

On the Asymptotic Capacity of Multi-Hop MIMO Ad Hoc Networks

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Abstract—Multi-input multi-output (MIMO) is a key technology to increase the capacity of wireless networks. Although there has been extensive work on MIMO at the physical and link layers, there is limited work on MIMO at the network layer (i.e., multi-hop MIMO network), particularly results on capacity scaling laws. In this paper, we investigate capacity scaling laws for MIMO ad hoc networks. Our goal is to find the achievable throughput of each node as the number of nodes in the network increases. We employ a MIMO network model that captures spatial multiplexing and interference cancellation. We show that for a MIMO network with n randomly located nodes, each equipped with α antennas and a rate of W on each data stream, the achievable throughput of each node is $\Theta(\frac{\alpha W}{\sqrt{n \ln n}})$.

Index Terms—MIMO, ad hoc networks, asymptotic capacity, scaling law

I. INTRODUCTION

By employing multiple antennas at both the transmitter and receiver, MIMO has brought significant benefits to wireless communications, such as increased link capacity [1], [2], [3], improved link diversity [4], and interference cancellation between conflicting links [5], [6]. Although there has been extensive work on MIMO at the physical and link layers, there is limited work on MIMO at the network layer (i.e., multi-hop MIMO network), particularly results on capacity scaling laws. The analysis of capacity scaling law studies how the achievable throughput of each node scales as the number of nodes in the network increases. Such investigation is considered critical to understand the fundamental behavior of large-sized networks. Capacity scaling law was first studied by Gupta and Kumar [7] on single-antenna ad hoc networks. Subsequently, the research community has extended this seminal work to other types of wireless networks, such as multi-channel multi-radio (MC-MR) ad hoc networks [8], [9], ultra-wide band ad hoc networks [10], [11], and cognitive radio networks [12], [13].

However, to date, there is very limited work [14], [15] on capacity scaling laws for MIMO ad hoc networks. In [14], Bolcskei *et al.* considered a MIMO source and destination pair, assisted by a set of relay nodes, and studied how the capacity between this source-destination pair scales with respect to the number of relay nodes. In [15], Chen and Gans studied the

capacity of a MIMO ad hoc network with a set of simultaneous one-hop source-destination pairs. In this work, routing is not considered due to one-hop communications. Apart from these results, capacity scaling laws for multi-hop MIMO ad hoc networks remain unexplored.

In this paper, we aim to characterize asymptotic capacity for multi-hop MIMO ad hoc networks. Although there are many schemes to exploit the benefits of antenna arrays at a node, we focus on the so-called *zero-forcing beamforming* (ZF-BF) scheme [5], [6], which captures the two key characteristics of MIMO: *spatial multiplexing* and *interference cancellation*. For asymptotic study, we analyze both the lower bound and the upper bound. We show that although a capacity lower bound can be obtained by extending the work of Gupta and Kumar [7], a tight capacity upper bound is a much harder problem. We propose to partition the network area into small squares cleverly so that the maximum data rate that can be received by the nodes inside the small square can be computed exactly. By taking the sum of data rates from all small squares, we can obtain the maximum data rate the whole network can support. Based on this result, we develop a tight capacity upper bound for our problem. Our main result in this paper is the following: for a MIMO network with n randomly located nodes, each equipped with α antennas and a rate of W on each data stream, we show that the capacity upper and lower bounds have the same order, and the achievable throughput of each node is $\Theta(\frac{\alpha W}{\sqrt{n \ln n}})$.

The remainder of this paper is organized as follows. In Section II, we present a model for MIMO network that will be used in our asymptotic capacity study. In Section III, we analyze asymptotic capacity bounds. Section IV presents some numerical results. Section V concludes this paper.

II. MIMO NETWORK MODELING

In this section, we present a model for MIMO ad hoc networks which we will use in our analysis of asymptotic capacity. This model captures MIMO's spatial multiplexing and interference cancellation capabilities at the physical layer.

A. Spatial Multiplexing and Interference Cancellation

There are a number of mechanisms at the physical layer to enable spatial multiplexing and interference cancellation, such as V-Blast (Vertical-Bell labs layered space time) [1], ZF-BF (zero-forcing beamforming) [5], [6], DPC (dirty paper coding) [16], among others. Spatial multiplexing refers that a transmitter can send several independent data streams to its intended receiver simultaneously on a link. Interference cancellation refers that by properly devising the transmission and reception vectors, the interference between several conflicting

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TABLE I
NOTATION.

Symbol	Definition
D	The mean distance between each node and its destination
$d_{i,j}$	The distance between node i and j
\mathbf{H}_{lk}	The channel coefficient matrix between nodes Tx(l) and Rx(k)
n	The number of nodes in the network
Rx(l)	The receiver of link l
$r(n)$	The common transmission range of all nodes
S_{li}	The signal of data stream i on link l
\hat{S}_{li}	The recovered signal of data stream i on link l
Tx(l)	The transmitter of link l
$UB(n)$	Capacity upper bound
\mathbf{u}_{li}	The transmission vector for transmitting signal S_{li}
\mathbf{v}_{li}	The reception vector for receiving signal S_{li}
W	The maximum data rate that a single data stream can support
z_l	The number of active data streams on link l
α	The number of antennas at each node
Δ	The parameter to set the interference range
$\lambda(n)$	The per-node throughput of a random multi-hop MIMO ad hoc network with n nodes
$\pi(\cdot)$	An ordering function of all nodes in the network

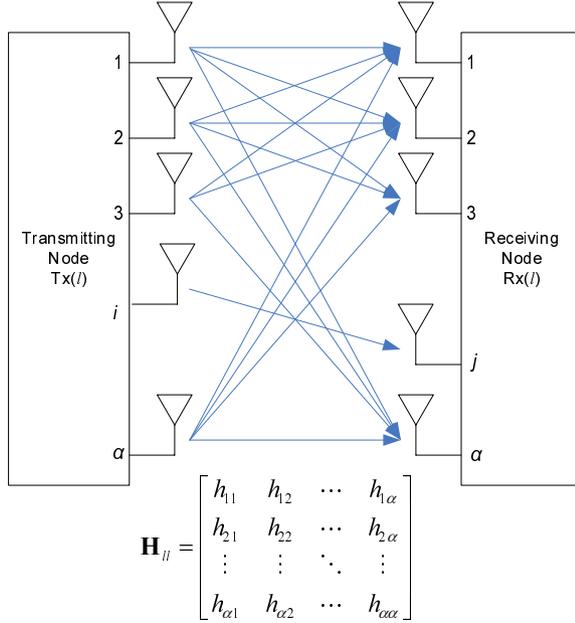


Fig. 1. A spatial multiplexing link.

links can be cancelled out. In this paper, for the ease of the mathematic modeling, we employ the simple but yet powerful ZFBF for spatial multiplexing and interference cancellation. Table I lists the notation in this paper.

Spatial multiplexing. We assume that the number of antennas at each node is α and the network is deployed in a rich scattering environment, so that the degree of freedom (DoF) at each node is approximately equal to its number of antennas (α). Suppose we want to activate z_l data streams on a link l (see Fig. 1). Denote Tx(l) and Rx(l) the transmitter and receiver of link l , respectively. Denote S_{li} the signal of data stream i ($1 \leq i \leq z_l$) at transmitter Tx(l) and \mathbf{u}_{li} the $\alpha \times 1$ transmission vector of signal S_{li} , respectively. Denote \mathbf{H}_{ll} the channel coefficient matrix between nodes Tx(l) and Rx(l). Although there are different MIMO channel models in

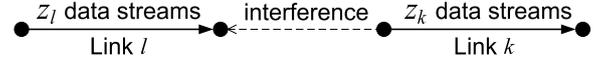


Fig. 2. Two interfering MIMO links.

the literature, e.g., MIMO Rician [17] and Rayleigh fading channels [5], [15], our results do not depend on the specific channel models. We assume that our channel matrix remains constant during a certain transmission period. Moreover, we assume that the channel matrix is of full rank, which is justifiable under rich scattering environment. Thus, during a transmission period, channel matrix \mathbf{H} is regarded as a full rank $\alpha \times \alpha$ constant matrix. Note that although the channel matrix may change for different transmission periods, the asymptotic results (Section III) will still hold under each transmission period, i.e., our results remain valid over time.

To transmit all z_l data streams, Tx(l) sends the combined signal $\sum_{j=1}^{z_l} \mathbf{u}_{lj} S_{lj}$ through its α antennas. The signal at receiver Rx(l)'s antennas will be $(\sum_{j=1}^{z_l} \mathbf{u}_{lj} S_{lj})^T \mathbf{H}_{ll}$. Receiver Rx(l) uses an $\alpha \times 1$ reception vector \mathbf{v}_{li} to recover the signal of data stream i . Then the recovered signal \hat{S}_{li} for data stream i is $\hat{S}_{li} = (\sum_{j=1}^{z_l} \mathbf{u}_{lj} S_{lj})^T \mathbf{H}_{ll} \mathbf{v}_{li} = \mathbf{u}_{li}^T \mathbf{H}_{ll} \mathbf{v}_{li} S_{li} + \sum_{1 \leq j \leq z_l, j \neq i} \mathbf{u}_{lj}^T \mathbf{H}_{ll} \mathbf{v}_{li} S_{lj}$. By choosing appropriate \mathbf{u} and \mathbf{v} , we can ensure that the recovered signal \hat{S}_{li} achieves a unit gain ($\mathbf{u}_{li}^T \mathbf{H}_{ll} \mathbf{v}_{li} = 1$) and zero interference ($\mathbf{u}_{lj}^T \mathbf{H}_{ll} \mathbf{v}_{li} = 0, j \neq i$) such that the data stream i can be successfully recovered. Thus, we have the following constraints to make all z_l data streams successful.

$$\mathbf{u}_{li}^T \mathbf{H}_{ll} \mathbf{v}_{li} = 1 \quad (1 \leq i \leq z_l), \quad (1)$$

$$\mathbf{u}_{lj}^T \mathbf{H}_{ll} \mathbf{v}_{li} = 0 \quad (1 \leq i, j \leq z_l, j \neq i). \quad (2)$$

Interference cancellation. Consider two links k and l and assume that the transmission on link k will interfere with the reception at link l (see Fig. 2). Suppose that we want to have z_k data streams on link k and z_l data streams on link l . Denote \mathbf{H}_{kl} the channel coefficient matrix between nodes Tx(k) and Rx(l). To ensure these simultaneous transmission of data streams possible, we must satisfy both the spatial multiplexing constraints in (1) and (2) for each link and the following interference cancellation constraints.

$$\mathbf{u}_{ki}^T \mathbf{H}_{kl} \mathbf{v}_{lj} = 0 \quad (1 \leq i \leq z_k, 1 \leq j \leq z_l).$$

The above constraints guarantee that the interference coming from each data stream of node Tx(k) is cancelled out for each data stream at node Rx(l).

B. Mathematical Modeling

In this paper, we consider a random multi-hop MIMO ad hoc network with n nodes, where each node, equipped with α antennas, is randomly located in a unit square area. Each node acts as a source node and transmits data to a randomly chosen destination node. The per-node throughput $\lambda(n)$ is defined as the minimum data rate that can be sent from each source to its destination via multi-hop routing. Our goal is to find the maximum asymptotic per-node throughput $\lambda(n)$.

We represent the random multi-hop MIMO ad hoc network by a directed graph, denoted by $\mathcal{G} = \{\mathcal{N}, \mathcal{L}\}$, where \mathcal{N}

and \mathcal{L} are the set of nodes and all possible MIMO links, respectively. We use the so-called protocol model [7] to deal with interference in the network. Under protocol model, each node in the network has a transmission range $r(n)$ and a node can only transmit data to the nodes within its transmission range. An ordered node pair (i, j) is said to be a link if node j is within i 's transmission range, i.e., $\mathcal{L} = \{(i, j) : d_{i,j} \leq r(n), i, j \in \mathcal{N}, i \neq j\}$, where $d_{i,j}$ is the distance. Likewise, each node also has an interference range $(1+\Delta)r(n)$ and when a node is transmitting, the other nodes (other than its intended receiver) within its interference range cannot be receiving data at the same time, where Δ is a non-negative constant. Under this protocol model, it has been shown by Gupta and Kumar in [7] that we need to set the transmission range $r(n) > \sqrt{\frac{\ln n}{\pi n}}$ to maintain network connectivity with high probability when $n \rightarrow \infty$. Due to the use of protocol model and our goal of keeping our analysis tractable, the impact of fading channel on network connectivity [18] is not considered in this study.

If one link is active and no interference cancellation scheme is used in the network, it will interfere with all its nearby links whose receiver are within the interference range of the transmitting node. When no interference cancellation is employed, denote \mathcal{I}_l^+ the set of links that are interfered by link $l \in \mathcal{L}$ and \mathcal{I}_l^- the set of links that interfere with link $l \in \mathcal{L}$.

MIMO physical layer. In this study, we focus on spatial multiplexing and interference cancellation to characterize MIMO physical layer behavior. Further, we employ DoF to represent MIMO resources at a node. A detailed discussion of DoF allocation for spatial multiplexing and interference cancellation is given in [19]. Simply put, when there is no interference, we need to allocate z_l DoFs at both transmitter $\text{Tx}(l)$ and receiver $\text{Rx}(l)$ to achieve z_l data streams on link l . When interference is present in the network, it is necessary to have an ordered list for all nodes and allocate DoFs sequentially to achieve interference cancellation. Denote $\pi(\cdot)$ the mapping between a node and its order in the node list. For two links l and k with z_l and z_k data streams on each link, $l \in \mathcal{L}, k \in \mathcal{I}_l^+$, we know that if $\pi(\text{Tx}(l)) > \pi(\text{Rx}(k))$ (i.e., node $\text{Tx}(l)$ is after node $\text{Rx}(k)$ in the ordered node list), then node $\text{Tx}(l)$ will be responsible for cancelling the interference from l to k and will thus consume z_k DoFs; if $\pi(\text{Tx}(l)) < \pi(\text{Rx}(k))$ (i.e., node $\text{Tx}(l)$ is before node $\text{Rx}(k)$ in the ordered node list), then node $\text{Rx}(k)$ will be responsible for cancelling the interference from l to k and will consume z_l DoFs. Then, a link l can support z_l active data streams in the network if and only if the following two constraints are satisfied.

- 1) DoF constraint at transmitter $\text{Tx}(l)$: The total number of DoFs transmitter $\text{Tx}(l)$ uses for spatial multiplexing and interference cancellation cannot exceed the number of available DoFs at node $\text{Tx}(l)$, i.e.,

$$z_l + \sum_{k \in \mathcal{I}_l^+} z_k \leq \alpha. \quad (3)$$

This constraint shows that the DoF consumption at transmitter $\text{Tx}(l)$ includes two parts: spatial multiplexing

(first term on the LHS) and interference cancellation (second term on the LHS). The total DoF allocation at transmitter $\text{Tx}(l)$ cannot exceed the total DoFs.

- 2) DoF constraint at receiver $\text{Rx}(l)$: The total number of DoFs that transmitter $\text{Rx}(l)$ allocates for spatial multiplexing and interference cancellation cannot exceed the number of available DoFs at node $\text{Rx}(l)$, i.e.,

$$z_l + \sum_{k \in \mathcal{I}_l^-} z_k \leq \alpha. \quad (4)$$

Routing and scheduling. In this paper, we assume that a node's transmitter is limited to a transmission range $r(n)$. When a source node cannot transmit data to its destination node in one hop, multi-hop routing is needed to relay the data. To avoid potential interference among active links, we employ TDMA to schedule conflict links into different time slots.

III. ASYMPTOTIC CAPACITY BOUNDS

In this section, we analyze the asymptotic capacity bounds for multi-hop MIMO ad hoc networks and the main result of this section is summarized as follows.

The capacity of a random multi-hop MIMO ad hoc network with n nodes is $\lambda(n) = \Theta(\frac{\alpha W}{\sqrt{n \ln n}})$ with high probability when $n \rightarrow \infty$.

The above capacity bound is determined by finding a capacity lower bound and a capacity upper bound and showing that they have the same order.

Lower bound analysis. For the capacity lower bound, it is only necessary to find a feasible routing and scheduling scheme for the underlying network. For this purpose, we can simply consider the trivial case where all DoF resource at a node is allocated for spatial multiplexing (i.e., no DoF will be used for interference cancellation). That is, when a link is active, it will use all its DoFs at the transmitter and the receiver to carry α data streams on that link. This simple case corresponds to scaling the capacity lower bound for a single-antenna ad hoc network by a factor of α . In [7], Gupta and Kumar showed that a capacity lower bound for a single-antenna ad hoc networks is $\Omega(\frac{W}{\sqrt{n \ln n}})$ by constructing a feasible routing and scheduling scheme. Thus, by adopting the same routing and scheduling scheme in our MIMO ad hoc networks as in [7], a capacity lower bound of $\Omega(\frac{\alpha W}{\sqrt{n \ln n}})$ can be obtained.

Upper bound analysis. The capacity upper bound analysis is more challenging and is the main contribution of this paper. Note that here we cannot simply scale the capacity upper bound result in [7] by a factor of α by just considering spatial multiplexing and neglecting interference cancellation. Doing so will neglect the potential increase of network capacity by joint consideration of using DoFs for spatial multiplexing and interference cancellation. Therefore, we must consider *both* spatial multiplexing and interference cancellation for upper bound analysis. As a result, the upper bound analysis in [7] cannot be extended over here.

To derive an upper bound for multi-hop MIMO ad hoc networks, we propose a novel partitioning method on the

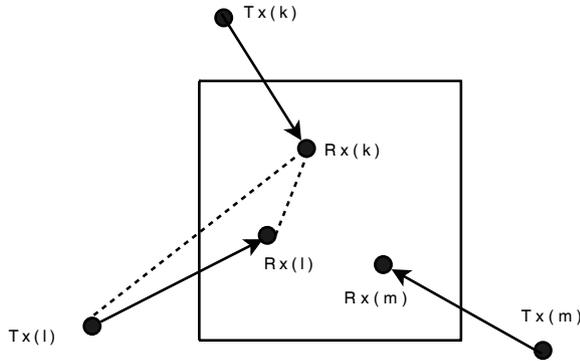


Fig. 3. The receivers in a square with side length $1/\lceil \frac{\sqrt{2}}{\Delta r(n)} \rceil$.

network area. We partition the unit square for the network into small squares, with the size of each small square being cleverly chosen so that the maximum data rate that can be received by the nodes inside the small square can be computed exactly. By taking the sum of data rates from all small squares, we can obtain the maximum data rate the whole network can support. Based on this result, we can further derive a capacity upper bound for the entire network.

Now the key problem in our analysis is how to set the size of each small square. This is because in a MIMO network, several receiving nodes can be active within close vicinity by using interference cancellation. If the size of each small square is set too large, then the maximum number of data streams that can be received by the nodes inside the square cannot be computed exactly. On the other hand, if the size of each small square is set too small, then the maximum number of data streams that can be received by the nodes inside the square is likely to be over-estimated, leading to a loose upper bound. We show that when the length of each side of small square is set to $1/\lceil \frac{\sqrt{2}}{\Delta r(n)} \rceil$, we can precisely determine the maximum number of data streams that can be received by nodes inside the small square, regardless of the number of receiving nodes in the square. We formally state this result in the following lemma.

Lemma 1: For a square with side length $1/\lceil \frac{\sqrt{2}}{\Delta r(n)} \rceil$, the maximum number of total data streams that can be received by nodes inside the square at any time slot for any routing scheme is no greater than α regardless of the number of receiving nodes inside the square.

Proof: Suppose there are K active links with their respective receivers being in this square. If $K = 1$, the theorem holds trivially, since the number of incoming data streams of a receiver cannot exceed the number of antennas at the receiver. Now we show the result also holds when $K \geq 2$.

We first show that when interference cancellation is not employed, any two links with receiving nodes inside the square will interfere with each other. Note that the distance between any two receivers inside this square is at most $\sqrt{2} \cdot \frac{\Delta r(n)}{\sqrt{2}} = \Delta \cdot r(n)$. Referring to Fig. 3, for two links l and k with their receivers $Rx(l)$ and $Rx(k)$ inside the square, we have $d_{Rx(l), Rx(k)} \leq \Delta \cdot r(n)$. Since $d_{Tx(l), Rx(l)} \leq r(n)$ (recall that $r(n)$ is transmission range) based on the triangle inequality, we have $d_{Tx(l), Rx(k)} \leq d_{Rx(l), Rx(k)} + d_{Tx(l), Rx(l)} \leq (1 + \Delta)r(n)$. Since

$(1 + \Delta)r(n)$ is the interference range of $Tx(l)$, this shows that $Tx(l)$ will interfere with $Rx(k)$. Similarly, we can prove that the transmitter $Tx(k)$ of link k will interfere receiver $Rx(l)$ of link l .

Denote the set of these K links as $\mathcal{K} = \{1, \dots, K\}$ and the number of active data streams on link l as $z_l, l \in \mathcal{K}$. We have shown that all these active links will interfere with each other. Thus, in order to make them all active simultaneously, interference cancellation is necessary. Based on the MIMO model we discussed earlier, we need an ordered list for all the $2K$ nodes on these K links to determine interference cancellation. Depending on whether the last node in the ordered list is a transmitter or receiver, we have two cases.

Case i. The last node in the ordered list is a receiver. Without loss of generality, assume that receiver $Rx(m)$ of link m is the last node in this list. To have z_m data streams on link m , based on (4), we have the following constraint on $Rx(m)$.

$$z_m + \sum_{k \in \mathcal{I}_m^-} z_k \leq \alpha, \quad (5)$$

where we recall $\pi(\cdot)$ is the mapping function between a node and its order in the node list. Since any two links interfere with each other in this small square, we have $\mathcal{I}_m^- = \mathcal{K} \setminus \{m\}$. Further, since $Rx(m)$ is the last node in this list, we have $\pi(Rx(m)) > \pi(Tx(k))$, for all $k \in \mathcal{K} \setminus \{m\}$. Therefore, (5) can be written as $z_m + \sum_{k \in \mathcal{K} \setminus \{m\}} z_k \leq \alpha$, which is $\sum_{k \in \mathcal{K}} z_k \leq \alpha$.

Thus, we have shown that the sum of data streams that can be received by nodes inside the small square is no greater than α regardless of the size of the set \mathcal{K} .

Case ii. The last node in the ordered list is a transmitter. In this case, based on (3) and following the same token as the above discussion, we can get the same result as *Case i*.

Combining the two cases, the proof is complete. \blacksquare

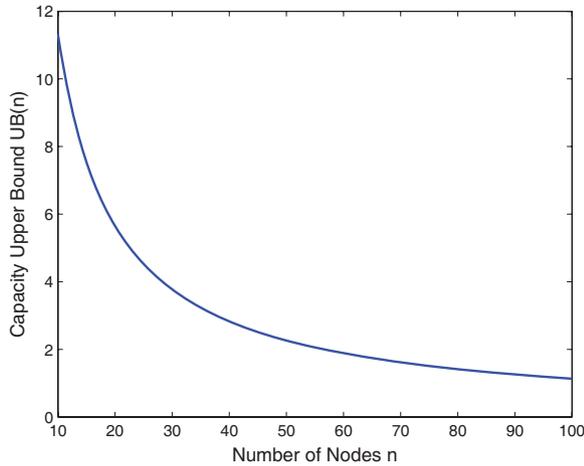
Based on Lemma 1, we can now compute the maximum data rate that can be supported in the unit square network by taking the sum of the data rates among all small squares. Since the side length of each small square is $1/\lceil \frac{\sqrt{2}}{\Delta r(n)} \rceil$, the total number of small squares in the unit square is $\lceil \frac{\sqrt{2}}{\Delta r(n)} \rceil^2$. From Lemma 1, we know that the maximum number of data streams inside a small square is α . Thus the total data rate that each square can support is at most αW . So the maximum data rate that can be supported in the network is $\lceil \frac{\sqrt{2}}{\Delta r(n)} \rceil^2 \alpha W$.

We are now ready to derive a capacity upper bound for MIMO ad hoc network, which is stated in the following theorem.

Theorem 1: For a random multi-hop MIMO ad hoc network, a capacity upper bound for all possible routing and scheduling schemes is $\lambda(n) = O(\frac{\alpha W}{\sqrt{n \ln n}})$ with high probability when $n \rightarrow \infty$.

Proof: Let D be the average length of source-destination lines. Since multi-hop routing is employed, we have that the average number of each source-destination pair is at least $\frac{D}{r(n)}$. Note that there are n source-destination pairs. Thus, the required transmission rate over the entire network is at least $\frac{D}{r(n)} n \lambda(n)$.

When TDMA is used to schedule conflict links into different time slots, the average rate over all time slots in the entire

Fig. 4. Capacity upper bound $UB(n)$.

network is at least $\frac{D}{r(n)}n\lambda(n)$. Since the maximum data rate that can be supported in the network at any time slot is $\lceil \frac{\sqrt{2}}{\Delta \cdot r(n)} \rceil^2 \alpha W$, we have $\frac{D}{r(n)}n\lambda(n) \leq \lceil \frac{\sqrt{2}}{\Delta \cdot r(n)} \rceil^2 \alpha W < (\frac{\sqrt{2}}{\Delta \cdot r(n)} + 1)^2 \alpha W$, which can be rewritten as

$$\lambda(n) < \frac{2\alpha W}{\Delta^2 D n r(n)} + \frac{2\sqrt{2}\alpha W}{\Delta D n} + \frac{\alpha W r(n)}{D n}. \quad (6)$$

It has been shown in [7] that to maintain the connectivity of the network, we need $r(n) > \sqrt{\frac{\ln n}{\pi n}}$. It can be verified that the right-hand-side of (6) is a non-increasing function of $r(n)$. By substituting $r(n) = \sqrt{\frac{\ln n}{\pi n}}$ into (6), we get

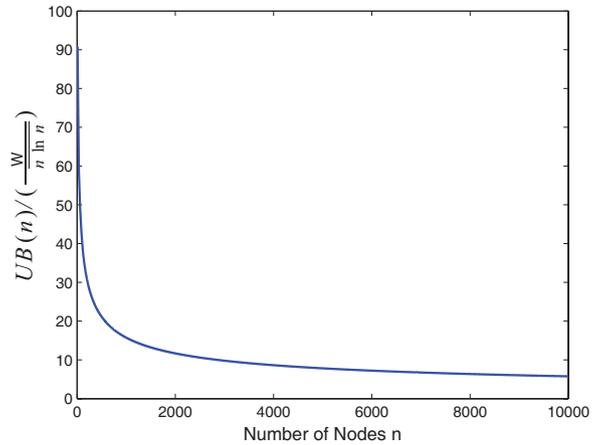
$$\lambda(n) < \frac{2\alpha W \sqrt{\pi}}{\Delta^2 D \sqrt{n \ln n}} + \frac{2\sqrt{2}\alpha W}{\Delta^2 D n} + \frac{\alpha W \sqrt{\ln n}}{D n \sqrt{\pi n}} = O\left(\frac{\alpha W}{\sqrt{n \ln n}}\right). \quad (7)$$

The upper bound in Theorem 1 is tight, because it has the same order as that of the capacity lower bound we obtained at the beginning of this section. Combining the capacity lower and upper bounds, we can see that the capacity of a random multi-hop MIMO ad hoc network with n nodes is $\Theta\left(\frac{\alpha W}{\sqrt{n \ln n}}\right)$.

IV. NUMERICAL RESULTS

In previous section, our theoretical results show that by using spatial multiplexing and interference cancellation, MIMO can have a constant improvement α on asymptotic capacity compared to the results of Gupta and Kumar [7]. However, our results also show that MIMO cannot fundamentally improve the asymptotic capacity of multi-hop wireless networks, since it still has the same order as the results of Gupta and Kumar.

In this section, we will present some numerical results to validate our theoretical findings. We set $\alpha = 4$, $W = 1$, and $\Delta = 1$. By running 1000 instances, we obtain the average length of source-destination lines $D = 0.52$. Denote the capacity upper bound (in Equation (7)) as $UB(n) = \frac{2\alpha W \sqrt{\pi}}{\Delta^2 D \sqrt{n \ln n}} + \frac{2\sqrt{2}\alpha W}{\Delta^2 D n} + \frac{\alpha W \sqrt{\ln n}}{D n \sqrt{\pi n}}$. We show that $UB(n)$ decreases as n increases in Fig. 4. Then, we validate $\lambda(n) = O\left(\frac{\alpha W}{\sqrt{n \ln n}}\right)$ by showing $UB(n)/\left(\frac{\alpha W}{\sqrt{n \ln n}}\right)$ converges to a constant when n goes to infinity. This result is shown in Fig. 5.

Fig. 5. The normalized capacity upper bound $UB(n)/\left(\frac{\alpha W}{\sqrt{n \ln n}}\right)$.

V. CONCLUSION

In this paper, we studied capacity scaling laws for MIMO ad hoc networks, i.e., the achievable throughput of each node as the number of nodes in the network increases. Our analysis was based on a MIMO network model that employs zero-forcing beamforming, a powerful physical layer technique that is capable of spatial multiplexing and interference cancellation. Based on this model, we obtained the capacity lower bound and upper bound. The main contribution of this paper is the development of upper bound, which requires joint consideration of spatial multiplexing and interference cancellation. Our results showed that both lower bound and upper bound have the same order, thus assuring that our asymptotic capacity for MIMO ad hoc network is tight.

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