Abstract—Cognitive radio (CR) is an enabling technology for efficient use of available spectrum and promises unprecedented flexibility in multi-hop wireless networking. This paper explores networking related issues associated with CRs. Specifically, we consider how to maximize the rates of a set of user communication sessions in a multi-hop CR-based wireless network. Due to potential interference at the physical layer, we find that it is essential to follow a cross-layer approach, with joint optimization at physical (power control), link (frequency band scheduling), and network (flow routing) layers. We give a mathematical characterization of this cross-layer optimization problem. We develop a centralized solution procedure based on the branch-and-bound framework. Using numerical results, we demonstrate the efficacy of the solution procedure and offer quantitative understanding on the joint optimization at different layers.

Index Terms—Cognitive radio, multi-hop wireless network, interference, cross-layer optimization.

I. INTRODUCTION

Cognitive radio (CR) is a revolution in radio technology that is enabled by advances in RF design, signal processing, and communications software [14]. It promises unprecedented flexibility in radio communications and efficient use of spectrum. The potential of CR has been recognized by the commercial sector as well as the military (e.g., JTRS program [10]) and public safety communications (e.g., SAFECOM [15]).

Our goal in this paper is to optimize network level performance of multi-hop CR networks. It is now well understood that network performance for such networks is tightly coupled with lower layer behaviors [17]. For instance, maximizing user throughput at the network level not only depends on flow routing, but also depends on the algorithms at link layer (e.g., frequency band assignment) and physical layer (e.g., power control). As a result, an optimal solution at the network level must be developed with joint consideration of multiple layers.

Recently, there is a growing interest in gaining understanding on multi-hop CR networks (see, e.g., [17], [18]). However, most of these work are based on the so-called “protocol interference model” [8]. Under such model, the notions of transmission range and interference range are used to determine the feasibility of successful transmission and the existence of interference. It is now well understood that such binary decision on interference modeling has its limitation. On the other hand, the so-called “physical model” is widely accepted as an accurate characterization of interference. Under physical model, a transmission is successful if and only if signal-to-interference-and-noise-ratio (SINR) at the intended receiver exceeds a certain threshold so that the transmitted signal can be decoded with an acceptable bit error rate (BER). Further, capacity calculation is based on SINR (via Shannon’s formula), which takes into account of interference due to simultaneous transmission at other nodes. Unfortunately, although physical model is accurate, there is much difficulty in carrying out analysis with physical model due to the computational complexity it involves, particularly when it comes to cross-layer optimization in a multi-hop network environment.

In this paper, we investigate networking problem for multi-hop CR networks. We employ the physical model for interference modeling and study the problem via cross-layer optimization approach. In particular, we consider how to maximize the rates of a set of user communication sessions, with joint consideration at physical layer (via power control), link layer (via frequency band scheduling), and network layer (flow routing). We give a mathematical characterization for these layers and formulate the problem into a mixed integer nonlinear program (MINLP). We develop a centralized solution to this complex optimization problem based on branch-and-bound (BB) framework and a reformulation-linearization technique (RLT) [16]. The solution we develop is guaranteed to be within a factor of $(1 - \varepsilon)$ from the optimum, where $\varepsilon$ is a small parameter reflecting our desired accuracy.

The remainder of this paper is organized as follows. In Section II, we give a mathematical characterization of power control, scheduling, and routing for multi-hop CR networks. We also present the problem formulation in this study. In Section III, we present a solution based on branch-and-bound framework. Section IV presents numerical results for the cross-layer solution. Section V reviews related work and Section VI concludes this paper.

II. MATHEMATICAL MODELS

We consider a CR-based ad hoc network with a set of nodes $\mathcal{N}$. For a node $i \in \mathcal{N}$, the set of available frequency bands $\mathcal{M}_i$ depends on its location and may not be identical to the available frequency bands at other nodes. We assume that the bandwidth of each frequency band (channel) is $W$. Denote $\mathcal{M}$ the set of all frequency bands present in the network, i.e., $\mathcal{M} = \bigcup_{i \in \mathcal{N}} \mathcal{M}_i$. Denote $\mathcal{M}_{ij} = \mathcal{M}_i \cap \mathcal{M}_j$, which is the set of frequency bands that is common on both nodes $i$ and $j$ and thus can be used for transmission between these two nodes. In the rest of this section, we present mathematical...
A Cross-Layer Approach to Multi-Hop Networking with Cognitive Radios

Cognitive radio (CR) is an enabling technology for efficient use of available spectrum and promises unprecedented flexibility in multi-hop wireless networking. This paper explores networking related issues associated with CRs. Specifically, we consider how to maximize the rates of a set of user communication sessions in a multi-hop CR-based wireless network. Due to potential interference at the physical layer, we find that it is essential to follow a cross-layer approach, with joint optimization at physical (power control), link (frequency band scheduling) and network (flow routing) layers. We give a mathematical characterization of this cross-layer optimization problem. We develop a centralized solution procedure based on the branch-and-bound framework. Using numerical results, we demonstrate the efficacy of the solution procedure and offer quantitative understanding on the joint optimization at different layers.
TABLE I
NOTATION.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \mathcal{N} )</td>
<td>The set of nodes in the network</td>
</tr>
<tr>
<td>( \mathcal{M}_i )</td>
<td>The set of available bands at node ( i \in \mathcal{N} )</td>
</tr>
<tr>
<td>( \mathcal{M} )</td>
<td>The set of frequency bands in the network</td>
</tr>
<tr>
<td>( \mathcal{M}_{ij} )</td>
<td>The set of frequency bands on link ( i \to j )</td>
</tr>
<tr>
<td>( W )</td>
<td>Bandwidth of a frequency band</td>
</tr>
<tr>
<td>( \mathcal{L} )</td>
<td>The set of active user communication sessions</td>
</tr>
<tr>
<td>( s(l), d(l) )</td>
<td>Source and destination nodes of session ( l \in \mathcal{L} )</td>
</tr>
<tr>
<td>( r(l) )</td>
<td>Minimum rate requirement of session ( l )</td>
</tr>
<tr>
<td>( P_{\text{max}} )</td>
<td>The maximum transmission power at a transmitter</td>
</tr>
<tr>
<td>( \eta )</td>
<td>Ambient Gaussian noise density</td>
</tr>
<tr>
<td>( g_{ij} )</td>
<td>Propagation gain from node ( i ) to node ( j )</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>The minimum required SINR</td>
</tr>
<tr>
<td>( T^m_i )</td>
<td>The set of nodes that can transmit to (and receive from) node ( i ) on band ( m )</td>
</tr>
<tr>
<td>( T_i )</td>
<td>The set of nodes that can transmit to (and receive from) node ( i )</td>
</tr>
<tr>
<td>( T^m_j )</td>
<td>The set of nodes that may make interference on band ( m ) at node ( j )</td>
</tr>
<tr>
<td>( x^m_{ij} )</td>
<td>Binary indicator to mark whether or not band ( m ) is used by link ( i \to j )</td>
</tr>
<tr>
<td>( f_{ij}(l) )</td>
<td>Data rate for session ( l ) on link ( i \to j )</td>
</tr>
<tr>
<td>( Q )</td>
<td>The number of transmission power levels</td>
</tr>
<tr>
<td>( q^m_i )</td>
<td>The transmission power level from node ( i ) to node ( j ) on band ( m )</td>
</tr>
<tr>
<td>( t^m_{ij} )</td>
<td>The transmission power level at node ( i ) on band ( m )</td>
</tr>
<tr>
<td>( s^m_{ij} )</td>
<td>The SINR from node ( i ) to node ( j ) on band ( m )</td>
</tr>
</tbody>
</table>

Notation for branch-and-bound procedure:

| \( \varepsilon \) | A small positive constant reflecting our desired accuracy in the final solution |
| \( \Omega_z \) | The set of all possible solutions in problem \( z \)                          |
| \( LB_{z, UB_z} \) | The lower and upper bounds of problem \( z \)                               |
| \( LB, UB \) | The maximum lower and upper bounds among all problems                        |
| \( \psi_z \) | The \((1 - \varepsilon)\) optimal solution                                    |

characterization of each layer in a multi-hop CR network. Table I lists all notation in this paper.

A. Scheduling and Power Control

Scheduling for transmission at each node in the network can be done either in time domain or frequency domain. In this paper, we consider scheduling in frequency domain in the form of assigning frequency bands (channels). To maximize the capacity, there may still be concurrent transmissions within the same channel (and thus interference).

Denote

\[
x^m_{ij} = \begin{cases} 1 & \text{if node } i \text{ transmits data to node } j \text{ on band } m, \\ 0 & \text{otherwise}. \end{cases}
\]

(1)

Due to interference, a node \( i \) can use a band \( m \) for transmitting to one node \( j \) or receiving from one node \( k \). That is,

\[
\sum_{k \in T^m_k} x^m_{ki} + \sum_{j \in T^m_j} x^m_{ij} \leq 1,
\]

(2)

where \( T^m_i \) is the set of all possible nodes that node \( i \) can transmit to (and receive from) on band \( m \) in the network.

For power control, we assume that the transmission power at a node can be tuned to a finite number of levels between 0 and \( P_{\text{max}} \). To model this discrete version of power control, we introduce an integer parameter \( Q \) that represents the total number of power levels to which a transmitter can be adjusted, i.e., \( 0, \frac{1}{Q} P_{\text{max}}, \frac{2}{Q} P_{\text{max}}, \ldots, P_{\text{max}} \). Denote \( q^m_{ij} \in \{0, 1, 2, \ldots, Q\} \) the integer power level. Clearly, when node \( i \) does not transmit data to node \( j \) on band \( m \), \( q^m_{ij} \) should be 0. Under the maximum allowed transmission power level \( Q \), we have

\[
q^m_{ij} \begin{cases} \leq Q & \text{if } x^m_{ij} = 1, \\ = 0 & \text{otherwise}. \end{cases}
\]

With joint consideration of \( x^m_{ij} \) and \( q^m_{ij} \), the above relationship can be re-written as

\[
q^m_{ij} \leq Qx^m_{ij},
\]

(3)

which shows that the coupling relationship between power control and scheduling.

As discussed earlier, to maximize capacity, there may be concurrent transmissions by different nodes on the same band. Under physical model, a transmission is successful if and only if the SINR at the receiving node exceeds a certain threshold, say \( \alpha \). Mathematically, for a transmission from node \( i \) to node \( j \) on band \( m \), when there is interference from concurrent transmissions on the same band, the SINR is

\[
x^m_{ij} = \frac{g_{ij} q^m_{ij} P_{\text{max}}}{\eta W + \sum_{k \in \mathcal{N}} \sum_{h \in T^m_k} g_{kh} q^m_{kh} P_{\text{max}}},
\]

where \( \eta \) is the ambient Gaussian noise density, \( g_{ij} \) is the propagation gain from node \( i \) to node \( j \).

Based on our model for scheduling, if \( x^m_{ij} = 1 \), then \( x^m_{ki} = 0 \) for \( k \in T^m_j \) and \( q^m_{kj} = 0 \) for \( k \in T^m_j \). As a result, by (3), \( q^m_{ki} = 0 \) and \( q^m_{kj} = 0 \). Then we have

\[
s^m_{ij} = \frac{g_{ij} q^m_{ij} P_{\text{max}}}{\eta W + \sum_{k \in \mathcal{N}} \sum_{h \in T^m_k} g_{kh} q^m_{kh}},
\]

Denote \( t^m_k = \sum_{h \in T^m_k} q^m_{kh} \). We have

\[
s^m_{ij} = \frac{g_{ij} q^m_{ij}}{\eta W + \sum_{k \in \mathcal{N}} \sum_{h \in T^m_k} g_{kh} t^m_k}. \tag{4}
\]

Note that this SINR computation also holds when \( q^m_{ij} = 0 \), i.e., when there is no transmission from node \( i \) to node \( j \) on band \( m \).

Recall that under physical model, a transmission from node \( i \) to node \( j \) on band \( m \) is successful if and only if \( s^m_{ij} \geq \alpha \). Then by (1), we can couple \( x^m_{ij} \) and \( s^m_{ij} \) as follows.

\[
x^m_{ij} = \begin{cases} 1 & \text{if } s^m_{ij} \geq \alpha, \\ 0 & \text{otherwise}. \end{cases}
\]

which can be written into the following equivalent relationship.

\[
s^m_{ij} \geq \alpha x^m_{ij}.
\]

B. Routing

In a multi-hop ad hoc network, we assume there is a set of \( \mathcal{L} \) active user communication (unicast) sessions. Denote \( s(l) \) and \( d(l) \) the source and destination nodes of session \( l \in \mathcal{L} \) and \( r(l) \) the minimum rate requirement (in b/s) of session \( l \). In our study, we aim to maximize a scaling factor \( K \) for all
session rates. That is, what is the maximum factor $K$ such that a rate of $K \cdot r(l)$ can be transmitted from $s(l)$ to $d(l)$ for each session $l \in \mathcal{L}$ in the network.

To route these data flows from its source node to destination node, multi-hop relaying is necessary, due to limited transmission power at each node. Further, for optimality and flexibility, it is desirable to allow flow splitting and multi-path routing. This is because a single path flow routing for a session is overly restrictive and is unlikely to guarantee optimal solution.

Mathematically, this can be modeled as follows. Denote $f_{ij}(l)$ the data rate on link $(i, j)$ that is attributed to session $l$, where $i \in \mathcal{N}, j \in \mathcal{T}_i = \bigcup_{m \in \mathcal{M}_i} T_i^m$. If node $i$ is the source node of session $l$, i.e., $i = s(l)$, then

$$\sum_{j \in \mathcal{T}_i} f_{ij}(l) = r(l)K . \tag{5}$$

If node $i$ is an intermediate relay node for session $l$, i.e., $i \neq s(l)$ and $i \neq d(l)$, then

$$j \neq s(l) \sum_{j \in \mathcal{T}_i} f_{ij}(l) = \sum_{k \in \mathcal{T}_i} f_{ki}(l) . \tag{6}$$

If node $i$ is the destination node of session $l$, i.e., $i = d(l)$, then

$$\sum_{k \in \mathcal{T}_i} f_{ki}(l) = r(l)K . \tag{7}$$

It can be easily verified that once (5) and (6) are satisfied, (7) must also be satisfied. As a result, it is sufficient to list only (5) and (6) in the formulation.

In addition to the above flow balance equations at each node $i \in \mathcal{N}$ for session $l \in \mathcal{L}$, the aggregated flow rates on each radio link cannot exceed this link’s capacity. For a link $i \rightarrow j$, we have

$$s(l) \neq j, d(l) \neq i \sum_{l \in \mathcal{L}} f_{ij}(l) \leq \sum_{m \in \mathcal{M}_{ij}} W \log_2(1 + s^m_{ij}) . \tag{8}$$

The constraint in (8) further illustrates the coupling relationship among flow routing, power control, and scheduling.

### C. Problem Formulation

Putting together all the constraints for scheduling, power control, and flow routing, we have the following complete problem formulation.

Max $\sum_{i \in \mathcal{T}_i^m} x^m_{ij} + \sum_{j \in \mathcal{T}_i^m} x^m_{ij} \leq 1 \quad (i \in \mathcal{N}, m \in \mathcal{M}_i) \tag{9}

q_{ij} - Q x_{ij}^m \leq 0 \quad (i \in \mathcal{N}, m \in \mathcal{M}_i, j \in \mathcal{T}_i^m) \tag{9}

\sum_{j \in \mathcal{T}_i^m} q_{ij}^m - t^m_i = 0 \quad (i \in \mathcal{N}, m \in \mathcal{M}_i) \tag{9}

\frac{\eta WQ}{T_{\text{max}}} s^m_{ij} + \sum_{k \in \mathcal{N}} g_{ij} t_k^m s^m_{ij} - g_{ij} q_{ij}^m = 0 \quad (i \in \mathcal{N}, m \in \mathcal{M}_i, j \in \mathcal{T}_i^m) \tag{10}

\alpha x^m_{ij} - s^m_{ij} \leq 0 \quad (i \in \mathcal{N}, m \in \mathcal{M}_i, j \in \mathcal{T}_i^m) \tag{10}

\sum_{i \in \mathcal{L}} f_{ij}(l) - \sum_{m \in \mathcal{M}_{ij}} W \log_2(1 + s^m_{ij}) \leq 0 \quad (i \in \mathcal{N}, j \in \mathcal{T}_i) \tag{11}

\sum_{j \in \mathcal{T}_i} f_{ij}(l) - r(l)K = 0 \quad (l \in \mathcal{L}, i = s(l)) \tag{12}

j \neq s(l) \sum_{j \in \mathcal{T}_i} f_{ij}(l) - \sum_{i \in \mathcal{T}_k} f_{ki}(l) = 0 \quad (l \in \mathcal{L}, i \in \mathcal{N}, i \neq s(l), d(l)) \tag{13}

x_{ij}^m \in \{0, 1\}, q_{ij}^m \in \{0, 1, 2, \cdots, Q\}, t^m_i, s^m_{ij} \geq 0 \quad (i \in \mathcal{N}, m \in \mathcal{M}_i, j \in \mathcal{T}_i^m) \tag{14}

K, f_{ij}(l) \geq 0 \quad (l \in \mathcal{L}, i \in \mathcal{N}, i \neq d(l), j \in \mathcal{T}_m, j \neq s(l)) \tag{15}
provide much improvement on LB and we can thus remove this problem from further consideration.

In the rest of this section, we present the key components in the branch-and-bound framework, which are problem specific and far from trivial.

**B. Linear Relaxation**

During each iteration of the branch-and-bound procedure, we need a linear relaxation technique to obtain an upper bound of the objective function.

For the polynomial term \( t_k^{m} x_{ij} \) in the problem formulation, we apply a novel method based on Reformulation-Linearization Technique (RLT) [16]. That is, we introduce a new variable \( u_{ij}^{m} \); replace \( t_k^{m} x_{ij} \) by \( u_{ij}^{m} \), and add RLT constraints on these variables. Suppose \( t_k^{m} \) and \( s_{ij}^{m} \) are bounded by \( (t_k^{m})_L \leq t_k^{m} \leq (t_k^{m})_U \) and \( (s_{ij}^{m})_L \leq s_{ij}^{m} \leq (s_{ij}^{m})_U \), respectively. Thus, we have:

\[
(t_k^{m})_L \cdot s_{ij}^{m} + (s_{ij}^{m})_U \cdot t_k^{m} - u_{ij}^{m} \leq (t_k^{m})_L \cdot (s_{ij}^{m})_U ,
\]

\[
(t_k^{m})_U \cdot s_{ij}^{m} + (s_{ij}^{m})_L \cdot t_k^{m} - u_{ij}^{m} \geq (t_k^{m})_U \cdot (s_{ij}^{m})_L ,
\]

\[
(t_k^{m})_L \cdot s_{ij}^{m} + (s_{ij}^{m})_U \cdot t_k^{m} - u_{ij}^{m} \geq (t_k^{m})_L \cdot (s_{ij}^{m})_U ,
\]

\[
(t_k^{m})_U \cdot s_{ij}^{m} + (s_{ij}^{m})_L \cdot t_k^{m} - u_{ij}^{m} \leq (t_k^{m})_U \cdot (s_{ij}^{m})_L .
\]

For the log term, we propose to employ three tangential supports, which is a convex hull linear relaxation. We first analyze the bounds for \( 1 + s_{ij}^{m} \). Then, we introduce a variable \( c_{ij}^{m} = \ln(1 + s_{ij}^{m}) \) and consider how to get a linear relaxation for \( y = \ln x \) over \( x_L \leq x \leq x_U \). This function can be bounded by four segments (or a convex hull), where segments I, II, and III are tangential supports and segment IV is the chord (see Fig. 1). In particular, three tangent segments are at \((x_L, \ln x_L)\), \((\beta, \ln \beta)\), and \((x_U, \ln x_U)\), where \( \beta = \frac{x_U \cdot x_U - \ln x_L}{x_U - x_L} \) is the horizontal location for the point intersects extended tangent segments I and III; segment IV is the segment that joins points \((x_L, \ln x_L)\) and \((x_U, \ln x_U)\). The convex region defined by the four segments can be described by the following four linear constraints:

\[
x_L \cdot y - x \leq x_L (\ln x_L - 1) ,
\]

\[
\beta \cdot y - x \leq \beta (\ln \beta - 1) ,
\]

\[
x_U \cdot y - x \leq x_U (\ln x_U - 1) ,
\]

\[
(Ux - x_L)(y + \ln x_L - \ln x_U) \geq x_U \ln x_U - x_L \ln x_U .
\]

As a result, the non-polynomial (log) term can also be relaxed into linear constraints.

Based on the above linear relaxation techniques, we can relax the original problem into a linear program (LP).

**C. Local Search of Feasible Solution**

A linear relaxation for a problem \( z \) can be solved in polynomial time. Denote the relaxation solution as \( \hat{z} \), which provides an upper bound to problem \( z \) but may not be feasible. We now show how to obtain a feasible solution \( \psi \) based on \( \hat{z} \).

Denote \( x \) and \( q \) as the vector for variables \( x_{ij}^{m} \) and \( q_{ij}^{m} \), respectively. To obtain a feasible solution, we need to determine the integer values for \( x \) and \( q \) in solution \( \hat{z} \) such that (2), (4), (9), (10) hold. All other variables are based on \( x \) and \( q \). Initially, each \( q_{ij}^{m} \) is set to the smallest value \((q_{ij}^{m})_L\) in its value set and \( x_{ij}^{m} \) is fixed as 0 or 1 if its value set only has one element 0 or 1, respectively. Based on these \( q_{ij}^{m} \), we can compute the capacity \( \sum_{m \in M_{ij}} W \log_2 \left( 1 + \frac{g_{ij}^{m} q_{ij}^{m}}{\sum_{m \notin M_{ij}} \sum_{k \notin L_{ij}} g_{ij}^{m} t_k^{m}} \right) \) for each link \( i \to j \). The requirement on a link \( i \to j \) is \( \sum_{m \notin M_{ij}} g_{ij}^{m} t_k^{m} \leq (q_{ij}^{m})_U \). Thus, we can compute \( k_{ij} \), the ratio between the capacity and the requirement. The objective value for the current \( x \) and \( q \) is \( K \cdot \min\{k_{ij} : i \in N, j \in T_i\} \). Thus, we aim to increase the minimum \( k_{ij} \). We always try to increase the smallest \( k_{ij} \) by increasing some \( q_{ij}^{m} \) under the constraint \((q_{ij}^{m})_L \leq (q_{ij}^{m})_U \). When we cannot further increase the smallest \( k_{ij} \), we are done. The pseudocode of this local search algorithm is given in Fig. 2.
TABLE II
LOCATION AND AVAILABLE FREQUENCY BANDS AT EACH NODE FOR A 20-NODE 5-SESSION NETWORK.

<table>
<thead>
<tr>
<th>Node</th>
<th>Location</th>
<th>Available Bands</th>
<th>Node</th>
<th>Location</th>
<th>Available Bands</th>
<th>Node</th>
<th>Location</th>
<th>Available Bands</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(0.1, 9.9)</td>
<td>1, 2, 3, 4, 7, 8, 9, 10</td>
<td>8</td>
<td>(22.6, 40.9)</td>
<td>1, 2, 3, 5, 7, 9, 10</td>
<td>15</td>
<td>(44.7, 24)</td>
<td>1, 2, 3, 4, 5, 6, 7, 8, 9, 10</td>
</tr>
<tr>
<td>2</td>
<td>(29.3, 31.7)</td>
<td>2, 3, 4, 5, 7, 8, 10</td>
<td>9</td>
<td>(35.3, 10.3)</td>
<td></td>
<td>16</td>
<td>(49.9, 43.8)</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>(3.3, 1)</td>
<td>1, 2, 4, 5, 6</td>
<td>10</td>
<td>(31.9, 19.6)</td>
<td>1, 2, 3, 4, 5, 6, 7, 8, 9, 10</td>
<td>17</td>
<td>(46.4, 16.8)</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>(11.8, 40.1)</td>
<td>1, 2, 3, 4, 6, 9, 10</td>
<td>11</td>
<td>(28.1, 25.6)</td>
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<td>18</td>
<td>(11.5, 12.2)</td>
<td>2, 5, 6, 10</td>
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<td>5</td>
<td>(15.8, 17.7)</td>
<td>1, 2, 3, 5, 8, 9</td>
<td>12</td>
<td>(32.3, 18)</td>
<td>1, 2, 3, 4, 6, 7, 8, 9, 10</td>
<td>19</td>
<td>(28.2, 14.8)</td>
<td>4, 5, 6, 7, 8, 9, 10</td>
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<tr>
<td>6</td>
<td>(16.3, 19.5)</td>
<td>3, 5, 6, 8, 9</td>
<td>13</td>
<td>(47.2, 2.6)</td>
<td>3, 5, 10</td>
<td>20</td>
<td>(2.5, 14.5)</td>
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<tr>
<td>7</td>
<td>(0.6, 27.4)</td>
<td>1, 4, 8, 9, 10</td>
<td>14</td>
<td>(44.7, 15)</td>
<td>2, 3, 6, 7, 8</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

TABLE III
SOURCE NODE, DESTINATION NODE, AND MINIMUM RATE REQUIREMENT OF EACH SESSION IN THE 20-NODE 5-SESSION NETWORK.

<table>
<thead>
<tr>
<th>Session</th>
<th>Source Node</th>
<th>Dest. Node</th>
<th>Min. Rate Req.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>16</td>
<td>10</td>
<td>9</td>
</tr>
<tr>
<td>2</td>
<td>18</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>13</td>
<td>11</td>
<td>4</td>
</tr>
<tr>
<td>4</td>
<td>15</td>
<td>17</td>
<td>3</td>
</tr>
<tr>
<td>5</td>
<td>15</td>
<td>6</td>
<td>2</td>
</tr>
</tbody>
</table>

D. Selection of Partition Variables

Based on the impact on the objective value, variables in $x$ are more important than variables in $q$. Thus, we should first select one of $x$ variables as the branch variable. In particular, for the relaxation solution $\hat{\psi}_z$, the relaxation error of a discrete variable $x_{ij}^m$ is $\min\{x_{ij}^m \Delta - \hat{\psi}_{ij}^m\}$, where $\hat{\psi}_{ij}^m$ is the value of variable $x_{ij}^m$ in solution $\hat{\psi}_z$. We choose an $x_{ij}^m$ with the maximum relaxation error among all $x$ variables and let its value set in problems $z_1$ and $z_2$ be $\{0\}$ and $\{1\}$, respectively. Since the value set for this $x_{ij}^m$ only has one element, this $x_{ij}^m$ can be replaced by a constant in the new problem. As a result, some constraints may also be removed.

It should be noted that we may pose more limitations on other variables based on the new value set of $x_{ij}^m$. That is, if the new value set of $x_{ij}^m$ is $\{0\}$, then we have $q_{ij}^m = 0$ based on (9). If the new value set of $x_{ij}^m$ is $\{1\}$, then we have $x_{ih}^m = 0$ for $h \neq j$ and $x_{ih}^m = 0$ for $k \in T_i$ based on (2).

When none of the $x$ variables can be partitioned (i.e., each of their value sets has only one element), we select one of $q$ variables for partitioning. In particular, in the relaxation solution $\hat{\psi}_z$, the relaxation error of $q_{ij}^m$ is $\min\{q_{ij}^m - \hat{\psi}_{ij}^m, 1 - \hat{\psi}_{ij}^m\}$, where $\hat{\psi}_{ij}^m$ is the value of variable $q_{ij}^m$ in solution $\hat{\psi}_z$. Assuming the value set of $q_{ij}^m$ in problem $z$ is $\{q_0, q_1, \cdots, q_K\}$, its value set in problems $z_1$ and $z_2$ will be $\{q_0, q_1, \cdots, q_K\}$ and $\{q_0, q_1, \cdots, q_K\} + 1, \{q_0, q_1, \cdots, q_K\} + 2, \cdots, \{q_0, q_1, \cdots, q_K\}$, respectively. Again, based on the new value set of $q_{ij}^m$, we may impose additional limitations on other variables. In particular, if the new value set of $q_{ij}^m$ is $\{0\}$, then we have $x_{ij}^m = 0$ based on (10). If the new value set of $q_{ij}^m$ does not include 0, then we have $x_{ij}^m = 1$ based on (9).

Note that when all possible partition variables in $x$ and $q$ can no longer be partitioned (i.e., all values are assigned), the other variables can be solved via an LP.

IV. Numerical Results

In this section, we present numerical results on the proposed solution. Our goals are to demonstrate the efficacy of the solution procedure and offer quantitative understanding on the joint optimization at different layers.

A. Simulation Setting

For the ease of exposition, we normalize all units for distance, bandwidth, rate, and power based on (8) with appropriate dimensions. We consider a 20-node CR networks with each node located in a 50x50 area. We assume there are $|M| = 10$ frequency bands in the network and each band has a bandwidth of $W = 50$. At each CR node, only a subset of these bands is available. Table II gives the details of the location of each node and the set of available bands at each node. We assume there are 5 user communication sessions, each with a minimum rate requirement within $[1, 10]$. The source node, destination node, and minimum rate requirement of each session are given in Table III.

We assume the propagation gain is $g_{ij} = d_{ij}^{-4}$ and the SINR threshold $\alpha = 3$ [7]. The maximum transmission power at each node is $P_{max} = 4.8 \cdot 10^5 \cdot q_i W$. We assume that power control can be done in $Q = 10$ levels.

For our proposed branch-and-bound solution procedure, we set $c$ to 0.1, which guarantees that the solution is within 90% optimal.

B. Results and Observations

For the 20-node network with 5 sessions, the transmission power levels on their respective frequency bands in the final solution are:

- **Band 1**: $q_{13}^1 = 1, q_{16,12}^1 = 7$;
- **Band 2**: $q_{5,2}^2 = 2$;
- **Band 3**: $q_{13,14}^3 = 2$;
- **Band 4**: $q_{11,7}^4 = 7, q_{2,10}^4 = 2$;
- **Band 5**: $q_{11,10}^5 = 1$;
- **Band 6**: $q_{15,19}^6 = 9$;
- **Band 7**: $q_{14,17}^7 = 1, q_{20,1}^7 = 1$;
- **Band 8**: $q_{12,11}^8 = 3$;
- **Band 9**: $q_{12,8}^9 = 1, q_{19,6}^9 = 3$;
- **Band 10**: $q_{18,20}^9 = 1$.

Note that the same frequency band may be used by concurrent transmissions, e.g., both node 7 → 3 and node 16 → 12 are transmitting on band 1. To minimize interference, our solution has placed these concurrent transmissions sufficiently apart and set the optimal transmission power less than the maximum.

Figure 3 shows the routing topology in the final solution. The flow rates are:

- **Session 1**: $f_{2,10}(1) = 103.30, f_{8,2}(1) = 103.30, f_{11,10}(1) = 15.86, f_{12,8}(1) = 103.30, f_{12,11}(1) = 15.86, f_{16,12}(1) =$
In this paper, we investigated cross-layer optimization problem for multi-hop CR networks, with joint consideration of solutions at physical, link, and network layers. We gave a mathematical characterization for power control, scheduling, and routing under physical interference model. We developed a centralized solution procedure based on the branch-and-bound framework. Using numerical results, we demonstrated the efficacy of the solution procedure and offered quantitative understanding on the joint optimization at different layers.

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REFERENCES


