An Analytical Model for Interference Alignment in Multi-hop MIMO Networks

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Abstract—Interference alignment (IA) is a powerful technique to handle interference in wireless networks. Since its inception, IA has become a central research theme in the wireless communications community. Due to its intrinsic nature of being a physical layer technique, IA has been mainly studied for point-to-point or single-hop scenario. There is a lack of research of IA from networking perspective in the context of *multi-hop* wireless networks. The goal of this paper is to make such an advance by bringing IA technique to multi-hop MIMO networks. We develop an IA model consisting of a set of constraints at a transmitter and a receiver that can be used to determine IA for a subset of interfering streams. We further prove the feasibility of this IA model by showing that a DoF vector can be supported free of interference at the physical layer as long as it satisfies the constraints in our IA model. Based on the proposed IA model, we develop an IA design space for a multi-hop MIMO network. To study how IA performs in a multi-hop MIMO network, we compare the performance of a network throughput optimization problem based on our developed IA design space against the same problem when IA is not employed. Simulation results show that the use of IA can significantly decrease the DoF consumption for IC, thereby improving network throughput.

Index Terms—Interference alignment, modeling and optimization, multi-hop MIMO network.

1 Introduction

Interference management is a fundamental problem in wireless networks. Interference alignment is a major advance in interference management in recent years that offers a new direction to handle mutual interference among different users. The basic idea of IA is to construct signals at transmitters so that these signals overlap (align in the same direction) at their unintended receivers while they are resolvable at their intended receivers. It was shown in [3] that IA can achieve $K/2$ degrees of freedom (DoF) in the K-user interference channel based on the assumption of arbitrary large time or frequency diversity. It was also shown in [6], [16] that IA can significantly increase the user throughput in practical MIMO WLAN. Given its huge potential in increasing network DoFs, IA has brought tremendous attention in the wireless communications community.

Due to its intrinsic nature of being a physical layer technique, most of the IA results are limited to point-to-point or single-hop scenario. There is a lack of advance of IA technique from networking perspective, especially in the context of *multi-hop* wireless networks. Extending IA from a single-hop to multi-hop network does not appear to be straightforward, as the transmission and interference patterns in a multi-hop network are much more complex and can easily become intractable. In [15], Li et al. made the first attempt to explore IA in a multi-hop MIMO network. There, the idea of IA was discussed in several example scenarios to illustrate its benefits. However, the key concept of IA (i.e., constructing signals at transmitters so that these signals overlap at their unintended receivers while remaining resolvable at their intended receivers) was not incorporated into their problem formulation and solution procedure. In [28] and [29], Zeng et al. studied IA in cellular and multi-hop networks from networking perspective. But their IA results were limited to single-antenna networks and cannot be applied to multi-hop MIMO networks.

The lack of results of IA in multi-hop MIMO networks underscores both the technical barrier in this area and the critical need to close this gap. The goal of this paper is to make a concrete step toward advancing IA technique in multi-hop MIMO networks. We study IA in its most basic form [3], i.e., the construction of transmit data streams so that (i) they overlap at their unintended receivers and (ii) they remain resolvable at their intended receivers. The construction of transmit data streams requires the design of precoding vector for each data stream at its transmitter. Since the interfering streams are overlapping at the receivers, one can use fewer DoFs to cancel these interfering streams. As a result, the DoF resources consumed for IC can be reduced and thus more DoF resources can be available to transport data streams. Although our IA design may not be immediately used in practical multi-hop networks due to its requirement of central controller and global channel state information (CSI), it can serve as a proof of concept and offer theoretical insights and guidance in the design of practical IA schemes. The main contributions

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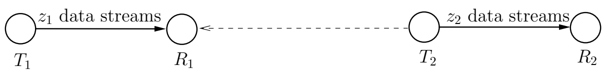


Fig. 1. SM and IC in MIMO. A solid line with arrow represents intended transmission while a dashed line with arrow represents interference.

of this paper are summarized as follows.

- We develop an analytical IA model for a multi-hop MIMO network. Our model consists of a set of constraints at each transmitter to determine the subset of interfering streams used for IA and a set of constraints at each receiver to determine the alignment pattern of the interfering streams.
- We prove the feasibility of the proposed analytical IA model. Specifically, we show that a DoF vector can be supported at the physical layer as long as it satisfies the constraints in our IA model. This is done by constructing the precoding and decoding vectors for each data stream such that all data streams in this DoF vector can be transported free of interference.
- Based on the analytical IA model, we develop a set of constraints across multiple layers of a multi-hop MIMO network. Collectively, these constraints characterize an IA design space for a multi-hop MIMO network. Based on this space, IA can be jointly exploited with upper-layer scheduling for a target network performance objective.
- To evaluate the performance of our IA design space, we study a network throughput optimization problem and compare the results against those for the same problem when IA is not employed. We show that the use of IA can conserve DoF resources in the network and increase throughput significantly.

The remainder of this paper is organized as follows. Section 2 offers some essential background on IA in MIMO networks. Section 3 discusses the challenges of applying IA in multi-hop networks. Section 4 presents a new analytical IA model for MIMO networks and Section 5 proves the feasibility of this model. In Section 6, we apply the IA model to a multi-hop MIMO network and develop a cross-layer design space for IA. In Section 7, we apply our IA design space to study a throughput maximization problem and demonstrate the benefits of IA in a multi-hop MIMO network. Section 8 presents related work and Section 9 concludes this paper.

2 Preliminaries: IA in MIMO

In this section, we review MIMO in terms of its DoF resources for spatial multiplexing (SM) and interference cancellation (IC). We also review how IA can help conserve DoF consumption required for IC. The notation in this paper is listed in Table 2 (in supplemental material).

MIMO's DoF Resources for SM and IC. The concept of DoF was originally defined to represent the maximum multiplexing gain of an MIMO channel by the information theory community (see e.g., [27]). It was then extended by the networking research community to characterize a node's spatial freedom provided by its multiple antennas (see e.g., [8], [21], [24]). Typically, the total number of DoFs at a node is equal to the number of antennas at this node, and represents the total available resources at this node that can be used for SM and IC. SM refers to the use of one or multiple DoFs (both at transmit and receive nodes) for data stream transmission/reception, with each DoF corresponding to one independent data stream. IC refers to the use of one or more DoFs to cancel interference, with each DoF being responsible for canceling one interfering stream. IC can be done either at a transmit node (to cancel interference to a receive node) or a receive node (to cancel interference from a transmit node). For example, consider the two links in Fig. 1. To transmit z_1 data streams on link (T_1, R_1), both nodes T_1 and R_1 need to consume z_1 DoFs for SM. Similarly, to transmit z_2 data streams on link (T_2, R_2), both nodes T_2 and R_2 need to consume z_2 DoFs for SM. The interference from T_2 to R_1 can be canceled by either R_1 or T_2. If R_1 cancels this interference, it needs to consume z_2 DoFs. If T_2 cancels this interference, it needs to consume z_1 DoFs.

IA in MIMO. In the context of MIMO, IA refers to a construction of data streams at transmitters so that (i) they overlap (align) at their unintended receivers and (ii) they remain resolvable at their intended receivers [3]. The construction of transmit data streams is equivalent to the design of precoding vector for each data stream at each transmitter. Since the interfering streams are overlapped at a receiver, one can use fewer DoFs to cancel these interfering streams. As a result, the DoF resources consumed for IC will be reduced and thus more DoF resources become available for data transport. To perform IA in an MIMO network, CSI is required at both transmitter and receiver sides.

We use the following example to illustrate the benefits of IA in MIMO networks. Consider the 4-link network as shown in Fig. 2. Assume that each node is equipped with three antennas. Suppose that there are 2 data streams on link (T_1, R_1), 2 data streams on link (T_2, R_2), and 1 data stream on link (T_3, R_3). At transmitter T_i, denote $\mathbf{u}_i^k$ as the precoding vector for its outgoing data stream k. Denote $\mathbf{H}_{ji}$ as the channel matrix between receiver R_j and transmitter T_i. We assume that $\mathbf{H}_{ji}$ is of full rank.

When IA is not employed, R_4 needs to consume 5 DoFs to cancel the interference from transmitters T_1, T_2, and T_3 [8], [24]. Since there are only 3 DoFs available at receiver R_4, it is not possible to cancel all 5 interfering streams, let alone to receive any data stream from T_4. But when IA is used (see Fig. 2), we can align the 5 interfering streams into 2 dimensions, which can be canceled by R_4 with only 2 DoFs. Hence, R_4 still has 1 remaining DoF, allowing it to receive 1 data stream from transmitter T_4.

We give one possible approach to construct the 5 precoding vectors at T_1, T_2, and T_3, respectively. To show that the 5 interfering streams can indeed be aligned into 2 dimensions at receiver R_4, we denote $\mathbf{a} := \mathbf{b}$ if there

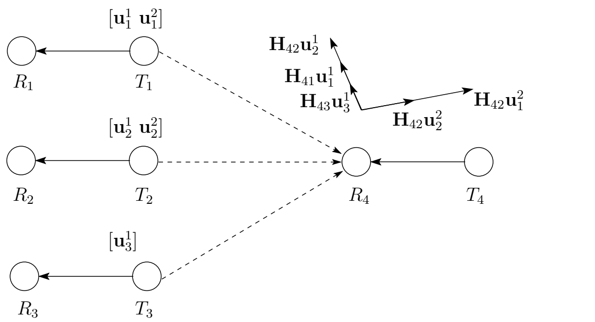


Fig. 2. An illustration of IA at node R_4. A solid line with arrow represents intended transmission while a dashed line with arrow represents interference.

exists a nonzero complex number c for vectors $\mathbf{a}$ and $\mathbf{b}$ such that $\mathbf{a} = c \cdot \mathbf{b}$. We begin by constructing the precoding vectors at transmitter T_1 by letting $\mathbf{u}_1^1 := \mathbf{e}_1$ and $\mathbf{u}_1^2 := \mathbf{e}_2$, where $\mathbf{e}_k$ is a vector with the k-th element being 1 and all the other elements being 0. For the two precoding vectors $[\mathbf{u}_2^1\ \mathbf{u}_2^2]$ at transmitter T_2, we align the interfering stream corresponding to $\mathbf{u}_2^1$ to the interfering stream corresponding to $\mathbf{u}_1^1$ at receiver R_4. This can be done by letting $\mathbf{H}_{42}\mathbf{u}_2^1 := \mathbf{H}_{41}\mathbf{u}_1^1$ and thus $\mathbf{u}_2^1 := \mathbf{H}_{42}^{-1}\mathbf{H}_{41}\mathbf{u}_1^1$. Similarly, we can align the interfering stream corresponding to $\mathbf{u}_2^2$ to the interfering stream corresponding to $\mathbf{u}_1^2$ at receiver R_4. This is done by letting $\mathbf{H}_{42}\mathbf{u}_2^2 := \mathbf{H}_{41}\mathbf{u}_1^2$ and thus $\mathbf{u}_2^2 := \mathbf{H}_{42}^{-1}\mathbf{H}_{41}\mathbf{u}_1^2$. Finally, for the precoding vector $\mathbf{u}_3^1$ at transmitter T_3, we can align its interfering stream to the interfering stream corresponding to $\mathbf{u}_1^1$ at receiver R_4. This is done by letting $\mathbf{H}_{43}\mathbf{u}_3^1 := \mathbf{H}_{41}\mathbf{u}_1^1$ and thus $\mathbf{u}_3^1 := \mathbf{H}_{43}^{-1}\mathbf{H}_{41}\mathbf{u}_1^1$. As a result of IA, the 5 interfering streams are aligned into only 2 dimensions and can be canceled with 2 DoFs (instead of 5 DoFs) by receiver R_4.

Note that in this example, we only illustrate how to achieve IA at one receiver. In a multi-hop network, the goal is to accomplish IA at as many receivers as possible so as to maximally harvest the benefits of IA. This requires careful coordination at network level and is a much harder problem, as we elaborate in the next section.

3 IA IN MULTI-HOP NETWORKS: WHERE ARE THE CHALLENGES

As discussed in Section 1, although there is a flourish of research on IA in the point-to-point or single-hop scenarios, results on extending IA to a multi-hop network remain very limited. This is because there are a number of new challenges, which we summarize as follows.

(i) How to coordinate IA among a large number of nodes in an MIMO network is a very hard problem. In particular, for each pair of nodes, one needs to decide which subset of interfering streams for IA and how to align them successfully at the receiver. While performing IA, one must also ensure that the desired data streams at each intended receiver remain resolvable. The answers to these questions require the development of a new IA model, as we shall present in this paper.

(ii) Proving the feasibility of an IA model is not trivial task. One must show that any DoF vector can be supported in the network as long as it satisfies the constraints in the underlying IA model. Specifically, one needs to show that for each data stream characterized by the DoF vector, there exist a precoding vector at its transmitter and a decoding vector at its receiver so that this data stream can be transported free of interference. As we will see in Section 5, constructing such a precoding vector and decoding vector for each data stream is very challenging.

(iii) In a multi-hop environment, an IA scheme is also coupled with the upper layer scheduling and routing algorithms. The upper layer algorithms determine the set of transmitters, the set of receivers, the set of links, and the number of data streams on each link, which vary from each time slot. Thus, an IA scheme must be jointly designed with upper layer scheduling and routing algorithms, which is again a challenging problem.

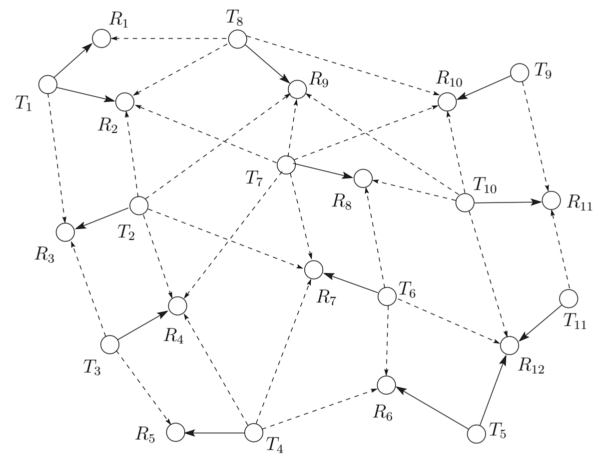


Fig. 3. An MIMO network. A solid line with arrow represents intended transmission while a dashed line with arrow represents interference.

In this paper, we address challenges (i) and (ii) in Sections 4 and 5, respectively. Challenge (iii) is addressed in Section 6.

4 MODELING IA IN MIMO NETWORKS

Consider a multi-hop MIMO network in Fig. 3. Each node is equipped with $N_{\rm A}$ antennas. We assume that the network is static and the CSI is available at both transmitter and receiver sides. We also assume that scheduling is done in the time domain, with each time frame having K time slots. To develop an IA model, we focus on one time slot t ($1 \le t \le K$). Denote $N_{\rm T}$ as the number of transmitters and $N_{\rm R}$ as the number of receivers in the time slot.[1] Denote $\mathcal{L}$ as the set of links in

1. When there is no ambiguity, we omit the time slot index t in this section and the next section.

the network with $L = |\mathcal{L}|$. Denote $\varphi = (z_1, z_2, \cdots, z_L)$ as the DoF vector in the network, where z_l is the number of data streams on link $l \in \mathcal{L}$.[2] At a transmitter, its different data streams may go to different receivers (see, e.g., T_1 in Fig. 3). For transmitter T_i, denote λ_i as the number of outgoing data streams and thus we have $\lambda_i = \sum_{l \in \mathcal{L}_i^{\text{out}}} z_l$, where $\mathcal{L}_i^{\text{out}}$ is the set of outgoing links from transmitter T_i. Similarly, at a receiver, it may receive desired data streams from multiple transmitters (see, e.g., R_{12} in Fig. 3). For receiver R_j, denote μ_j as the number of its incoming data streams and thus we have $\mu_j = \sum_{l \in \mathcal{L}_j^{\text{in}}} z_l$, where $\mathcal{L}_j^{\text{in}}$ is the set of its incoming links to receiver R_j.

Consider a node pair (T_i, R_j). Denote s_{ij}^k as the transmission of stream k ($1 \le k \le \lambda_i$) from transmitter T_i to receiver R_j. If this stream k is intended to receiver R_j, then s_{ij}^k is a *data stream* for receiver R_j. Otherwise, stream s_{ij}^k is an *interfering stream* for receiver R_j. Denote $\mathcal{S}_{ij}$ as the set of *data streams* from transmitter T_i to receiver R_j, with $\sigma_{ij} = |\mathcal{S}_{ij}|$. Denote $\mathcal{A}_{ij}$ as the set of *interfering streams* from transmitter T_i to receiver R_j, with $\alpha_{ij} = |\mathcal{A}_{ij}|$. Thus, we have $\sigma_{ij} + \alpha_{ij} = \lambda_i$. Note that without IA, receiver R_j needs to consume α_{ij} DoFs to cancel the interfering streams from transmitter T_i [8], [24].

For receiver R_j, to reduce its DoF consumption for IC, we can align a subset of its interfering streams to the other interfering streams by properly constructing their precoding vectors (as illustrated by Fig. 2). Among the interfering streams in $\mathcal{A}_{ij}$, denote $\mathcal{B}_{ij}$ as the subset of interfering streams that are aligned to the other interfering streams at receiver R_j, with $\beta_{ij} = |\mathcal{B}_{ij}|$. Then the number of "effective" interfering streams from transmitter T_i to receiver R_j is decreased from α_{ij} to $\alpha_{ij} - \beta_{ij}$.

The question to ask is how to perform IA among the nodes in the network so that

- **(C-1):** each interfering stream in $\mathcal{B}_{ij}$'s can be successfully aligned at the unintended receivers;
- **(C-2):** each data stream remains resolvable at its intended receiver.

Sections 4.1 and 4.2 address this question by exploring constraints at a transmitter and at a receiver, respectively.

4.1 IA Constraints at a Transmitter

Consider a transmitter T_i as shown in Fig. 4. Based on the definitions of $\mathcal{A}_{ij}$ and $\mathcal{B}_{ij}$, we know $\mathcal{B}_{ij} \subseteq \mathcal{A}_{ij}$. Thus, we have the following constraints at transmitter T_i:

$$\beta_{ij} \le \alpha_{ij}, \quad j \in \mathcal{I}_i, 1 \le i \le N_{\text{T}}, \tag{1}$$

where $\mathcal{I}_i$ is the set of nodes within the interference range of node i.

At transmitter T_i in Fig. 4, there are λ_i precoding vectors corresponding to λ_i outgoing streams. Since each outgoing stream interferes with all the unintended receivers within transmitter T_i's interference range, the corresponding precoding vector determines the direction

2. The activity of link l is determined by the value of z_l. When $z_l = 0$, link l is inactive.

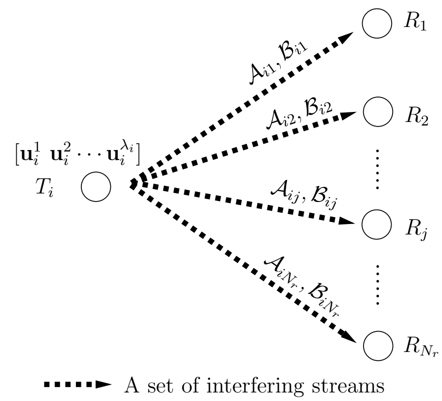


Fig. 4. IA constraints at transmitter T_i. In this figure, data streams from T_i to the receivers are not shown.

of one interfering stream for each of those receivers. For instance, precoding vector $\mathbf{u}_i^1$ determines the directions of the outgoing stream at receivers R_1, R_2, $\cdots$, R_{N_r}, one of which is the intended receiver and the rest are unintended receivers. However, among the $N_r - 1$ directions for interfering streams, only one of them can be successfully aligned to a particular direction for IA by constructing $\mathbf{u}_i^1$. Therefore, for the interfering streams from transmitter T_i, at most λ_i interfering streams can be successfully used for IA at their receivers, since there are λ_i precoding vectors at transmitter T_i. Mathematically, we have the following constraints at transmitter T_i:

$$\sum_{j \in \mathcal{I}_i} \beta_{ij} \le \lambda_i, \quad 1 \le i \le N_{\text{T}}. \tag{2}$$

At transmitter T_i, the DoF consumption is only for SM. Specifically, the number of DoFs consumed at transmitter T_i is equal to the number of its outgoing data streams (i.e., λ_i). Since the DoFs consumed at a node cannot exceed its total DoFs, we have the following constraints at transmitter T_i:

$$\lambda_i \le N_{\text{A}}, \quad 1 \le i \le N_{\text{T}}. \tag{3}$$

4.2 IA Constraints at a Receiver

Consider a receiver R_j in Fig. 5. To ensure (C-1) and (C-2) at receiver R_j, we have the following conditions:

- Based on our definition of $\mathcal{B}_{ij}$, the interfering streams in each $\mathcal{B}_{ij}$ should not occupy "effective" directions at receiver R_j. Therefore, at receiver R_j, each interfering stream in $\cup_{i \in \mathcal{I}_j} \mathcal{B}_{ij}$ can only be aligned to an interfering stream in $\cup_{i \in \mathcal{I}_j} (\mathcal{A}_{ij} \backslash \mathcal{B}_{ij})$.
- To ensure the resolvability of the data streams at each receiver, we must have that any interfering stream in $\mathcal{B}_{ij}$ cannot be aligned to an interfering stream in $\mathcal{A}_{ij}$. To show the reason, suppose that s_{ij}^k in $\mathcal{B}_{ij}$ is aligned to $s_{ij}^{k'}$ in $\mathcal{A}_{ij}$ at receiver R_j. Then, we have $\mathbf{u}_i^k := \mathbf{H}_{ji}^{-1} \mathbf{H}_{ji} \mathbf{u}_i^{k'} := \mathbf{u}_i^{k'}$. This means that $\mathbf{u}_i^k$ and $\mathbf{u}_i^{k'}$ are linearly dependent and consequently these two streams are not resolvable at their intended receivers.

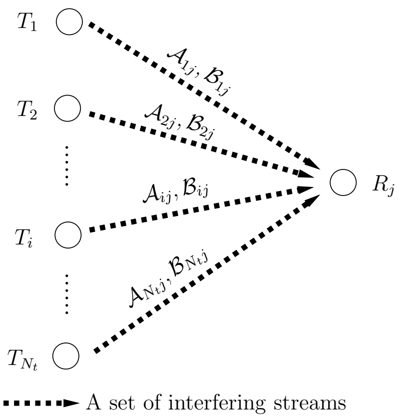


Fig. 5. IA constraints at receiver R_j. Data streams from the transmitters to receiver R_j are not shown.

- To ensure the resolvability of the data streams at each receiver, we must ensure that any two interfering streams in $\mathcal{B}_{ij}$ cannot be aligned to the same (a third) interfering stream. To show the reason, suppose that both s_{ij}^k and $s_{ij}^{k'}$ in $\mathcal{B}_{ij}$ are aligned to $s_{i'j}^l$ at receiver R_j. Then, we have $\mathbf{u}_i^k := \mathbf{H}_{ji}^{-1}\mathbf{H}_{ji'}\mathbf{u}_{i'}^l$ and $\mathbf{u}_i^{k'} := \mathbf{H}_{ji}^{-1}\mathbf{H}_{ji'}\mathbf{u}_{i'}^l$. We therefore have $\mathbf{u}_i^k := \mathbf{u}_i^{k'}$, indicating that $\mathbf{u}_i^k$ and $\mathbf{u}_i^{k'}$ are linearly dependent. This means that these two streams are not resolvable at their intended receivers.

We shall show that the above three conditions are all satisfied if the following constraints are satisfied at each receiver R_j:

$$\beta_{ij} \le \sum_{k\in\mathcal{I}_j}^{k\neq i}(\alpha_{kj} - \beta_{kj}), \qquad i \in \mathcal{I}_j, 1 \le j \le N_{\mathrm{R}}. \quad (4)$$

At each receiver R_j, its DoFs are consumed for SM and IC. Specifically, the number of DoFs consumed for SM is equal to the number of its incoming data streams (i.e., μ_j); the number of DoFs consumed for IC is equal to the number of "effective" interfering streams at this receiver (i.e., $\sum_{i\in\mathcal{I}_j}(\alpha_{ij}-\beta_{ij})$). Since the DoFs consumed for SM and IC cannot exceed its total DoFs, we have the following constraints at receiver R_j:

$$\mu_j + \sum_{k\in\mathcal{I}_j}(\alpha_{kj} - \beta_{kj}) \le N_{\mathrm{A}}, \qquad 1 \le j \le N_{\mathrm{R}}. \quad (5)$$

Collectively, constraints (1)–(5) characterize an analytical IA model for an MIMO network. A question about this model is its feasibility: For a DoF vector $\varphi = (z_1, z_2, \cdots, z_L)$ that meets these IA constraints, is it also feasible? We answer this question in the following section.

5 Feasibility of the IA Model

To prove the feasibility of the proposed analytical IA model, we must first clarify what we mean by "feasibility." The following definition clarifies this issue.

Definition 1: Suppose that a stream k from transmitter T_i is intended to receiver R_j. $\mathbf{u}_i^k$ is its precoding vector at transmitter T_i and $\mathbf{v}_j^l$ is its decoding vector at receiver R_j. Then, DoF vector $\varphi = (z_1, z_2, \cdots, z_L)$ is feasible if there exist precoding vector $\mathbf{u}_i^k$ and decoding vector $\mathbf{v}_j^l$ for each stream k from transmitter T_i, $1 \le i \le N_{\mathrm{T}}$, $1 \le k \le \lambda_i$, such that

$$(\mathbf{v}_j^l)^T\mathbf{H}_{ji}\mathbf{u}_i^k = 1, \quad (6a)$$

$$(\mathbf{v}_j^l)^T\mathbf{H}_{ji'}\mathbf{u}_{i'}^{k'} = 0, \quad (6b)$$

for $1 \le k' \le \lambda_{i'}, i' \in \mathcal{I}_j, (i', k') \neq (i, k)$.

Note that (6a) and (6b) are bilinear constraints and how to develop a general solution to a set of bilinear equations remains open [13].

Simply put, we say a DoF vector is feasible if there exist precoding and decoding vectors for each stream so that the stream can be decoded at its intended receiver free of interference. The following theorem is the main result of this section.

Theorem 1: A DoF vector $\varphi = (z_1, z_2, \cdots, z_L)$ is feasible if it satisfies constraints (1)–(5) in the IA model.

It is worth pointing out that for a given DoF vector $\varphi = (z_1, z_2, \cdots, z_L)$, the values of α_{ij}, λ_i, and μ_j in the IA constraints are fixed (i.e., $\lambda_i = \sum_{l\in\mathcal{L}_i^{\mathrm{out}}} z_l$, $\mu_j = \sum_{l\in\mathcal{L}_j^{\mathrm{in}}} z_l$, and $\alpha_{ij} = \sum_{l\in\mathcal{L}_i^{\mathrm{out}}}^{\mathrm{Rx}(l)\neq j} z_l$), while the values of β_{ij} depend on the specific IA scheme that one designs. In the rest of this section, we prove Theorem 1 by construction.

5.1 Proof of Theorem 1: A Roadmap

As for notation, we use calligraphic uppercase letter to denote a set of data/interfering streams and use boldface uppercase letter to denote the set of its corresponding precoding vectors. For a set of data streams in $\mathcal{S}_{ij}$, denote $\mathbf{S}_{ij}$ as the corresponding set of precoding vectors. For a set of interfering streams in $\mathcal{A}_{ij}$, denote $\mathbf{A}_{ij}$ as the corresponding set of precoding vectors. For a set of interfering streams in $\mathcal{B}_{ij}$, denote $\mathbf{B}_{ij}$ as the corresponding set of precoding vectors. Mathematically, we have

$$\mathbf{S}_{ij} = \{\mathbf{u}_i^k : s_{ij}^k \in \mathcal{S}_{ij}\},$$
$$\mathbf{A}_{ij} = \{\mathbf{u}_i^k : s_{ij}^k \in \mathcal{A}_{ij}\},$$
$$\mathbf{B}_{ij} = \{\mathbf{u}_i^k : s_{ij}^k \in \mathcal{B}_{ij}\}.$$

Accordingly, we have $|\mathbf{S}_{ij}| = \sigma_{ij}$, $|\mathbf{A}_{ij}| = \alpha_{ij}$, and $|\mathbf{B}_{ij}| = \beta_{ij}$.

Consider receiver R_j shown in Fig. 5. Denote $\mathbf{D}_j^{\mathrm{S}}$ as the set of *data stream* directions at receiver R_j. Denote $\mathbf{D}_j^{\mathrm{I}}$ as the set of *interfering stream* directions at receiver R_j. Then we have

$$\mathbf{D}_j^{\mathrm{S}} = \cup_{i\in\mathcal{I}_j}\{\mathbf{H}_{ji}\mathbf{u}_i^k : \mathbf{u}_i^k \in \mathbf{S}_{ij}\},$$
$$\mathbf{D}_j^{\mathrm{I}} = \cup_{i\in\mathcal{I}_j}\{\mathbf{H}_{ji}\mathbf{u}_i^k : \mathbf{u}_i^k \in \mathbf{A}_{ij}\}.$$

The following lemma shows a sufficient condition for DoF vector φ to be feasible.

Lemma 1: A DoF vector $\varphi = (z_1, z_2, \cdots, z_L)$ is feasible if there exists a precoding vector $\mathbf{u}_i^k$ for each stream k from transmitter T_i ($1 \le i \le N_{\mathrm{T}}$, $1 \le k \le \lambda_i$), such that

$$\dim(\mathbf{D}_j^{\mathrm{S}} \cup \mathbf{D}_j^{\mathrm{I}}) = \mu_j + \dim(\mathbf{D}_j^{\mathrm{I}}), \qquad 1 \le j \le N_{\mathrm{R}}, \quad (7)$$

where μ_j is the number of incoming data streams at receiver R_j and $\mu_j = \sum_{l\in\mathcal{L}_j^{\text{in}}} z_l$ for $1 \le j \le N_{\text{R}}$.

Proof: We show DoF vector φ is feasible by arguing that if (7) is satisfied, then we can find a decoding vector $\mathbf{v}_j^l$ for each stream k from transmitter T_i such that (6a) and (6b) are satisfied. Specifically, we show that the following linear system is consistent if (7) is satisfied.

$$(\mathbf{v}_j^l)^T \mathbf{H}_{ji}\mathbf{u}_i^k = 1,$$
$$(\mathbf{v}_j^l)^T \mathbf{H}_{ji'}\mathbf{u}_{i'}^{k'} = 0, \quad i' \in \mathcal{I}_j, 1 \le k' \le \lambda_{i'}, (i',k') \ne (i,k).$$

where $\mathbf{v}_j^l$ is variable vector and $\mathbf{H}$'s and $\mathbf{u}$'s are given.

Based on the definition of $\mathbf{D}_j^{\text{S}}$ and $\mathbf{D}_j^{\text{I}}$, we know

$$\mathbf{D}_j^{\text{S}} \cup \mathbf{D}_j^{\text{I}} = \{\mathbf{H}_{ji'}\mathbf{u}_{i'}^{k'} : i' \in \mathcal{I}_j, 1 \le k' \le \lambda_{i'}\}.$$

It is easy to see that $\mathbf{D}_j^{\text{S}} \cup \mathbf{D}_j^{\text{I}}$ is the set of coefficient-vectors of this linear system. Moreover, this system has N_{A} free variables and at most N_{A} linearly independent equations. If we can show that vector $\mathbf{H}_{ji}\mathbf{u}_i^k$ is not a linear combination of other vectors in $\mathbf{D}_j^{\text{S}} \cup \mathbf{D}_j^{\text{I}}$, then this system is consistent. We prove this point by contradiction.

Suppose that $\mathbf{H}_{ji}\mathbf{u}_i^k$ is a linear combination of other vectors in $\mathbf{D}_j^{\text{S}} \cup \mathbf{D}_j^{\text{I}}$. Since $\mathbf{H}_{ji}\mathbf{u}_i^k \in \mathbf{D}_j^{\text{S}}$, we have

$$\dim(\mathbf{D}_j^{\text{S}} \cup \mathbf{D}_j^{\text{I}}) < |\mathbf{D}_j^{\text{S}}| + \dim(\mathbf{D}_j^{\text{I}}) = \mu_j + \dim(\mathbf{D}_j^{\text{I}}).$$

But this contradicts the given condition in (7). Therefore, we conclude that the linear system is consistent. □

Intuitively, Lemma 1 tells us that at each receiver R_j, if a data stream lies in an independent direction (i.e., not within the subspace spanned by other data/interfering streams), then this data stream is resolvable. Lemma 1 offers another route for checking the feasibility of a given DoF vector: instead of checking the existence of both precoding and decoding vectors that satisfy (6a) and (6b) in Definition 1, one only needs to check the existence of the precoding vectors that satisfy (7) in Lemma 1.

We give a roadmap for our proof of Theorem 1.

- *Step 1 (Designing An IA Scheme):* Based on the constraints in the IA model, we propose an IA scheme for the network. The objective of this scheme is to ensure that at each receiver R_j, the interfering streams in $\cup_{i\in\mathcal{I}_j}\mathcal{B}_{ij}$ can be successfully aligned to the interfering streams in $\cup_{i\in\mathcal{I}_j}\mathcal{A}_{ij}\backslash\mathcal{B}_{ij}$. We achieve this objective by addressing two questions: (i) How to select β_{ij} interfering streams from $\mathcal{A}_{ij}$ for $\mathcal{B}_{ij}$ at each transmitter T_i? (ii) How to align the interfering streams in $\mathcal{B}_{ij}$ to other interfering streams at each receiver R_j? Details are given in Section 5.2.
- *Step 2 (Constructing Precoding Vectors):* Based on the IA scheme proposed in Step 1, we present an approach to construct the precoding vectors at the transmitters. Specifically, we divide the precoding vectors into two groups: $\mathbf{B}$ and $\mathbf{U}\backslash\mathbf{B}$. For a precoding vector $\mathbf{u}_i^k$ in $\mathbf{U}\backslash\mathbf{B}$, we set $\mathbf{u}_i^k := \mathbf{e}_k$. For the precoding vectors in $\mathbf{B}$, we construct them based on the IA scheme in Step 1. Details are given in Section 5.3.
- *Step 3 (Resolving Intended Signals):* We show that the constructed precoding vectors in Step 2 satisfy (7) in Lemma 1, thereby concluding that DoF vector $\varphi = (z_1, z_2, \cdots, z_L)$ is feasible. Details are given in Section 5.4.

5.2 Step 1: Designing An IA Scheme

Based on the constraints in the IA model, we propose an IA scheme at a transmitter and a receiver. The goal of this IA scheme is that at each receiver R_j, the interfering streams in $\cup_{i\in\mathcal{I}_j}\mathcal{B}_{ij}$ can be successfully aligned to the interfering streams in $\cup_{i\in\mathcal{I}_j}\mathcal{A}_{ij}\backslash\mathcal{B}_{ij}$. We present the IA scheme by addressing the following two questions: (i) At each transmitter T_i, how to select a subset of β_{ij} interfering streams for $\mathcal{B}_{ij}$ from the α_{ij} interfering streams in $\mathcal{A}_{ij}$? (ii) At each receiver R_j, how to align the interfering streams in $\mathcal{B}_{ij}$ to others interfering streams?

Selecting interfering streams for $\mathcal{B}_{ij}$. Consider transmitter T_i in Fig. 4. To ensure that the interfering streams in $\cup_{j\in\mathcal{I}_i}\mathcal{B}_{ij}$ can be successfully aligned to particular directions at their receivers, each of them must be corresponding to a unique precoding vector at transmitter T_i. Mathematically, this requirement can be interpreted as

$$\mathbf{B}_{ij_1} \cap \mathbf{B}_{ij_2} = \emptyset, \quad j_1, j_2 \in \mathcal{I}_i, j_1 \ne j_2, 1 \le i \le N_{\text{T}}. \quad (8)$$

Then, we have the following lemma.

Lemma 2: For any β_{ij} that meets constraints (1) and (2), we can select β_{ij} interfering streams for $\mathcal{B}_{ij}$ (from the α_{ij} interfering streams in $\mathcal{A}_{ij}$) so that (8) is satisfied.

Proof: Proving Lemma 2 is equivalent to solving the precoding vector selection problem (PVS-Problem) as follows:

PVS-Problem: For transmitter T_i and its neighboring receivers as shown in Fig. 4, select β_{ij} precoding vectors from $\mathbf{U}_i = \{\mathbf{u}_i^k : 1 \le k \le \lambda_i\}$ for $\mathbf{B}_{ij}$, $j \in \mathcal{I}_i$, such that

$$\mathbf{B}_{ij} \subseteq \mathbf{A}_{ij}, \qquad j \in \mathcal{I}_i^{\text{T}}, \quad (9a)$$

$$\mathbf{B}_{ij_1} \cap \mathbf{B}_{ij_2} = \emptyset, \quad j_1, j_2 \in \mathcal{I}_i, j_1 \ne j_2. \quad (9b)$$

where $\mathbf{U}_i = \cup_{k\in\mathcal{I}_i}\mathbf{S}_{ik}$, $\mathbf{A}_{ij} = \cup_{k\in\mathcal{I}_i}^{k\ne j}\mathbf{S}_{ik}$, $|\mathbf{S}_{ij}| = \sigma_{ij}$, $\sum_{j\in\mathcal{I}_i}\sigma_{ij} = \lambda_i$, and $\sum_{j\in\mathcal{I}_i}\beta_{ij} \le \lambda_i$.

We solve the PVS-Problem by two steps. First, we propose a greedy algorithm to select precoding vectors for $\mathbf{B}_{ij}$ (for each $j \in \mathcal{I}_i$). Second, we show that the resulting $\mathbf{B}_{ij}$ satisfies constraints (9a) and (9b).

A greedy algorithm. Without loss of generality, we index the receivers within $\mathcal{I}_i$ from 1 to J, where $J = |\mathcal{I}_i|$. We select precoding vectors for $\mathbf{B}_{ij}$ $(1 \le j \le J)$ sequentially. Specifically, we first select β_{i1} precoding vectors for $\mathbf{B}_{i1}$, and then select β_{i2} precoding vectors for $\mathbf{B}_{i2}$, and so forth. In each iteration j, we select β_{ij} precoding vectors for $\mathbf{B}_{ij}$ as follows: For each k within $j < k < J$, we move $(|\mathbf{S}_{ik}| - \sum_{k'=k+1}^{J}\beta_{ik'})^+$ precoding vectors from $\mathbf{S}_{ik}$ to $\mathbf{B}_{ij}$, where $(\cdot)^+ = \max\{\cdot, 0\}$. After that, if $\mathbf{B}_{ij}$ does not have enough precoding vectors, we move the precoding vectors from $\cup_{k\in\mathcal{I}_i}^{k\ne j}\mathbf{S}_{ik}$ to $\mathbf{B}_{ij}$ until $\mathbf{B}_{ij}$ has enough precoding vectors. A pseudo-code for this algorithm is given in Fig. 6.

Algorithm: Solving PVS-Problem at transmitter T_i.

$J = |\mathcal{I}_i|$;
for $j := 1$ to J {
  $\tilde{\mathbf{S}}_{ij} = \mathbf{S}_{ij}$; $\mathbf{B}_{ij} = \emptyset$; $\tilde{\beta}_{ij} = \beta_{ij}$; $\tilde{\sigma}_{ij} = \sigma_{ij}$; }
for $j := 1$ to J {
  for $k := j+1$ to $J-1$ {
    if $\tilde{\beta}_{ij} == 0$ {break;}
    $d = (\tilde{\sigma}_{ik} - \sum_{k'=k+1}^{J} \tilde{\beta}_{ik'})^+$;
    move $\min\{\tilde{\beta}_{ij}, d\}$ precoding vectors from $\tilde{\mathbf{S}}_{ik}$ to $\mathbf{B}_{ij}$;
    $\tilde{\beta}_{ij} := \tilde{\beta}_{ij} - \min\{\tilde{\beta}_{ij}, d\}$;
    $\tilde{\sigma}_{ik} := \tilde{\sigma}_{ik} - \min\{\tilde{\beta}_{ij}, d\}$; }
  for $k := 1$ to J {
    if $\tilde{\beta}_{ij} == 0$ {break;}
    if $k == j$ {continue;}
    move $\min\{\tilde{\beta}_{ij}, \tilde{\sigma}_{ik}\}$ precoding vectors from $\tilde{\mathbf{S}}_{ik}$ to $\mathbf{B}_{ij}$;
    $\tilde{\beta}_{ij} := \tilde{\beta}_{ij} - \min\{\tilde{\beta}_{ij}, \tilde{\sigma}_{ik}\}$;
    $\tilde{\sigma}_{ik} := \tilde{\sigma}_{ik} - \min\{\tilde{\beta}_{ij}, \tilde{\sigma}_{ik}\}$; }}

Fig. 6. A pseudo-code for solving PVS-Problem at transmitter T_i.

Algorithm analysis. Two observations on the algorithm are in order. First, the resulting solution meets (9a), because all precoding vectors in $\mathbf{B}_{ij}$ are selected from $\cup_{k\in\mathcal{I}_i}^{k\neq j}\mathbf{S}_{ik}$ and $\mathbf{A}_{ij} = \cup_{k\in\mathcal{I}_i}^{k\neq j}\mathbf{S}_{ik}$. Second, the resulting solution meets (9b), because each precoding vector in $\mathbf{U}_i$ is selected for only one $\mathbf{B}_{ij}$. Therefore, if we can show that the algorithm can successfully select β_{ij} precoding vectors for $\mathbf{B}_{ij}$ in each iteration j, then the PVS-Problem is solved.

Consider the precoding vector selection for $\mathbf{B}_{ij}$ in iteration j. In our algorithm (see Fig. 6), any precoding vectors in $\cup_{k\in\mathcal{I}_i}^{k\neq j}\tilde{\mathbf{S}}_{ik}$ can be moved to $\mathbf{B}_{ij}$. Therefore, if we can show that $\tilde{\beta}_{ij} \le \sum_{k\in\mathcal{I}_i}^{k\neq j}\tilde{\sigma}_{ik}$ at the beginning of each iteration j, then the PVS-Problem is solved. We now argue that this is true in different cases.

Case I. $\tilde{\sigma}_{ij} - \sum_{k=j+1}^{J}\tilde{\beta}_{ik} \le 0$ at the beginning of iteration j. In this case, we have

$$\begin{aligned}
&\sum_{k\in\mathcal{I}_i}^{k\neq j}\tilde{\sigma}_{ik} - \tilde{\beta}_{ij} = \sum_{k\in\mathcal{I}_i}\tilde{\sigma}_{ik} - \tilde{\sigma}_{ij} - \tilde{\beta}_{ij} \\
&\overset{(a)}{=} \lambda_i - \sum_{k=1}^{j-1}\beta_{ik} - \tilde{\sigma}_{ij} - \tilde{\beta}_{ij} \overset{(b)}{=} \lambda_i - \tilde{\sigma}_{ij} - \sum_{k=1}^{j}\beta_{ik} \\
&\overset{(c)}{\ge} \sum_{k=j+1}^{J}\beta_{ik} - \tilde{\sigma}_{ij} \overset{(d)}{=} \sum_{k=j+1}^{J}\tilde{\beta}_{ik} - \tilde{\sigma}_{ij} \ge 0,
\end{aligned}$$

where (a) follows from the fact that $\sum_{k\in\mathcal{I}_i}\tilde{\sigma}_{ik} = \sum_{k\in\mathcal{I}_i}\sigma_{ik} - \sum_{k=1}^{j-1}\beta_{ik} = \lambda_i - \sum_{k=1}^{j-1}\beta_{ik}$ at the beginning of iteration j; (b) and (d) follow from the fact that $\tilde{\beta}_{ik} = \beta_{ik}$ for $j \le k \le J$ at the beginning of iteration j; (c) follows from constraint (2).

Case II. $\tilde{\sigma}_{ij} - \sum_{k=j+1}^{J}\tilde{\beta}_{ik} > 0$ at the beginning of iteration j. In this case, if there exists a j' such that $j' < j$ and $\tilde{\sigma}_{ij'} = \sum_{k=j'+1}^{J}\tilde{\beta}_{ik}$, then it is easy to see that

$$\sum_{k\in\mathcal{I}_i}^{k\neq j}\tilde{\sigma}_{ik} - \tilde{\beta}_{ij} \ge \tilde{\sigma}_{ij'} - \beta_{ij} \ge 0.$$

Otherwise (i.e., there does not exist such a j'), all precoding vectors in $\cup_{k=1}^{j-1}\mathbf{B}_{ik}$ are from $\tilde{\mathbf{S}}_{ij}$. Then, we have

$$\sum_{k\in\mathcal{I}_i}^{k\neq j}\tilde{\sigma}_{ik} - \tilde{\beta}_{ij} \overset{(a)}{=} \sum_{k\in\mathcal{I}_i}^{k\neq j}\sigma_{ik} - \tilde{\beta}_{ij} \overset{(b)}{=} \alpha_{ij} - \tilde{\beta}_{ij} = \alpha_{ij} - \beta_{ij} \overset{(c)}{\ge} 0,$$

where (a) follows from the fact that $\tilde{\sigma}_{ik} = \sigma_{ik}$ for $j \in \mathcal{I}_i$ and $k \neq j$; (b) follows from the fact that $\sum_{k\in\mathcal{I}_i}^{k\neq j}\sigma_{ik} = \lambda_i - \sigma_{ij} = \alpha_{ij}$; (c) follows from constraint (1).

Combining the two cases, we conclude that the PVS-Problem is solved and Lemma 2 is proved. □

Lemma 2 ensures that each interfering stream in $\mathcal{B}_{ij}$ corresponds to a unique precoding vector and, therefore, each interfering stream in $\mathcal{B}_{ij}$ can be aligned to any particular direction by constructing its corresponding precoding vector.

Aligning the interfering streams in $\mathcal{B}_{ij}$. Consider receiver R_j in Fig. 5. For each $i \in \mathcal{I}_j$, we use the following algorithm to align the interfering streams in $\mathcal{B}_{ij}$ at receiver R_j.

Algorithm 1: At receiver R_j, each interfering stream in $\mathcal{B}_{ij}$ is aligned to a unique interfering stream in $\cup_{k\in\mathcal{I}_j}^{k\neq i}(\mathcal{A}_{kj}\backslash\mathcal{B}_{kj})$, $i \in \mathcal{I}_j$.

Based on (4), we know that there are more interfering streams in $\cup_{k\in\mathcal{I}_j}^{k\neq i}(\mathcal{A}_{kj}\backslash\mathcal{B}_{kj})$ than those in $\mathcal{B}_{ij}$. Therefore, each interfering stream in $\mathcal{B}_{ij}$ can be successfully aligned to a unique interfering stream in $\cup_{k\in\mathcal{I}_j}^{k\neq i}(\mathcal{A}_{kj}\backslash\mathcal{B}_{kj})$.

For an interfering stream $s_{ij}^k \in \mathcal{B}_{ij}$, in order to align it to an interfering stream $s_{i'j}^{k'} \in \cup_{k\in\mathcal{I}_j}^{k\neq i}(\mathcal{A}_{kj}\backslash\mathcal{B}_{kj})$ at receiver R_j, we should construct its corresponding precoding vector by $\mathbf{u}_i^k := \mathbf{H}_{ji}^{-1}\mathbf{H}_{ji'}\mathbf{u}_{i'}^{k'}$, which we denote as $\mathbf{u}_i^k \xrightarrow{j} \mathbf{u}_{i'}^{k'}$. The following lemma shows that there is a unique mapping for each precoding vector in $\mathbf{B}_{ij}$.

Lemma 3: For each $\mathbf{u}_i^k$ in $\mathbf{B}_{ij}$, there exists one and only one $\mathbf{u}_{i'}^{k'}$, such that $\mathbf{u}_i^k \xrightarrow{j} \mathbf{u}_{i'}^{k'}$ with $\mathbf{u}_{i'}^{k'} \in \mathbf{A}_{i'j}\backslash\mathbf{B}_{i'j}$ and $i' \neq i$.

Lemma 3 is proved by the following two facts. First, each interfering stream in $\mathcal{B}_{ij}$ is associated with a unique precoding vector (Lemma 2). Second, each interfering stream in $\mathcal{B}_{ij}$ is aligned to an interfering stream in $\cup_{k\in\mathcal{I}_j}^{k\neq i}(\mathcal{A}_{kj}\backslash\mathcal{B}_{kj})$ (according to Alg. 1).

5.3 Step 2: Constructing Precoding Vectors

We now explain how to construct the precoding vector for each stream based on the IA scheme in Section 5.2. Denote $\mathbf{U}$ as the set of all precoding vectors in the network. Denote $\mathbf{B}$ as the set of the precoding vectors that correspond to the interfering streams for alignment. Mathematically, we have

$$\begin{aligned}
&\mathbf{U} = \{\mathbf{u}_i^k : 1 \le k \le \lambda_i, 1 \le i \le N_{\mathrm{T}}\}, \\
&\mathbf{B} = \cup_{j\in\mathcal{I}_i, 1\le i\le N_{\mathrm{T}}}\mathbf{B}_{ij}.
\end{aligned}$$

To construct the precoding vectors in $\mathbf{U}$, we divide $\mathbf{U}$ into two groups: $\mathbf{B}$ and $\mathbf{U}\backslash\mathbf{B}$. We first construct the precoding vectors in $\mathbf{U}\backslash\mathbf{B}$ and then construct the precoding vectors in $\mathbf{B}$.

For each precoding vector in $\mathbf{U}\backslash\mathbf{B}$, we construct it as follows:

$$\mathbf{u}_i^k := \mathbf{e}_k, \qquad \text{for } \mathbf{u}_i^k \in \mathbf{U}\backslash\mathbf{B}, \tag{10}$$

where $\mathbf{e}_k$ is a vector with the k-th element being 1 and all the others being 0.

For the precoding vectors in $\mathbf{B}$, their construction is more complicated. We describe their construction as follows. Based on Lemma 3, we know that if $\mathbf{u}_{i_1}^{k_1} \in \mathbf{B}$, then there exists a precoding vector $\mathbf{u}_{i_2}^{k_2}$ such that $\mathbf{u}_{i_1}^{k_1} \xrightarrow{j_1} \mathbf{u}_{i_2}^{k_2}$ (i.e., $\mathbf{u}_{i_1}^{k_1} := \mathbf{H}_{j_1 i_1}^{-1}\mathbf{H}_{j_1 i_2}\mathbf{u}_{i_2}^{k_2}$). To construct $\mathbf{u}_{i_1}^{k_1}$, we first need to construct $\mathbf{u}_{i_2}^{k_2}$. If $\mathbf{u}_{i_2}^{k_2} \in \mathbf{U}\backslash\mathbf{B}$, we know that $\mathbf{u}_{i_2}^{k_2}$ has already been constructed by (10). Otherwise (i.e., $\mathbf{u}_{i_2}^{k_2} \in \mathbf{B}$), we construct $\mathbf{u}_{i_2}^{k_2}$ in the same way as $\mathbf{u}_{i_1}^{k_1}$, i.e., there exists a precoding vector $\mathbf{u}_{i_3}^{k_3}$ such that $\mathbf{u}_{i_2}^{k_2} \xrightarrow{j_2} \mathbf{u}_{i_3}^{k_3}$. Following the same token, we can establish a chain as follows:

$$\mathcal{C}: \mathbf{u}_{i_1}^{k_1} \xrightarrow{j_1} \mathbf{u}_{i_2}^{k_2} \xrightarrow{j_2} \cdots\cdots \xrightarrow{j_{M-2}} \mathbf{u}_{i_{M-1}}^{k_{M-1}} \xrightarrow{j_{M-1}} \mathbf{u}_{i_M}^{k_M}, \tag{11}$$

where $i_m \neq i_{m+1}$ for $m = 1, 2, \cdots, M-1$.

Chain $\mathcal{C}$ terminates if any of the following two cases occurs.

- *Case I:* $\mathbf{u}_{i_M}^{k_M}$ has already been constructed.
- *Case II:* $\mathbf{u}_{i_M}^{k_M}$ appears twice in chain $\mathcal{C}$.

It is easy to see that chain $\mathcal{C}$ will terminate, either by case I or case II. We now show how to construct the precoding vectors in chain $\mathcal{C}$ for the two cases, respectively.

Case I. In this case, chain $\mathcal{C}$ terminates because $\mathbf{u}_{i_M}^{k_M}$ has already been constructed. We can conclude: (i) All other precoding vectors in chain $\mathcal{C}$ have not been constructed. (ii) All precoding vectors in this chain are unique. Thus, we can construct the precoding vectors in chain $\mathcal{C}$ sequentially in the *backward* direction as follows:

$$\mathbf{u}_{i_{M-1}}^{k_{M-1}} := \mathbf{H}_{j_{M-1} i_{M-1}}^{-1}\mathbf{H}_{j_{M-1} i_M}\mathbf{u}_{i_M}^{k_M}.$$
$$\mathbf{u}_{i_{M-2}}^{k_{M-2}} := \mathbf{H}_{j_{M-2} i_{M-2}}^{-1}\mathbf{H}_{j_{M-2} i_{M-1}}\mathbf{u}_{i_{M-1}}^{k_{M-1}}.$$

Following the same token, we construct all the precoding vectors in chain $\mathcal{C}$.

Case II. In this case, chain $\mathcal{C}$ terminates because $\mathbf{u}_{i_M}^{k_M}$ appears twice. We can conclude: (i) All precoding vectors in chain $\mathcal{C}$ have not been constructed. (ii) All precoding vectors in chain $\mathcal{C}$ are unique except $\mathbf{u}_{i_M}^{k_M}$. (iii) There exists $\hat{m}$ such that $(i_{\hat{m}}, k_{\hat{m}}) = (i_M, k_M)$ and $1 \leq \hat{m} < M$.

To construct the precoding vectors in chain $\mathcal{C}$, we divide chain $\mathcal{C}$ into two sub-chains $\mathcal{C}_1$ and $\mathcal{C}_2$:

$$\mathcal{C}_1: \mathbf{u}_{i_1}^{k_1} \xrightarrow{j_1} \mathbf{u}_{i_2}^{k_2} \xrightarrow{j_2} \cdots \xrightarrow{j_{\hat{m}-2}} \mathbf{u}_{i_{\hat{m}-1}}^{k_{\hat{m}-1}} \xrightarrow{j_{\hat{m}-1}} \mathbf{u}_{i_{\hat{m}}}^{k_{\hat{m}}},$$
$$\mathcal{C}_2: \mathbf{u}_{i_{\hat{m}}}^{k_{\hat{m}}} \xrightarrow{j_{\hat{m}}} \mathbf{u}_{i_{\hat{m}+1}}^{k_{\hat{m}+1}} \xrightarrow{j_{\hat{m}+1}} \cdots \xrightarrow{j_{M-2}} \mathbf{u}_{i_{M-1}}^{k_{M-1}} \xrightarrow{j_{M-1}} \mathbf{u}_{i_M}^{k_M},$$

where $(i_{\hat{m}}, k_{\hat{m}}) = (i_M, k_M)$.

For these two sub-chains, we first construct the precoding vectors in $\mathcal{C}_2$ and then construct the precoding vectors in $\mathcal{C}_1$.

Based on the relationships among the vectors in chain $\mathcal{C}_2$, we have:

$$\begin{aligned}
\mathbf{u}_{i_{\hat{m}}}^{k_{\hat{m}}} &:= \mathbf{H}_{j_{\hat{m}} i_{\hat{m}}}^{-1}\mathbf{H}_{j_{\hat{m}} i_{\hat{m}+1}}\mathbf{u}_{i_{\hat{m}+1}}^{k_{\hat{m}+1}}, \\
\mathbf{u}_{i_{\hat{m}+1}}^{k_{\hat{m}+1}} &:= \mathbf{H}_{j_{\hat{m}+1} i_{\hat{m}+1}}^{-1}\mathbf{H}_{j_{\hat{m}+1} i_{\hat{m}+2}}\mathbf{u}_{i_{\hat{m}+2}}^{k_{\hat{m}+2}}, \\
&\vdots \\
\mathbf{u}_{i_{M-2}}^{k_{M-2}} &:= \mathbf{H}_{j_{M-2} i_{M-2}}^{-1}\mathbf{H}_{j_{M-2} i_{M-1}}\mathbf{u}_{i_{M-1}}^{k_{M-1}}, \\
\mathbf{u}_{i_{M-1}}^{k_{M-1}} &:= \mathbf{H}_{j_{M-1} i_{M-1}}^{-1}\mathbf{H}_{j_{M-1} i_M}\mathbf{u}_{i_M}^{k_M}.
\end{aligned} \tag{12}$$

Given that $(i_{\hat{m}}, k_{\hat{m}}) = (i_M, k_M)$, we have

$$\mathbf{u}_{i_{\hat{m}}}^{k_{\hat{m}}} = \mathbf{u}_{i_M}^{k_M}. \tag{13}$$

(12) and (13) form a linear equation system, where $\mathbf{H}$'s are given matricies and $\mathbf{u}$'s are variables. It can be verified that a solution to $\mathbf{u}_{i_M}^{k_M}$ in the system is

$$\mathbf{u}_{i_M}^{k_M} := \text{eigvec}\left(\prod_{m=\hat{m}}^{M-1}(\mathbf{H}_{j_m i_m}^{-1}\mathbf{H}_{j_m i_{m+1}})\right), \tag{14}$$

where $\text{eigvec}(\cdot)$ is an eigenvector of the square matrix. Once we obtain $\mathbf{u}_{i_M}^{k_M}$, we can sequentially construct all of the other precoding vectors in sub-chain $\mathcal{C}_2$ by (12).

After constructing the precoding vectors in sub-chain $\mathcal{C}_2$, we construct the precoding vectors in sub-chain $\mathcal{C}_1$. Since $\mathbf{u}_{i_{\hat{m}}}^{k_{\hat{m}}}$ has already been constructed, we can construct the other precoding vectors in sub-chain $\mathcal{C}_1$ following the same token in Case I.

It is easy to see that, in the end, all precoding vectors in $\mathbf{U}$ will be constructed.

5.4 Step 3: Resolving Intended Signals

We now show that the constructed precoding vectors in Step 2 satisfy (7) in Lemma 1. First, we present the following lemma.

Lemma 4: The constructed precoding vectors at each transmitter are linearly independent, i.e., $\dim\{\mathbf{u}_i^k : 1 \leq k \leq \lambda_i\} = \lambda_i$ *for* $1 \leq i \leq N_{\text{T}}$.

Proof: Consider transmitter T_i and its neighboring receivers in Fig. 4. Let $\mathbf{U}_i = \{\mathbf{u}_i^k : 1 \leq k \leq \lambda_i\}$ and $\mathbf{B}_i = \cup_{j\in\mathcal{I}_i}\mathbf{B}_{ij}$. Then we divide the precoding vectors in $\mathbf{U}_i$ into two groups: $\mathbf{U}_i\backslash\mathbf{B}_i$ and $\mathbf{B}_i$. Recall that in our precoding vector construction, we construct $\mathbf{u}_i^k$ by $\mathbf{u}_i^k := \mathbf{e}_k$ if $\mathbf{u}_i^k \in \mathbf{U}_i\backslash\mathbf{B}_i$ and construct $\mathbf{u}_i^k$ by $\mathbf{u}_i^k := \mathbf{H}_{ji}^{-1}\mathbf{H}_{ji'}\mathbf{u}_{i'}^{k'}$ ($i \neq i'$) if $\mathbf{u}_i^k \in \mathbf{B}_i$. This indicates that the precoding vectors in $\mathbf{U}_i\backslash\mathbf{B}_i$ are independent of the channel matrices and the precoding vectors in $\mathbf{B}_i$ are dependent on the channel matrices. Given that the channel matrices are independent Gaussian random matrices, we have

$$\begin{aligned}
\dim(\mathbf{U}_i) &= \dim(\mathbf{U}_i\backslash\mathbf{B}_i) + \dim(\mathbf{B}_i) \\
&= |\mathbf{U}_i\backslash\mathbf{B}_i| + \dim(\cup_{j\in\mathcal{I}_i}\mathbf{B}_{ij}),
\end{aligned} \tag{15}$$

almost surely.

Now we analyze the dimension of $\cup_{j\in\mathcal{I}_i}\mathbf{B}_{ij}$. Consider two precoding vectors $\mathbf{u}_i^k \in \mathbf{B}_{ij_1}$ and $\mathbf{u}_i^{k'} \in \mathcal{B}_{ij_2}$ with $j_1 \neq j_2$. In our precoding vector construction, $\mathbf{u}_i^k$ is set to $\mathbf{u}_i^k := \mathbf{H}_{j_1 i}^{-1}\mathbf{H}_{j_1 i_1}\mathbf{u}_{i_1}^{k_1}$ and $\mathbf{u}_i^{k'}$ is set to $\mathbf{u}_i^{k'} := \mathbf{H}_{j_2 i}^{-1}\mathbf{H}_{j_2 i_2}\mathbf{u}_{i_2}^{k_2}$ for some i_1, k_1, i_2, and k_2. Hence, $\mathbf{u}_i^k$ is dependent on $\mathbf{H}_{j_1 i}$ and $\mathbf{u}_i^{k'}$ is dependent on $\mathbf{H}_{j_2 i}$. Given that $\mathbf{H}_{j_1 i}$ and $\mathbf{H}_{j_2 i}$ are two independent Gaussian random matrices, we have

$$\dim(\cup_{j\in\mathcal{I}_i}\mathbf{B}_{ij}) = \sum_{j\in\mathcal{I}_i}\dim(\mathbf{B}_{ij}), \tag{16}$$

almost surely.

We now analyze the dimension of $\mathbf{B}_{ij}$. Based on (11), each precoding vector $\mathbf{u}_i^k \in \mathbf{B}_{ij}$ is constructed in the form of

$$\mathbf{u}_i^k = \left(\prod_{m=1}^{M-1}(\mathbf{H}_{j_m i_m}^{-1}\mathbf{H}_{j_m i_{m+1}})\right)\mathbf{u}_{i_M}^{k_M},$$

where $(i_1, k_1) = (i, k)$, $M \geq 2$, and $\mathbf{u}_{i_M}^{k_M}$ is constructed either by (10) or (14). Let $\mathbf{G}_i^k = \prod_{m=1}^{M-1}(\mathbf{H}_{j_m i_m}^{-1}\mathbf{H}_{j_m i_{m+1}})$. We call $\mathbf{G}_i^k$ the "effective channel" for $\mathbf{u}_i^k$. We divide the precoding vectors in $\mathbf{B}_{ij}$ into subsets such that the precoding vectors in the same subset have the same "effective channel". Denote the subsets as $\mathbf{B}_{ij}^n$, $1 \leq n \leq N_{ij}$. Since $\mathbf{H}_{ij}$'s are independent Gaussian random matrices, any the "effective channels" are independent random matrices. Thus, we have

$$\dim(\mathbf{B}_{ij}) = \sum_{n=1}^{N_{ij}}\dim(\mathbf{B}_{ij}^n). \tag{17}$$

For each $\mathbf{u}_i^k \in \mathbf{B}_{ij}^n$, it is determined by its corresponding precoding vector $\mathbf{u}_{i_M}^{k_M}$ and $\mathbf{u}_{i_M}^{k_M}$ is constructed either by (10) or (14). Denote $\tilde{\mathbf{B}}_{ij}^n$ as the set of precoding vectors $\mathbf{u}_{i_M}^{k_M}$ corresponding to the precoding vectors in $\mathbf{B}_{ij}^n$. Then we have $\dim(\tilde{\mathbf{B}}_{ij}^n) = |\tilde{\mathbf{B}}_{ij}^n|$ based on three facts: (i) the precoding vectors in $\tilde{\mathbf{B}}_{ij}^n$ are at the same transmitter; (ii) the precoding vectors constructed by (10) are linearly independent; (iii) there are N_{A} linearly independent solutions (eigenvectors) to (14). Thus, we have

$$\dim(\mathbf{B}_{ij}^n) = \dim(\tilde{\mathbf{B}}_{ij}^n) = |\tilde{\mathbf{B}}_{ij}^n| = |\mathbf{B}_{ij}^n|, \tag{18}$$

where the first equation follows from the fact that the "effective channel" has full rank.

Based on (17) and (18), we have

$$\dim(\mathbf{B}_{ij}) = \sum_{n=1}^{N_{ij}}\dim(\mathbf{B}_{ij}^n) = \sum_{n=1}^{N_{ij}}|\mathrm{B}_{ij}^n| = |\mathrm{B}_{ij}|. \tag{19}$$

Based on (15), (16), and (19), we have

$$\begin{aligned}\dim(\mathbf{U}_i) &= |\mathbf{U}_i\backslash\mathbf{B}_i| + \dim(\cup_{j\in\mathcal{I}_i}\mathbf{B}_{ij})\\ &= |\mathbf{U}_i\backslash\mathbf{B}_i| + \cup_{j\in\mathcal{I}_i}\dim(\mathbf{B}_{ij})\\ &= |\mathbf{U}_i\backslash\mathbf{B}_i| + \cup_{j\in\mathcal{I}_i}|\mathbf{B}_{ij}|\\ &= \lambda_i.\end{aligned}$$

Therefore, Lemma 4 is proved. □

Denote $\mathbf{D}_j^{\text{I,eff}}$ as the set of "effective" interfering stream directions at receiver R_j. Denote $\mathbf{D}_j^{\text{I,algn}}$ as the set of interfering stream directions for alignment at receiver R_j. Mathematically, we have

$$\begin{aligned}\mathbf{D}_j^{\text{I,eff}} &= \cup_{i\in\mathcal{I}_j}\{\mathbf{H}_{ji}\mathbf{u}_i^k : \mathbf{u}_i^k \in \mathbf{A}_{ij}\backslash\mathbf{B}_{ij}\},\\ \mathbf{D}_j^{\text{I,algn}} &= \cup_{i\in\mathcal{I}_j}\{\mathbf{H}_{ji}\mathbf{u}_i^k : \mathbf{u}_i^k \in \mathbf{B}_{ij}\}.\end{aligned}$$

Based on the precoding vector construction procedure, we know that for each $\mathbf{H}_{ji}\mathbf{u}_i^k \in \mathbf{D}_j^{\text{I,algn}}$, there exists a $\mathbf{H}_{ji'}\mathbf{u}_{i'}^{k'} \in \mathbf{D}_j^{\text{I,eff}}$ such that $\mathbf{H}_{ji}\mathbf{u}_i^k := \mathbf{H}_{ji'}\mathbf{u}_{i'}^{k'}$. Thus we have

$$\text{span}(\mathbf{D}_j^{\text{I,algn}}) \subseteq \text{span}(\mathbf{D}_j^{\text{I,eff}}). \tag{20}$$

For the number of vectors in $\mathbf{D}_j^{\text{S}} \cup \mathbf{D}_j^{\text{I,eff}}$, we have

$$|\mathbf{D}_j^{\text{S}} \cup \mathbf{D}_j^{\text{I,eff}}| = \mu_j + \sum_{i\in\mathcal{I}_j}(\alpha_{ij} - \beta_{ij}) \leq N_{\text{A}}, \tag{21}$$

where the inequality follows from (5).

The dimension of signal and interference space at receiver R_j is:

$$\begin{aligned}&\dim(\mathbf{D}_j^{\text{S}} \cup \mathbf{D}_j^{\text{I}}) \overset{(a)}{=} \dim(\mathbf{D}_j^{\text{S}} \cup \mathbf{D}_j^{\text{I,eff}})\\ &= \dim\cup_{i\in\mathcal{I}_j}\{\mathbf{H}_{ji}\mathbf{u}_i^k : \mathbf{u}_i^k \in \mathbf{S}_{ij}\cup\mathbf{A}_{ij}\backslash\mathbf{B}_{ij}\}\\ &\overset{(b)}{=} \sum_{i\in\mathcal{I}_j}\dim\{\mathbf{H}_{ji}\mathbf{u}_i^k : \mathbf{u}_i^k \in \mathbf{S}_{ij}\cup\mathbf{A}_{ij}\backslash\mathbf{B}_{ij}\}\\ &\overset{(c)}{=} \sum_{i\in\mathcal{I}_j}\dim(\mathbf{S}_{ij}\cup\mathbf{A}_{ij}\backslash\mathbf{B}_{ij})\\ &\overset{(d)}{=} \sum_{i\in\mathcal{I}_j}|\mathbf{S}_{ij}\cup\mathbf{A}_{ij}\backslash\mathbf{B}_{ij}|\\ &\overset{(e)}{=} \sum_{i\in\mathcal{I}_j}|\mathbf{S}_{ij}| + \sum_{i\in\mathcal{I}_j}|\mathbf{A}_{ij}\backslash\mathbf{B}_{ij}|\\ &= \mu_j + \sum_{i\in\mathcal{I}_j}(\alpha_{ij} - \beta_{ij}),\end{aligned} \tag{22}$$

where (a) follows from (20); (b) follows from two facts: (i) The number of elements in $\mathbf{D}_j^{\text{S}} \cup \mathbf{D}_j^{\text{I,eff}}$ is bounded by N_{A} as shown in (21); (ii) The matrices in $\{\mathbf{H}_{ji} : i \in \mathcal{I}_j\}$ are Gaussian random matrices and are independent of each other; (c) follows from our assumption that $\mathbf{H}_{ji}$ is of full rank, which is usually the case in practical networks; (d) follows from Lemma 4; (e) follows from $\mathbf{S}_{ij} \cap \mathbf{A}_{ij} = \emptyset$ and $\mathbf{B}_{ij} \subseteq \mathbf{A}_{ij}$.

Similarly, the dimension of interference subspace at receiver R_j is:

$$\dim(\mathbf{D}_j^{\text{I}}) = \dim(\mathbf{D}_j^{\text{I,eff}}) = \sum_{i\in\mathcal{I}_j}(\alpha_{ij} - \beta_{ij}). \tag{23}$$

Based on (22) and (23), we have

$$\dim(\mathbf{D}_j^{\text{S}} \cup \mathbf{D}_j^{\text{I}}) = \mu_j + \dim(\mathbf{D}_j^{\text{I}}), \tag{24}$$

which indicates that the constructed precoding vectors satisfy (7) in Lemma 1.

6 An IA Design Space for Multi-hop Networks

In this section, we apply this new analytical IA model to develop a set of cross-layer constraints that can characterize an IA design space for a multi-hop MIMO network. Denote $\mathcal{N}$ as the set of nodes in the network with $N = |\mathcal{N}|$, each of which is equipped with N_{A} antennas. Denote $\mathcal{F}$ the set of sessions in the network with $F = |\mathcal{F}|$. Denote $r(f)$ as the data rate of session $f \in \mathcal{F}$. Denote

$\mathrm{src}(f)$ and $\mathrm{dst}(f)$ as the source and the destination nodes of session $f \in \mathcal{F}$, respectively. To transport data flow f from $\mathrm{src}(f)$ to $\mathrm{dst}(f)$, we allow flow splitting inside the network for better load balancing and network resource utilization. We assume that a time frame consists of K time slots.

Half Duplex Constraints. We assume that a node cannot transmit and receive in the same time slot. Denote $x_i(t)$ as a binary variable to indicate whether node $i \in \mathcal{N}$ is a transmitter in time slot t, i.e., $x_i(t) = 1$ if node i is a transmitter in time slot t and 0 otherwise. Similarly, denote $y_i(t)$ as a binary variable to indicate whether node $i \in \mathcal{N}$ is a receiver in time slot t. Then the half duplex constraints can be written as

$$x_i(t) + y_i(t) \le 1, \quad (1 \le i \le N, 1 \le t \le K). \tag{25}$$

Node Activity Constraints. Denote $z_l(t)$ as the number of data streams on link $l \in \mathcal{L}$ in time slot t. If node i is a transmitter, we have $1 \le \sum_{l \in \mathcal{L}_i^{\mathrm{out}}} z_l(t) \le N_{\mathrm{A}}$. Otherwise (i.e., node i is either a receiver or inactive), we have $\sum_{l \in \mathcal{L}_i^{\mathrm{out}}} z_l(t) = 0$. Combining the two cases, we have the following constraints:

$$x_i(t) \le \sum_{l \in \mathcal{L}_i^{\mathrm{out}}} z_l(t) \le N_{\mathrm{A}} \cdot x_i(t), \quad (1 \le i \le N, 1 \le t \le K). \tag{26}$$

Similarly, by considering whether or not node i is a receiver, we have the following constraints:

$$y_j(t) \le \sum_{l \in \mathcal{L}_j^{\mathrm{in}}} z_l(t) \le N_{\mathrm{A}} \cdot y_j(t), \quad (1 \le j \le N, 1 \le t \le K). \tag{27}$$

General IA Constraints at a Node. In Section 4 and 5, we developed IA constraints at a transmitter and a receiver. Here, we can rewrite these constraints at a node based on the node status variables.

Based on (1) in our IA model, if node i is a transmitter and node j is a receiver in time slot t, we have $\beta_{ij}(t) \le \alpha_{ij}(t)$ for each $j \in \mathcal{I}_i$. Otherwise (i.e., $x_i(t) = 0$ or $y_j(t) = 0$), we have $\beta_{ij}(t) = 0$ and $\alpha_{ij}(t) = 0$. Combining these two cases, constraint (1) can be rewritten as

$$\beta_{ij}(t) \le \alpha_{ij}(t), \quad (j \in \mathcal{I}_i, 1 \le i \le N, 1 \le t \le K). \tag{28}$$

Based on (2) in our IA model, if node i is a transmitter in time slot t, we have $\sum_{j \in \mathcal{I}_i} \beta_{ij}(t) \le \sum_{l \in \mathcal{L}_i^{\mathrm{out}}} z_l(t)$ as $\lambda_i = \sum_{l \in \mathcal{L}_i^{\mathrm{out}}} z_l(t)$. Otherwise (i.e., $x_i = 0$), we have $\sum_{j \in \mathcal{I}_i} \beta_{ij}(t) = 0$ and $\sum_{l \in \mathcal{L}_i^{\mathrm{out}}} z_l(t) = 0$. Combining these two cases, constraint (2) can be rewritten as

$$\sum_{j \in \mathcal{I}_i} \beta_{ij}(t) \le \sum_{l \in \mathcal{L}_i^{\mathrm{out}}} z_l(t), \quad (1 \le i \le N, 1 \le t \le K). \tag{29}$$

Based on (3) in our IA model, if node i is a transmitter in time slot t, we have $\sum_{l \in \mathcal{L}_i^{\mathrm{out}}} z_l(t) \le N_{\mathrm{A}}$ as $\lambda_i = \sum_{l \in \mathcal{L}_i^{\mathrm{out}}} z_l(t)$. Otherwise (i.e., $x_i = 0$), we have $\sum_{l \in \mathcal{L}_i^{\mathrm{out}}} z_l(t) = 0$. Combining these two cases, constraint (3) can be rewritten as

$$\sum_{l \in \mathcal{L}_i^{\mathrm{out}}} z_l(t) \le N_{\mathrm{A}} \cdot x_i(t), \quad (1 \le i \le N, 1 \le t \le K). \tag{30}$$

Based on (4) in our IA model, if node j is a receiver in time slot t, we have $\beta_{ij}(t) \le \sum_{k \in \mathcal{I}_j}^{k \ne i} [\alpha_{kj}(t) - \beta_{kj}(t)]$ for each $i \in \mathcal{I}_j$. Otherwise (i.e., $y_j = 0$), we have $\beta_{ij}(t) = 0$ and $\alpha_{ij}(t) = 0$ for each $i \in \mathcal{I}_j$. Combining these two cases, constraint (4) can be rewritten as

$$\beta_{ij}(t) \le \sum_{k \in \mathcal{I}_j}^{k \ne i} [\alpha_{kj}(t) - \beta_{kj}(t)], \quad (i \in \mathcal{I}_j, 1 \le j \le N, 1 \le t \le K). \tag{31}$$

Based on (5) in our IA model, if node j is a receiver in time slot t, we have $\sum_{l \in \mathcal{L}_j^{\mathrm{in}}} z_l(t) + \sum_{i \in \mathcal{I}_j} [\alpha_{ij}(t) - \beta_{ij}(t)] \le N_{\mathrm{A}}$. Otherwise (i.e., $y_j = 0$), we have $z_l(t) = 0$ for $l \in \mathcal{L}_j^{\mathrm{in}}$ and $\alpha_{ij}(t) = \beta_{ij}(t) = 0$ for each $i \in \mathcal{I}_j$. Combining these two cases, constraint (5) can be rewritten as

$$\sum_{l \in \mathcal{L}_j^{\mathrm{in}}} z_l(t) + \sum_{i \in \mathcal{I}_j} [\alpha_{ij}(t) - \beta_{ij}(t)] \le N_{\mathrm{A}} \cdot y_j(t), \quad (1 \le j \le N, 1 \le t \le K). \tag{32}$$

Finally, we characterize the relationship between $\alpha_{ij}(t)$ and $z_l(t)$. If node i is a transmitter and node j is a receiver in time slot t, we have $\alpha_{ij}(t) = \sum_{l \in \mathcal{L}_i^{\mathrm{out}}}^{\mathrm{Rx}(l) \ne j} z_l(t)$, where $\mathrm{Rx}(l)$ is the receiver of link l. Otherwise (i.e., $x_i(t) = 0$ or $y_j(t) = 0$), we have $\alpha_{ij}(t) = 0$. In general, we have the following constraints:

$$\alpha_{ij}(t) = y_j(t) \cdot \sum_{l \in \mathcal{L}_i^{\mathrm{out}}}^{\mathrm{Rx}(l) \ne j} z_l(t), \quad (j \in \mathcal{I}_i, 1 \le i \le N, 1 \le t \le K). \tag{33}$$

Link Capacity Constraints. Denote $r_l(f)$ as the amount of data rate on link l that is attributed to session $f \in \mathcal{F}$. For ease of calculation, we assume that fixed modulation and coding scheme (MCS) is used for data transmission and one data stream in one time slot corresponds to one unit data rate. Then the average rate of link l over K time slots is $\frac{1}{K} \sum_{t=1}^{K} z_l(t)$. Since the aggregate data rates cannot exceed the average link rate, we have

$$\sum_{f=1}^{F} r_l(f) \le \frac{1}{K} \sum_{t=1}^{K} z_l(t), \quad (1 \le l \le L). \tag{34}$$

Flow Routing Constraints. At each node, flow conservation must be observed. At a source node, we have

$$\sum_{l \in \mathcal{L}_i^{\mathrm{out}}} r_l(f) = r(f), \quad (i = \mathrm{src}(f), 1 \le f \le F). \tag{35}$$

At an intermediate relay node, we have

$$\sum_{l \in \mathcal{L}_i^{\mathrm{in}}} r_l(f) = \sum_{l \in \mathcal{L}_i^{\mathrm{out}}} r_l(f), \quad (1 \le i \le N, i \ne \mathrm{src}(f), i \ne \mathrm{dst}(f), 1 \le f \le F). \tag{36}$$

At a destination node, we have

$$\sum_{l \in \mathcal{L}_i^{\mathrm{in}}} r_l(f) = r(f), \quad (i = \mathrm{dst}(f), 1 \le f \le F). \tag{37}$$

It can be easily verified that if (35) and (36) are satisfied, then (37) is also satisfied. Therefore, it suffices to include only (35) and (36).

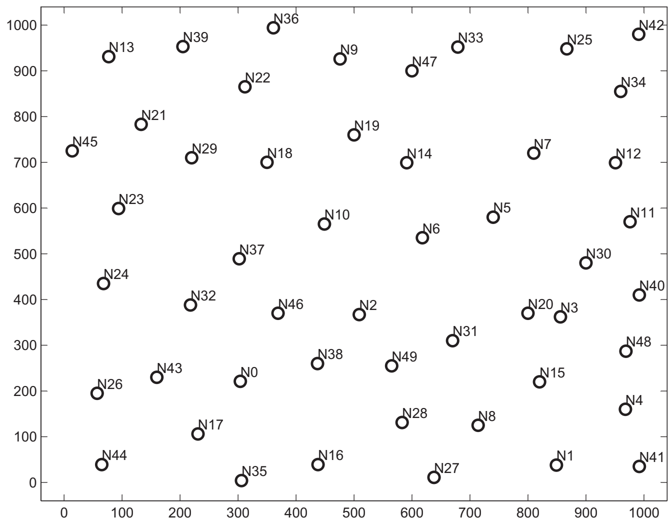


Fig. 7. A 50-node network topology.

Collectively, constraints (25)–(36) define an IA design space for cross-layer throughput maximization problems in a multi-hop MIMO network. In particular, (29), (32), and (33) are coupling constraints that involve both the physical layer and the link layer; (34) is coupling constraints that involve the physical layer, the link layer, and the network layer.

7 Performance Evaluation

In this section, we apply our IA design space developed in Section 6 to study a throughput maximization problem in a multi-hop MIMO network. Our goals are twofold. First, we want to see how IA is performed in a network setting through a case study. Second, we want to have a quantitative comparison between our IA design space and the case when IA is not used, thereby affirming the significant advantage of exploiting IA in a network environment.

7.1 A Throughput Maximization Problem

We define the objective function to be maximization of the minimum rate among all sessions.[3] Denote the objective variable as $r_{\min}$. Then we have

$$r_{\min} \le r(f), \quad 1 \le f \le F. \tag{38}$$

We have the following formulation:

OPT-IA$^{\mathbf{raw}}$:	Max	$r_{\min}$
	S.t.	Half duplex constraints: (25);
		Node activity constraints: (26), (27);
		General IA constraints: (28)–(33);
		Link capacity constraints: (34);
		Flow routing constraints: (35), (36);
		Min rate constraints: (38);

3. Other objective functions such as maximizing sum of weighted rates or a proportional scaling of all session rates belongs to the same category of linear function and can be solved following the same token.

where $x_i(t)$ and $y_i(t)$ are binary variables; $z_l(t)$, $\alpha_{ij}(t)$, and $\beta_{ij}(t)$ are non-negative integer variables; $r(f)$ and $r_l(f)$ are non-negative variables; N_{A}, N, L, F, K, and B are constants.

Among the constraints, (33) is nonlinear. We can apply the *Reformulation Linearization Technique* (RLT) [20] to linearize (33). By analyzing the relationship between $\alpha_{ij}(t)$ and $\sum_{l \in \mathcal{L}_i^{\text{out}}}^{\mathrm{Rx}(l) \neq j} z_l(t)$ in (33), we construct two new sets of constraints (39) and (40) to replace (33):

$$0 \le \sum_{l \in \mathcal{L}_i^{\text{out}}}^{\mathrm{Rx}(l) \neq j} z_l(t) - \alpha_{ij}(t) \le (1 - y_j(t)) \cdot B, \quad (j \in \mathcal{I}_i, 1 \le i \le N, 1 \le t \le K), \tag{39}$$

$$0 \le \alpha_{ij}(t) \le y_j(t) \cdot B, \quad (j \in \mathcal{I}_i, 1 \le i \le N, 1 \le t \le K), \tag{40}$$

where B is a fixed integer (e.g., $B = N_{\mathrm{A}}$). It can be verified that the combination of (39) and (40) is equivalent to (33) in terms of maximizing $r_{\min}$.

By replacing nonlinear constraint (33) with (39) and (40), we have the following problem formulation:

OPT-IA:	Max	$r_{\min}$
	S.t.	(25)–(32), (34)–(36), (38)–(40);

where $x_i(t)$ and $y_i(t)$ are binary variables; $z_l(t)$, $\alpha_{ij}(t)$, and $\beta_{ij}(t)$ are non-negative integer variables; $r(f)$ and $r_l(f)$ are non-negative variables; N_{A}, N, L, F, K, and B are constants.

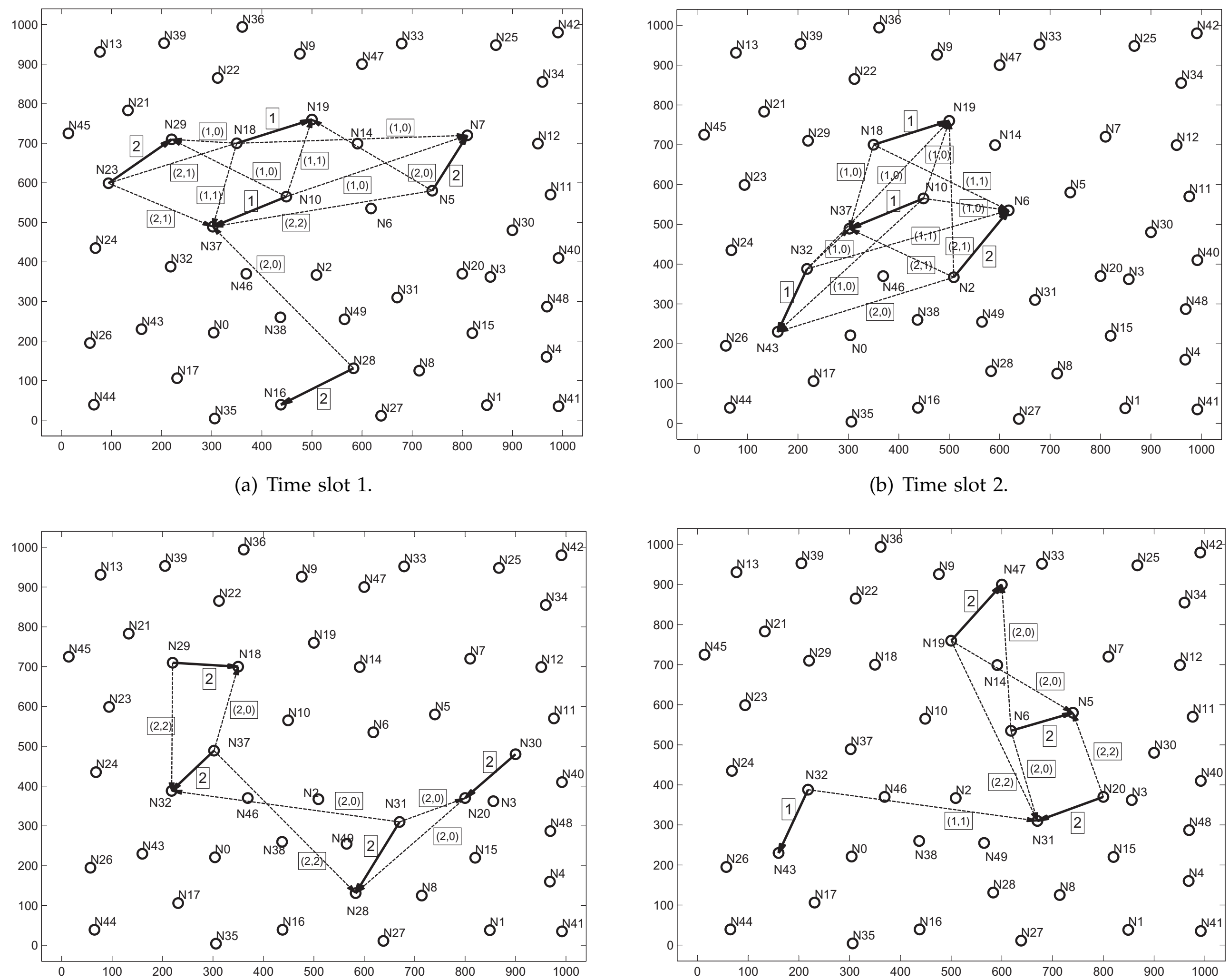
OPT-IA is a mixed integer linear programming (MILP). Although the theoretical worst-case complexity of solving a general MILP problem is exponential [5], [18], there exist highly efficient approximation algorithms (e.g., branch-and-bound with cutting planes [19]) and heuristic algorithms (e.g., sequential fixing algorithm [9], [10]). For small to moderate network size, an off-the-shelf solver such as CPLEX [30] may also be effective. Since the main goal of this paper is to present a new analytical IA model multi-hop MIMO networks (rather than developing a solution procedure), we will employ CPLEX solver for numerical results.

7.2 A Case Study

Without loss of generality, we normalize all units for distance, data rate, bandwidth, time and power with appropriate dimensions. We consider a randomly generated multi-hop MIMO network with 50 nodes (see Fig. 7), which are distributed in a 1000×1000 square region. Each node in the network is equipped with 4 antennas. We assume that all nodes have the same transmission range 250 and interference range 500.

In this network, there are 4 active sessions: N_{10} to N_{43}, N_{23} to N_{47}, N_{30} to N_{16}, and N_2 to N_7 in Fig. 7. For ease of illustration, we assume that there are 4 time slots in a time frame. By solving OPT-IA, we obtain the optimal objective (i.e., the maximum throughput) of 0.50.

Fig. 8 shows the transmission/reception pattern, interference pattern, and IA scheme in each time slot. Specifically, a solid line with arrow represents a directed link (with the number of data streams on this link shown



(a) Time slot 1.
(b) Time slot 2.
(c) Time slot 3.
(d) Time slot 4.

Fig. 8. Transmission/reception pattern, interference pattern, and IA scheme in each time slot. A solid arrow line represents a directed link (with the number of data streams on this link (i.e., z_l) shown in a box). A dashed arrow link represents interference, with the total number of interfering streams and the number of subset interfering streams chosen for IA shown in a box, i.e., $(\alpha_{ij}, \beta_{ij})$.

in a box, i.e., z_l). A dashed line with arrow represents interference, with the total number of interfering streams and the number of subset interfering streams chosen for IA shown in a box, i.e., $(\alpha_{ij}, \beta_{ij})$. For example, in Fig. 8(a), on the dashed line between N_5 and N_{37}, $(2, 2)$ in the box represents that $\alpha_{5,37} = 2$ and $\beta_{5,37} = 2$, i.e., there are two interfering streams from node N_5 to node N_{37} and both of them are selected for IA at node N_{37} in our solution.

As an example to illustrate how IA is performed in the network, let's take a look at N_{37} in time slot 1 (see Fig. 8(a)). At node N_{37}, there is a total of 7 interfering streams (from transmitting nodes N_5, N_{18}, N_{23}, and N_{28}). In our solution, we find that for the 2 interfering streams from node N_5, both of them are aligned to the interfering streams from node N_{28}. Similarly, for the interfering stream from node N_{18}, it has been aligned to an interfering stream from node N_{28}. For the 2 interfering streams from node N_{23}, one of them has been aligned to the interfering streams from node N_{28}. That is, for the 7 interfering streams at node N_{37}, 4 of them have been successfully aligned to the remaining 3 interfering streams. As a result, node N_{37} only needs to consume 3 DoFs to cancel the 7 interfering streams.

Table 1 summarizes the savings of DoFs in IC due to IA at each receiving node in each time slot. To abbreviate notation in the table, denote $P(N_j)$ as the total number of interfering streams at node N_j, i.e., $P(N_j) = \sum_{i \in \mathcal{I}_j} \alpha_{ij}$. Denote $Q(N_j)$ as the total number of DoFs that are consumed by node N_j for IC, i.e., $Q(N_j) = \sum_{i \in \mathcal{I}_j} (\alpha_{ij} - \beta_{ij})$. Then the difference between $P(N_j)$ and $Q(N_j)$ is the saving in DoFs at node N_j due to IA. Note that savings in DoFs are directly translated into improvement of network throughput.

To compare to the case when IA is not applied in the network, we formulate a throughput maximization

TABLE 1
A comparison between $P(N_j)$ and $Q(N_j)$. $P(N_j)$ is the number of interfering streams at node N_j and $Q(N_j)$ is the total number of DoFs consumed for IC at node N_j.

Time slot 1			Time slot 2		
Rx	$P(Rx)$	$Q(Rx)$	Rx	$P(Rx)$	$Q(Rx)$
N_7	2	2	N_6	3	1
N_{16}	0	0	N_{19}	4	3
N_{19}	5	3	N_{37}	4	3
N_{29}	2	2	N_{43}	3	3
N_{37}	7	3			

Time slot 3			Time slot 4		
Rx	$P(Rx)$	$Q(Rx)$	Rx	$P(Rx)$	$Q(Rx)$
N_{18}	2	2	N_5	4	2
N_{20}	2	2	N_{31}	5	2
N_{28}	4	2	N_{43}	0	0
N_{32}	4	2	N_{47}	2	2

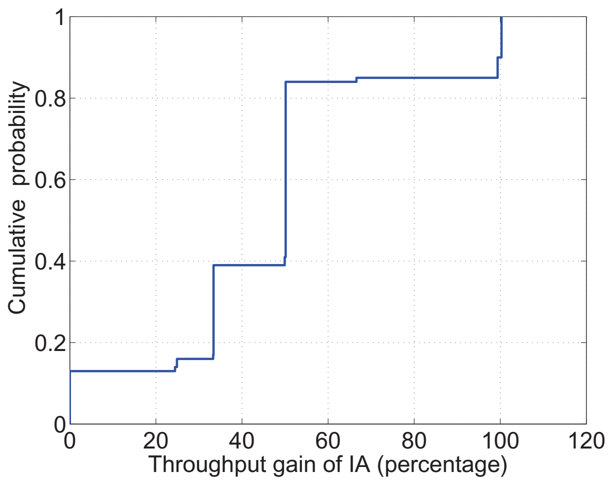


Fig. 9. CDF of the comparison between OPT-IA and OPT-base.

problem and denote it as OPT-base (see supplemental material for details). By solving OPT-base with CPLEX, we find that the objective is only 0.25 (comparing to 0.50 under OPT-IA).

7.3 Complete Results

The case study discussed in the last section gives results from one 50-node network instance. In this section, we perform the same drill for 100 network instances. Here, a time frame has 6 time slots. For each network instance, we compute the throughput gain of IA by $(\hat{r}^*_{\min} - \tilde{r}^*_{\min})/\tilde{r}^*_{\min}$, where $\hat{r}^*_{\min}$ and $\tilde{r}^*_{\min}$ are the optimal objective values of OPT-IA and OPT-base, respectively. Fig. 9 presents the CDF of the throughput gain of IA. We see that the CDF curve is not smooth but follows staircase shape. This is because the optimal objective values of OPT-IA and OPT-base are discrete. On average over the 100 random network instances, the throughput gain of IA is about 48%.

It is worth pointing out that it is unfair to compare our results with the $K/2$ DoF result in [3]. This is because the $K/2$ result in [3] was achieved based on the assumption of *infinitely large* time or frequency diversity while the IA design in our paper is based on *practical (finite)* spatial diversity from multiple antennas (with no symbol extensions). It was shown by Bresler et al. in [2] that the total DoFs available in the K-user interference channel, using only spatial diversity from multiple antennas, is at most 2, which is in sharp contrast to the $K/2$ result in [3].

8 Related Work

The concept of IA was coined in a seminar paper by Jafar and Shamai for the two-user X channel [12]. Since then, results for IA have been developed for a variety of channels and networks in increasingly sophisticated forms, such as the K-user interference channel [2], [3], the cellular network [22], [23], the MIMO Y channel [14], ergodic capacity in fading channel [17], the X network with arbitrary number of users, and the complex interference channel. A distributed IA scheme was proposed by Gomadam et al. in [7]. The feasibility of IA in signal vector space for K-user MIMO interference channel was studied by Yetis et al. in [26], and blind IA (no CSI at transmitter) was studied in [25]. A tutorial on IA from information theory perspective is [11].

In wireless communications and networking communities, efforts on IA have been mainly invested in validations on small toy networks [1], [4], [6], [16]. In [1], Al-Ali et al. studied IA in vehicular cognitive radio networks with the objectives of reducing the overhead of direct database queries and improving the accuracy of spectrum sensing for mobile vehicles. In [4], El Ayach et al. did an experimental study of IA in MIMO-OFDM interference channels and showed that IA achieves the theoretical throughput gains. In [6], Gollakotta et al. demonstrated that the combination of IA and IC increases the average throughput by 1.5 times on the downlink and 2 times on the uplink in a 2×2 MIMO WLAN. In [16], Lin et al. proposed a distributed random access protocol (called 802.11n$^+$) based on IA and demonstrated that the system can double the average network throughput in a small network with three pairs of nodes. None of these prior efforts have made advances to extend IA technique in a network setting as we have done in this paper.

9 Conclusions

The goal of this paper is to make a concrete step forward in advancing IA technique in multi-hop MIMO networks. We developed an analytical IA model consisting of a set of constraints at a transmitter and a receiver. We also proved the feasibility of the IA model by showing that each DoF vector satisfying the constraints in the IA model is feasible at the physical layer. We anticipate that this IA model or its variants will be adopted by the networking community to study IA in a multi-hop network environment.

Based on this IA model, we characterized an IA design space for cross-layer throughput maximization problems in a multi-hop MIMO network. As an application of this IA design space, we studied a network throughput optimization problem and compared performance objective with our IA model against that without IA. Simulation results showed that the use of IA in a multi-hop MIMO network can significantly reduce DoF consumption for IC at the receivers, thereby improving network throughput.

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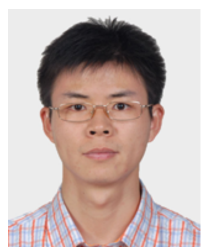

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