A CROSS-LAYER APPROACH TO MULTI-HOP NETWORKING WITH COGNITIVE RADIOS

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***Abstract*—Cognitive radio (CR) is an enabling technology for efficient use of available spectrum and promises unprecedented flexibility in multi-hop wireless networking. This paper explores networking related issues associated with CRs. Specifically, we consider how to maximize the rates of a set of user communication sessions in a multi-hop CR-based wireless network. Due to potential interference at the physical layer, we find that it is essential to follow a cross-layer approach, with joint optimization at physical (power control), link (frequency band scheduling), and network (flow routing) layers. We give a mathematical characterization of this cross-layer optimization problem. We develop a centralized solution procedure based on the branch-and-bound framework. Using numerical results, we demonstrate the efficacy of the solution procedure and offer quantitative understanding on the joint optimization at different layers.**

***Index Terms*—Cognitive radio, multi-hop wireless network, interference, cross-layer optimization.**

I. INTRODUCTION

Cognitive radio (CR) is a revolution in radio technology that is enabled by advances in RF design, signal processing, and communications software [14]. It promises unprecedented flexibility in radio communications and efficient use of spectrum. The potential of CR has been recognized by the commercial sector as well as the military (e.g., JTRS program [10]) and public safety communications (e.g., SAFECOM [15]).

Our goal in this paper is to optimize network level performance of multi-hop CR networks. It is now well understood that network performance for such networks is tightly coupled with lower layer behaviors [17]. For instance, maximizing user throughput at the network level not only depends on flow routing, but also depends on the algorithms at link layer (e.g., frequency band assignment) and physical layer (e.g., power control). As a result, an optimal solution at the network level must be developed with joint consideration of multiple layers.

Recently, there is a growing interest on gaining understanding on multi-hop CR networks (see, e.g., [17], [18]). However, most of these work are based on the so-called "protocol interference model" [8]. Under such model, the notions of transmission range and interference range are used to determine the feasibility of successful transmission and the existence of interference. It is now well understood that such binary decision on interference modeling has its limitation. On the other hand, the so-called "physical model" is widely accepted as an accurate characterization of interference. Under physical model, a transmission is successful if and only if signal-to-interference-and-noise-ratio (SINR) at the intended receiver exceeds a certain threshold so that the transmitted signal can be decoded with an acceptable bit error rate (BER). Further, capacity calculation is based on SINR (via Shannon's formula), which takes into account of interference due to simultaneous transmission at other nodes. Unfortunately, although physical model is accurate, there is much difficulty in carrying out analysis with physical model due to the computational complexity it involves, particularly when it comes to cross-layer optimization in a multi-hop network environment.

In this paper, we investigate networking problem for multi-hop CR networks. We employ the physical model for interference modeling and study the problem via cross-layer optimization approach. In particular, we consider how to maximize the rates of a set of user communication sessions, with joint consideration at physical layer (via power control), link layer (via frequency band scheduling), and network layer (flow routing). We give a mathematical characterization for these layers and formulate the problem into a mixed integer nonlinear program (MINLP). We develop a centralized solution to this complex optimization problem based on branch-and-bound (BB) framework and a reformulation-linearization technique (RLT) [16]. The solution we develop is guaranteed to be within a factor of $(1-\varepsilon)$ from the optimum, where ε is a small parameter reflecting our desired accuracy.

The remainder of this paper is organized as follows. In Section II, we give a mathematical characterization of power control, scheduling, and routing for multi-hop CR networks. We also present the problem formulation in this study. In Section III, we present a solution based on branch-and-bound framework. Section IV presents numerical results for the cross-layer solution. Section V reviews related work and Section VI concludes this paper.

II. MATHEMATICAL MODELS

We consider a CR-based ad hoc network with a set of nodes $\mathcal{N}$. For a node $i \in \mathcal{N}$, the set of available frequency bands $\mathcal{M}_i$ depends on its location and may not be identical to the available frequency bands at other nodes. We assume that the bandwidth of each frequency band (channel) is W. Denote $\mathcal{M}$ the set of all frequency bands present in the network, i.e., $\mathcal{M} = \bigcup_{i\in\mathcal{N}} \mathcal{M}_i$. Denote $\mathcal{M}_{ij} = \mathcal{M}_i \bigcap \mathcal{M}_j$, which is the set of frequency bands that is common on both nodes i and j and thus can be used for transmission between these two nodes. In the rest of this section, we present mathematical

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14. ABSTRACT

Cognitive radio (CR) is an enabling technology for efficient use of available spectrum and promises unprecedented flexibility in multi-hop wireless networking. This paper explores networking related issues associated with CRs. Specifically, we consider how to maximize the rates of a set of user communication sessions in a multi-hop CR-based wireless network. Due to potential interference at the physical layer, we find that it is essential to follow a cross-layer approach, with joint optimization at physical (power control), link (frequency band scheduling) and network (flow routing) layers. We give a mathematical characterization of this cross-layer optimization problem. We develop a centralized solution procedure based on the branchand- bound framework. Using numerical results, we demonstrate the efficacy of the solution procedure and offer quantitative understanding on the joint optimization at different layers.

15. SUBJECT TERMS

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TABLE I
NOTATION.

Symbol	Definition
$\mathcal{N}$	The set of nodes in the network
$\mathcal{M}_i$	The set of available bands at node $i \in \mathcal{N}$
$\mathcal{M}$	The set of frequency bands in the network
$\mathcal{M}_{ij}$	The set of frequency bands on link $i \to j$
W	Bandwidth of a frequency band
$\mathcal{L}$	The set of active user communication sessions
$s(l), d(l)$	Source and destination nodes of session $l \in \mathcal{L}$
$r(l)$	Minimum rate requirement of session l
K	Rate scaling factor for all sessions
$P_{\max}$	The maximum transmission power at a transmitter
η	Ambient Gaussian noise density
g_{ij}	Propagation gain from node i to node j
α	The minimum required SINR
$\mathcal{T}_i^m$	The set of nodes that can transmit to (and receive from) node i on band m
$\mathcal{T}_i$	The set of nodes that can transmit to (and receive from) node i
$\mathcal{I}_j^m$	The set of nodes that may make interference on band m at node j
x_{ij}^m	Binary indicator to mark whether or not band m is used by link $i \to j$
$f_{ij}(l)$	Data rate for session l on link $i \to j$
Q	The number of transmission power levels
q_{ij}^m	The transmission power level from node i to node j on band m
t_i^m	The transmission power level at node i on band m
s_{ij}^m	The SINR from node i to node j on band m
Notation for branch-and-bound procedure	
ε	A small positive constant reflecting our desired accuracy in the final solution
Ω_z	The set of all possible solutions in problem z
LB_z, UB_z	The lower and upper bounds of problem z
ψ_z	The local search solution for problem z
LB, UB	The maximum lower and upper bounds among all problems
ψ_ε	The $(1-\varepsilon)$ optimal solution

characterization of each layer in a multi-hop CR network. Table I lists all notation in this paper.

A. Scheduling and Power Control

Scheduling for transmission at each node in the network can be done either in time domain or frequency domain. In this paper, we consider scheduling in frequency domain in the form of assigning frequency bands (channels). To maximize the capacity, there may still be concurrent transmissions within the same channel (and thus interference).

Denote

$$x_{ij}^m = \begin{cases} 1 & \text{If node } i \text{ transmits data to node } j \text{ on band } m, \\ 0 & \text{otherwise.} \end{cases} \quad (1)$$

Due to interference, a node i can use a band m for transmitting to one node j or receiving from one node k. That is,

$$\sum_{k \in \mathcal{T}_i^m} x_{ki}^m + \sum_{j \in \mathcal{T}_i^m} x_{ij}^m \le 1 , \quad (2)$$

where $\mathcal{T}_i^m$ is the set of all possible nodes that node i can transmit to (and receive from) on band m in the network.

For power control, we assume that the transmission power at a node can be tuned to a finite number of levels between 0 and $P_{\max}$. To model this discrete version of power control, we introduce an integer parameter Q that represents the total number of power levels to which a transmitter can be adjusted, i.e., $0, \frac{1}{Q}P_{\max}, \frac{2}{Q}P_{\max}, \cdots, P_{\max}$. Denote $q_{ij}^m \in \{0, 1, 2, \cdots, Q\}$ the integer power level. Clearly, when node i does not transmit data to node j on band m, q_{ij}^m should be 0. Under the maximum allowed transmission power level Q, we have

$$q_{ij}^m \begin{cases} \le Q & \text{If } x_{ij}^m = 1, \\ = 0 & \text{otherwise.} \end{cases}$$

With joint consideration of x_{ij}^m and q_{ij}^m, the above relationship can be re-written as

$$q_{ij}^m \le Q x_{ij}^m , \quad (3)$$

which shows that the coupling relationship between power control and scheduling.

As discussed earlier, to maximize capacity, there may be concurrent transmissions by different nodes on the same band. Under physical model, a transmission is successful if and only if the SINR at the receiving node exceeds a certain threshold, say α. Mathematically, for a transmission from node i to node j on band m, when there is interference from concurrent transmissions on the same band, the SINR is

$$\begin{aligned} s_{ij}^m &= \frac{g_{ij} \frac{q_{ij}^m}{Q} P_{\max}}{\eta W + \sum_{k \in \mathcal{N}}^{k \ne i,j} \sum_{h \in \mathcal{T}_k^m}^{h \ne i,j} g_{kj} \frac{q_{kh}^m}{Q} P_{\max}} \\ &= \frac{g_{ij} q_{ij}^m}{\frac{\eta W Q}{P_{\max}} + \sum_{k \in \mathcal{N}}^{k \ne i,j} \sum_{h \in \mathcal{T}_k^m}^{h \ne i,j} g_{kj} q_{kh}^m} , \end{aligned}$$

where η is the ambient Gaussian noise density, g_{ij} is the propagation gain from node i to node j.

Based on our model for scheduling, if $x_{ij}^m = 1$, then $x_{ki}^m = 0$ for $k \in \mathcal{T}_i^m$ and $x_{kj}^m = 0$ for $k \in \mathcal{T}_j^m$. As a result, by (3), $q_{ki}^m = 0$ and $q_{kj}^m = 0$. Then we have

$$s_{ij}^m = \frac{g_{ij} q_{ij}^m}{\frac{\eta W Q}{P_{\max}} + \sum_{k \in \mathcal{N}}^{k \ne i,j} \sum_{h \in \mathcal{T}_k^m} g_{kj} q_{kh}^m} .$$

Denote $t_k^m = \sum_{h \in \mathcal{T}_k^m} q_{kh}^m$. We have

$$s_{ij}^m = \frac{g_{ij} q_{ij}^m}{\frac{\eta W Q}{P_{\max}} + \sum_{k \in \mathcal{N}}^{k \ne i,j} g_{kj} t_k^m} . \quad (4)$$

Note that this SINR computation also holds when $q_{ij}^m = 0$, i.e., when there is no transmission from node i to node j on band m.

Recall that under physical model, a transmission from node i to node j on band m is successful if and only if $s_{ij}^m \ge \alpha$. Then by (1), we can couple x_{ij}^m and s_{ij}^m as follows.

$$x_{ij}^m = \begin{cases} 1 & \text{If } s_{ij}^m \ge \alpha, \\ 0 & \text{otherwise.} \end{cases}$$

which can be written into the following equivalent relationship.

$$s_{ij}^m \ge \alpha x_{ij}^m .$$

B. Routing

In a multi-hop ad hoc network, we assume there is a set of $\mathcal{L}$ active user communication (unicast) sessions. Denote $s(l)$ and $d(l)$ the source and destination nodes of session $l \in \mathcal{L}$ and $r(l)$ the minimum rate requirement (in b/s) of session l. In our study, we aim to maximize a scaling factor K for all session rates. That is, what is the maximum factor K such that a rate of $K \cdot r(l)$ can be transmitted from $s(l)$ to $d(l)$ for *each* session $l \in \mathcal{L}$ in the network.

To route these data flows from its source node to destination node, multi-hop relaying is necessary, due to limited transmission power at each node. Further, for optimality and flexibility, it is desirable to allow flow splitting and multi-path routing. This is because a single path flow routing for a session is overly restrictive and is unlikely to guarantee optimal solution.

Mathematically, this can be modeled as follows. Denote $f_{ij}(l)$ the data rate on link (i,j) that is attributed to session l, where $i \in \mathcal{N}, j \in \mathcal{T}_i = \bigcup_{m \in \mathcal{M}_i} \mathcal{T}_i^m$. If node i is the source node of session l, i.e., $i = s(l)$, then

$$\sum_{j \in \mathcal{T}_i} f_{ij}(l) = r(l)K \ . \tag{5}$$

If node i is an intermediate relay node for session l, i.e., $i \neq s(l)$ and $i \neq d(l)$, then

$$\sum_{j \in \mathcal{T}_i}^{j \neq s(l)} f_{ij}(l) = \sum_{k \in \mathcal{T}_i}^{k \neq d(l)} f_{ki}(l) \ . \tag{6}$$

If node i is the destination node of session l, i.e., $i = d(l)$, then

$$\sum_{k \in \mathcal{T}_i} f_{ki}(l) = r(l)K \ . \tag{7}$$

It can be easily verified that once (5) and (6) are satisfied, (7) must also be satisfied. As a result, it is sufficient to list only (5) and (6) in the formulation.

In addition to the above flow balance equations at each node $i \in \mathcal{N}$ for session $l \in \mathcal{L}$, the aggregated flow rates on each radio link cannot exceed this link's capacity. For a link $i \rightarrow j$, we have

$$\sum_{l \in \mathcal{L}}^{s(l) \neq j, d(l) \neq i} f_{ij}(l) \leq \sum_{m \in \mathcal{M}_{ij}} W \log_2(1 + s_{ij}^m) \ . \tag{8}$$

The constraint in (8) further illustrates the coupling relationship among flow routing, power control, and scheduling.

C. Problem Formulation

Putting together all the constraints for scheduling, power control, and flow routing, we have the following complete problem formulation.

$$\begin{aligned}
\text{Max} \quad & K \\
\text{s.t.} \quad & \sum_{i \in \mathcal{T}_k^m} x_{ki}^m + \sum_{j \in \mathcal{T}_i^m} x_{ij}^m \leq 1 && (i \in \mathcal{N}, m \in \mathcal{M}_i) \\
& q_{ij}^m - Q x_{ij}^m \leq 0 && (i \in \mathcal{N}, m \in \mathcal{M}_i, j \in \mathcal{T}_i^m) \quad (9) \\
& \sum_{j \in \mathcal{T}_i^m} q_{ij}^m - t_i^m = 0 && (i \in \mathcal{N}, m \in \mathcal{M}_i) \\
& \frac{\eta W Q}{P_{\max}} s_{ij}^m + \sum_{k \in \mathcal{N}}^{k \neq i,j} g_{kj} t_k^m s_{ij}^m - g_{ij} q_{ij}^m = 0 \\
& && (i \in \mathcal{N}, m \in \mathcal{M}_i, j \in \mathcal{T}_i^m) \\
& \alpha x_{ij}^m - s_{ij}^m \leq 0 && (i \in \mathcal{N}, m \in \mathcal{M}_i, j \in \mathcal{T}_i^m) \quad (10) \\
& \sum_{l \in \mathcal{L}}^{s(l) \neq j, d(l) \neq i} f_{ij}(l) - \sum_{m \in \mathcal{M}_{ij}} W \log_2(1 + s_{ij}^m) \leq 0 \\
& && (i \in \mathcal{N}, j \in \mathcal{T}_i) \\
& \sum_{j \in \mathcal{T}_i} f_{ij}(l) - r(l)K = 0 && (l \in \mathcal{L}, i = s(l)) \\
& \sum_{j \in \mathcal{T}_i}^{j \neq s(l)} f_{ij}(l) - \sum_{i \in \mathcal{T}_k}^{k \neq d(l)} f_{ki}(l) = 0 && (l \in \mathcal{L}, i \in \mathcal{N}, i \neq s(l), d(l)) \\
& x_{ij}^m \in \{0,1\}, q_{ij}^m \in \{0,1,2,\cdots,Q\}, t_i^m, s_{ij}^m \geq 0 \\
& && (i \in \mathcal{N}, m \in \mathcal{M}_i, j \in \mathcal{T}_i^m) \\
& K, f_{ij}(l) \geq 0 \quad (l \in \mathcal{L}, i \in \mathcal{N}, i \neq d(l), j \in \mathcal{T}_i, j \neq s(l)) \ ,
\end{aligned}$$

where $Q, \eta, W, \alpha, P_{\max}, g_{ij}$, and $r(l)$ are all constants and $K, x_{ij}^m, q_{ij}^m, t_i^m, s_{ij}^m$, and $f_{ij}(l)$ are all optimization variables. This formulation is a mixed integer non-linear program (MINLP), which is NP-hard in general [6].

III. A Centralized Solution

A. Overview

For the complex MINLP problem, we employ the so-called *branch-and-bound* framework [13] to develop a solution. Under branch-and-bound, we aim to provide a $(1-\varepsilon)$-optimal solution, where ε is a small positive constant reflecting our desired accuracy in the final solution.

To start with, branch-and-bound employs some *relaxation* technique to obtains a linear relaxation for the original problem. The solution to this relaxed problem provides an upper bound (UB) to our objective function. With the relaxation solution as a starting point, branch-and-bound uses a *local search* algorithm to find a feasible solution to the original problem, which provides a lower bound (LB) for the objective function. If the obtained lower and upper bounds are close to each other, i.e., $LB \geq (1-\varepsilon)UB$, then the current feasible solution is $(1-\varepsilon)$ optimal and we are done.

Otherwise, branch-and-bound replaces the original problem with two sub-problems, say problem 1 and problem 2. This is accomplished by choosing an appropriate variable for partitioning and dividing the range of the partition variable into smaller ranges. For problems 1 and 2, branch-and-bound performs relaxation and local search on each problem and we obtain upper bounds UB_1 and UB_2 and lower bounds LB_1 and LB_2 for problems 1 and 2, respectively. Since the relaxations in problems 1 and 2 are both tighter than that in the original problem, we have $\max\{UB_1, UB_2\} \leq UB$ and $\max\{LB_1, LB_2\} \geq LB$. Then the upper bound of the original problem is updated as $UB = \max\{UB_1, UB_2\}$ and the lower bound is updated as $LB = \max\{LB_1, LB_2\}$. As a result, we now have smaller gap between UB and LB. Then we either have a $(1-\varepsilon)$ optimal solution (if $LB \geq (1-\varepsilon)UB$) or choose a problem with the maximum upper bound and perform partition for this problem.

An important property of branch-and-bound is that we may remove some problems from further consideration during the iterations. In particular, if we find a problem z with $LB \geq (1-\varepsilon)UB_z$, then we can conclude that this problem cannot

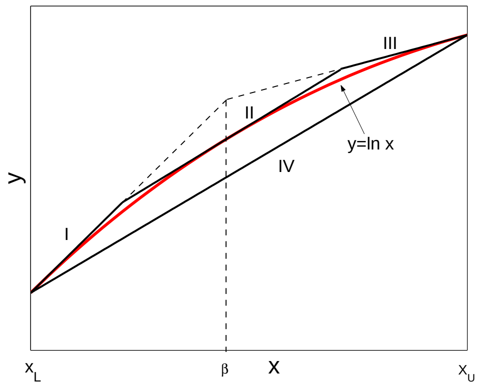

Fig. 1. A convex hull for $y = \ln x$.

Local Search Algorithm

Initialization:

1. Set $q_{ij}^m = (q_{ij}^m)_L$ and x_{ij}^m as 0 or 1 if its value set only have one element 0 or 1, respectively.
2. Compute the requirement $\sum_{l \in \mathcal{L}}^{s(l) \neq j, d(l) \neq i} \hat{f}_{ij}(l)$.

Iteration:

3. Compute the capacity and the ratio k_{ij} between the capacity and the requirement for each link $i \to j$.
4. Suppose link $i \to j$ has the smallest k_{ij}. We will try to increase its capacity as follows.
5. If we can increase q_{ij}^m on a used band {
6. Suppose band m has the largest $\hat{q}_{ij}^m$ among these bands.
7. Increase q_{ij}^m under the constraints of $q_{ij}^m \leq (q_{ij}^m)_U$ and new $k_{ij} \leq 1$. }
8. else, if we can increase q_{ij}^m on an available but currently unused band {
9. Suppose band m has the largest $\hat{q}_{ij}^m$ among these bands.
10. Increase q_{ij}^m under the constraints of $q_{ij}^m \leq (q_{ij}^m)_U$ and new $k_{ij} \leq 1$.
11. We also need to set $x_{ij}^m = 1$, $x_{ih}^m = 0$ for $h \in \mathcal{T}_i, h \neq j$, $x_{ki}^m = 0$ for $k \in \mathcal{T}_i$. }
12. else the iteration terminates since the smallest k_{ij} cannot be increased.

Fig. 2. Pseudocode of proposed local search algorithm.

provide much improvement on LB and we can thus remove this problem from further consideration.

In the rest of this section, we present the key components in the branch-and-bound framework, which are problem specific and far from trivial.

B. Linear Relaxation

During each iteration of the branch-and-bound procedure, we need a linear relaxation technique to obtain an upper bound of the objective function.

For the polynomial term $t_k^m s_{ij}^m$ in the problem formulation, we apply a novel method based on *Reformulation-Linearization Technique* (RLT) [16]. That is, we introduce a new variable u_{ijk}^m; replace $t_k^m s_{ij}^m$ by u_{ijk}^m; and add RLT constraints on these variables. Suppose t_k^m and s_{ij}^m are bounded by $(t_k^m)_L \leq t_k^m \leq (t_k^m)_U$ and $(s_{ij}^m)_L \leq s_{ij}^m \leq (s_{ij}^m)_U$, respectively. Thus, we have $[t_k^m - (t_k^m)_L] \cdot [s_{ij}^m - (s_{ij}^m)_L] \geq 0$, $[t_k^m - (t_K^m)_L] \cdot [(s_{ij}^m)_U - s_{ij}^m] \geq 0$, $[(t_k^m)_U - t_k^m] \cdot [s_{ij}^m - (s_{ij}^m)_L] \geq 0$, and $[(t_k^m)_U - t_k^m] \cdot [(s_{ij}^m)_U - s_{ij}^m] \geq 0$. From the above relationships and substituting $u_{ijk}^m = t_k^m s_{ij}^m$, we have the following RLT constraints for u_{ijk}^m.

$$
\begin{aligned}
&(t_k^m)_L \cdot s_{ij}^m + (s_{ij}^m)_L \cdot t_k^m - u_{ijk}^m \leq (t_k^m)_L \cdot (s_{ij}^m)_L \ , \\
&(t_k^m)_U \cdot s_{ij}^m + (s_{ij}^m)_L \cdot t_k^m - u_{ijk}^m \geq (t_k^m)_U \cdot (s_{ij}^m)_L \ , \\
&(t_k^m)_L \cdot s_{ij}^m + (s_{ij}^m)_U \cdot t_k^m - u_{ijk}^m \geq (t_k^m)_L \cdot (s_{ij}^m)_U \ , \\
&(t_k^m)_U \cdot s_{ij}^m + (s_{ij}^m)_U \cdot t_k^m - u_{ijk}^m \leq (t_k^m)_U \cdot (s_{ij}^m)_U \ .
\end{aligned}
$$

For the log term, we propose to employ three tangential supports, which is a convex hull linear relaxation. We first analyze the bounds for $1 + s_{ij}^m$. Then, we introduce a variable $c_{ij}^m = \ln(1 + s_{ij}^m)$ and consider how to get a linear relaxation for $y = \ln x$ over $x_L \leq x \leq x_U$. This function can be bounded by four segments (or a convex hull), where segments I, II, and III are tangential supports and segment IV is the chord (see Fig. 1). In particular, three tangent segments are at $(x_L, \ln x_L)$, $(\beta, \ln \beta)$, and $(x_U, \ln x_U)$, where $\beta = \frac{x_L \cdot x_U \cdot (\ln x_U - \ln x_L)}{x_U - x_L}$ is the horizontal location for the point intersects extended tangent segments I and III; segment IV is the segment that joins points $(x_L, \ln x_L)$ and $(x_U, \ln x_U)$. The convex region defined by the four segments can be described by the following four *linear* constraints.

$$
\begin{aligned}
x_L \cdot y - x &\leq x_L(\ln x_L - 1) \ , \\
\beta \cdot y - x &\leq \beta(\ln \beta - 1) \ , \\
x_U \cdot y - x &\leq x_U(\ln x_U - 1) \ , \\
(x_U - x_L)y + (\ln x_L - \ln x_U)x &\geq x_U \ln x_L - x_L \ln x_U \ .
\end{aligned}
$$

As a result, the non-polynomial (log) term can also be relaxed into linear constraints.

Based on the above linear relaxation techniques, we can relax the original problem into a linear program (LP).

C. Local Search of Feasible Solution

A linear relaxation for a problem z can be solved in polynomial time. Denote the relaxation solution as $\hat{\psi}_z$, which provides an upper bound to problem z but may not be feasible. We now show how to obtain a feasible solution ψ_z based on $\hat{\psi}_z$.

Denote $\mathbf{x}$ and $\mathbf{q}$ as the vector for variables x_{ij}^m and q_{ij}^m, respectively. To obtain a feasible solution, we need to determine the integer values for $\mathbf{x}$ and $\mathbf{q}$ in solution ψ_z such that (2), (4), (9), (10) hold. All other variables are based on $\mathbf{x}$ and $\mathbf{q}$. Initially, each q_{ij}^m is set to the smallest value $(q_{ij}^m)_L$ in its value set and x_{ij}^m is fixed as 0 or 1 if its value set only has one element 0 or 1, respectively. Based on these q_{ij}^m's, we can compute the capacity $\sum_{m \in \mathcal{M}_{ij}} W \log_2 \left(1 + \frac{g_{ij} q_{ij}^m}{\frac{\eta W Q}{P_{\max}} + \sum_{k \in \mathcal{N}}^{k \neq i, j} g_{kj} t_k^m}\right)$ for each link $i \to j$. The requirement on a link $i \to j$ is $\sum_{l \in \mathcal{L}}^{s(l) \neq j, d(l) \neq i} \hat{f}_{ij}(l)$. Thus, we can compute k_{ij}, the ratio between the capacity and the requirement. The objective value for the current $\mathbf{x}$ and $\mathbf{q}$ is $K \cdot \min\{k_{ij} : i \in \mathcal{N}, j \in \mathcal{T}_i\}$. Thus, we aim to increase the minimum k_{ij}. We always try to increase the smallest k_{ij} by increasing some q_{ij}^m under the constraint $q_{ij}^m \leq (q_{ij}^m)_U$. When we cannot further increase the smallest k_{ij}, we are done. The pseudocode of this local search algorithm is given in Fig. 2.

TABLE II
LOCATION AND AVAILABLE FREQUENCY BANDS AT EACH NODE FOR A 20-NODE 5-SESSION NETWORK.

Node	Location	Available Bands	Node	Location	Available Bands	Node	Location	Available Bands
1	$(0.1, 9.9)$	1, 2, 3, 4, 7, 8, 9, 10	8	$(22.6, 40.9)$	1, 2, 3, 5, 7, 9, 10	15	$(44.7, 24)$	1, 2, 3, 4, 5, 6, 7, 8, 9, 10
2	$(29.2, 31.7)$	1, 2, 3, 4, 5, 7, 8, 10	9	$(35.3, 10.3)$	2, 9	16	$(47.9, 43.8)$	1, 3
3	$(3, 31.1)$	1, 4, 5, 6	10	$(31.9, 19.6)$	1, 2, 3, 4, 5, 6, 7, 8, 9, 10	17	$(46.4, 16.8)$	1, 7, 9
4	$(11.8, 40.1)$	1, 2, 3, 4, 6, 9, 10	11	$(28.1, 25.6)$	1, 2, 3, 4, 5, 6, 7, 8, 9, 10	18	$(11.5, 12.2)$	2, 5, 6, 10
5	$(15.8, 9.7)$	1, 2, 3, 5, 6, 8, 9	12	$(32.3, 38)$	1, 8, 9, 10	19	$(28.2, 14.8)$	4, 5, 6, 7, 8, 9, 10
6	$(16.3, 19.5)$	3, 5, 6, 8, 9	13	$(47.2, 2.6)$	3, 5, 10	20	$(2.5, 14.5)$	1, 7, 10
7	$(0.6, 27.4)$	1, 4, 8, 9, 10	14	$(44.7, 15)$	2, 3, 6, 7, 8			

TABLE III
SOURCE NODE, DESTINATION NODE, AND MINIMUM RATE REQUIREMENT OF EACH SESSION IN THE 20-NODE 5-SESSION NETWORK.

Session l	Source Node $s(l)$	Dest. Node $d(l)$	Min. Rate Req. $r(l)$
1	16	10	9
2	18	3	1
3	12	11	4
4	13	17	3
5	15	6	2

D. Selection of Partition Variables

Based on the impact on the objective value, variables in $\mathbf{x}$ are more important than variables in $\mathbf{q}$. Thus, we should first select one of $\mathbf{x}$ variables as the branch variable. In particular, for the relaxation solution $\hat{\psi}_z$, the relaxation error of a discrete variable x_{ij}^m is $\min\{\hat{x}_{ij}^m, 1 - \hat{x}_{ij}^m\}$, where $\hat{x}_{ij}^m$ is the value of variable x_{ij}^m in solution $\hat{\psi}_z$. We choose an x_{ij}^m with the maximum relaxation error among all $\mathbf{x}$ variables and let its value set in problems z_1 and z_2 be $\{0\}$ and $\{1\}$, respectively. Since the value set for this x_{ij}^m only has one element, this x_{ij}^m can be replaced by a constant in the new problem. As a result, some constraints may also be removed.

It should be note that we may pose more limitations on other variables based on the new value set of x_{ij}^m. That is, if the new value set of x_{ij}^m is $\{0\}$, then we have $q_{ij}^m = 0$ based on (9). If the new value set of x_{ij}^m is $\{1\}$, then we have $x_{ih}^m = 0$ for $h \in \mathcal{T}_i, h \neq j$ and $x_{ki}^m = 0$ for $k \in \mathcal{T}_i$ based on (2).

When none of the $\mathbf{x}$ variables can be partitioned (i.e., each of their value sets has only one element), we select one of $\mathbf{q}$ variables for partitioning. In particular, in the relaxation solution $\hat{\psi}_z$, the relaxation error of q_{ij}^m is $\min\{\hat{q}_{ij}^m - \lfloor \hat{q}_{ij}^m \rfloor, \lfloor \hat{q}_{ij}^m \rfloor + 1 - \hat{q}_{ij}^m\}$, where $\hat{q}_{ij}^m$ is the value of variable q_{ij}^m in solution $\hat{\psi}_z$. Assuming the value set of q_{ij}^m in problem z is $\{q_0, q_1, \cdots, q_K\}$, its value set in problems z_1 and z_2 will be $\{q_0, q_1, \cdots, \lfloor \hat{q}_{ij}^m \rfloor\}$ and $\{\lfloor \hat{q}_{ij}^m \rfloor + 1, \lfloor \hat{q}_{ij}^m \rfloor + 2, \cdots, q_K\}$, respectively. Again, based on the new value set of q_{ij}^m, we may impose additional limitations on other variables. In particular, if the new value set of q_{ij}^m is $\{0\}$, then we have $x_{ij}^m = 0$ based on (10). If the new value set of q_{ij}^m does not include 0, then we have $x_{ij}^m = 1$ based on (9).

Note that when all possible partition variables in $\mathbf{x}$ and $\mathbf{q}$ can no longer be partitioned (i.e., all values are assigned), the other variables can be solved via an LP.

IV. NUMERICAL RESULTS

In this section, we present numerical results on the proposed solution. Our goals are to demonstrate the efficacy of the solution procedure and offer quantitative understanding on the joint optimization at different layers.

A. Simulation Setting

For the ease of exposition, we normalize all units for distance, bandwidth, rate, and power based on (8) with appropriate dimensions. We consider a 20-node CR networks with each node located in a 50x50 area. We assume there are $|\mathcal{M}| = 10$ frequency bands in the network and each band has a bandwidth of $W = 50$. At each CR node, only a subset of these bands is available. Table II gives the details of the location of each node and the set of available bands at each node. We assume there are 5 user communication sessions, each with a minimum rate requirement within $[1, 10]$. The source node, destination node, and minimum rate requirement of each session are given in Table III.

We assume the propagation gain is $g_{ij} = d_{ij}^{-4}$ and the SINR threshold $\alpha = 3$ [7]. The maximum transmission power at each node is $P_{\text{max}} = 4.8 \cdot 10^5 \cdot \eta \cdot W$. We assume that power control can be done in $Q = 10$ levels.

For our proposed branch-and-bound solution procedure, we set ε to 0.1, which guarantees that the solution is within 90% optimal.

B. Results and Observations

For the 20-node network with 5 sessions, the transmission power levels on their respective frequency bands in the final solution are:
Band 1: $q_{7,3}^1 = 1$, $q_{16,12}^1 = 7$;
Band 2: $q_{8,2}^2 = 2$;
Band 3: $q_{13,14}^3 = 2$;
Band 4: $q_{1,7}^4 = 7$, $q_{2,10}^4 = 2$;
Band 5: $q_{11,10}^5 = 1$;
Band 6: $q_{15,19}^6 = 9$;
Band 7: $q_{14,17}^7 = 1$, $q_{20,1}^7 = 1$;
Band 8: $q_{12,11}^8 = 3$;
Band 9: $q_{12,8}^9 = 1$, $q_{19,6}^9 = 3$;
Band 10: $q_{18,20}^{10} = 1$.
Note that the same frequency band may be used by concurrent transmissions, e.g., both node $7 \rightarrow 3$ and node $16 \rightarrow 12$ are transmitting on band 1. To minimize interference, our solution has placed these concurrent transmissions sufficiently apart and set the optimal transmission power less than the maximum.

Figure 3 shows the routing topology in the final solution. The flow rates are
Session 1: $f_{2,10}(1) = 103.30$, $f_{8,2}(1) = 103.30$, $f_{11,10}(1) = 15.86$, $f_{12,8}(1) = 103.30$, $f_{12,11}(1) = 15.86$, $f_{16,12}(1) =$

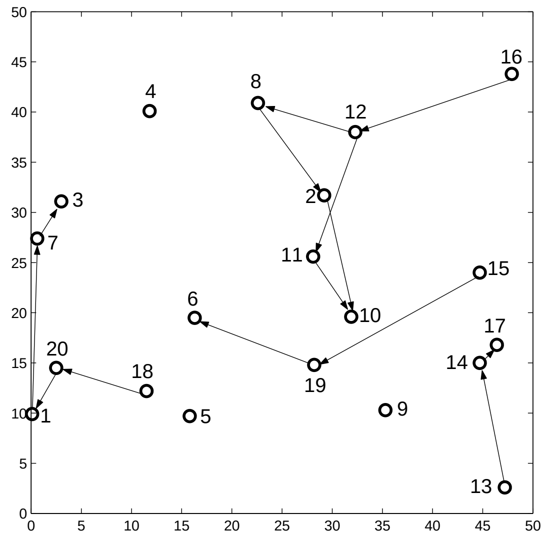

Fig. 3. The routing topology for the 20-node 5-session network.

119.16;
Session 2: $f_{1,7}(2) = 13.24$, $f_{7,3}(2) = 13.24$, $f_{18,20}(2) = 13.24$, $f_{20,1}(2) = 13.24$;
Session 3: $f_{12,11}(3) = 52.96$;
Session 4: $f_{13,14}(4) = 39.72$, $f_{14,17}(4) = 39.72$;
Session 5: $f_{15,19}(5) = 26.48$, $f_{19,6}(5) = 26.48$.
We can see that, to maximized the achieved capacity, multi-path routing are used for session 1.

Under this solution, the achieved data rate for sessions 1 to 5 is $119.16, 13.24, 52.96, 39.72, 26.48$, respectively, which corresponds to a scaling factor of 13.24.

V. Related Work

There has been substantial research efforts on multi-hop wireless networks based on the protocol interference model (see, e.g., [1], [9], [11]). The controversy surrounding (or arguments against) the protocol model is that a binary decision of whether or not interference exists (based on interference range) does not accurately captures physical layer characteristics. As a result, the accuracy (and validity) of results developed under protocol model remains unclear.

As discussed in Section I, physical model is widely accepted as an accurate characterization of interference. Unfortunately, although physical model is accurate, there is much difficulty in carrying out analysis with physical model due to the computational complexity it involves, particularly when it comes to cross-layer optimization in a multi-hop network environment. As a result, various simplifications have been employed in recent investigations. In [2], Behzad and Rubin studied the special case that the same power level are used at each node and found that the maximum transmission power should be used. However, for the general case where each node can adjust its transmission power independently, a general solution is not available. In [5], Elbatt and Ephremides proposed a two-step approach with the aim of using a minimum power vector while supporting as many users as possible; routing is given a priori instead of being part of the optimization problem. In [3], Chen and Lee proposed a layered (de-coupled) approach for QoS scheduling and power control; while in [4], Cruz and Santhanam proposed a two-step approach to minimize a power cost function that first optimizes link scheduling and power control, and then optimizes routing. Due to de-coupling in the solution procedure, these approaches only yield sub-optimal solutions.

In the area of multi-hop CR networks, although there has been some recent efforts (e.g., [17], [18]), none of them has made significant progress in addressing joint optimization across multiple layers via the physical interference model.

On another line of research, various efforts have been made to study asymptotic behavior (or scaling laws) of wireless networks (see, e.g., [8], [12]). These efforts differ from ours in this paper, which focuses on designing optimal cross-layer algorithms for finite sized network.

VI. Conclusion

In this paper, we investigated cross-layer optimization problem for multi-hop CR networks, with joint consideration of solutions at physical, link, and network layers. We gave a mathematical characterization for power control, scheduling, and routing under physical interference model. We developed a centralized solution procedure based on the branch-and-bound framework. Using numerical results, we demonstrated the efficacy of the solution procedure and offered quantitative understanding on the joint optimization at different layers.

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