A General Model for DoF-based Interference Cancellation in MIMO Networks with Rank-deficient Channels

Yongce Chen[†] Yan Huang[†] Yi Shi[†] Y. Thomas Hou[†] Wenjing Lou[†] Sastry Kompella[‡]
[†]Virginia Tech, Blacksburg, VA, USA
[‡]U.S. Naval Research Laboratory, Washington, DC, USA

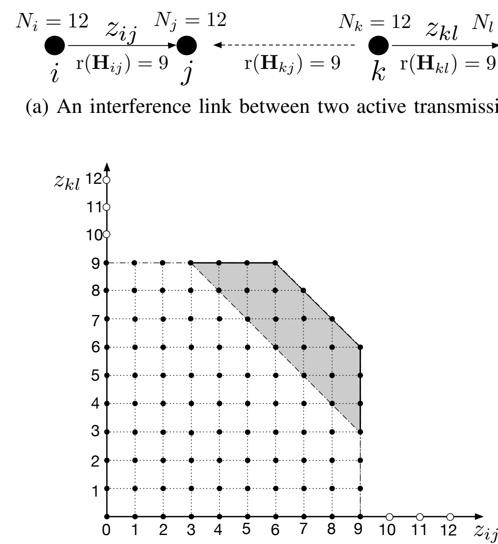
***Abstract*—In recent years, degree-of-freedom (DoF) based models were proven to be very successful in studying MIMO-based wireless networks. However, most of these studies assume channel matrix is of full-rank. Such assumption, although attractive, quickly becomes problematic as the number of antennas increases and propagation environment is not close to ideal. In this paper, we address this problem by developing a general theory for DoF-based model under rank-deficient conditions. We start with a fundamental understanding on how MIMO's DoFs are consumed for spatial multiplexing (SM) and interference cancellation (IC) in the presence of rank deficiency. Based on this understanding, we develop a general DoF model that can be used for identifying DoF region of a multi-link MIMO network and for studying DoF scheduling in MIMO networks. Specifically, we found that shared DoF consumption at transmit and receive nodes is critical for optimal allocation of DoF for IC. The results of this paper serve as an important tool for future research of many-antenna based MIMO networks.**

***Index Terms*—MIMO, rank deficiency, degree of freedom (DoF), interference cancellation.**

I. Introduction

Degree-of-freedom (DoF) based models are powerful tools to characterize MIMO's spatial multiplexing (SM) and interference cancellation (IC) capabilities [1–10]. The concept of DoF was first introduced by the information theory community to represent the multiplexing gain of a MIMO channel [11]. In high signal-to-noise ratio (SNR) region, the channel capacity grows linearly with the number of SM gain [11, 12]. This concept was then extended by the wireless networking community to characterize a node's spatial freedom. A node can also use its DoFs for IC. By employing zero-forcing (ZF) precoding technique, one can create interference-free signals through beamforming in the null space of interference signals [13]. Based on DoF concept, the so-called DoF region can be used to characterize the performance envelope of SM for a set of links that transmit simultaneously (free of interference) [14]. Although not without limitations, DoF-based models have served the wireless networking community well. It is as a simple and tractable tool to analyze MIMO's SM and IC capability without getting tangled with the complexity of matrix-based representations.

However, existing DoF-based models in the literature do suffer from one serious limitation. They typically assume the channel matrix is of full-rank (see, e.g., [1–10]) which is one would encounter when the number of antennas is small and the propagation environment is ideal (i.e., rich scattering). But such assumption quickly falls apart as the number of antennas increases and the propagation environment is not close to ideal. As expected, a rank-deficient channel will hinder MIMO's SM capability and undermine the validity of existing DoF-based IC models, which all assume full-rank channels. With FCC's recent interest toward communications in midband spectrum (between 3.7 and 24 GHz) [15], which is the spectrum where we expect to see *many-antenna* MIMO (typically ranges from 12 to 64), issues associated with rank-deficiency will become significant and critical.



(a) An interference link between two active transmissions.

(b) DoF regions under different IC models

Fig. 1: A motivating example showing different DoF regions for a two-link network.

We use an example to demonstrate issues associated with rank-deficient channels and motivate the need of our research in this paper. Consider two active transmission in Fig. 1(a), where Tx node i transmits z_{ij} data streams to Rx node j while Tx node k transmits z_{kl} data streams to Rx node l. Rx node j is interfered with by Tx node k. Suppose all the nodes have 12 antennas. Denote $\mathbf{H}_{ij}, \mathbf{H}_{kl}$ and $\mathbf{H}_{kj}$ as channel matrices of $i \to j$, $k \to l$ and $k \to j$, respectively and let the ranks of $\mathbf{H}_{ij}, \mathbf{H}_{kl}$ and $\mathbf{H}_{kj}$ all be 9 (< 12, i.e., rank-deficient). Under these rank-deficient channels, SM on links $i \to j$ and $k \to$

j are now each upper limited to 9 (instead of 12). So it is infeasible to have z_{ij} or z_{kl} to carry 12 data streams as under full-rank assumption. To find the DoF region of the two links (i.e., feasible data streams that can be carried on links $i \rightarrow j$ and $k \rightarrow j$ simultaneously), we need to consider how the interference (from Tx node k to Rx node j) is cancelled. It was well understood that for full-rank channels, DoF consumption for IC is most efficiently done by either Tx node k or Rx node j, but *not* both nodes. That is, either Rx node j (consuming z_{kl} DoFs) or Tx node k (consuming z_{ij} DoFs) can be used to cancel the interference from node k to j [3, 4, 7, 9, 10]. This will result in a DoF region that is bounded by the inner pentagon (dash lines) in Fig. 1(b). However, as we shall show in this paper, such unilateral DoF consumption for IC (at either Tx node or Rx node, but not both) is *inefficient* for general rank-deficient channels. In fact, to maximize efficiency, DoF consumption must be shared between Tx node k and Rx node j to cancel the interference from k to j. We will show that through shared DoF consumption by both Tx node k and Rx node j for IC, a larger DoF region can be achieved, as shown in the outer pentagon in Fig. 1(b), where the shaded area is the gain in feasible DoF region.

The existence of rank-deficient channels for MIMOs calls for a new and more general DoF based MIMO model. Unfortunately, to date, there is hardly any research result available on this important problem in the wireless networking community. Most research on MIMO that used DoF model assumed full-rank channels [1–10]. In [3], Blough *et al.* showed that it is sufficient to consume DoFs at either transmitter or receiver to cancel the interference between the two nodes, but not both. Most IC DoF-based models (e.g. [4, 7, 9, 10]) were developed along this approach. As we shall show in this paper, such (unilateral) IC scheme for full-rank channels is a special case of the general IC scheme for rank-deficient channels, i.e., when the channels are of full ranks.

In information theory community, there has been some active research activities to understand MIMO's behavior under rank-deficient channels [16–19]. The focus there has been to derive closed-form expressions of achievable/outer-bound DoF region for specific link topology and rank settings. These results are not useful for DoF resource scheduling in a MIMO network, which is the primary interest in the wireless networking community. Some representative research includes point-to-point MIMO [16], 3-link MIMO with symmetric antenna and rank [16–18], K-link MIMO with symmetric antenna and rank [16], $2\times2\times2$ link topology [19]. None of these research efforts can be used for DoF scheduling for arbitrary network topology and general rank-deficient conditions.

The goal of this paper is to explore this important area by developing a unified theory on DoF consumption for SM and IC under general channel rank conditions. Specifically, starting from a single transmission link and a single interference link under arbitrary rank condition, we offer an rigorous analysis on how DoFs are consumed at each node for SM and IC. We show that full-rank assumption is a special case of rank deficiency, thus concluding that previous results on IC models are in fact special case under our model. We further extend the general DoF model to analyze multi-link MIMO networks and show how to perform DoF scheduling in MIMO networks. Using numerical studies, we demonstrate the efficacy of our general DoF IC model under rank-deficient conditions.

TABLE I: Notation

Symbol	Definition
d^{T}_{i*}	Total number of DoFs consumed by Tx node i for IC.
d^{R}_{*j}	Total number of DoFs consumed by Rx node j for IC.
d^{R}_{ij}	Number of DoFs consumed by Rx node j to cancel interference from Tx node i to Rx node j.
d^{T}_{ij}	Number of DoFs consumed by Tx node i to cancel interference from Tx node i to Rx node j.
$\mathbf{H}_{ij}$	Channel matrix from Tx node i to Rx node j
$\mathcal{I}_i$	Set of nodes within node i's interference range
$\mathcal{K}$	Set of nodes in the network
N_i	Number of antennas at node i
r_{ij}	Rank of $\mathbf{H}_{ij}$
$\mathcal{T}_i$	Set of nodes within node i's transmission range
$\mathbf{U}_i$	Weight matrix at Tx node i
$\mathbf{V}_j$	Weight matrix at Rx node j
z_{i*}	Total number of outgoing data streams at Tx i
z_{*j}	Total number of incoming data streams at Rx j
z_{ij}	Number of data streams from Tx node i to Rx node j
$\mathbf{1}^{\mathrm{R}}_{ij}$	A binary variable to indicate whether Rx node j consumes DoFs for IC from i to j
$\mathbf{1}^{\mathrm{T}}_{ij}$	A binary variable to indicate whether Tx node i consumes DoFs for IC from i to j

The remainder of this paper is organized as follows. In Section II, we present a general model on DoF consumption with rank deficiency. In Section III, we revisit previous DoF models under full-rank conditions and show that they are special case under our general theory. In Section IV, we develop a DoF scheduling model for multi-link MIMO networks. Section V presents case studies and demonstrate the efficacy of our DoF model for rank deficiency channels. Section VI concludes this paper.

II. DoF Consumption with Rank Deficiency: A General Model

A. DoF Consumption at Node

The concept of DoF originally represents the multiplexing gain of a MIMO channel. For a multi-link network, the DoF of the network also represents the number of interference-free data streams that can be transmitted reliably over the network. This DoF concept was then extended to characterize a node's spatial freedom by its multiple antennas. To concretize our analysis, we formally define a node's DoFs mathematically.

Assume node i has N_i antennas. Denote $\mathbf{x}_{ij} \in \mathbb{C}^{N_i\times1}$ as the weight vector at node i for the j-th stream, where $\mathbb{C}^{m\times n}$ denotes complex set with dimension $m\times n$. With N_i antennas, there can be at most N_i streams. Assume node i transmits or receives n_s streams (where $n_s \le N_i$). Then its weight matrix $\mathbf{X}_i = [\mathbf{x}_{i1}, \mathbf{x}_{i2}, ..., \mathbf{x}_{in_s}] \in \mathbb{C}^{N_i\times n_s}$.

The number of a node's remaining DoFs can be determined by the set of constraints on its weight matrix. Initially, when there is no constraint on $\mathbf{X}_i$, each of its elements is undetermined and can be set arbitrarily. There is a feasible region (a space) that includes all possible values by such an unconstrained matrix. The initial DoFs of this feasible region is equal to the number of rows $\mathbf{X}_i$ (or the number of antennas

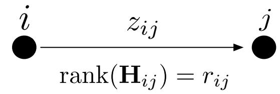


Fig. 2: Spatial multiplexing on a link.

at the node), i.e., N_i, since N_i is the maximum number of dimensions spanned by $\mathbf{x}_{i1}, \mathbf{x}_{i2}, ..., \mathbf{x}_{in_s}$. Thus, a node's initial available DoFs is the number of antennas at this node.

To perform SM and IC, a node's weight matrix must satisfy certain constraints to achieve interference-free transmission. Thus some DoFs at the node will be consumed for SM and IC. The number of consumed DoFs at a node is directly tied to the number of constraints imposed on its weight matrix. Assume some constraints are imposed on $\mathbf{X}_i$ in the form $\mathbf{AX}_i = \mathbf{B}$, where $\mathbf{A} \in \mathbb{C}^{M \times N_i}$ and $\mathbf{B} \in \mathbb{C}^{M \times n_s}$. That is, M linear constraints are imposed on each $\mathbf{x}_{ij}$. Denote $\mathbf{\Phi}$ as the union solution space of each $\mathbf{x}_{ij}$ to problem $\mathbf{AX}_i = \mathbf{B}$, i.e., $\mathbf{\Phi} = \{\boldsymbol{\phi}_1 \cup \boldsymbol{\phi}_2 \cup \cdots \cup \boldsymbol{\phi}_{n_s} | \mathbf{A}[\boldsymbol{\phi}_1 \ \boldsymbol{\phi}_2 \ \cdots \ \boldsymbol{\phi}_{n_s}] = \mathbf{B}\}$. Then the remaining available DoF is defined as the free dimension of $\mathbf{X}_i$, namely dim($\mathbf{\Phi}$).

Lemma 1 *Suppose node i has N_i antennas and its weight matrix $\mathbf{X}_i$ is constrained by $\mathbf{AX}_i = \mathbf{B}$. If rank$([\mathbf{A}\ \mathbf{B}])$ = rank$(\mathbf{A})$, then the number of consumed DoFs at node i is equal to rank($[\mathbf{A}\ \mathbf{B}]$), and the remaining available DoFs at node i is $N_i -$rank($[\mathbf{A}\ \mathbf{B}]$). If rank$([\mathbf{A}\ \mathbf{B}]) \neq$ rank$(\mathbf{A})$, then there is no feasible solution to $\mathbf{AX}_i = \mathbf{B}$.*

Proof Initially, all the elements in $\mathbf{X}_i$ are undetermined and can be set arbitrarily. N_i is the maximum number of dimensions spanned by $\mathbf{x}_{i1}, \mathbf{x}_{i2}, ..., \mathbf{x}_{in_s}$, i.e., the number of initial available DoFs provided by $\mathbf{X}_i$ is N_i.

Let $\mathbf{B} = [\mathbf{b}_1, \mathbf{b}_2, ..., \mathbf{b}_{n_s}]$. For any $\mathbf{A} \in \mathbb{C}^{M \times N_i}$ and $\mathbf{b}_j \in \mathbb{C}^{M \times 1}$, where $j \in \{1, ..., n_s\}$, if rank$([\mathbf{A}\ \mathbf{b}_j])$ = rank$(\mathbf{A})$, then the set of solutions to non-homogeneous linear system $\mathbf{Ax}_{ij} = \mathbf{b}_j$ is an affine subspace of $\mathbb{C}^{N_i \times 1}$, denoted as $\mathbf{\Phi}_j$. Since the solution dimension of a non-homogeneous linear system is the same as its corresponding homogeneous linear system, we have dim$(\mathbf{\Phi}_j)$ = dim(nullspace($\mathbf{A}$)) = $N_i -$ rank$([\mathbf{A}\ \mathbf{b}_j])$. Note that non-homogeneous linear systems $\mathbf{Ax}_{ij} = \mathbf{b}_j$ $(j = 1, ..., n_s)$ are sharing the same corresponding homogeneous linear system $\mathbf{Ax}_{ij} = \mathbf{0}$ (so share the same homogeneous solutions). We can conclude dim$(\mathbf{\Phi})$ = dim$(\mathbf{\Phi}_j)$ = $N_i -$ rank$([\mathbf{A}\ \mathbf{B}])$. If rank$([\mathbf{A}\ \mathbf{b}_j]) \neq$ rank$(\mathbf{A})$, then there is no feasible solution to $\mathbf{Ax}_{ij} = \mathbf{b}_j$. Consequently, if rank$([\mathbf{A}\ \mathbf{B}]) \neq$ rank$(\mathbf{A})$, then there is no feasible solution to $\mathbf{AX}_i = \mathbf{B}$. ∎

Lemma 1 shows one DoF is consumed for each linear independent constraint imposed on $\mathbf{X}_i$. The number of linear independent constraints on $\mathbf{X}_i$ is equal to rank$([\mathbf{A}\ \mathbf{B}])$, which gives the remaining available DoFs of $\mathbf{X}_i$ to be $N_i -$ rank$([\mathbf{A}\ \mathbf{B}])$. In the following two sections, we study DoF consumption by SM and IC under general rank deficient channels.

B. DoF Consumption for SM under Rank-deficient Channels

Consider the single transmission link in Fig. 2, where the number of data streams transmitted from Tx node i to Rx node j is z_{ij}, and rank$(\mathbf{H}_{ij}) = r_{ij}$. Then some DoFs at Tx node i and Rx node j will be consumed for SM. As expected, the number of data streams transmitted on channel $\mathbf{H}_{ij}$ cannot exceed the rank of this channel.

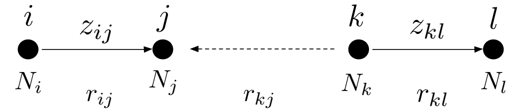


Fig. 3: Interference cancellation between two nodes.

Lemma 2 *For transmission on a single link where node i is a transmitter and node j is a receiver, z_{ij} data streams can be transmitted free of interference only if $z_{ij} \leq r_{ij}$. Further, the number of DoFs consumed by SM at node i and node j are both z_{ij}.*

Proof Denote $\mathbf{U}_i$ and $\mathbf{V}_j$ as the weight matrices at Tx node i and Rx node j, respectively. To ensure interference-free transmission of z_{ij} data streams, the following constraint must be satisfied:

$$\underset{z_{ij} \times N_i}{\mathbf{U}_i^T} \cdot \underset{N_i \times N_j}{\mathbf{H}_{ij}} \cdot \underset{N_j \times z_{ij}}{\mathbf{V}_j} = \mathbf{I}_{z_{ij}}, \tag{1}$$

where $\mathbf{I}_{z_{ij}}$ denotes identity matrix with dimension $z_{ij} \times z_{ij}$.

We first consider the DoF consumption at Rx node j. We have

$$\text{rank}\left(\left[\underset{z_{ij} \times N_i}{\mathbf{U}_i^T} \cdot \underset{N_i \times N_j}{\mathbf{H}_{ij}} \quad \mathbf{I}_{z_{ij}}\right]\right) = z_{ij}. \tag{2}$$

Note that rank$(\mathbf{H}_{ij})$ must be at least z_{ij}. Otherwise, rank$(\mathbf{U}_i^T \mathbf{H}_{ij}) \leq \min\{\text{rank}(\mathbf{U}_i^T), \text{rank}(\mathbf{H}_{ij})\} < z_{ij} =$ rank$([\mathbf{U}_i^T \mathbf{H}_{ij}\ \mathbf{I}_{z_{ij}}])$, and then Eq. (1) has no solution. This means z_{ij} data streams can be transmitted only if $z_{ij} \leq r_{ij}$ is satisfied. By Eq. (1), (2) and Lemma 1, the number of DoFs consumed by SM at Rx node j is z_{ij}.

Following the same token, one can show that at Tx node i, the number of DoFs consumed for SM is also z_{ij}. ∎

C. DoF Consumption for IC under Rank-deficient Channels

Consider a single-interference case as shown in Fig. 3, Tx nodes i and k are transmitting z_{ij} and z_{kl} data streams to Rx nodes j and l, respectively, where $z_{ij} \geq 1, z_{kl} \geq 1$, and Rx node j is interfered with by Tx node k, rank$(\mathbf{H}_{kj}) = r_{kj}$. Suppose channel matrix $\mathbf{H}_{kj}$ is of general rank condition, (i.e., $\mathbf{H}_{kj}$ may be rank deficient). Then how to cancel the interference from k to j so that data streams z_{ij} can be received at Rx node j free of interference?

Denote $\mathbf{1}_{kj}^{\text{R}}$ and $\mathbf{1}_{kj}^{\text{T}}$ as binary variables with the following definitions: $\mathbf{1}_{kj}^{\text{R}} = 1$ if Rx node j consumes DoFs for IC and $\mathbf{1}_{kj}^{\text{R}} = 0$ if it does not; $\mathbf{1}_{kj}^{\text{T}} = 1$ if Tx node k consumes DoFs for IC and $\mathbf{1}_{kj}^{\text{T}} = 0$ if it does not. Then following theorem shows how the interference is cancelled by consuming DoFs at Tx node k and Rx node j.

Theorem 1 *For the single-interference case, let Tx node k consume d_{kj}^{T} DoFs and Rx node j consume d_{kj}^{R} DoFs for IC. Then interference from Tx node k to Rx node j is cancelled if*

$$d_{kj}^{\mathrm{R}}\mathbf{1}_{kj}^{\mathrm{R}} + d_{kj}^{\mathrm{T}}\mathbf{1}_{kj}^{\mathrm{T}} = \min\left\{z_{kl}\mathbf{1}_{kj}^{\mathrm{R}} + z_{ij}\mathbf{1}_{kj}^{\mathrm{T}}, \quad r_{kj}\right\}, \quad (3a)$$

$$\left(\mathbf{1}_{kj}^{\mathrm{R}}, \mathbf{1}_{kj}^{\mathrm{T}}\right) \neq (0,0). \quad (3b)$$

We offer a sketch of proof here. The details can be found by [20].

A Sketch of Proof To guarantee interference-free transmission, the constraint $\mathbf{U}_k^T \cdot \mathbf{H}_{kj} \cdot \mathbf{V}_j = \mathbf{0}$ must be satisfied. Theorem 1 can be proved by enumerating all possibilities of $\left(\mathbf{1}_{kj}^{\mathrm{R}}, \mathbf{1}_{kj}^{\mathrm{T}}\right)$. As a first case: only Rx node j consumes DoFs for IC, i.e., $\left(\mathbf{1}_{kj}^{\mathrm{R}}, \mathbf{1}_{kj}^{\mathrm{T}}\right) = (1,0)$. This means we impose constraint $\mathbf{U}_k^T \cdot \mathbf{H}_{kj} \cdot \mathbf{V}_j = \mathbf{0}$ on $\mathbf{V}_j$. We have rank $\left(\left[\mathbf{U}_k^T \cdot \mathbf{H}_{kj} \quad \mathbf{0}\right]\right) \leq \min\{z_{kl}, r_{kj}\}$. Since $\mathbf{H}_{kj}$ is generic, without "special treatment" (third case) on $\mathbf{U}_k$, we have to consider the upper bound $\min\{z_{kl}, r_{kj}\}$ to guarantee interference-free transmission. Thus according to Lemma 1, the number of DoFs consumed for IC at Rx node j is $\min\{z_{kl}, r_{kj}\}$. As a second case: only Tx node k consumes DoFs for IC, i.e., $\left(\mathbf{1}_{kj}^{\mathrm{R}}, \mathbf{1}_{kj}^{\mathrm{T}}\right) = (0,1)$. Similar to the first case, the number of DoFs consumed for IC at Tx node j is $\min\{z_{ij}, r_{kj}\}$. For the third case: let both Tx node k and Rx node j consume DoFs for IC. i.e., $\left(\mathbf{1}_{kj}^{\mathrm{R}}, \mathbf{1}_{kj}^{\mathrm{T}}\right) = (1,1)$. Obviously, if $z_{kl} + z_{ij} \leq r_{kj}$, Theorem 1 is trivial and can be proved based on the same analysis as the first two cases. Now consider $z_{kl} + z_{ij} > r_{kj}$. According to Sylvester's rank inequality, we have rank $\left(\left[\mathbf{U}_k^T \cdot \mathbf{H}_{kj}\right]\right) \geq z_{kl} + r_{kj} - N_k$. We can force the rank of $\left[\mathbf{U}_k^T \cdot \mathbf{H}_{kj}\right]$ to be at most $r', (z_{kl} + r_{kj} - N_k \leq r')$, by adding $r_{kj} - r'$ linear independent constraints on $\mathbf{U}_k^T$. As a consequence, $r_{kj} - r'$ DoFs are consumed at node k. Next, since the rank of $\left[\mathbf{U}_k^T \cdot \mathbf{H}_{kj} \quad \mathbf{0}\right]$ is at most r', we can use r' DoFs at node j to force $\mathbf{U}_k^T\mathbf{H}_{kj}\mathbf{V}_j = \mathbf{0}$ according to Lemma 1. Thus we have $d_{kj}^{\mathrm{R}} + d_{kj}^{\mathrm{T}} = r_{kj}$.

Theorem 1 shows that to cancel an interference from Tx node k to Rx node j, DoFs can be consumed at Tx node k and/or Rx node j. The required number of DoFs consumed at Tx node k and Rx node j are related to the number of data streams and rank of the interference channel. By enumerating all possibilities of $\left(\mathbf{1}_{kj}^{\mathrm{R}}, \mathbf{1}_{kj}^{\mathrm{T}}\right)$ in (3b), IC can be done by one of the following scenarios:

- $\left(\mathbf{1}_{kj}^{\mathrm{R}}, \mathbf{1}_{kj}^{\mathrm{T}}\right) = (1,0)$, i.e., only Rx node j consumes DoFs for IC and the number of DoFs Rx node j consumes is $\min\{z_{kl}, r_{kj}\}$.
- $\left(\mathbf{1}_{kj}^{\mathrm{R}}, \mathbf{1}_{kj}^{\mathrm{T}}\right) = (0,1)$, i.e., only Tx node k consumes DoFs for IC and the number of DoFs Tx node k consumes is $\min\{z_{ij}, r_{kj}\}$.
- $\left(\mathbf{1}_{kj}^{\mathrm{R}}, \mathbf{1}_{kj}^{\mathrm{T}}\right) = (1,1)$, i.e., both Tx node k and Rx node j consume DoFs for IC. If $z_{kl} + z_{ij} \leq r_{kj}$, then $d_{kj}^{\mathrm{R}} + d_{kj}^{\mathrm{T}} = z_{kl} + z_{ij}$. That is, a total of $z_{kl} + z_{ij}$ DoFs are used for IC, which is more than that in the previous two cases (either transmitter or receiver). On the other hand, if $r_{kj} < z_{kl} + z_{ij}$, it is possible to design $\mathbf{U}_k^T$ and $\mathbf{V}_j^T$ such that $(r_{kj} - x)$ DoFs are consumed at Tx node k to guarantee the rank of $\left[\mathbf{U}_k^T \cdot \mathbf{H}_{kj}\right]$ is at most x. Then Rx node j will consume x DoFs to cancel this interference. Thus we have $d_{kj}^{\mathrm{R}} + d_{kj}^{\mathrm{T}} = r_{kj}$. This shows that a shared DoF consumption between Tx and Rx for IC is most efficient under rank-deficient conditions.

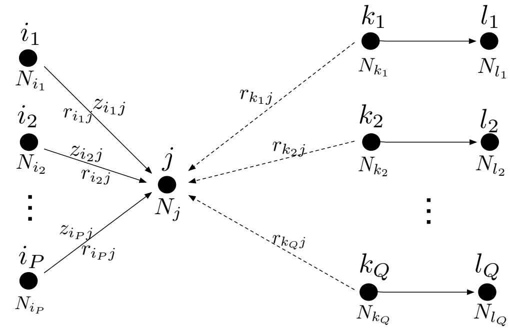


Fig. 4: Additivity of DoF consumption at receiver.

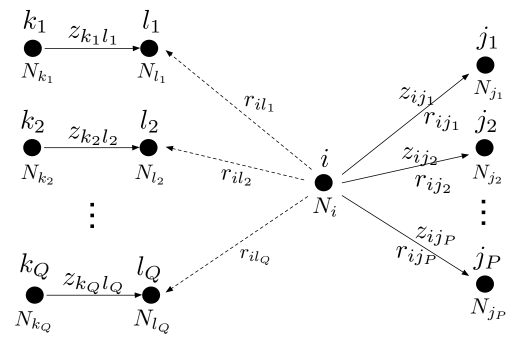


Fig. 5: Additivity of DoF consumption at transmitter.

As an example, let's re-visit the motivating example in Section I (see Fig. 1). First, $(z_{ij}, z_{kl}) = (5,7)$ is a feasible solution and can be realized by the first case, i.e., $\left(\mathbf{1}_{kj}^{\mathrm{R}}, \mathbf{1}_{kj}^{\mathrm{T}}\right) = (1,0)$, because Rx node j can consume 5 DoFs for SM and 7 DoFs for IC, and Tx node k uses 7 DoFs for SM. Second, $(z_{ij}, z_{kl}) = (5,7)$ can also be designed under second case, i.e., $\left(\mathbf{1}_{kj}^{\mathrm{R}}, \mathbf{1}_{kj}^{\mathrm{T}}\right) = (0,1)$, where Tx node k consumes 7 DoFs for SM and 5 DoFs for IC, and Rx node j consumes 5 DoFs for SM. Following the same token, we can find a feasible region of the inner pentagon in Fig. 1(b). However, for $(z_{ij}, z_{kl}) = (8,7)$, it is impossible to have only Tx node k or Rx node j alone to cancel this interference. But if we let Rx node j consumes 8 DoFs for SM and 4 DoFs for IC, and Tx node k consumes 7 DoFs for SM and 5 DoFs for IC, then the condition in the third case (i.e., $\left(\mathbf{1}_{kj}^{\mathrm{R}}, \mathbf{1}_{kj}^{\mathrm{T}}\right) = (1,1)$) will be satisfied and we have a feasible solution. That is, under rank-deficient condition, a shared DoF consumption between both Tx node k and Rx node j can offer more feasible solutions than unilateral IC by only Tx node or Rx node. The outer pentagon in Fig. 1(b) shows the extra feasible DoF region.

D. Extension to Multiple Links and Additivity Property

The results in the previous two sections show DoF consumption for SM on a single link and IC between a Tx node and a Rx node. Using these results as basic building blocks, we explore DoF consumption for the general multiple-link case in this section. Consider Fig. 4, where Tx nodes $i_1, i_2, ..., i_P$ are transmitting $z_{i_1 j}, z_{i_2 j}, ..., z_{i_P j}$ data streams to Rx node j, respectively. Denote $z_{*j} = \sum_{m=1}^{P} z_{i_m j}$. Rx node j is also interfered with by Tx nodes $k_1, k_2, .., k_Q$ simultaneously. Suppose Tx nodes $k_1, k_2, .., k_Q$ are transmitting $z_{k_1 l_1}, z_{k_2 l_2}, ..., z_{k_Q l_Q}$ data streams to their respective receivers. Suppose the number of consumed DoFs at Rx node j for cancelling interference from k_n to j is $d^{\mathrm{R}}_{k_n j}$, where $d^{\mathrm{R}}_{k_n j}\mathbf{1}^{\mathrm{R}}_{k_n j} + d^{\mathrm{T}}_{k_n j}\mathbf{1}^{\mathrm{T}}_{k_n j} = \min\left\{z_{k_n l_n}\mathbf{1}^{\mathrm{R}}_{k_n j} + z_{*j}\mathbf{1}^{\mathrm{T}}_{k_n j},\ r_{k_n j}\right\}$, $\left(\mathbf{1}^{\mathrm{R}}_{k_n j}, \mathbf{1}^{\mathrm{T}}_{k_n j}\right) \neq (0,0)$, $k_n = k_1, k_2, ..., k_Q$. The following Lemma shows the required DoF consumption at Rx node j.

Lemma 3 *In a general multi-link case for a Rx node j, the number of consumed DoFs for SM and IC at Rx node j is additive and constrained by channel ranks. If $z_{i_m j} \leq r_{i_m j}$ for $m = 1, 2, ..., P$ are satisfied, then the number of consumed DoFs for SM at Rx node j is z_{*j}, i.e., $\sum_{m=1}^{P} z_{i_m j}$. The number of consumed DoFs for IC at Rx node j is $d^{\mathrm{R}}_{*j} = \sum_{n=1}^{Q} d^{\mathrm{R}}_{k_n j}$. The total number of consumed DoFs for SM and IC at Rx node j is $z_{*j} + d^{\mathrm{R}}_{*j}$.*

Proof Supposing Rx node j consumes $d^{\mathrm{R}}_{k_n j}$ DoFs to cancel interference from Tx node k_n to Rx node j, $n = 1, 2, ..., Q$. According to Lemma 1 and Theorem 1, we have $\mathrm{rank}([\mathbf{U}^T_{k_1}\mathbf{H}_{k_1 j}]) \leq d^{\mathrm{R}}_{k_1 j}$, $\mathrm{rank}([\mathbf{U}^T_{k_2}\mathbf{H}_{k_2 j}]) \leq d^{\mathrm{R}}_{k_2 j}$,..., $\mathrm{rank}([\mathbf{U}^T_{k_Q}\mathbf{H}_{k_Q j}]) \leq d^{\mathrm{R}}_{k_Q j}$. Now considering general multi-link case for a Rx node j. Weight matrix $\mathbf{V}_j$ of node j must satisfy

$$\begin{bmatrix} \mathbf{U}^T_{i_1}\mathbf{H}_{i_1 j} \\ \mathbf{U}^T_{i_2}\mathbf{H}_{i_2 j} \\ \vdots \\ \mathbf{U}^T_{i_P}\mathbf{H}_{i_P j} \\ \mathbf{U}^T_{k_1}\mathbf{H}_{k_1 j} \\ \mathbf{U}^T_{k_2}\mathbf{H}_{k_2 j} \\ \vdots \\ \mathbf{U}^T_{k_Q}\mathbf{H}_{k_Q j} \end{bmatrix} \mathbf{V}_j = \begin{bmatrix} \mathbf{I}_{z_{i_1 j}} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{I}_{z_{i_2 j}} & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{I}_{z_{i_P j}} \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} \end{bmatrix} \tag{4}$$

Thus we have

$$\begin{aligned} &\mathrm{rank}\left(\begin{bmatrix} \mathbf{U}^T_{i_1}\mathbf{H}_{i_1 j} & \mathbf{I}_{z_{i_1 j}} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{U}^T_{i_2}\mathbf{H}_{i_2 j} & \mathbf{0} & \mathbf{I}_{z_{i_2 j}} & \cdots & \mathbf{0} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \mathbf{U}^T_{i_P}\mathbf{H}_{i_P j} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{I}_{z_{i_P j}} \\ \mathbf{U}^T_{k_1}\mathbf{H}_{k_1 j} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{U}^T_{k_2}\mathbf{H}_{k_2 j} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \mathbf{U}^T_{k_Q}\mathbf{H}_{k_Q j} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} \end{bmatrix}\right) \\ &\leq z_{i_1 j} + z_{i_2 j} ... + z_{i_P j} + d^{\mathrm{R}}_{k_1 j} + d^{\mathrm{R}}_{k_2 j} + ... + d^{\mathrm{R}}_{k_Q j} \\ &\leq z_{*j} + d^{\mathrm{R}}_{*j}. \end{aligned} \tag{5}$$

The first z_{*j} rows are with full rank z_{*j}; the remaining rows may have rank lower than d^{R}_{*j} and we consider the upper bound. According to Eq. (4), (5) and Lemma 1, the number of DoFs consumed for SM and IC at Rx node j is $z_{*j} + d^{\mathrm{R}}_{*j}$. Note that $z_{i_m j} \leq r_{i_m j}, m = 1, 2, ..., P$ must be satisfied or otherwise Eq. (4) has no feasible solution. ∎

Next we consider the case of SM and IC at Tx node i as shown in Fig. 5, where Tx node i is transmitting $z_{ij_1}, z_{ij_2}, ..., z_{ij_P}$ data streams to Rx nodes $j_1, j_2, ..., j_P$, respectively. Denote $z_{i*} = \sum_{m=1}^{P} z_{ij_m}$. Tx node i is also interfering with Rx nodes $l_1, l_2, .., l_Q$. Suppose Rx nodes $l_1, l_2, .., l_Q$ are receiving $z_{k_1 l_1}, z_{k_2 l_2}, ..., z_{k_Q l_Q}$ data streams from Tx nodes $k_1, k_2, ..., k_Q$, respectively.. Supposing the number of consumed DoFs at Tx node i for cancelling interference from i to l_n is $d^{\mathrm{T}}_{il_n}$, where $d^{\mathrm{R}}_{il_n}\mathbf{1}^{\mathrm{R}}_{il_n} + d^{\mathrm{T}}_{il_n}\mathbf{1}^{\mathrm{T}}_{il_n} = \min\left\{z_{i*}\mathbf{1}^{\mathrm{R}}_{il_n} + z_{k_n l_n}\mathbf{1}^{\mathrm{T}}_{il_n},\ r_{il_n}\right\}$, $\left(\mathbf{1}^{\mathrm{R}}_{il_n}, \mathbf{1}^{\mathrm{T}}_{il_n}\right) \neq (0,0)$, $l_n = l_1, l_2, ..., l_Q$. The following Lemma shows the required DoF consumption at Tx node i.

Lemma 4 *In a general multi-link case for a Tx node i, the number of consumed DoFs for SM and IC at Tx node i is additive and constrained by channel ranks. If $z_{ij_m} \leq r_{ij_m}$ for $m = 1, 2, ..., P$, then the number of consumed DoFs for SM at Tx node i is $z_{i*} = \sum_{m=1}^{P} z_{ij_m}$. The number of consumed DoFs for IC at Tx node i is $d^{\mathrm{T}}_{i*} = \sum_{n=1}^{Q} d^{\mathrm{T}}_{il_n}$. The total number of consumed DoFs for SM and IC at Tx node i is $z_{i*} + d^{\mathrm{T}}_{i*}$.*

The proof of Lemma 4 is similar to Lemma 3 and is omitted to conserve space.

III. A Special Case: Full-rank Channels

In this section, we show that, under full-rank condition, our general DoF model degenerates into the well-known unilateral DoF consumption model in the literature (e.g., [1, 3, 4, 7, 9, 10]). Therefore, existing full-rank DoF model is a special case of our model.

For SM, consider a single link transmission (see Fig. 2). If the channel matrix is of full rank, i.e. $\mathrm{rank}(\mathbf{H}_{ij}) = \min\{N_i, N_j\}$, then at most $\min\{N_i, N_j\}$ data streams can be transmitted over this link, and the number of DoFs consumed by SM at Tx node i and Rx node j are both z_{ij}.

For IC, consider the single-interference link (see Fig. 3). Now we suppose channel matrix $\mathbf{H}_{kj}$ is of full rank, i.e., $\mathrm{rank}(\mathbf{H}_{kj}) = \min\{N_k, N_j\}$.

Scheme 1: IC by Tx or Rx node, but not both: As shown in the literature (e.g. [3, 4, 7, 9, 10]), IC can is done unilaterally at either Rx node j by consuming z_{kl} DoFs, or at Tx node k by consuming z_{ij} DoFs. Without loss of generality, assume $N_k \leq N_j$, then we have $r_{kj} = \min\{N_k, N_j\} = N_k$.

Case 1: Rx node j consumes DoFs for IC. Since $r_{kj} = N_k \geq z_{kl}$, this is consistent to Theorem 1 where $\left(\mathbf{1}^{\mathrm{R}}_{kj}, \mathbf{1}^{\mathrm{T}}_{kj}\right) = (1, 0)$.

Case 2: Tx node k consumes DoFs for IC. Since $z_{ij} \geq 1, z_{kl} \geq 1$, the available DoFs at Tx node k for IC is no more than $N_k - 1$. Consequently the number of data streams that can be received at Rx node j is no more than $N_k - 1$, i.e., $z_{ij} \leq N_k - 1$. We have $z_{ij} < r_{kj}$. This is consistent to Theorem 1 where $\left(\mathbf{1}^{\mathrm{R}}_{kj}, \mathbf{1}^{\mathrm{T}}_{kj}\right) = (0, 1)$.

Scheme 2: IC by both Tx and Rx nodes: In this case, interference is cancelled at Rx node j by consuming z_{kl} DoFs, and at Tx node k by consuming z_{ij} DoFs as in [1]. Obviously $z_{ij} + z_{kl} \geq \min\left\{z_{kl}\mathbf{1}^{\mathrm{R}}_{kj} + z_{ij}\mathbf{1}^{\mathrm{T}}_{kj}, r_{kj}\right\}$. We can let $d^{\mathrm{R}}_{kj} = z_{kl}, d^{\mathrm{T}}_{kj} = z_{ij}, \left(\mathbf{1}^{\mathrm{R}}_{kj}, \mathbf{1}^{\mathrm{T}}_{kj}\right) = (1, 1)$, which will satisfy the sufficient condition in Theorem 1. Although feasible, this scheme uses more DoFs than necessary and is considered wasteful.

The next question to ask is: in full-rank case, is it possible for Tx and Rx nodes to share DoF consumption for IC such that Tx node k consumes fewer than z_{ij} DoFs and Rx node j consumes fewer than z_{kl} DoFs as in rank-deficient case? The answer to this question is given in the following Lemma.

Lemma 5 *In full-rank case, to cancel interference from Tx node k to Rx node j (as shown in Fig. 3), it is infeasible to have Tx node k consume fewer than z_{ij} DoFs and Rx node j consume fewer than z_{kl} DoFs, where $z_{ij} \geq 1, z_{kl} \geq 1$.*

We offer a proof sketch here. Details can be found in [20].

A Sketch of Proof Suppose Tx node k consumes x DoFs and Rx node j consumes y DoFs to cancel interference from Tx node k to Rx node j, where $x < z_{ij}$ and $y < z_{kl}$. We must have $\text{rank}\left(\left[\mathbf{V}_j^T \cdot \mathbf{H}_{kj}^T \quad \mathbf{0}\right]\right) = x < z_{ij}$, and $\text{rank}\left(\left[\mathbf{U}_k^T \cdot \mathbf{H}_{kj} \quad \mathbf{0}\right]\right) = y < z_{kl}$. However, according to Sylvester's rank inequality, we have $\text{rank}\left(\left[\mathbf{V}_j^T \cdot \mathbf{H}_{kj}^T \quad \mathbf{0}\right]\right) \geq z_{ij} + \min\{N_k, N_j\} - N_j$, and $\text{rank}\left(\left[\mathbf{U}_k^T \cdot \mathbf{H}_{kj} \quad \mathbf{0}\right]\right) \geq z_{kl} + \min\{N_k, N_j\} - N_k$. This is a contradiction.

Lemma 5 shows that in full-rank case, there is no benefit to have both Tx node and Rx node consume DoFs for IC, as doing so will incur more DoF consumption than necessary. That is why existing DoF IC models only consider using DoFs unilaterally at either Tx or Rx node for IC, but not both. But under rank deficient conditions, situation is different, as we have shown in Theorem 1.

IV. DoF Scheduling in a Network

In previous sections, we establish a general model for DoF consumption under rank-deficient conditions. In this section, we apply this model for DoF scheduling in a general rank-deficient MIMO network.

Node Activity and SM Constraints We assume each node in the network is half-duplex, i.e., a node can be either a Tx node, a Rx node, or idle at any time. Define a binary variable $x_i(t)$ to indicate whether or not node i is a Tx node at time t, i.e., $x_i(t) = 1$ if node i is transmitting at time t and 0 otherwise. Likewise, denote $y_i(t)$ as a binary variable to indicate whether or not node i is a Rx node at time t, i.e., $y_i(t) = 1$ if node i is receiving at time t and 0 otherwise. Then half-duplex constraint can be modeled as:

$$x_i(t) + y_i(t) \leq 1, \quad i \in \mathcal{K}, \tag{6}$$

where $\mathcal{K}$ is the set of nodes in the network.

Denote $\mathcal{T}_i$ as the set of nodes that are within node i's transmission range. If node i is an active Tx node (i.e., $x_i(t) = 1$), then the total number of DoFs used for transmission cannot exceed the total number of antennas N_i at this node, i.e.,

$$x_i(t) \leq \sum_{j \in \mathcal{T}_i} z_{ij}(t) \leq N_i x_i(t), \quad i \in \mathcal{K}. \tag{7}$$

Similarly, if a node j is an active Rx node (i.e., $y_j(t) = 1$), then the total number of DoFs used for transmission cannot exceed the total number of antennas N_j at this node, i.e.,

$$y_j(t) \leq \sum_{i \in \mathcal{T}_j} z_{ij}(t) \leq N_j y_j(t), \quad j \in \mathcal{K}. \tag{8}$$

Further, considering channel rank condition, the number of data streams that can be sent over a channel must satisfy the following constraint (Lemma 2):

$$z_{ij}(t) \leq r_{ij}(t), \quad i \in \mathcal{K},\ j \in \mathcal{K},\ i \neq j. \tag{9}$$

IC Constraints For interference from Tx node i to Rx node j, denote d_{ij}^{T} as the number of DoFs consumed at Tx node i and d_{ij}^{R} as the number of DoFs consumed at Rx node j to cancel the interference from i to j. Denote $\mathcal{I}_i$ as the set of nodes within node i's interference range. By Theorem 1, for every interference from Tx node i to Rx node j, the following constraints must be satisfied:

$$\text{If } x_i(t) = 1 \text{ and } y_j(t) = 1, \text{ then}$$

$$d_{ij}^{\mathrm{T}}(t)\mathbf{1}_{ij}^{\mathrm{T}}(t) + d_{ij}^{\mathrm{R}}(t)\mathbf{1}_{ij}^{\mathrm{R}}(t) = \min\left\{\mathbf{1}_{ij}^{\mathrm{R}}(t)\sum_{k \in \mathcal{T}_i}^{k \neq j} z_{ik}(t) + \mathbf{1}_{ij}^{\mathrm{T}}(t)\sum_{k \in \mathcal{T}_j}^{k \neq i} z_{kj}(t),\ r_{ij}(t)\right\}, \tag{10a}$$

$$\left(\mathbf{1}_{ij}^{\mathrm{T}}(t), \mathbf{1}_{ij}^{\mathrm{R}}(t)\right) \neq (0, 0),\ i \in \mathcal{K}, j \in \mathcal{I}_i \tag{10b}$$

We can relax Eq. (10a) by substituting equal sign as greater-than-equal sign. Then by incorporating $x_i(t)$ and $y_j(t)$ into (10), constraint (10) can be re-written as

$$[2 - x_i(t) - y_j(t)]r_{ij}(t) + x_i(t)y_j(t)\left(d_{ij}^{\mathrm{T}}(t)\mathbf{1}_{ij}^{\mathrm{T}}(t) + d_{ij}^{\mathrm{R}}(t)\mathbf{1}_{ij}^{\mathrm{R}}(t)\right) \geq \min\left\{\mathbf{1}_{ij}^{\mathrm{R}}(t)\sum_{k \in \mathcal{T}_i}^{k \neq j} z_{ik}(t) + \mathbf{1}_{ij}^{\mathrm{T}}(t)\sum_{k \in \mathcal{T}_j}^{k \neq i} z_{kj}(t),\ r_{ij}(t)\right\}, \tag{11a}$$

$$\mathbf{1}_{ij}^{\mathrm{T}}(t) + \mathbf{1}_{ij}^{\mathrm{R}}(t) \geq x_i(t) + y_j(t) - 1,\ i \in \mathcal{K}, j \in \mathcal{I}_i. \tag{11b}$$

By employing the *Reformulated-Linearization Technique* (RLT) [21], Eq. (11a) can be reformulated as a set of mixed integer linear (MIL) constraints [20]. We omit the details to conserve space.

Node's DoF Constraints A node can use its DoFs for both SM and IC, as long as the total number of consumed DoFs does not exceed the total available DoFs at the node. If node i is an active Tx node, by Lemmas 3 and 4, we have

$$\text{If } x_i(t) = 1, \text{ then } \sum_{k \in \mathcal{T}_i} z_{ik}(t) + \sum_{j \in \mathcal{I}_i} d_{ij}^{\mathrm{T}}(t) \leq N_i, \quad i \in \mathcal{K}. \tag{12}$$

Similarly, if node j is an active Rx node, we have

$$\text{If } y_j(t) = 1, \text{ then } \sum_{k \in \mathcal{T}_j} z_{kj}(t) + \sum_{i \in \mathcal{I}_j} d_{ij}^{\mathrm{R}}(t) \leq N_j, \quad j \in \mathcal{K}. \tag{13}$$

For constraint (12), it can be reformulated by incorporating binary variable $x_i(t)$ into the expression as follows:

$$\sum_{k \in \mathcal{T}_i} z_{ik}(t) + \sum_{j \in \mathcal{I}_i} d_{ij}^{\mathrm{T}}(t) \leq N_i x_i(t) + (1 - x_i(t))B_i, \quad i \in \mathcal{K}, \tag{14}$$

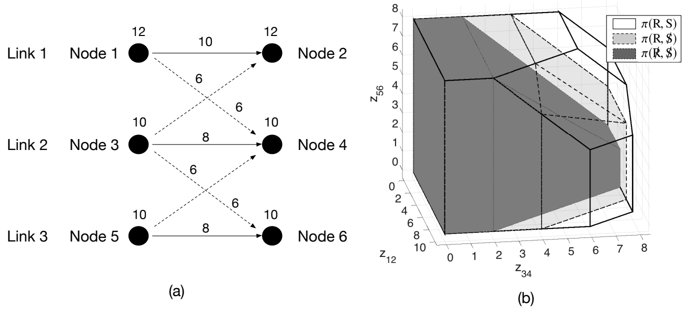


Fig. 6: A study of DoF region for a three-link example. (a) Transmission and interference topology, number of antennas at each node, and rank of each link. (b) DoF region obtained under different models.

where B_i is a large constant, which can be set as $B_i = \sum_{j\in\mathcal{I}_i} N_j$ to ensure that B_i is an upper bound of $\sum_{j\in\mathcal{I}_i} d^{\mathrm{T}}_{ij}(t)$.

Similarly, constraint (13) can be reformulated as follows:

$$\sum_{k\in\mathcal{T}_j} z_{kj}(t) + \sum_{i\in\mathcal{I}_j} d^{\mathrm{R}}_{ij}(t) \le N_j y_j(t) + (1-y_j(t))B_j, \quad j \in \mathcal{K}. \quad (15)$$

To recap, (6)-(9), (11), (14) and (15) serve as a set of feasibility constraints for SM and IC. When outfitted with a proper objective function (based on a specific application, e.g., the example in Section V-B), we have an optimization problem involving a DoF scheduling model with rank-deficient channels.

V. Case Studies

In this section, we use examples to illustrate the DoF regions obtained by our general model and compare them to those obtained by other models. We also apply our general model for DoF scheduling in MIMO networks and demonstrate its efficacy. For ease of comparison, we define three models with the following notation:

- *Rank-blind non-shared DoF consumption model, denoted as* $\pi(\not{\mathrm{R}},\not{\mathrm{S}})$. Under this model, IC is done unilaterally either by Tx node or Rx node, as in [3, 4, 7, 9, 10].
- *Rank-aware non-shared DoF consumption model, denoted as* $\pi(\mathrm{R},\not{\mathrm{S}})$. Under this model, IC is done unilaterally either by Tx node or Rx node. But the number of DoFs consumed for IC takes into consideration of channel rank deficiency.
- *Rank-aware shared DoF consumption model, denoted as* $\pi(\mathrm{R},\mathrm{S})$. This is our general model, where DoF consumption for IC is shared between Tx node and Rx node. This is the most efficient IC model under general rank-deficient channels.

A. Comparison of DoF Regions

We now perform numerical study of DoF regions for some examples. We will show that our general DoF model not only ensures feasibility, but also has the potential to expand DoF region. We have already illustrated these for the two-link example in Fig. 1. Here, we will consider a few more examples.

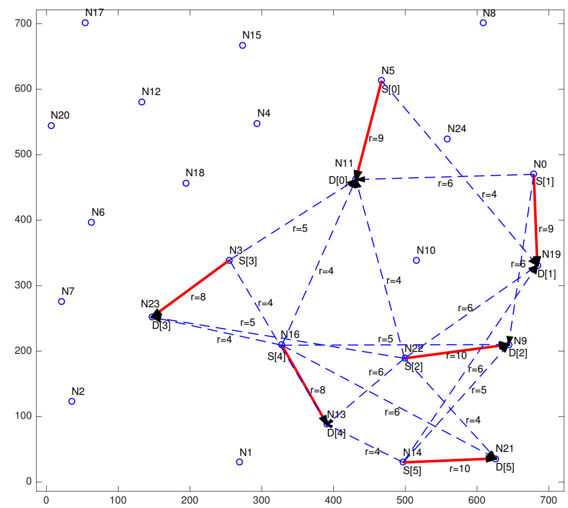


Fig. 7: Topology of a 25-node network

Fig. 6(a) shows a three-link example, where links 1 and 2 are interfering with each other and links 2 and 3 are interfering with each other. Suppose that the number of antennas at both the Tx and the Rx of link 1 are 12, the number of antennas at both the Tx and the Rx of link 2 and link 3 are 10. Also, suppose the rank of each channel is given as shown in Fig. 6(a).

By examining all possible solutions under our general DoF model, the DoF region obtained by $\pi(\mathrm{R},\mathrm{S})$ is shown in Fig. 6(b) (a polyhedron). We also show the DoF regions of $\pi(\not{\mathrm{R}},\not{\mathrm{S}})$ and $\pi(\mathrm{R},\not{\mathrm{S}})$ in Fig. 6(b), both of which are polyhedrons and are inside the DoF region of $\pi(\mathrm{R},\mathrm{S})$. The DoF region by $\pi(\mathrm{R},\mathrm{S})$ is 51.2% and 14.3% larger than those under $\pi(\not{\mathrm{R}},\not{\mathrm{S}})$ and $\pi(\mathrm{R},\not{\mathrm{S}})$, respectively.

We have also compared DoF regions among $\pi(\not{\mathrm{R}},\not{\mathrm{S}})$, $\pi(\mathrm{R},\not{\mathrm{S}})$, and $\pi(\mathrm{R},\mathrm{S})$ for 4 and 5-link cases and have the same conclusion, i.e., $\pi(\mathrm{R},\mathrm{S})$ offers the largest feasible DoF region [20]. Note that when the number of links are more than three, a higher dimensional visualization is needed to present the envelope of DoF region and calculate hypervolume within the respective DoF regions.

B. DoF Scheduling for Multi-link Networks

To show how the general rank-deficient DoF model ($\pi(\mathrm{R},\mathrm{S})$) can be used for studying MIMO networks, we study a throughput maximization problem. Suppose we want to maximize the minimum rate c_{min} for a set of links $\mathcal{L}$ in a multi-link MIMO network. For ease of exposition, we assume that one data stream corresponds to one unit data rate, and use normalized unit for distance. The transmission and interference ranges are 180 and 360, respectively. The problem formulation becomes a mixed integer linear programming (MILP) as follows:

$$\begin{aligned} \text{maximize} \quad & c_{min} \\ \text{s.t.} \quad & \text{Node activity and SM constraints: (6) – (9);} \\ & \text{IC constraints: (11);} \\ & \text{Node's DoF constraints: (14), (15).} \end{aligned}$$

Consider a multi-link MIMO network topology in Fig. 7. Each node in the network is equipped with 16 antennas. There are six links transmitting simultaneously. The rank of each transmitting or interfering channel is indicated by r next to the channel. We use an off-the-shelf solver CPLEX to solve this MILP. We find the optimal objective value is 8. On the other hand, the optimal objective values obtained by $\pi(\not{R}, \not{S})$ and $\pi(R, \not{S})$ are 4 and 6, respectively. That is, c_{min} under $\pi(R, S)$ is 100% and 33.3% more than those under $\pi(\not{R}, \not{S})$ and $\pi(R, \not{S})$, respectively.

We have also generated other random topologies and all results are consistent, i.e., c_{min} under $\pi(R, S)$ is larger than under both $\pi(\not{R}, \not{S})$ and $\pi(R, \not{S})$. This affirms the importance of shared DoF consumption for IC at both Tx node and Rx node when there is rank deficiency in the channel.

VI. Conclusions

DoF-based models have become prevalent in the research community to analyze the behavior and capabilities of MIMO-based wireless networks. However, most of existing DoF-based models assume channel matrix is of full-rank, which is no longer valid as more and more antennas are employed at a node. This paper addresses this fundamental limitation in existing DoF-based models by considering rank deficiency. We develop a general model on how DoFs are consumed at transmitter and receiver for SM and IC under rank-deficient conditions. In particular, we show that a shared DoF consumption at both transmitter and receiver for IC is the most efficient and can achieve a larger DoF region than having only transmitter or receiver unilaterally consume DoFs for IC. Further, we show that DoF consumption under existing full rank assumption is a special case of our DoF model for rank-deficient channels. Based on the general DoF model, we also explored DoF allocation (scheduling) in a multi-link MIMO network. Our findings in this paper pave the way for further research on MIMO-based wireless networks with rank deficient channels.

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