \hat{b}_{ij}^m - \hat{u}_{ij}^m|, |\hat{p}_{ij}^m \hat{b}_{ij}^m - \hat{v}_{ij}^m|$,
40. or $|\hat{\lambda}^{(m)} \hat{p}_{ij}^m - \hat{w}_{ij}^m|$.
41. Relaxation:
42. Solve the RLT relaxation for Problems z_1 and z_2 and obtain their
43. upper bounds UB_{z1} and UB_{z2}.
44. Remove Problem z from the problem list L.
45. If $(1 - \varepsilon) UB_{z1} > LB$, add Problem z_1 into the problem list L.
46. If $(1 - \varepsilon) UB_{z2} > LB$, add Problem z_2 into the problem list L.
47. If $L = \emptyset$, we stop with the ε-optimal solution ψ^*.
48. Otherwise, go to Line 15 for next iteration.

Figure 1: A solution procedure to the RFP-2 problem based on branch-and bound and RLT.

0. **Power Update Algorithm**
1. Choose a node i that meets one of the following requirements. If no
2. such node exists, we are done and the updated power vector is $\mathbf{p}$.
3. First, identify a node such that all nodes that receive data from
4. this node already have their transmission power updated.
5. If no such node exists, choose the node among all nodes that do not
6. have their transmission power updated and is closest to the base-station.
7. If there is no sub-band used for both transmission and receiving at node i
8. under $\hat{\mathbf{p}}$, we do not need to update its power and go to Line 1.
9. Otherwise, define Γ_{out} as the total bandwidth used by node i for sending
10. data and Γ_{in-out} as the total bandwidth used by node i only for
11. receiving data.
12. If $\Gamma_{out} > \Gamma_{in-out}$, node i tries to release some sub-bands used for both
13. transmission and receiving under $\hat{\mathbf{p}}$ and reduce the total used bandwidth to
14. $(\Gamma_{out} + \Gamma_{in-out})/2$ in the following order.
15. First it releases sub-band m with $\sum_{j \in \mathcal{S}_i} (p_{ij}^m)_L = 0$, in
16. non-decreasing order of $\sum_{j \in \mathcal{S}_i} p_{ij}^m$.
17. Second it reduces the transmission power in sub-band m, i.e., from
18. p_{ij}^m to $(p_{ij}^m)_L$, in non-decreasing order of $\sum_{j \in \mathcal{S}_i} (p_{ij}^m)_L$.
19. If node i decides to use a sub-band, all nodes should not use this sub-band
20. to send data to node i.
21. Go to Line 1.

Figure 2: An algorithm to obtain $\mathbf{p}$ from $\hat{\mathbf{p}}$.

The details of the proposed branch-and-bound solution procedure with RLT are given in Fig. 1. Note that in Line 27 of the Feasibility Check Algorithm (see Fig. 1), the method chooses a partitioning variable based on the maximum relaxation error. Clearly, $\lambda^{(m)}$ is a key variable in the problem formulation. As a result, the algorithm will run much more efficiently if we give the highest priority to $\lambda^{(m)}$ when it comes to choosing a partitioning variable.[1]

Local Search Algorithm. In the branch-and-bound procedure, we need to find a solution to the original problem from the solution to the relaxation problem (see Line 18 in Fig. 1). In particular, we obtain a solution from $\hat{\mathbf{\Lambda}}$ and $\hat{\mathbf{p}}$. We now show how to obtain such a solution.

We can let $\mathbf{\Lambda} = \hat{\mathbf{\Lambda}}$. Note that in RFP-2, we introduced the notion of a self-interference parameter to remove the binary variables in RFP. Then in $\hat{\mathbf{p}}$, it is possible that $p_{il}^m > 0$ and $p_{ji}^m > 0$ for a certain node i within some sub-band m. Therefore, it is necessary to find a new $\mathbf{p}$ from $\hat{\mathbf{p}}$ such that no node is allowed to transmit and receive within the same sub-band. An algorithm that achieves this purpose is shown in Fig. 2. The basic idea is to split the total bandwidth used at node i into two groups of equal bandwidth: one group for transmission and the other group for receiving.

After we obtain $\mathbf{\Lambda}$ and $\mathbf{p}$, we can compute b_{ij} from Eqs. (4) and (5). Then, we solve the following simple LP for K.

$$
\begin{array}{lll}
\text{Maximize} & K & \\
\text{subject to} & f_{ij} \le b_{ij} & (1 \le i \le N, j \in \mathcal{S}_i) \\
& \displaystyle\sum_{j \in \mathcal{S}_i} f_{ij} - \sum_{j \in \mathcal{R}_i} f_{ji} - r_i K = 0 & (1 \le i \le N) \\
& K, f_{ij} \ge 0 & (1 \le i \le N, j \in \mathcal{S}_i) \ .
\end{array}
$$

If an LP solution provides a $K \ge 1$, then this rate vector $\mathbf{r}$ is feasible.

[1] In our implementation of the algorithm, we give the highest priority to $\lambda^{(m)}$, the second highest priority to p_{ij}^m, and consider q_j^m last when we choose a partitioning variable. This does not hamper the convergence property of the algorithm [14].

4. A FAST HEURISTIC ALGORITHM FOR LARGE-SIZED SENSOR NETWORKS

In the last section, we presented an algorithm to determine the rate feasibility and a corresponding solution for scheduling, power control, and routing if the rate vector is found feasible. Since the size of the LP is in the order of $O(|\mathcal{S}| \cdot N \cdot M)$, where $|\mathcal{S}|$ is a typical number of one-hop neighbors, the algorithm can only handle small-sized network. For example, in our numerical results, we found that, with a Dell Precision 340 (2.0 GHz Pentium 4), the size of the network that can be handled by the solution procedure in Fig. 1 is on the order of 100 nodes. For large-sized networks, the algorithm in Fig. 1 has excessive storage and computational requirements that is beyond the capability of an ordinary desktop PC. Therefore, a new solution approach would be desirable for large-sized networks.

Our Approach. Our approach is based on the following observations for sensor networks. Although the total number of sensor nodes (N) in the network is large, the number of simultaneous events that produce sensing data are limited. That is, we assume that the number of source sensor nodes that actually producing sensing data (i.e., the number of nodes $s \in \mathcal{N}$ with $r_s > 0$) is not a large number. Further, since we only have a single base-station as the sink node for all data generated in the network, the nodes that are close to the base-station will be "bottleneck" nodes for the entire network. That is, the burden on a node near the base-station is clearly much higher than that on a node far away from the base-station.

Based on these observations, we partition the network into two parts as shown in Fig. 3(a): a set of nodes $\mathcal{H}_0^c$ that lie within a circle centered around the base-station and the remaining set of nodes, i.e., $\mathcal{N} - \mathcal{H}_0^c$ that lie outside the circle. The size of $|\mathcal{H}_0^c|$ is determined by the maximum size that can be handled by the Feasibility Check Algorithm in Fig. 1 (e.g., 100). For convenience, we call the set of nodes in $\mathcal{H}_0^c$ as the network *core* and the set of nodes in $\mathcal{N} - \mathcal{H}_0^c$ as the network *edge.*

The partitioning of the network into the core $\mathcal{H}_0^c$ and the edge $\mathcal{N} - \mathcal{H}_0^c$ has also effectively partitioned the source rate vector $\mathbf{r}$ into $\mathbf{r}^c$ and $\mathbf{r} - \mathbf{r}^c$, corresponding to the source bit rates generated within the network core $\mathcal{H}_0^c$ and outside the core, respectively. Now, our objective is to determine whether it is feasible to transport *both* $\mathbf{r}^c$ and $\mathbf{r} - \mathbf{r}^c$ to the base-station. The feasibility test for $\mathbf{r}^c$ can be done when we apply the Feasibility Check Algorithm in Fig. 1 for the core network $\mathcal{H}_0^c$. The tricky part is how to test feasibility for $\mathbf{r} - \mathbf{r}^c$ at the *same* time.

Since not all nodes in $\mathcal{H}_0^c$ can receive data directly from nodes outside $\mathcal{H}_0^c$, we further identity a set of nodes in $\mathcal{H}_0^c$ as $\mathcal{H}_0^d$, where each node in $\mathcal{H}_0^d$ can receive data directly from nodes outside of $\mathcal{H}_0^c$. Intuitively, $\mathcal{H}_0^d$ consists of nodes at the edge within the set of $\mathcal{H}_0^c$ (see Fig. 3(b)).

Our approximation algorithm is based on the following idea. For each node $i \in \mathcal{H}_0^d$, denote f_i^{in} as the rate of incoming flows to node i (for data generated outside of $\mathcal{H}_0^c$). We can set up a new optimization problem with the objective of maximizing $\sum_{i \in \mathcal{H}_0^d} f_i^{in}$, i.e., the total incoming rates to $\mathcal{H}_0^d$ from nodes outside the network core, subject to the constraint that the rate vector $\mathbf{r}^c$ must also be delivered to the base-station, among other constraints. During the iterations of this optimization problem, for the current value of $\sum_{i \in \mathcal{H}_0^d} f_i^{in}$, if $\sum_{j \notin \mathcal{H}_0^c} r_j \le \sum_{i \in \mathcal{H}_0^d} f_i^{in}$, then we need to check whether it is possible to "re-connect" source sensor nodes $\mathbf{r} - \mathbf{r}^c$ to the nodes $i \in \mathcal{H}_0^d$ in the network core. If such a "connection" is possible, we declare the entire rate vector $\mathbf{r}$ as feasible and we have found a solution. Otherwise, we move on to the next iteration of maximizing $\sum_{i \in \mathcal{H}_0^d} f_i^{in}$.

Algorithmic Details. For node $i \in \mathcal{H}_0^c$, denote $\mathcal{R}_i^c$ as the set of nodes in $\mathcal{H}_0^c$ that can directly send data to node i, $\mathcal{I}_i^c$ as the set of nodes in $\mathcal{H}_0^c$ that can produce interference at node i, and $\mathcal{S}_i^c$ as the set of nodes in $\mathcal{H}_0^c$ to which node i can send data in one hop. Then for node $i \in \mathcal{H}_0^d$, we have the following flow balance:

$$\sum_{j \in \mathcal{S}_i^c} f_{ij} - \sum_{j \in \mathcal{R}_i^c} f_{ji} - f_i^{in} = r_i \qquad \text{for } i \in \mathcal{H}_0^d \ .$$

For node $i \in \mathcal{H}_0^d$, we have

$$\begin{aligned} b_{ij}^m &= W\lambda^{(m)} \log_2 \left(1 + \frac{g_{ij} p_{ij}^m}{\eta W \lambda^{(m)} + \sum_{k \in \mathcal{I}_j, l \in \mathcal{S}_k}^{(k,l) \neq (i,j)} g_{kj} p_{kl}^m}\right) \\ &= W\lambda^{(m)} \cdot \log_2 \Bigg(1 + \\ &\qquad \frac{g_{ij} p_{ij}^m}{\eta W \lambda^{(m)} + \sum_{k \in \mathcal{I}_j^c, l \in \mathcal{S}_k}^{(k,l) \neq (i,j)} g_{kj} p_{kl}^m + \sum_{k \in \mathcal{I}_j - \mathcal{I}_j^c, l \in \mathcal{S}_k}^{(k,l) \neq (i,j)} g_{kj} p_{kl}^m}\Bigg) \\ &\geq W\lambda^{(m)} \cdot \log_2 \Bigg(1 + \\ &\qquad \frac{g_{ij} p_{ij}^m}{\eta W \lambda^{(m)} + \sum_{k \in \mathcal{I}_j^c, l \in \mathcal{S}_k^c}^{(k,l) \neq (i,j)} g_{kj} p_{kl}^m + \sum_{k \in \mathcal{I}_j}^{k \notin \mathcal{I}_j^c} g_{kj} p_{max} \lambda^{(m)}}\Bigg) \ . \end{aligned}$$

The last inequality holds because of Eq. (3).

We now have the following problem formulation.

Revised-RFP:

Maximize $\quad \sum_{i \in \mathcal{H}_0^d} f_i^{in}$

subject to $\quad \sum_{m=1}^{M} \lambda^{(m)} = 1$

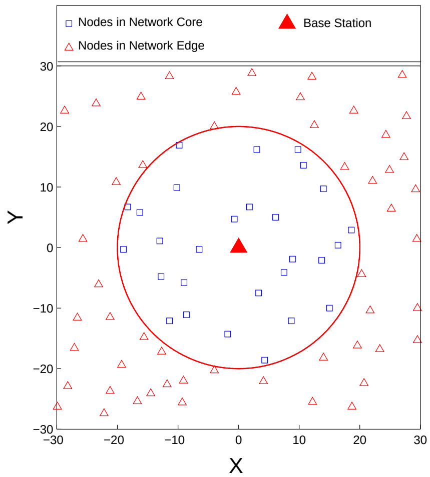
$$\sum_{j \in \mathcal{S}_i^c} p_{ij}^m - p_{max} \lambda^{(m)} \leq 0 \quad (i \in \mathcal{H}_0^c, 1 \leq m \leq M)$$

$$\sum_{i \in \mathcal{R}_j^c} p_{ij}^m \leq |\mathcal{R}_j| p_{max} \lambda^{(m)} x_j^m \quad (j \in \mathcal{H}_0^c, 1 \leq m \leq M)$$

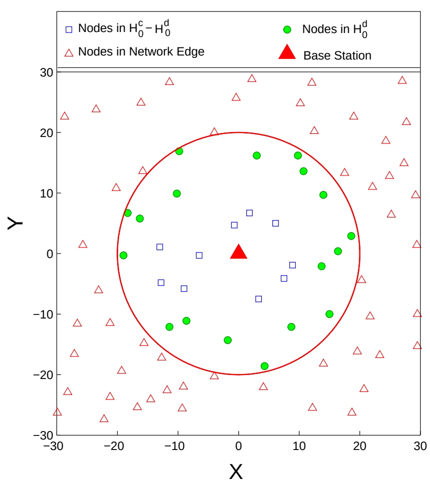
$$\sum_{l \in \mathcal{S}_j^c} p_{jl}^m \leq p_{max} \lambda^{(m)} (1 - x_j^m) \quad (j \in \mathcal{H}_0^c, 1 \leq m \leq M)$$

$$b_{ij}^m = W\lambda^{(m)} \log_2 \left(1 + \frac{g_{ij} p_{ij}^m}{\eta W \lambda^{(m)} + \sum_{k \in \mathcal{I}_j^c, l \in \mathcal{S}_k^c}^{(k,l) \neq (i,j)} g_{kj} p_{kl}^m}\right)$$
$$(i \in \mathcal{H}_0^c - \mathcal{H}_0^d, j \in \mathcal{S}_i^c, 1 \leq m \leq M)$$

$$b_{ij}^m \geq W\lambda^{(m)} \cdot \log_2 \Bigg(1 + \frac{g_{ij} p_{ij}^m}{\eta W \lambda^{(m)} + \sum_{k \in \mathcal{I}_j^c, l \in \mathcal{S}_k^c}^{(k,l) \neq (i,j)} g_{kj} p_{kl}^m + \sum_{k \in \mathcal{I}_j}^{k \notin \mathcal{I}_j^c} g_{kj} p_{max} \lambda^{(m)}}\Bigg)$$
$$(i \in \mathcal{H}_0^d, j \in \mathcal{S}_i^c, 1 \leq m \leq M)$$



(a) Network core and network edge.



(b) Nodes in $\mathcal{H}_0^d$ within the network core.

Figure 3: Illustration of network partitioning strategy.

$$
\begin{aligned}
\sum_{m=1}^{M} b_{ij}^{m} - f_{ij} \geq 0 & \qquad (i \in \mathcal{H}_0^c, j \in \mathcal{S}_i) \\
\sum_{j \in \mathcal{S}_i^c} f_{ij} - \sum_{j \in \mathcal{R}_i^c} f_{ji} = r_i & \qquad (i \in \mathcal{H}_0^c - \mathcal{H}_0^d) \\
\sum_{j \in \mathcal{S}_i^c} f_{ij} - \sum_{j \in \mathcal{R}_i^c} f_{ji} - f_i^{in} = r_i & \qquad (i \in \mathcal{H}_0^d) \\
\lambda_{min} \leq \lambda^{(m)} \leq \lambda_{max} & \qquad (1 \leq m \leq M) \\
f_i^{in} \geq 0 & \qquad (i \in \mathcal{H}_0^d) \\
x_j^m = 0 \text{ or } 1 & \qquad (j \in \mathcal{H}_0^c, 1 \leq m \leq M) \\
p_{ij}^m, b_{ij}^m, f_{ij} \geq 0 & \qquad (i \in \mathcal{H}_0^c, j \in \mathcal{S}_i^c, 1 \leq m \leq M) \ .
\end{aligned}
$$

Problem Revised-RFP is of the same form as the original RFP problem and thus can be solved by following the same solution procedure described in Section 3.

During the iteration, if we have $\sum_{j \notin \mathcal{H}_0^c} r_j \leq \sum_{i \in \mathcal{H}_0^d} f_i^{in}$, we will need to check whether it is possible to "re-connect" source sensor nodes $\mathbf{r} - \mathbf{r}^c$ to those nodes $i \in \mathcal{H}_0^d$ corresponding to their f_i^{in}-values. The first step is similar to the solution procedure to Problem RFP-2. We need to find a solution to Revised-RFP from the solution to the relaxation problem. As expected, this step is similar to the local search algorithm at the end of Section 3.2. That is, we let $\mathbf{\Lambda} = \hat{\mathbf{\Lambda}}$ and obtain $\mathbf{p}$ from $\hat{\mathbf{p}}$ via a similar procedure as in the Power Update Algorithm (see Fig. 2). The only difference resides in Line 9 of Fig. 2. For a node $i \in \mathcal{H}_0^d$, since the solution to the Revised-RFP problem does not show how scheduling is done by nodes $j \notin \mathcal{H}_0^c$ when they send data to node i, we aim that the total bandwidth of sub-bands used by node i for transmission is no more than half of the entire spectrum W in $\mathbf{p}$, which will give a balanced distribution of bandwidth between transmission and receiving. From $\mathbf{\Lambda}$ and $\mathbf{p}$, we can compute b_{ij} from Eqs. (4) and (5). Then, we solve the following LP:

$$
\begin{aligned}
\text{Maximize} \quad & \sum_{i \in \mathcal{H}_0^d} f_i^{in} \\
\text{subject to} \quad & f_{ij} \leq b_{ij} \qquad (i \in \mathcal{H}_0^c, j \in \mathcal{S}_i) \\
& \sum_{j \in \mathcal{S}_i^c} f_{ij} - \sum_{j \in \mathcal{R}_i^c} f_{ji} = r_i \qquad (i \in \mathcal{H}_0^c - \mathcal{H}_0^d) \\
& \sum_{j \in \mathcal{S}_i^c} f_{ij} - \sum_{j \in \mathcal{R}_i^c} f_{ji} - f_i^{in} = r_i \qquad (i \in \mathcal{H}_0^d) \\
& f_i^{in} \geq 0 \qquad (i \in \mathcal{H}_0^d) \\
& f_{ij} \geq 0 \qquad (i \in \mathcal{H}_0^c, j \in \mathcal{S}_i) \ .
\end{aligned}
$$

For the second step, we find a solution for nodes not in $\mathcal{H}_0^c$ that have a positive data rate. Figure 4 shows such an algorithm.

We briefly discuss how our algorithm discussed in this and previous sections can be implemented in practice. Since our algorithm requires global topology, a natual approach is to implement it at the base-station. That is, the base-station runs the feasibility check and if feasible, it broadcasts the solution to all the nodes in the network so as to instruct them on how to relay data flows to the base-station.

```
0.  Routing Algorithm for Nodes Outside the Network Core
1.  Main function:
2.      Identify a node that has not found a routing solution and is farthest
3.      from the network core.
4.      If no such a node exists, we are done. Otherwise, denote this node as s.
5.      Node s chooses a destination in H_0^d in the order of non-decreasing
6.      distance.
7.      If all nodes in H_0^d have been considered in previous iterations, we
8.      declare that we cannot find a solution.
9.      Otherwise, denote the destination node as t.
10.     ret = Rst(s, t, min{r_s, f_t^in}); //Rst() returns how much data rate
11.                                       // is transmitted to node t.
12.     If r_s > ret, update r_s = r_s − ret and go to Line 5.
13.     Otherwise, we are done with node s and go to Line 2.
14. //RST() attempts to send data rate req from node s to node t and returns
15. //the data rate that can be routed successfully.
16. double Rst(s, t, req) {
17.     For each next-hop node k, node s uses function Lsk(s, k) to find how
18.     much data can be sent to node k.
19.     Node s tries to find the nearest next-hop node k (to node t) that can
20.     receive data rate req from s. In this case, let r_sk = req.
21.     If no such a node exists, node s tries to find the next-hop k that can
22.     receive the maximum data rate r_sk from s.
23.     Node s uses a subset of available sub-bands to send r_sk data to node k.
24.     If k = t, return r_sk. Otherwise, node s calls ret = Rst(k, t, r_sk).
25.     If ret < r_sk, node s reduces its data rate r_sk = ret by releasing
26.     some used sub-bands. }
27. //Lsk() computes and returns the available link capacity from node s to
28. //node k.
29. double Lsk(s, k) {
30.     For each sub-band, node s first checks whether it is available (cannot
31.     send and receive within the same sub-band).
32.         Moreover, node s checks if neighboring links can maintain the
33.         same transmission rate by increasing their transmission power
34.         (subject to maximum power limit).
35.     For each available sub-band, node s then computes the maximum
36.     available capacity and returns the sum of them. }
```

Figure 4: An algorithm to obtain a solution for nodes not in $\mathcal{H}_0^c$.

5. AN UPPER BOUND FOR THE MAXIMUM RATE AT THE BASE-STATION

Since we consider a single base-station within a UWB-based sensor network, the maximum rate that the base-station can receive must be an upper bound for total bit rates of any feasible rate vector. In this section, we calculate this maximum rate that can be received at the base-station. Our result shows that this rate occurs if and only if all source nodes belong to the set of one-hop neighboring nodes from the base-station, denoted as $\mathcal{H}_0^1$. This result is intuitive since any remote source sensor node will decrease the transmission rate of a node within $\mathcal{H}_0^1$ (since a node cannot transmit and receive at the same time) and thus decrease the rate that can be received by the base-station. As a result, when the set of source sensor nodes are distributed in arbitrary locations within the network, the maximum rate at the base-station calculated in this section is usually not achievable and it is necessary to solve a decision problem, as we have done in the previous two sections.

Denote C_0 as the maximum rate that can be received by the base-station from nodes in the network, i.e., $C_0 = \max \sum_{i=1}^{N} r_i$ over all feasible $\mathbf{r}$ vectors. In Lemma 1, we show that for any feasible rate vector $\mathbf{r}$ with $C = \sum_{i=1}^{N} r_i$, we can find a feasible $\hat{\mathbf{r}}$ vector achieving at least the same aggregate rate C but with $\hat{r}_i = 0$ for $i \notin \mathcal{H}_0^1$, i.e., a one-hop solution. As a result, to calculate C_0, it is sufficient to consider r_i only for $i \in \mathcal{H}_0^1$ and its one-hop solution. In Theorem 1, we calculate a feasible rate vector $\hat{\mathbf{r}}^*$ that achieves the maximum aggregate rate C_0 at the base-station. In Theorem 2, we show that the rate vector that achieves C_0 is unique.

LEMMA 1. *For any feasible rate vector* $\mathbf{r}$ *with aggregate rate* $C = \sum_{i=1}^{N} r_i$ *sent to the base-station, there exists a feasible vector* $\hat{r}$ *with the same aggregate rate* C *and such that (1)* $\hat{r}_i > 0$ *only if* $i \in \mathcal{H}_0^1$ *and (2)* $\hat{f}_{ij} > 0$ *and* $\hat{p}_{ij}^m > 0$ *only if* $i \in \mathcal{H}_0^1$ *and* $j = 0$.

Proof. The proof is based on construction. That is, for any given feasible vector $\mathbf{r}$ with aggregate rate C that does not meet the one-hop property in the lemma, we will repeatedly move the data generation locations towards the base-station and remove the corresponding transmissions until we get a one-hop solution, i.e., a feasible rate vector $\hat{\mathbf{r}}$ with the same aggregate rate C and such that (1) $\hat{r}_i > 0$ only if $i \in \mathcal{H}_0^1$ and (2) $\hat{f}_{ij} > 0$ and $\hat{p}_{ij}^m > 0$ only if $i \in \mathcal{H}_0^1$ and $j = 0$.

Suppose that we have a feasible rate vector $\mathbf{r}$ with aggregate rate C. The termination of the following operations requires the routing solution be cycle-free. A flow cycle $(i_1, i_2, \cdots i_k)$ has positive rates on each of its link, i.e., $f_{i_j, i_{j+1}} > 0$ for $1 \leq j \leq k-1$ and $f_{i_k, i_1} > 0$. If a routing solution has a flow cycle, we first should remove this cycle by decreasing each of these flow rates by f_{min}, where f_{min} is the minimum flow rate among these links. Once we have a cycle-free routing solution, we maintain the same vector $\mathbf{\Lambda}$ for spectrum allocation and repeat the following operation until we get a one-hop solution. If there is a positive flow f_{ij} and $j \neq 0$, we define new r_i, r_j, and f_{ij} as follows: $r_i = r_i - f_{ij}$, $r_j = r_j + f_{ij}$, and $f_{ij} = 0$. This loop will terminate when we cannot find such a flow f_{ij} and $j \neq 0$. At this point, we can update $p_{ij}^m = 0$ for all $f_{ij} = 0$.

Define the final values of $\lambda^{(m)}, r_i, f_{ij}$, and p_{ij}^m-values as $\hat{\lambda}^{(m)}, \hat{r}_i$, $\hat{f}_{ij}$, and $\hat{p}_{ij}^m$, respectively. Clearly, the flow balance at each node still holds. Moreover, $\hat{f}_{i0} = f_{i0}$ and $\hat{p}_{i0}^m = p_{i0}^m$ for $i \in \mathcal{H}_0^1$. Therefore, the total data rates received by the base-station will remain the same, i.e., $C = \sum_{i=1}^N r_i = \sum_{i \in \mathcal{H}_0^1} \hat{r}_i$ and that (1) $\hat{r}_i > 0$ only if $i \in \mathcal{H}_0^1$; and (2) $\hat{f}_{ij} > 0$ and $\hat{p}_{ij}^m > 0$ only if $i \in \mathcal{H}_0^1$ and $j = 0$.

We now show that the rate vector $\hat{\mathbf{r}}$ is feasible. Since the flow balance holds throughout the iterative transformation, we only need to show that $\hat{b}_{ij} \geq \hat{f}_{ij}$. If $\hat{f}_{ij} = 0$, $\hat{b}_{ij} \geq \hat{f}_{ij}$ holds trivially. If $\hat{f}_{ij} > 0$, then $i \in \mathcal{H}_0^1$ and $j = 0$. Since $\hat{p}_{i0}^m = p_{i0}^m$ and other $\hat{p}_{jk}^m \leq p_{ik}^m$, we have $\hat{b}_{i0}^m \geq b_{i0}^m$. Because $b_{i0} \geq f_{i0}$ in the original solution and $\hat{f}_{i0} = f_{i0}$, node i can send data $\hat{f}_{i0}$ to the base-station under $\hat{\lambda}^{(m)}$ and $\hat{p}_{ij}^m$-values. That is, $(\hat{\lambda}^{(m)}, \hat{p}_{ij}^m, \hat{f}_{ij})$ is a feasible solution for $\hat{r}_i$-values. □

Based on Lemma 1, to calculate C_0, it is sufficient to consider only one-hop solutions. The following theorem gives the value of C_0 and a feasible rate vector $\hat{\mathbf{r}}^*$ that achieves C_0.

THEOREM 1. *The rate vector* $\hat{\mathbf{r}}^*$ *with*

$$\hat{r}_i^* = W \cdot \log_2 \left(1 + \frac{g_{i0}\pi_{max}}{\eta g_{nom} + \pi_{max} \sum_{k \in \mathcal{H}_0^1}^{k \neq i} g_{k0}} \right)$$

for $i \in \mathcal{H}_0^1$ *and* $\hat{r}_i^* = 0$ *for* $i \notin \mathcal{H}_0^1$ *is feasible and achieves the maximum aggregate rate* C_0 *that can be received by the base-station. That is,*

$$C_0 = \sum_{i=1}^N \hat{r}_i^* = W \sum_{i \in \mathcal{H}_0^1} \log_2 \left(1 + \frac{g_{i0}\pi_{max}}{\eta g_{nom} + \pi_{max} \sum_{k \in \mathcal{H}_0^1}^{k \neq i} g_{k0}} \right) .$$

Proof. First, we show that the rate vector $\hat{\mathbf{r}}^*$ stated in the theorem is feasible. That is, there exists a scheduling and power control policy such that the rate vector $\hat{\mathbf{r}}^*$ can be successfully sent to the base-station. As an existence proof for such a policy, let $M = 1$ and $\lambda^{(1)} = 1$, i.e., the simple case where the single sub-band is the same as the entire spectrum. Let each node $i \in \mathcal{H}_0^1$ send its data to the base-station via one hop with power $p_{i0}^1 = p_{max}$. The link capacity b_{i0} can be computed from Eqs. (4) and (5). It is easy to verify that $\hat{r}_i^* = b_{i0}$, i.e., node i can send all its data to the base-station. Therefore, the set of $\hat{r}_i^*$ stated in the theorem is feasible.

We now show that the rate vector $\hat{\mathbf{r}}^*$ achieves the maximum aggregate rate. That is, there does not exist another feasible vector $\hat{\mathbf{r}}$ with a one-hop solution such that $\sum_{i=1}^N \hat{r}_i > \sum_{i=1}^N \hat{r}_i^*$.

Our proof goes as follows. Since we are only considering a one-hop solution, the search space now only consists of scheduling and power control. For any given scheduling policy (the value of M and allocation of the spectrum into M sub-bands), we show that the achievable bit rate increases with transmission power of each node. This suggests that to achieve C_0, each node should use the maximum transmission power in each sub-band. Subsequently, we compute the maximum achievable data rate C_0 at the base-station.

For any given scheduling policy, we now consider a sub-band m ($1 \leq m \leq M$). Since $\sum_{j \in \mathcal{S}_i} p_{ij}^m \leq p_{max}\lambda^{(m)}$ in Eq. (3), we must have $p_{i0}^m \leq p_{max}\lambda^{(m)}$. Then

$$\frac{g_{i0}p_{i0}^m}{\eta W \lambda^{(m)}} \leq \frac{g_{i0}p_{max}\lambda^{(m)}}{\eta W \lambda^{(m)}} = \frac{g_{i0}p_{max}}{\eta W} .$$

But according to Eq. (2), $p_{max} = \frac{W\pi_{max}}{g_{nom}}$, we have

$$\frac{g_{i0}p_{i0}^m}{\eta W \lambda^{(m)}} \leq \frac{g_{i0}}{\eta W} \cdot \frac{W\pi_{max}}{g_{nom}} = \frac{g_{i0}\pi_{max}}{g_{nom}\eta} .$$

For UWB, we have $\frac{\pi_{max}}{\eta} \ll 1$ (e.g., on the order of 10^{-2} [13]). Since g_{i0} and g_{nom} are comparable, we have

$$\frac{g_{i0}p_{i0}^m}{\eta W \lambda^{(m)}} \ll 1 \qquad \text{for } i \in \mathcal{H}_0^1 . \tag{11}$$

Denoting $\delta = \max_{i \in \mathcal{H}_0^1} \frac{g_{i0}p_{i0}^m}{\eta W \lambda^{(m)}}$, we have $\delta \ll 1$. We now consider the total rate ($\sum_{i \in \mathcal{H}_0^1} b_{i0}^m$) received by the base-station in sub-band m. According to Eq. (4) and the fact that we are considering a one-hop solution, we have

$$b_{i0}^m = W\lambda^{(m)} \log_2 \left(1 + \frac{p_{i0}^m g_{i0}}{\eta W \lambda^{(m)} + \sum_{k \in \mathcal{H}_0^1}^{k \neq i} g_{k0}p_{k0}^m} \right) .$$

Using Eq. (11), we have

$$b_{i0}^m \approx \frac{W\lambda^{(m)}}{\ln 2} \frac{g_{i0}p_{i0}^m}{\eta W \lambda^{(m)} + \sum_{k \in \mathcal{H}_0^1}^{k \neq i} g_{k0}p_{k0}^m} .$$

Thus,

$$\frac{\partial(\sum_{i \in \mathcal{H}_0^1} b_{i0}^m)}{\partial(g_{i0}p_{i0}^m)} = \frac{W\lambda^{(m)}}{\ln 2} \cdot \left[\frac{1}{\eta W \lambda^{(m)} + \sum_{k \in \mathcal{H}_0^1}^{k \neq i} g_{k0}p_{k0}^m} - \sum_{j \in \mathcal{H}_0^1}^{j \neq i} \frac{g_{j0}p_{j0}^m}{(\eta W \lambda^{(m)} + \sum_{k \in \mathcal{H}_0^1}^{k \neq j} g_{k0}p_{k0}^m)^2} \right] .$$

To simplify notation, denote $a = \eta W \lambda^{(m)}$ and $b = \sum_{k \in \mathcal{H}_0^1} g_{k0}p_{k0}^m$.

Thus, we have

$$
\begin{aligned}
\frac{\partial(\sum_{i\in\mathcal{H}_0^1} b_{i0}^m)}{\partial(g_{i0}p_{i0}^m)} &= \frac{W\lambda^{(m)}}{\ln 2}\left[\frac{1}{a+b-g_{i0}p_{i0}^m}\right. \\
&\quad \left. -\sum_{j\in\mathcal{H}_0^1}^{j\neq i}\frac{g_{j0}p_{j0}^m}{(a+b-g_{j0}p_{j0}^m)^2}\right] \\
&\geq \frac{W\lambda^{(m)}}{\ln 2}\left[\frac{1}{a+b}-\sum_{j\in\mathcal{H}_0^1}^{j\neq i}\frac{g_{j0}p_{j0}^m}{(a+b-\delta a)^2}\right] \\
&\geq \frac{W\lambda^{(m)}}{\ln 2}\left[\frac{1}{a+b}-\frac{b}{(a+b-\delta a)^2}\right] \\
&= \frac{W\lambda^{(m)}}{\ln 2}\frac{[(1-2\delta)(a^2+ab)+\delta^2a^2]}{(a+b)(a+b-\delta a)^2} > 0\,.
\end{aligned}
$$

The last inequality holds because $\delta \ll 1$. Since $(\sum_{i\in\mathcal{H}_0^1} b_{i0}^m)$ is an increasing function of p_{i0}^m, to maximize the achievable bit rate in sub-band m, we must have $p_{i0}^m = p_{max}\lambda^{(m)}$ for all nodes $i \in \mathcal{H}_0^1$ and all sub-bands m. Thus, for a set of feasible rates to maximize the rate at the base-station, the rate of any node $i \in \mathcal{H}_0^1$ must satisfy

$$
\begin{aligned}
\hat{r}_i &\leq \sum_{m=1}^{M} b_{i0}^m \\
&= \sum_{m=1}^{M} W\lambda^{(m)}\cdot\log_2\Bigg(1+ \\
&\quad \frac{g_{i0}\frac{W\pi_{max}}{g_{nom}}\lambda^{(m)}}{\eta W\lambda^{(m)}+\sum_{k\in\mathcal{H}_0^1}^{k\neq i} g_{k0}\frac{W\pi_{max}}{g_{nom}}\lambda^{(m)}}\Bigg) \\
&= W\sum_{m=1}^{M}\lambda^{(m)}\cdot\log_2\left(1+\frac{g_{i0}\pi_{max}}{\eta g_{nom}+\pi_{max}\sum_{k\in\mathcal{H}_0^1}^{k\neq i} g_{k0}}\right) \\
&= W\log_2\left(1+\frac{g_{i0}\pi_{max}}{\eta g_{nom}+\pi_{max}\sum_{k\in\mathcal{H}_0^1}^{k\neq i} g_{k0}}\right) \\
&= \hat{r}_i^*\,. \qquad (12)
\end{aligned}
$$

That is, for any rate vector $\hat{\mathbf{r}}$ to maximize the rate at the base-station, we must have $\hat{r}_i \leq \hat{r}_i^*$. The maximum aggregate rate that the base-station can receive is thus

$$
C_0 = \sum_{i\in\mathcal{H}_0^1}\hat{r}_i^* = W\sum_{i\in\mathcal{H}_0^1}\log_2\left(1+\frac{g_{i0}\pi_{max}}{\eta g_{nom}+\pi_{max}\sum_{k\in\mathcal{H}_0^1}^{k\neq i} g_{k0}}\right)\,.
$$

This completes the proof. □

COROLLARY 1.1. *Among all feasible rate vectors with one-hop solutions, i.e., (1) $\hat{r}_i > 0$ only if $i \in \mathcal{H}_0^1$ and (2) $\hat{f}_{ij} > 0$ and $\hat{p}_{ij}^m > 0$ only if $i \in \mathcal{H}_0^1$ and $j = 0$, the maximum aggregate rate C_0 that can be received by the base-station is achievable only if $\mathbf{r} = \hat{\mathbf{r}}^*$.*

Proof. This result follows by the proof of Theorem 1. Among all feasible rate vectors with one-hop solutions, we have shown that $\hat{r}_i \leq \hat{r}_i^*$ (Eq. (12)) for any rate vector $\hat{\mathbf{r}}$ to maximize the rate at the base-station and the maximum rate $C_0 = \sum_{i\in\mathcal{H}_0^1}\hat{r}_i^*$. Thus, among all feasible rate vectors with one-hop solutions, C_0 is achievable only if $\hat{\mathbf{r}} = \hat{\mathbf{r}}^*$. □

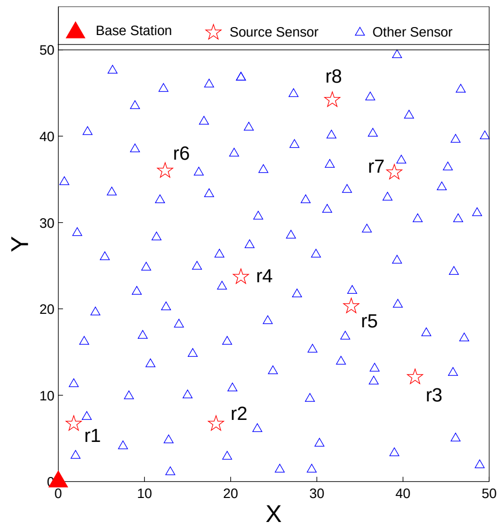


Figure 5: Network topology for a 100-node network.

In fact, the result in Theorem 1 can be further strengthened by the *uniqueness* of the rate vector that can achieve C_0. This is stated in the following theorem and the proof is omitted due to paper length limitation. Interested readers are referred to [16].

THEOREM 2. *The feasible rate vector $\mathbf{r}$ that can achieve C_0 is unique and $\mathbf{r} = \hat{\mathbf{r}}^*$.*

6. SIMULATION RESULTS

6.1 Simulation Setting

In this section, we present numerical results for our solution procedure and compare it with other possible approaches. Given that the total UWB spectrum is $W = 7.5$ GHz and that each sub-band is at least 500 MHz, we have that the maximum number of sub-bands is $M = 15$. The gain model for a link (i, j) is $g_{ij} = \min(d_{ij}^{-2}, 1)$ and the nominal gain is chosen as $g_{nom} = 0.02$. The power density limit π_{max} is assumed to be 1% of the white noise η.

We consider both a small network of 100 nodes (see Fig. 5) over a 50×50 area and a large network of 500 nodes (not shown) over a 100×100 area, where the distance is based on normalized length in Eq. (1). Both networks are generated at random. Under both networks, the base-station is located at the origin (lower left corner of the network). The details for each network will be elaborated shortly when we present the results.

6.2 Results

In this section, we investigate the impact of scheduling and routing. We are interested in comparing a cross-layer approach to a decoupled-layering approach to our problem. We do not explicitly show the impact of power control, since power level at a node is the single most important factor in wireless communications and directly determines both scheduling and routing. For example, if $p_{ij}^m > 0$, then node i uses sub-band m to send data to node j. As a result, scheduling (which sub-band is used) and routing (which link is used) are immediately determined. Although $\lambda^{(m)}$ is not known,

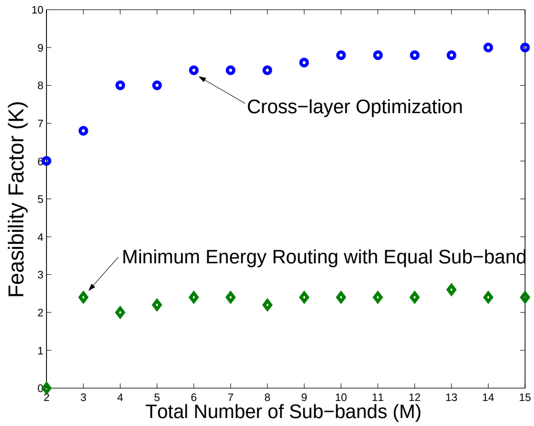


Figure 6: The maximum achievable K as a function of M for the 100-node network.

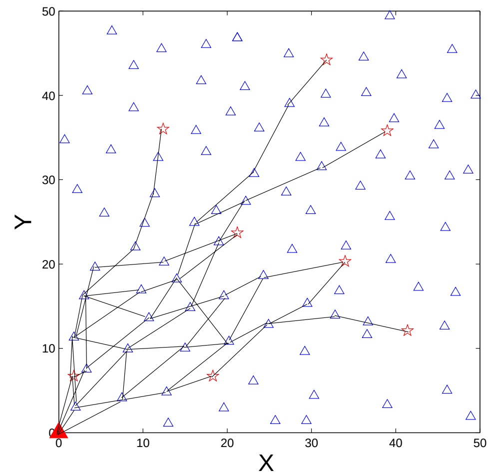
Table 2: Performance of feasibility factor K under different spectrum allocations with $M = 5$ for the 100-Node Network.

Spectrum Allocation	K	Rate
Optimal: (0.4256, 0.2339, 0.1660 0.1066 0.0679)	8.0	200
Equal: (0.20, 0.20, 0.20, 0.20, 0.20)	4.2	105
Random 1: (0.36, 0.23, 0.20, 0.11, 0.10)	2.8	70
Random 2: (0.27, 0.24, 0.21, 0.17, 0.11)	4.2	105

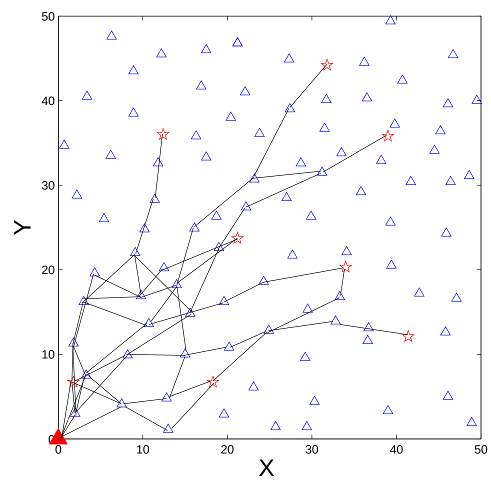
the lower bound of $\lambda^{(m)}$ is also given by $\sum_{j \in \mathcal{S}_i} p_{ij}^m / p_{max}$ (from Eq. (3)).

Impact of Scheduling. We first consider the 100-node network shown in Fig. 5. There are 8 source sensor nodes (marked as stars) in the network. The data rate are $r_1 = 5$, $r_2 = 2$, $r_3 = 2$, $r_4 = 4$, $r_5 = 5$, $r_6 = 3$, $r_7 = 3$, and $r_8 = 1$, with units defined in an appropriate manner. To show performance limits, we investigate the maximum feasible K (feasibility factor) under different approaches. Figure 6 (upper curve) shows the maximum achievable K for different M under our solution procedure. Clearly, K is a non-decreasing function of M, which states that the more sub-bands available, the larger traffic volume that the network can support. The physical explanation for this is that the more sub-bands available, the more opportunity for each node to avoid interference with other nodes within the same sub-band, and thus, this yields more capacity in the network. Also, note that there is a noticeable increase in K when M is small. But when $M \geq 4$, the increase in K is no longer significant. This suggests that for simplicity, we could just choose a small value (e.g., $M = 5$) for the number of sub-bands instead of the maximum $M = 15$.

To show the importance of joint optimization of link level scheduling and power control and network level routing, in Fig. 6, we also plot K as a function of M for a pre-defined routing strategy, namely, the minimum-energy routing with equal sub-band scheduling. Here, the energy cost is defined as g_{ij}^{-1} for link (i, j). Under this approach, we find a minimum-energy path for each source sensor node and determine which sub-band to use for each link and with how much power. When a node cannot find a feasible solution to send data to the next hop, it declares that the given rate vector



(a) $M = 5$.



(b) $M = 10$.

Figure 7: Optimal routing obtained via the cross-layer optimization solution procedure for the 100-node network.

Table 3: Performance of feasibility factor K under different spectrum allocations with $M = 10$ for the 100-node network.

Spectrum Allocation	K	Rate
Optimal: (0.1551, 0.1365, 0.1283, 0.0962, 0.0952, 0.0916, 0.0901, 0.0702, 0.0689, 0.0679)	8.8	220
Equal: (0.10, 0.10, 0.10, 0.10, 0.10, 0.10, 0.10, 0.10, 0.10, 0.10)	3.6	90
Random 1: (0.14, 0.13, 0.12, 0.11, 0.09, 0.09, 0.09, 0.08, 0.08, 0.07)	4.0	100
Random 2: (0.17, 0.13, 0.11, 0.10, 0.10, 0.09, 0.08, 0.08, 0.07, 0.07)	3.8	95

Table 4: Location and rate for each source sensor node in a 500-node network.

Source Node Index	Location	Rate	Source Node Index	Location	Rate
1	(3.3, 82.0)	1	11	(4.2, 55.6)	1
2	(7.5, 4.1)	4	12	(15, 10)	2
3	(16.3, 35.8)	2	13	(19, 22.6)	2
4	(19.2, 58.7)	2	14	(20.9, 77.3)	4
5	(31.6, 94.4)	1	15	(36.2, 44.5)	1
6	(39.4, 20.5)	3	16	(39.8, 37.2)	3
7	(42.5, 96.2)	2	17	(45.8, 12.6)	1
8	(60.1, 17.1)	4	18	(63.4, 71.1)	1
9	(65.0, 66.6)	2	19	(71.4, 21.3)	3
10	(74.4, 74.8)	2	20	(90.5, 28.7)	3

is infeasible. In Fig. 6, we find that the minimum-energy routing with equal sub-band scheduling approach is significantly inferior than the proposed cross-layer optimization approach.

Table 2 shows the results for K under different spectrum allocations for $M = 5$. The routes are the same as those obtained under optimal routing from our cross-layer optimal solution (see Fig. 7(a)) and are fixed in this study. The first optimal spectrum allocation is obtained from the cross-layer optimal solution. The second is an equal spectrum allocation and the following two are random spectrum allocations. Clearly, the cross-layer optimal spectrum allocation provides the best performance among all these spectrum allocations. It is important to realize that in addition to the number of sub-bands M, the way how the spectrum is allocated for a given M also has a profound impact on the performance. In Table 3, we perform the same study for $M = 10$ and obtain the same conclusion.

Impact of Routing. For the rest of this section, we consider a network of 500 nodes randomly deployed over a 100×100 area. Among these nodes, there are 20 source sensor nodes and the coordinates and bit rates for the source sensor nodes are listed in Table 4. We study the impact of routing on our cross-layer optimization problem under a given optimal schedule (obtained through our solution procedure). Table 5 shows the results in this study. In addition to our cross-layer optimal routing, we also consider the following two routing approaches, namely, minimum-energy routing and minimum-hop routing. The minimum-hop routing is similar to the minimum-energy routing, except the cost here is measured in the number of hops.

Table 5: Performance of feasibility factor K under different routing strategies for the 500-node network.

Routing Strategy	$M = 5$		$M = 10$	
	K	Rate	K	Rate
Optimal Routing	4.6	202.4	4.6	202.4
Minimum-Energy Routing	1.0	44.0	1.2	52.8
Minimum-Hop Routing	0.6	26.4	1.0	44.0

In Table 5, the spectrum allocation is chosen as the optimal spectrum allocation from our cross-layer optimal solution and is fixed. Specifically, for $M = 5$, we have $\mathbf{\Lambda} = (0.0758, 0.1144, 0.1234, 0.3045, 0.3819)$; for $M = 10$, we have $\mathbf{\Lambda} = (0.0692, 0.0719, 0.0758, 0.0781, 0.0853, 0.1047, 0.1084, 0.1144, 0.1234, 0.1688)$. Clearly, the cross-layer optimal routing outperforms both minimum-energy and minimum-hop routing approaches. Under minimum-hop routing, the routing solution prefers longer distance hops (with smaller number of hops) toward the base-station, which is likely to reduce link capacity due to the distance gain factor. On the other hand, minimum-energy routing prefers shorter distance hops. But by using short-distance links, more links are used in the routing, and thus will produce more interferences among the links. As a result, the capacity on the links will also be reduced. The minimum-hop routing and minimum-energy routing can be viewed as two extreme approaches, while the cross-layer optimal routing takes a balanced approach and thus yields the best result.

7. RELATED WORK

A good recent overview paper on UWB is given [11]. Physical layer issues associated with UWB-based multiple access communications can be found in [5, 6, 17] and references therein. In this section, we focus on related work addressing networking problems with UWB.

In [7], Negi and Rajeswaran first showed that, in contrast to previously published results, the throughput for UWB-based ad hoc networks increases with node density. This important result is mainly due to the large bandwidth and the ability of power and rate adaptation of UWB-based nodes, which alleviate interference. More importantly, this result demonstrates the significance of physical layer properties on network layer metrics such as network capacity. In [1], Baldi et al. considered the admission control problem based on a flexible cost function in UWB-based networks. Under their approach, a communication cost is attached to each path and the cost of a path is the sum of costs associated with the links it comprises. An admissibility test is then made based on the cost of a path. However, there is no explicit consideration of joint cross-layer optimization of scheduling, power control, and routing in this admissibility test. In [2], Cuomo et al. studied a multiple access scheme for UWB. Power control and rate allocation problems were formulated for both elastic bandwidth data traffic and guaranteed-service traffic. The impact of routing, however, was not addressed.

The most closely related research to our work are [8] and [12]. In [8], Negi and Rajeswaran studied how to maximize proportional rate allocation in a single-hop UWB network (each node can communicate to any other node in a single hop). The problem was formulated as a cross-layer optimization problem with similar scheduling and power control constraints as in this paper. In contrast, our focus in this paper is on an admissibility test for a rate vector in a sensor network and we consider a multi-hop network environment where routing is also part of the cross-layer optimization problem. As a result, the problem in this paper is more difficult. In [12], Radunovic and Le Boudec studied how to maximize the total log-utility of flow rates in multi-hop ad hoc networks. The cross-layer optimization space consists of scheduling, power control, and routing. As the optimization problem is NP-hard, the authors then studied a simple ring network as well as a small-sized network with pre-defined scheduling and routing policies. On the other hand, in this paper, we have developed a novel solution procedure to our cross-layer optimization problem, despite that it is highly complex. Further, due to differences in optimization objectives, we find that some of the results regarding power control and routing discussed in [12] are no longer applicable to our problem for sensor networks.

8. CONCLUSIONS

In this paper, we have studied the important problem of routing data traffic in a UWB-based sensor network. We followed a cross-layer optimization approach with joint consideration of link layer scheduling, power control, and network layer routing. Our contributions are three-fold. First, for small-sized networks, we developed a solution procedure based on a branch-and-bound approach and the RLT technique. Second, for large-sized networks, we designed an efficient heuristic algorithm by intelligently partitioning the network into core and edge components, where the problem associated with the core can be effectively addressed by the solution procedure for small-sized networks. Finally, we analyzed the maximum rate that a base-station can receive and gave a closed-form expression, which can be used as the first feasibility test for our problem. Our simulation results demonstrated the efficacy of our proposed solution procedure and substantiated the importance of cross-layer optimization for UWB-based sensor networks.

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