Variable Bit Rate Flow Routing in Wireless Sensor Networks

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***Abstract*— Since energy constraint is a fundamental issue for wireless sensor networks, network lifetime performance has become a key performance metric for such networks. In this paper, we consider a two-tier wireless sensor network and focus on the flow routing problem for the upper tier aggregation and forwarding nodes (AFNs). Specifically, we are interested in how to perform flow routing among the nodes when the bit rate from each source node is time-varying. We present an algorithm that can be used to construct a flow routing solution with the following properties: (1) If the average rate from each source node is known a priori, then flow routing solution obtained via such algorithm is optimal and offers provably maximum network lifetime performance; (2) If the average rate of each source node is unknown but is within a fraction (ε) of an estimated rate value, then network lifetime by the proposed flow routing solution is within $\frac{2\varepsilon}{1-\varepsilon}$ from the optimum. These results fill in an important gap in theoretical foundation for flow routing in energy-constrained sensor networks.**

***Index Terms*— Network lifetime, energy constraint, directional antenna, power control, flow routing, variable bit rate, wireless sensor networks.**

I. Introduction

Wireless sensor network is a special form of wireless networks dedicated to surveillance and monitoring applications. It is characterized by severe battery constraint on each node. In this paper, we consider a two-tier wireless sensor network for various sensing applications (see Fig. 1) [4], [6], [7], [10]. A key performance measure for a wireless sensor network is *network lifetime* [2], [6], [10]. For the two-tier wireless sensor networks considered in this paper, whenever a backbone node, also called *aggregation and forwarding node* (AFN), runs out of energy, the sensing coverage for that local area is lost. Therefore, the most stringent definition for network lifetime is the time until any backbone AFN fails.

To conserve energy consumption, we assume each AFN is equipped with directional antennas [18], [20], which are capable of forming multiple beams at each node for flow routing in the network. Further, we assume each beam's distance coverage can be controlled by the beam's transmission power. In this network setting, we investigate the flow routing problem for the upper-tier AFNs with the aim of maximizing network lifetime. When the bit rate generated by each AFN is a constant, the network lifetime optimization problem can be solved by a linear program (LP) (see e.g., [2]). But in practice, the bit rate generated by each AFN is hardly a constant. As a result, new algorithms addressing variable bit rates from source nodes need to be developed.

In this paper, we study this flow routing problem for variable bit rates from source nodes. The main result of this paper is an algorithm that can construct a flow routing solution for variable bit rate from each source node, with the following network lifetime performance guarantees:

- If the average of each AFN's time-varying bit rate is known a priori, then the flow routing solution is optimal and provides maximum network lifetime performance.
- If the average of each AFN's time-varying bit rate is unknown but can be estimated within an error bound of ε, then the network lifetime provided by the proposed flow routing solution is provably to be within $\frac{2\varepsilon}{1-\varepsilon}$ from the optimum.

The basic idea of the proposed algorithm is to employ a constant bit rate auxiliary problem as a reference and construct flow routing solution for the variable bit rate problem. Specifically, the constant bit rate of each source node in the auxiliary problem is defined as the average rate (either given or estimated) of each source in the variable bit rate problem. Since the auxiliary problem can be solved by a single LP as in [2], we can conduct detailed analysis of flow at each link and define a set of source nodes that contribute traffic on this link and their corresponding bit rates. Consequently, we can define a set of weights (or fractions) for the bit rate generated by each source node on a particular link. By using the same set of weights, we construct flow routing solution for the variable bit rate problem.

The remainder of this paper is organized as follows. In Section II, we present the network and energy models. In Section III, we present a flow routing algorithm for the time-varying bit rate problem. Subsequently, we show that if the average data rate of each AFN is known, the flow routing solution obtained via this algorithm offers provable maximum network lifetime. In Section IV, we consider the case when the average bit rate of each AFN is unknown. We show that as long as the unknown average rate is within a fraction (ε) of an estimated rate value, the network lifetime performance by the same flow routing solution is within $\frac{2\varepsilon}{1-\varepsilon}$ from the optimal. Section V reviews related work and Section VI concludes this paper.

II. Network and Energy Models

A. Network Model

We focus on a two-tier architecture for wireless sensor networks [4], [6], [7], [10]. Figures 1(a) and (b) show the

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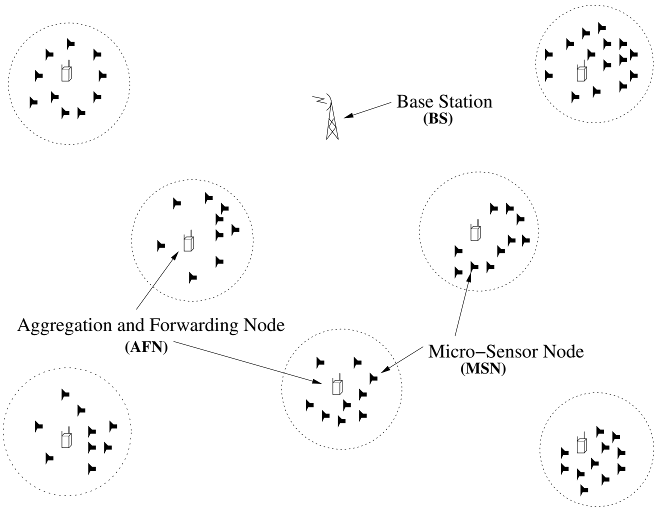
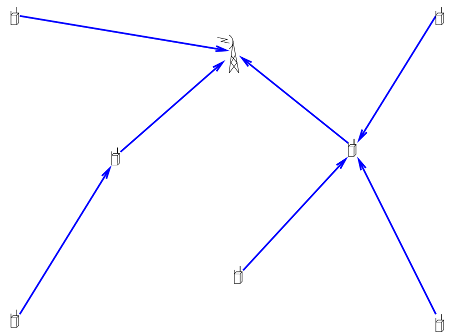


(a) Physical network topology.



(b) An instance of flow routing topology among the AFNs.

Fig. 1. Reference architecture for a two-tier wireless sensor network.

physical network topology and a flow routing among AFNs for such a network, respectively. As shown in these figures, we have three types of nodes in the network, namely, *micro sensor nodes* (MSNs), *aggregation and forwarding nodes* (AFNs), and a *base-station* (BS). The MSNs constitute the lower-tier of the network. They are deployed in groups (or clusters) at strategic locations for various sensing applications. The objective of an MSN is very simple: once triggered by an event, it starts to capture sensing data and sends it directly to its local AFN.

Within each cluster of MSNs, there is one AFN, which differs from an MSN in terms of physical structure and functions. The primary functions of an AFN are: 1) *data aggregation* (or "fusion") for flows coming from the local cluster of MSNs, and 2) *forwarding* (or relaying) the aggregated data to the next hop AFN toward the base-station. Due to energy consumption, each AFN has a limited lifetime. Upon the depletion of energy at an AFN, the *coverage* for the particular area is lost. In this paper, we define network lifetime as the time instance until any AFN runs out of energy.

The base-station is the *sink* node for data generated at all AFNs in the network. We assume that a base station is not energy constrained.

In summary, the function of the lower-tier MSNs is for data acquisition, while the upper-tier AFNs are used for data fusion and forwarding the aggregated data toward the base-station. Our focus in this paper is on the flow routing problem for the upper tier AFNs.

There are many benefits associated with such two-tier sensor network architecture. First, the two-tier architecture can take advantage of recent advances in *distributed source coding* (DSC) for sensor networks, e.g., [3], [11]. Note that there are correlations in data collected in neighboring MSNs. DSC enables the removal of such redundancy through a distributed compression algorithm which obviates the need for an MSN to communication with its neighboring MSNs. Second, the two-tier architecture alleviates the scalability problem associated with sensor networking. Third, the two-tier architecture effectively *decouples* information collection (at MSNs) and information fusion/forwarding (at AFNs). The MSNs on the lower tier can concentrate on sensing (with limited communication capability), while the AFNs on the upper tier can focus on data fusion and wireless networking. The data forwarding (routing) problem on the upper tier can now be addressed *separately* and *independently* from the data acquisition problem on the lower tier. This approach enables us to optimize each component for maximum efficiency, and also simplifies the control and management of the network.

B. Energy Model

For an AFN, the transmission power can be characterized by [20]

$$p_t(i,\ k) = c_{ik} \cdot f_{ik}\ ,$$

where $p_t(i,k)$ is the power dissipated when AFN i is transmitting to k, f_{ik} is the rate of the flow from node i to node k. c_{ik} is the power consumption cost for directional antenna over link (i,k) and $c_{ik} = \alpha + \frac{\theta}{360}\beta \cdot d_{ik}^n$, where α and β are constant coefficients, θ is the beam-width of the directional antenna and is a constant in this paper, d_{ik} is the distance between nodes i and k, n is the path loss index [14].

The power dissipation at the receiver of AFN i can be modeled as [20]

$$p_r(i) = \sum_{m \neq i} \rho \cdot f_{mi}\ ,$$

where f_{mi} (in b/s) is the incoming rate of received data stream from AFN m.

The above transmission and reception energy model assumes a contention-free MAC protocol, where interference from simultaneous transmission can be effectively minimized or avoided. For traffic pattern in this paper, a contention-free MAC protocol is fairly easy to design (see, e.g., [17]) and its discussion is beyond the scope of this paper. Table I lists the notation used in this paper.

III. Optimal Flow Routing with Given Average Bit Rate

Our main focus in this paper is the flow routing problem when the bit rate from each AFN is time-varying. We consider a sensor network with a set of $\mathcal{N}$ AFNs ($|\mathcal{N}| = N$). For each AFN $i \in \mathcal{N}$, denote its rate as $g_i(t)$ and its initial energy as e_i. Let $\bar{g}_i$ be the average of $g_i(t)$, i.e.,

$$\bar{g}_i = E[g_i(t)]\ . \tag{1}$$

TABLE I
NOTATION.

Symbol	Definition
$\mathcal{N}$	The set of AFNs in the network
N	$= \|\mathcal{N}\|$, the total number of AFNs in the network
e_i	Initial energy at AFN i
$g_i(t)$	Local generated rate at AFN i at time t
$\bar{g}_i$ (or $\hat{g}_i$)	Average (or estimated average) local generated rate at AFN i
P	The flow routing problem under $g_i(t)$, $i \in \mathcal{N}$
$\bar{P}$ (or $\hat{P}$)	The flow routing problem under $\bar{g}_i$ (or $\hat{g}_i$), $i \in \mathcal{N}$
$\bar{\pi}$ (or $\hat{\pi}$)	Optimal flow routing solution to problem $\bar{P}$ (or $\hat{P}$)
ψ (or π)	Flow routing solution to problem P from solution $\bar{\pi}$ (or $\hat{\pi}$)
$\bar{T}$ (or $\hat{T}$)	Maximum network lifetime for problem $\bar{P}$ (or $\hat{P}$)
T^* (or T)	Network lifetime for problem P from solution $\bar{\pi}$ (or $\hat{\pi}$)
ε	The maximum estimation error on data rate, i.e., $\|\bar{g}_i - \hat{g}_i\| \leq \varepsilon \hat{g}_i$ for $1 \leq i \leq N$.
ρ	Power consumption coefficient for receiving data
c_{ik} (or c_{iB})	Power consumption coefficient for transmitting data from AFN i to AFN k (or base-station B)
α, β	Constant terms in power consumption coefficient for transmitting data
θ	Beam-width of the directional antenna
n	Path loss index
d_{ik} (or d_{iB})	Distance from AFN i to AFN k (or base-station B)
$f_{ik}(t)$ (or $f_{iB}(t)$)	For problem P, rate from AFN i to AFN k (or base-station B) at time t
$\bar{f}_{ik}$ (or $\bar{f}_{iB}$)	For problem $\bar{P}$, rate from AFN i to AFN k (or base-station B)
$\hat{f}_{ik}$ (or $\hat{f}_{iB}$)	For problem $\hat{P}$, rate from AFN i to AFN k (or base-station B)
H	$= 1/T$
$\bar{H}$ (or $\hat{H}$)	$= 1/\bar{T}$ (or $= 1/\hat{T}$)
w_{ik}^s (or w_{iB}^s)	Percentage of data generated by AFN s transmitted from AFN i to AFN k (or base-station B)
$f_{ik}^s(t)$ (or $f_{iB}^s(t)$)	For problem P, data generated by AFN s and transmitted from AFN i to AFN k (or base-station B) at time t
$\bar{f}_{ik}^s$ (or $\bar{f}_{iB}^s$)	For problem $\bar{P}$, data generated by AFN s and transmitted from AFN i to AFN k (or base-station B)

Denote P as the flow routing problem under variable bit rate sources $g_i(t)$ and $\bar{P}$ as the flow routing problem under constant bit rate sources $\bar{g}_i$, $i \in \mathcal{N}$, with the same network configuration and the same initial energy on each AFN.

In this section, we study the optimal flow routing problem when $g_i(t)$ is time-varying but $\bar{g}_i$ is known, $i \in \mathcal{N}$. We show that the optimal flow routing solution for problem P with variable source bit rates can be obtained by studying an optimal flow routing solution for the auxiliary problem $\bar{P}$ with constant source bit rates $\bar{g}_i$, $i \in \mathcal{N}$.

A. Preliminary

We first consider the auxiliary flow routing problem $\bar{P}$ with constant source rates $\bar{g}_i$, $1 \leq i \leq N$, and formulate it as a linear programming (LP) problem. Denote $\bar{T}$ the network lifetime, i.e., time until any node runs out of energy. Denote $\bar{f}_{ik}$ and $\bar{f}_{iB}$ the flow rates from node i to node k and to the base-station B, respectively. We have the following flow balance equations and energy constraints for each node $i \in \mathcal{N}$.

$$\bar{g}_i + \sum_{m \neq i} \bar{f}_{mi} = \sum_{k \neq i} \bar{f}_{ik} + \bar{f}_{iB} \qquad (2)$$

$$\left(\sum_{m \neq i} \rho \bar{f}_{mi} + \sum_{k \neq i} c_{ik} \bar{f}_{ik} + c_{iB} \bar{f}_{iB} \right) \bar{T} \leq e_i \qquad (3)$$

Eq. (2) states that, at each node i, the data rate $\bar{g}_i$ generated by $i \in \mathcal{N}$, plus total received data rates from other nodes, is equal to its total outgoing (transmitted) bit rate. Inequality (3) states that at each node $i \in \mathcal{N}$, the energy required to receive and transmit its incoming and outgoing data rates cannot exceed its initial energy over the period of network lifetime $\bar{T}$. Our objective is to maximize the network lifetime $\bar{T}$ while both (2) and (3) are satisfied.

Denoting $\bar{H} = 1/\bar{T}$, we have the following LP.

LP-Basic

$$\begin{aligned} \text{Min} \quad & \bar{H} \\ \text{s.t.} \quad & \sum_{k \neq i} \bar{f}_{ik} + \bar{f}_{iB} - \sum_{m \neq i} \bar{f}_{mi} = \bar{g}_i \quad (1 \leq i \leq N)\,, \qquad (4) \\ & \sum_{m \neq i} \rho \bar{f}_{mi} + \sum_{k \neq i} c_{ik} \bar{f}_{ik} + c_{iB} \bar{f}_{iB} - e_i \bar{H} \leq 0 \quad (1 \leq i \leq N)\,, \qquad (5) \\ & \bar{f}_{ik}, \bar{f}_{iB}, \bar{H} \geq 0 \quad (1 \leq i, k \leq N, i \neq k)\,, \end{aligned}$$

where (4) follows from the flow balance equation (2) and (5) follows from the energy constraint (3). Note that $\bar{H}$, $\bar{f}_{mi}$, $\bar{f}_{ik}$, and $\bar{f}_{iB}$ are variables and $\bar{g}_i$, ρ, c_{ik}, c_{iB}, and e_i are all constants. This LP problem is similar to that in [2] and can be solved by standard techniques in polynomial time.

B. Optimal Flow Routing for Problem P

We now present an algorithm to obtain a flow routing solution ψ for problem P that achieves a network lifetime $T^* = \bar{T}$, where $\bar{T}$ is the maximum network lifetime under an optimal flow routing solution $\bar{\pi}$ (obtained via LP) to the constant bit rate auxiliary problem $\bar{P}$ in Section III-A. Then, we prove that T^* is also the maximum network lifetime for problem P.

The main idea of the algorithm is as follows. Note that in LP solution $\bar{\pi}$, we have found the aggregate flow rate on each link but without any knowledge of what source nodes have contributed traffic on this link. For variable bit rate problem P, it is necessary to identify the specific source nodes that contribute traffic on each link and the amount of such contribution. That is, we first identify that, in the flow routing solution $\bar{\pi}$, for each source node s, what is the fraction of data generated by node s that is being forwarded by node i to node k and to the base-station B, which we denote as w_{ik}^s and w_{iB}^s. Then for problem P, we keep the same w_{ik}^s and w_{iB}^s when constructing the flow routing solution ψ.

More formally, under $\bar{\pi}$, denote $\bar{f}_{ik}^s$ and $\bar{f}_{iB}^s$ the flow rates generated by node s that are being forwarded by node i to node k and to the base-station B, respectively. Then by definition, we have $w^s_{ik} = \frac{\bar{f}^s_{ik}}{\bar{g}_s}$ and $w^s_{iB} = \frac{\bar{f}^s_{iB}}{\bar{g}_s}$. Therefore,

$$\bar{f}_{ik} = \sum_{s \in \mathcal{N}, s \neq k} \bar{f}^s_{ik} = \sum_{s \in \mathcal{N}, s \neq k} w^s_{ik} \cdot \bar{g}_s \ , \quad (6)$$

$$\bar{f}_{iB} = \sum_{s=1}^{N} \bar{f}^s_{iB} = \sum_{s=1}^{N} w^s_{iB} \cdot \bar{g}_s \ . \quad (7)$$

The following algorithm constructs a flow routing solution for variable bit rate source nodes and is the main result of this paper.

Algorithm 1: **(Flow Routing with Variable Rates)**

1) We first define all w^s_{ik} and w^s_{iB}, $1 \leq i, k, s \leq N$, $i \neq k$, $s \neq k$, as follows.

   a) (The case of source node.) Identify a source node $s \in \mathcal{N}$ such that w^s_{sk} and w^s_{sB} have not been defined in previous iterations. If no such node exists, go to Step 1(b).
   For w^s_{sk}, $k \in \mathcal{N}$ and $k \neq s$, and w^s_{sB}, we have

$$w^s_{sk} = \frac{\bar{f}_{sk}}{\bar{f}_{sB} + \sum_{r \neq s} \bar{f}_{sr}} \ , \quad (8)$$

$$w^s_{sB} = \frac{\bar{f}_{sB}}{\bar{f}_{sB} + \sum_{r \neq s} \bar{f}_{sr}} \ . \quad (9)$$

   Go to Step 1(a).

   b) (The case of relay node.) Identify a source node $s \in \mathcal{N}$ and one of its relay node $i \in \mathcal{N}$ (not necessarily one hop relay), $i \neq s$, such that (i) for all incoming links (m, i) to node i, w^s_{mi} are already defined in previous iterations and (ii) w^s_{ik} and w^s_{iB} have not been defined in previous iterations. If no such (s, i) node pair exists, we have already defined all w^s_{ik} and w^s_{iB} and we can continue to Step 2.
   For w^s_{ik}, $k \in \mathcal{N}$ and $k \neq i, s$, and w^s_{iB}, we have

$$w^s_{ik} = \frac{\bar{f}_{ik}}{\bar{f}_{iB} + \sum_{r \neq i} \bar{f}_{ir}} \sum_{m \neq i} w^s_{mi} \ , \quad (10)$$

$$w^s_{iB} = \frac{\bar{f}_{iB}}{\bar{f}_{iB} + \sum_{r \neq i} \bar{f}_{ir}} \sum_{m \neq i} w^s_{mi} \ . \quad (11)$$

   Go to Step 1(b).

2) Define $f^s_{ik}(t)$ and $f^s_{iB}(t)$, i.e., the respective data rates generated by node s and forwarded from node i to node k and to the base-station B at time t under ψ, as follows:

$$f^s_{ik}(t) = w^s_{ik} \cdot g_s(t) \ , \quad (12)$$

$$f^s_{iB}(t) = w^s_{iB} \cdot g_s(t) \ . \quad (13)$$

   Then for $f_{ik}(t)$ and $f_{iB}(t)$, i.e., the respective flow rates from node i to node k and to the base-station B at time t under ψ, we have

$$f_{ik}(t) = \sum_{s \neq k} f^s_{ik}(t) = \sum_{s \neq k} w^s_{ik} \cdot g_s(t) \ , \quad (14)$$

$$f_{iB}(t) = \sum_{s=1}^{N} f^s_{iB}(t) = \sum_{s=1}^{N} w^s_{iB} \cdot g_s(t) \ . \quad (15)$$

Lemma 1: The flow routing solution ψ produced by Algorithm 1 is feasible and has network lifetime $\bar{T}$.

Proof: The feasibility part can be proved by showing that at each node, the flow balance property is maintained. To show that the network lifetime under ψ is $\bar{T}$, it is sufficient to show that the remaining energy on each node i under both ψ and $\bar{\pi}$ at time $\bar{T}$ is identical.

We first consider the part of incoming and outgoing data flows at node i that originates from source node s, $i, s \in \mathcal{N}$.

If $i = s$, i.e., node i is the source node s, we have

$$\begin{aligned} g_i(t) &= \left(\frac{\bar{f}_{iB}}{\bar{f}_{iB} + \sum_{r \neq i} \bar{f}_{ir}} + \sum_{k \neq i} \frac{\bar{f}_{ik}}{\bar{f}_{iB} + \sum_{r \neq i} \bar{f}_{ir}} \right) g_i(t) \\ &= \left(w^i_{iB} + \sum_{k \neq i} w^i_{ik} \right) g_i(t) = f^i_{iB}(t) + \sum_{k \neq i} f^i_{ik}(t) \ . \quad (16) \end{aligned}$$

The second equality holds by the definitions of w^s_{ik} and w^s_{iB} in (8) and (9). The last equality holds by the definitions of $f^s_{ik}(t)$ and $f^s_{iB}(t)$ in (12) and (13).

If $i \neq s$, i.e., node i is a relay node (not necessarily one hop away) for bit rate generated at s, we have

$$\begin{aligned} &\sum_{m \neq i} f^s_{mi}(t) = \sum_{m \neq i} w^s_{mi} \cdot g_s(t) \\ &= \left(\frac{\bar{f}_{iB}}{\bar{f}_{iB} + \sum_{r \neq i} \bar{f}_{ir}} + \sum_{k \neq i} \frac{\bar{f}_{ik}}{\bar{f}_{iB} + \sum_{r \neq i} \bar{f}_{ir}} \right) \sum_{m \neq i} w^s_{mi} \cdot g_s(t) \\ &= \left(w^s_{iB} + \sum_{k \neq i} w^s_{ik} \right) g_s(t) = f^s_{iB}(t) + \sum_{k \neq i} f^s_{ik}(t) \ . \quad (17) \end{aligned}$$

The first equality holds by the definitions of $f^s_{ik}(t)$ in (12). The third equality holds by (10) and (11). The last equality holds by (12) and (13).

To verify flow balance, we have

$$\begin{aligned} &g_i(t) + \sum_{m \neq i} f_{mi}(t) = g_i(t) + \sum_{s \neq i} \sum_{m \neq i} f^s_{mi}(t) \\ &= f^i_{iB}(t) + \sum_{k \neq i} f^i_{ik}(t) + \sum_{j \neq i} \left[f^s_{iB}(t) + \sum_{k \neq i} f^s_{ik}(t) \right] \\ &= \left[f^i_{iB}(t) + \sum_{s \neq i} f^s_{iB}(t) \right] + \left[\sum_{k \neq i} f^i_{ik}(t) + \sum_{j \neq i} \sum_{k \neq i} f^s_{ik}(t) \right] \\ &= \sum_{s=1}^{N} f^s_{iB}(t) + \sum_{s=1}^{N} \sum_{k \neq i} f^s_{ik}(t) = f_{iB}(t) + \sum_{k \neq i} f_{ik}(t) \ . \end{aligned}$$

The first equality holds by the definitions of $f_{ik}(t)$ in (14). The second equality holds by our results in (16) and (17). The last equality holds by (14) and (15). Therefore, flow balance on each node $i \in \mathcal{N}$ holds.

For energy constraint, we first consider a flow $f_{ik}(t)$, $1 \leq i, k \leq N$ and $i \neq k$. We have

$$\begin{aligned} &\int_{t=0}^{\bar{T}} f_{ik}(t) dt = \int_{t=0}^{\bar{T}} \sum_{s \neq k} w^s_{ik} g_s(t) dt \\ &= \sum_{s \neq k} w^s_{ik} \int_{t=0}^{\bar{T}} g_s(t) dt = \sum_{s \neq k} w^s_{ik} \bar{g}_s \bar{T} = \bar{f}_{ik} \bar{T} \ . \end{aligned}$$

The first equality holds by the definitions of $f_{ik}(t)$ in (14). The third equality holds by (1). The last equality by (6). Similarly, we can show that $\int_{t=0}^{\bar{T}} f_{iB}(t)dt = \bar{f}_{iB}\bar{T}$. Thus, the total energy consumed at node $s \in \mathcal{N}$ over time $\bar{T}$ is

$$\int_0^{\bar{T}} \left[\sum_{m \neq i} \rho f_{mi}(t) + \sum_{k \neq i} c_{ik} f_{ik}(t) + c_{iB} f_{iB}(t) \right] dt$$
$$= \sum_{m \neq i} \rho \bar{f}_{mi}\bar{T} + \sum_{k \neq i} c_{ik}\bar{f}_{ik}\bar{T} + c_{iB}\bar{f}_{iB}\bar{T} \ .$$

Therefore, at time $\bar{T}$ the energy consumption at each node $i \in \mathcal{N}$ under ψ for problem P is exactly the same as that under $\bar{\pi}$ for problem $\bar{P}$. Since at time $\bar{T}$, there is at least one node with zero remaining energy under $\bar{P}$, $\bar{T}$ is thus also the network lifetime for problem P. This completes the proof. ■

The following theorem shows that $\bar{T}$ is the *maximum* network lifetime for problem P. That is, there does not exist a flow routing solution that can produce a network lifetime greater than $\bar{T}$ for problem P.

Theorem 1: Suppose that for each $i \in \mathcal{N}$, we have $\bar{g}_i = E[g_i(t)]$. Calculate ψ, the time-varying flow routing solution to problem P, by Algorithm 1 based on problem $\bar{P}$ with average rates $\bar{g}_i$, $i \in \mathcal{N}$. Then the network lifetime $\bar{T}$ under ψ is the maximum network lifetime T^* for problem P.

Proof: It is sufficient to show that problems P and $\bar{P}$ have the same maximum network lifetime. First, since Algorithm 1 and Lemma 1 show that there is a solution for problem P with network lifetime $\bar{T}$, where $\bar{T}$ is the maximum network lifetime for problem $\bar{P}$, then the maximum network lifetime for problem P is no less than that of problem $\bar{P}$. We now show that the maximum network lifetime for problem $\bar{P}$ is also no less than that of problem P. Consequently, P and $\bar{P}$ must have the same maximum network lifetime.

To show that the maximum network lifetime for problem $\bar{P}$ is indeed no less than the maximum network lifetime for problem P, it is sufficient to prove that, for a given optimal flow routing solution ψ for P with the maximum network lifetime T^*, we can find a flow routing solution $\bar{\pi}$ for $\bar{P}$ with the same network lifetime T^*.

We now construct a flow routing solution $\bar{\pi}$ for $\bar{P}$ that has the same network lifetime T^*. For $\bar{f}_{ik}$, $1 \leq i, k \leq N$ and $i \neq k$, and $\bar{f}_{iB}$, we define

$$\bar{f}_{ik} = \frac{\int_0^{T^*} f_{ik}(t)dt}{T^*} \ , \quad (18)$$

$$\bar{f}_{iB} = \frac{\int_0^{T^*} f_{iB}(t)dt}{T^*} \ . \quad (19)$$

We show that through such construction, both the flow balance equation and energy constraint are satisfied for $\bar{P}$. Consequently, $\bar{\pi}$ is a feasible flow routing solution for $\bar{P}$.

To verify flow balance at node $i \in \mathcal{N}$, we have

$$\bar{g}_i + \sum_{m \neq i} \bar{f}_{mi} = \frac{1}{T^*} \left[\int_0^{T^*} g_i(t)dt + \sum_{m \neq i} \int_0^{T^*} f_{mi}(t)dt \right]$$
$$= \frac{1}{T^*} \left[\int_0^{T^*} f_{iB}(t)dt + \sum_{k \neq i} \int_0^{T^*} f_{ik}(t)dt \right] = \bar{f}_{iB} + \sum_{k \neq i} \bar{f}_{ik} \ .$$

The first equality holds by (1) and (18). The second equality holds due to the flow balance equation in solution ψ. The third equality holds due to (18) and (19).

Similarly, to verify energy constraint at node $i \in \mathcal{N}$, we have

$$\sum_{m \neq i} \rho \bar{f}_{mi} T^* + \sum_{k \neq i} c_{ik} \bar{f}_{ik} T^* + c_{iB} \bar{f}_{iB} T^*$$
$$= \int_0^{T^*} \left[\sum_{m \neq i} \rho f_{mi}(t) + \sum_{k \neq i} c_{ik} f_{ik}(t) + c_{iB} f_{iB}(t) \right] dt$$

by (18) and (19). Thus, at time T^*, the energy consumption at each node i under $\bar{\pi}$ for problem $\bar{P}$ is exactly the same as that under ψ for problem P, i.e., the network lifetime under $\bar{\pi}$ is also T^* for $\bar{P}$. This completes the proof. ■

In a nutshell, the procedure to obtain an optimal flow routing solution ψ to problem P has the following two steps: (1) First, for problem $\bar{P}$, we solve an LP problem and find an optimal flow routing solution $\bar{\pi}$. (2) Second, based on $\bar{\pi}$, we apply Algorithm 1 to get an optimal flow routing solution ψ for problem P.

IV. Near-Optimal Flow Routing with Estimated Average Rate

Our investigation in the last section assumes that we have precise knowledge of the average rates $\bar{g}_i$ for the time-varying source bit rates $g_i(t)$, $i \in \mathcal{N}$. In that case, the flow routing solution obtained by Algorithm 1 yields provably maximum network lifetime. But in practice, $\bar{g}_i$ may not be readily available and the best thing we can do is to make an estimate $\hat{g}_i$ for $\bar{g}_i$. Denote problem $\hat{P}$ as the corresponding flow routing problem with the same network topology and initial energy as those under problem P. Since we have rate information for problem $\hat{P}$, we can obtain its network lifetime $\hat{T}$ using a single LP similar to LP-Basic in Section III-A. Consequently, using Algorithm 1, we can obtain a flow routing solution π (for problem P) from this flow routing solution $\hat{\pi}$ (for problem $\hat{P}$). In this section, we show that as long as the true average is within a small ε of the estimated average, i.e.,

$$\frac{|\bar{g}_i - \hat{g}_i|}{\hat{g}_i} \leq \varepsilon \ , \quad (20)$$

for each AFN $i \in \mathcal{N}$, then the flow routing solution obtained via Algorithm 1 will yield a network lifetime T within $\frac{2\varepsilon}{1-\varepsilon}$ to the (unknown) maximum network lifetime T^* for problem P.

A. Analysis

Denote $\hat{T}$ the network lifetime to problem $\hat{P}$ under flow routing $\hat{\pi}$, $\hat{f}_{ik}$ and $\hat{f}_{iB}$ the rates of flows from node i to node k and to the base-station B, respectively. Although the average bit rates $\bar{g}_i$, $i \in \mathcal{N}$, are unknown, we denote fictitious $\bar{T}$, $\bar{f}_{ik}$, $\bar{f}_{iB}$, and $\bar{\pi}$ for reference purpose. The following lemma shows the difference between $\bar{T}$ and $\hat{T}$.

Lemma 2: For each $i \in \mathcal{N}$, suppose $\frac{|\bar{g}_i - \hat{g}_i|}{\hat{g}_i} \leq \varepsilon$ for some small ε. Then the network lifetime $\hat{T}$ for problem $\hat{P}$ does not deviate from the maximum network lifetime $\bar{T}$ (unknown) for problem $\bar{P}$ by a fraction of $\frac{|\bar{T} - \hat{T}|}{\bar{T}} \leq \varepsilon$.

Proof: The proof is based on *parametric analysis* in LP. First, since we have rate information for problem $\hat{P}$, we can obtain its network lifetime $\hat{T}$ using a single LP similar to LP-Basic in Section III-A. Now we can use the same LP formulation for network lifetime $\bar{T}$ (under problem $\bar{P}$), with the only difference being that each $\hat{g}_i$ is now replaced by $\bar{g}_i$, $1 \le i \le N$. Since $|\bar{g}_i - \hat{g}_i| \le \varepsilon \hat{g}_i$ for $1 \le i \le N$, we show how to obtain an upper bound of $|\bar{T} - \hat{T}|$ via parametric analysis, which will prove the lemma. The following paragraphs give the details of the proof.

Step 1. We first apply the LP-Basic formulation for $\hat{T}$ (under problem $\hat{P}$) and put it into the standard primal form as follows.

$$\begin{array}{lll} \text{Max} & -\hat{H} & \\ \text{s.t.} & \sum_{k\neq i} \hat{f}_{ik} + \hat{f}_{iB} - \sum_{m \neq i} \hat{f}_{mi} = \hat{g}_i & (1 \le i \le N) \\ \multicolumn{2}{l}{\sum_{m\neq i} \rho \hat{f}_{mi} + \sum_{k\neq i} c_{ik}\hat{f}_{ik} + c_{iB}\hat{f}_{iB} - e_i \hat{H} + \hat{\delta}_i = 0} & (1 \le i \le N) \\ & \hat{f}_{ik}, \hat{f}_{iB}, \hat{H}, \hat{\delta}_i \ge 0 & (1 \le i, k \le N, i \neq k) \end{array}$$

Now we have an LP for problem $\hat{P}$ in the standard primal form of Max $\mathbf{c}\hat{\mathbf{x}}$, s.t. $\mathbf{A}\hat{\mathbf{x}} = \hat{\mathbf{b}}$ and $\hat{\mathbf{x}} \ge 0$, where $\hat{\mathbf{x}}$ is a vector of variables $\hat{f}_{ik}$, $\hat{f}_{iB}$, $\hat{H}$, and $\hat{\delta}_i$; $\mathbf{c}$ is a vector having -1 corresponding to variable $\hat{H}$ and 0 corresponding to all other variables; $\mathbf{A}$ is a matrix containing left hand side coefficients in flow balance and energy constraints; $\hat{\mathbf{b}}$ is a column vector of right hand side in flow balance and energy constraints. The dual problem has the form of Min $\hat{\mathbf{v}}\hat{\mathbf{b}}$, s.t. $\hat{\mathbf{v}}\mathbf{A} \ge \mathbf{c}$ with $\hat{\mathbf{v}}$ being unrestricted in sign [1].

Similarly, we can cast the LP-Basic formulation for $\bar{T}$ (under problem $\bar{P}$) into the standard primal form of Max $\mathbf{c}\bar{\mathbf{x}}$, s.t. $\mathbf{A}\bar{\mathbf{x}} = \bar{\mathbf{b}}$ and $\bar{\mathbf{x}} \ge 0$. Note that both $\hat{\mathbf{b}}$ and $\bar{\mathbf{b}}$ are $2N$ column vector with the i-th element being

$$\begin{array}{lll} \hat{b}_i = \hat{g}_i \text{ for } 1 \le i \le N & \text{and} & \hat{b}_i = 0 \text{ for } N+1 \le i \le 2N, \\ \bar{b}_i = \bar{g}_i \text{ for } 1 \le i \le N & \text{and} & \bar{b}_i = 0 \text{ for } N+1 \le i \le 2N. \end{array} \quad (21)$$

Step 2. If $\hat{\mathbf{x}}^*$ and $\hat{\mathbf{v}}^*$ are the optimal primal and dual solutions for problem $\hat{P}$, respectively, then the following dual property exists,

$$\hat{\mathbf{v}}^* \hat{\mathbf{b}} = \mathbf{c}\hat{\mathbf{x}}^* = -\hat{H} . \quad (22)$$

Note that LP problems for $\hat{P}$ and $\bar{P}$ share the same vector $\mathbf{c}$ and matrix $\mathbf{A}$ in the LP formulation, but with different right hand side $\mathbf{b}$, i.e., $\mathbf{b} = \hat{\mathbf{b}}$ for problem $\hat{P}$ and $\mathbf{b} = \bar{\mathbf{b}}$ for problem $\bar{P}$. Thus, $-\bar{H}$ can be obtained through $-\hat{H}$ by changing $\hat{\mathbf{b}}$ in problem $\hat{P}$ to $\bar{\mathbf{b}}$. We now analyze the effect on the objective $\mathbf{cx}$ by changing the i-th element of right hand side vector b_i from $\hat{b}_i$ to $\bar{b}_i$, $1 \le i \le N$. Note that there is no need to change b_i for $N+1 \le i \le 2N$ since $\hat{b}_i = \bar{b}_i = 0$ for $N+1 \le i \le 2N$.

Case (i). If the LP problem for $\hat{P}$ is non-degenerate, then there exists an interval $[l_i, u_i]$ with $l_i \le \hat{b}_i$ and $u_i \ge \hat{b}_i$ such that $\left.\frac{\partial(\mathbf{cx})}{\partial b_i}\right|_{b_i} = \hat{v}_i^*$ for $b_i \in [l_i, u_i]$ [1]. Note that based on the nature of problem $\hat{P}$, we have $\hat{v}_i^* \le 0$. Therefore, a change in b_i from $\hat{b}_i$ to $\bar{b}_i$, the objective $\mathbf{cx}$ will change by

$$\left|\hat{v}_i^* \cdot (\bar{b}_i - \hat{b}_i)\right| = |\hat{v}_i^* \cdot (\bar{g}_i - \hat{g}_i)| \le |\hat{v}_i^*| \cdot \varepsilon \hat{g}_i = \varepsilon(-\hat{v}_i^* \hat{b}_i) ,$$

where the first and the third equalities hold by (21) and the second inequality holds by (20).

Case (ii). If the LP problem for problem $\hat{P}$ is degenerate, i.e., there are multiple optimal dual solutions at $\hat{b}_i$, then there exists an interval $[l_i, \hat{b}_i]$ with $l_i \le \hat{b}_i$ such that $\left.\frac{\partial^-(\mathbf{cx})}{\partial b_i}\right|_{b_i} = \min\{v_i^* : \text{over all optimal dual solutions } \mathbf{v}^*\}$ for $b_i \in [l_i, \hat{b}_i]$ [1]. Denote $\hat{\mathbf{v}}^*$ the optimal dual solution such that $\hat{v}_i^* = \min\{v_i^* : \text{over all optimal dual solutions } \mathbf{v}^*\}$. Thus, we have

$$\left.\frac{\partial^-(\mathbf{cx})}{\partial b_i}\right|_{b_i} = \hat{v}_i^* \quad \text{for } b_i \in [l_i, \hat{b}_i]. \quad (23)$$

For this optimal dual solution $\hat{\mathbf{v}}^*$, there also exists an interval $[\hat{b}_i, u_i]$ with $u_i \ge \hat{b}_i$ such that [1]

$$\begin{aligned} \left.\frac{\partial^+(\mathbf{cx})}{\partial b_i}\right|_{b_i} &= \max\{v_i^* : \text{over all optimal dual solutions } \mathbf{v}^*\} \\ &\ge \hat{v}_i^* \quad \text{for } b_i \in [\hat{b}_i, u_i]. \end{aligned} \quad (24)$$

Note that based on the nature of problem $\hat{P}$, we have $\left.\frac{\partial^+(\mathbf{cx})}{\partial b_i}\right|_{b_i} \le 0$. Then if $\bar{b}_i < \hat{b}_i$, i.e., we are decreasing $\hat{b}_i$ to $\bar{b}_i$, then based on (23), the objective $\mathbf{cx}$ will increase by $\left|\hat{v}_i^* \cdot (\bar{b}_i - \hat{b}_i)\right|$. On the other hand, if $\bar{b}_i > \hat{b}_i$, i.e., we are increasing $\hat{b}_i$ to $\bar{b}_i$, then based on (24), the objective $\mathbf{cx}$ will decrease by at most $\left|\hat{v}_i^* \cdot (\bar{b}_i - \hat{b}_i)\right|$. Combining the above two scenarios, we conclude that a change in b_i from $\hat{b}_i$ to $\bar{b}_i$, the objective $\mathbf{cx}$ will change (either increase or decrease) by at most $\left|\hat{v}_i^* \cdot (\bar{b}_i - \hat{b}_i)\right| = |\hat{v}_i^* \cdot (\bar{g}_i - \hat{g}_i)| \le |\hat{v}_i^*| \cdot \varepsilon \hat{g}_i = \varepsilon(-\hat{v}_i^* \hat{b}_i)$.

Therefore, regardless of whether problem $\hat{P}$ is non-degenerate or degenerate, we have

$$\begin{aligned} \left|\frac{1}{\bar{T}} - \frac{1}{\hat{T}}\right| &= |\bar{H} - \hat{H}| = |(-\mathbf{c}\bar{\mathbf{x}}) - (-\mathbf{c}\hat{\mathbf{x}})| \le \sum_{i=1}^{N} \varepsilon(-\hat{v}_i^* \hat{b}_i) \\ &= \varepsilon(-\hat{\mathbf{v}}^* \hat{\mathbf{b}}) = \varepsilon(-\mathbf{c}\hat{\mathbf{x}}) = \varepsilon \hat{H} = \frac{\varepsilon}{\hat{T}} , \end{aligned} \quad (25)$$

where the fourth equality holds by (21) and the fifth equality holds by (22). Therefore, we have $\frac{|\bar{T} - \hat{T}|}{\bar{T}} \le \varepsilon$. This completes the proof. ■

Due to the difference between $\bar{g}_i$ and $\hat{g}_i$, the network lifetime T under solution π for problem P will not be the same as $\hat{T}$ under solution $\hat{\pi}$ for problem $\hat{P}$. The following lemma shows the difference between $\hat{T}$ and T is also bounded.

Lemma 3: For each $i \in \mathcal{N}$, suppose $\frac{|\bar{g}_i - \hat{g}_i|}{\hat{g}_i} \le \varepsilon$ for some small ε. Then the network lifetime T under solution π for problem P does not deviated from the network lifetime $\hat{T}$ under solution $\hat{\pi}$ for problem $\hat{P}$ by a fraction of $\frac{|\hat{T} - T|}{T} \le \frac{\varepsilon(1+\varepsilon)}{1-\varepsilon}$.

Proof: From (20), we have $(1-\varepsilon)\hat{g}_i \le \bar{g}_i \le (1+\varepsilon)\hat{g}_i$ for $i \in \mathcal{N}$. For $\bar{g}_i \le (1+\varepsilon)\hat{g}_i$, we will show that $\hat{T} \le (1+\varepsilon)T$. For $\bar{g}_i \le (1-\varepsilon)\hat{g}_i$, we will show that $\hat{T} \le (1-\varepsilon)T$. As a result, we have $|\hat{T} - T| \le \varepsilon T$.

(i) We first consider $\bar{g}_i \le (1+\varepsilon)\hat{g}_i$. For $1 \le i, k \le N$ and $i \neq k$, we have

$$\begin{aligned} \frac{1}{T}\int_{t=0}^{T} f_{ik}(t)dt &= \frac{1}{T}\int_{t=0}^{T} \sum_{s \neq k} w_{ik}^s g_s(t) dt \\ &= \frac{1}{T}\sum_{s\neq k} w_{ik}^s \int_{t=0}^{T} g_s(t)dt = \frac{1}{T}\sum_{s\neq k} w_{ik}^s \bar{g}_s T \end{aligned}$$

$$\leq \quad (1+\varepsilon)\sum_{s\neq k} w_{ik}^s \hat{g}_s \ , \tag{26}$$

where the first equality holds by the definitions of $f_{ik}(t)$ in (14); the third equality holds by (1); and the last inequality holds by (20). Similarly, we also have

$$\frac{1}{T}\int_{t=0}^{T} f_{iB}(t)dt \quad \leq \quad (1+\varepsilon)\sum_{s=1}^{N} w_{iB}^s \hat{g}_s \ . \tag{27}$$

Thus, under π, for each node $i \in \mathcal{N}$, the average energy consumption rate is

$$\begin{aligned} & \frac{1}{T}\int_0^T \left[\sum_{m\neq i}\rho f_{mi}(t) + \sum_{k\neq i} c_{ik} f_{ik}(t) + c_{iB} f_{iB}(t)\right] dt \\ \leq \quad & (1+\varepsilon)\left(\sum_{m\neq i}\rho\sum_{s\neq i} w_{mi}^s \hat{g}_s + \sum_{k\neq i} c_{ik}\sum_{s\neq k} w_{ik}^s \hat{g}_s + c_{iB}\sum_{s=1}^{N} w_{iB}^s \hat{g}_s\right) \\ = \quad & (1+\varepsilon)\left(\sum_{m\neq i}\rho \hat{f}_{mi} + \sum_{k\neq i} c_{ik}\hat{f}_{ik} + c_{iB}\hat{f}_{iB}\right) \ , \end{aligned}$$

where the first inequality holds by (26) and (27) and the second equality holds by (6) and (7) (for problem $\hat{P}$). Note that $\left(\sum_{m\neq i}\rho \hat{f}_{mi} + \sum_{k\neq i} c_{ik}\hat{f}_{ik} + c_{iB}\hat{f}_{iB}\right)$ is the energy consumption rate on node i under $\hat{\pi}$. Thus, the average energy consumption rate for any node $i \in \mathcal{N}$ under π is no more than $(1+\varepsilon)$ of that under $\hat{\pi}$, i.e., node i under π has a lifetime at least $\frac{1}{1+\varepsilon}$ of that under $\hat{\pi}$. Therefore, the network lifetime T under π also will be at least $\frac{1}{1+\varepsilon}$ of that under $\hat{\pi}$, i.e. $T \geq \frac{1}{1+\varepsilon}\hat{T}$, or $\hat{T} \leq (1+\varepsilon)T$.

(ii) We now consider $\bar{g}_i \leq (1-\varepsilon)\hat{g}_i$. Following similar steps in (i), we can show that $\hat{T} \geq (1-\varepsilon)T$.

Combining (i) and (ii), we have $|\hat{T}-T| \leq \varepsilon T$. Therefore,

$$\frac{|\hat{T}-T|}{\bar{T}} \leq \frac{\varepsilon T}{\bar{T}} = \frac{\varepsilon T}{\hat{T}}\cdot\frac{\hat{T}}{\bar{T}} \leq \frac{\varepsilon}{1-\varepsilon}\cdot\frac{\bar{H}}{\hat{H}} \leq \frac{\varepsilon(1+\varepsilon)}{1-\varepsilon} \ .$$

The third inequality holds by the result in (ii). The last inequality holds by $\frac{\bar{H}}{\hat{H}} \leq 1+\varepsilon$, which comes from (25). This completes the proof. ■

The following theorem is the main result of this section.

Theorem 2: Suppose that for each $i \in \mathcal{N}$, *we have* $\frac{|\bar{g}_i-\hat{g}_i|}{\hat{g}_i} \leq \varepsilon$ *for some small* ε. *Calculate* π, *the time-varying flow routing solution to problem* P, *by using Algorithm 1 based on problem* $\hat{P}$ *with estimated average rates* $\hat{g}_i$, $i \in \mathcal{N}$. *Then the network lifetime* T *under* π *does not deviated from the maximum (unknown) network lifetime* T^* *by a fraction of* $\frac{T^*-T}{T^*} \leq \frac{2\varepsilon}{1-\varepsilon}$.

Proof: Since $T^* = \bar{T}$ by Theorem 1, we have

$$\begin{aligned} \frac{T^*-T}{T^*} \quad &= \quad \frac{\bar{T}-T}{\bar{T}} = \frac{\bar{T}-\hat{T}+\hat{T}-T}{\bar{T}} \\ &\leq \quad \frac{|\bar{T}-\hat{T}|}{\bar{T}} + \frac{|\hat{T}-T|}{\bar{T}} \leq \varepsilon + \frac{\varepsilon(1+\varepsilon)}{1-\varepsilon} = \frac{2\varepsilon}{1-\varepsilon} \ , \end{aligned}$$

where the fourth inequality follows from Lemmas 2 and 3. ■

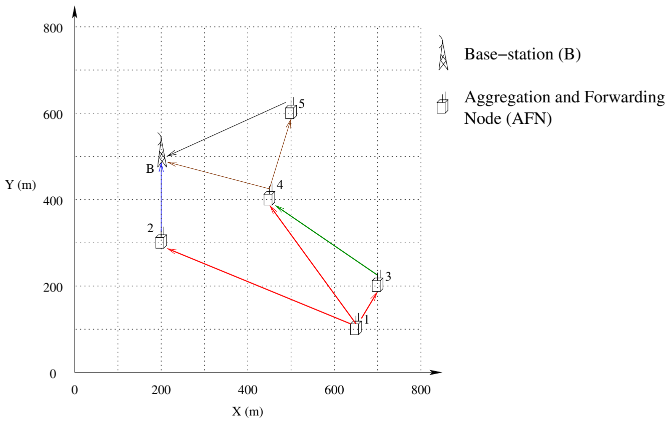


Fig. 2. Flow routing solution $\hat{\pi}$ for problem $\hat{P}$ in Example 1.

TABLE II

LOCATION, ESTIMATED AVERAGE BIT RATE, AND INITIAL ENERGY FOR THE 5-NODE EXAMPLE.

AFN i	$(x_i,\ y_i)(m)$	$\hat{g}_i$ (kb/s)	e_i (kJ)
1	(650, 100)	8	70
2	(200, 300)	9	65
3	(700, 200)	6	95
4	(450, 400)	4	80
5	(500, 600)	5	55

B. Numerical Example

We use the following example to illustrate how to apply Algorithm 1 to obtain a flow routing solution and verify the near-optimal performance of T. In the example, we set $\alpha = 50$ nJ/b, $\beta = 0.0013$ pJ/b/m^4, $\rho = 50$ nJ/b, and path loss index $n = 4$ [5]. For beam-width, we set $\theta = 30$.

Example 1: Referring to Fig. 2, suppose that we have 5 nodes with location (x_i, y_i) (in meters), estimated average bit rate $\hat{g}_i$ (in kb/s), and initial energy e_i (in kJ) listed in Table II. The base-station (B) is located at $(200, 500)$ m.

Assume that the true average rates are $\bar{g}_1 = 8.7$ kb/s, $\bar{g}_2 = 8.1$ kb/s, $\bar{g}_3 = 5.6$ kb/s, $\bar{g}_4 = 3.6$ kb/s, and $\bar{g}_5 = 5.5$ kb/s. Comparing with Table II, we find the maximum estimation error ε on average bit rate is 10%. From Theorem 2, we have an upper bound $\frac{T^*-T}{T^*} \leq \frac{2\varepsilon}{1-\varepsilon} = 22.22\%$. It turns out that the solution obtained by Algorithm 1 in this example has much better performance than the proved bound.

Now we compute $\frac{T^*-T}{T^*}$. By setting up an LP for problem $\bar{P}$, we have $\bar{T} = 84.24$, which is the maximum network lifetime for both problem $\bar{P}$ and problem P, i.e., $T^* = 84.24$.

Now we assume that the average rates are unknown and thus we need to solve problem $\hat{P}$. The optimal routing solution $\hat{\pi}$ (with $\hat{T} = 85.32$) for problem $\hat{P}$ is shown in Table III.

We now compute w_{ik}^s and w_{iB}^s by (10) and (11) as follows. First, we compute w_{sk}^s and w_{sB}^s for $1 \leq s, k \leq 5$ and $s \neq k$ by (8). For example,

$$w_{13}^1 = \frac{\hat{f}_{13}}{\hat{f}_{1B}+\sum_{r\neq 1}\hat{f}_{1r}} = \frac{4.6538}{0.6374+4.6538+2.7088} = 0.5817 \ .$$

Next, we compute w_{ik}^s and w_{iB}^s for $1 \leq i, k, s \leq 5$, $i \neq k$, and $s \neq k$ by (10). For example, since we have $w_{13}^1 = 0.5817$, we

TABLE III
INTER-NODAL FLOW RATES UNDER $\hat{\pi}$.

	$\hat{f}_{ik}$ (kb/s)					$\hat{f}_{iB}$ (kb/s)
i	$k=1$	$k=2$	$k=3$	$k=4$	$k=5$	B
1	0	0.6374	4.6538	2.7088	0	0
2	0	0	0	0	0	9.6374
3	0	0	0	10.6538	0	0
4	0	0	0	0	1.5183	15.8443
5	0	0	0	0	0	6.5183

can compute

$$w_{34}^1 = \frac{\hat{f}_{34}}{\hat{f}_{3B} + \sum_{r \neq 4} \hat{f}_{3r}} w_{13}^1 = \frac{10.6538}{10.6538} 0.5817 = 0.5817 \ .$$

Upon the completion of Step 1(a) and 1(b) in Algorithm 1, we have the following values for w_{ik}^s and w_{iB}^s:
$w_{12}^1 = 0.0797$, $w_{13}^1 = 0.5817$, $w_{14}^1 = 0.3386$, $w_{2B}^1 = 0.0797$,
$w_{34}^1 = 0.5817$, $w_{45}^1 = 0.0804$,
$w_{4B}^1 = 0.8399$, $w_{5B}^1 = 0.0804$;
$w_{2B}^2 = 1.0000$;
$w_{34}^3 = 1.0000$, $w_{45}^3 = 0.0874$, $w_{4B}^3 = 0.9126$, $w_{5B}^3 = 0.0874$;
$w_{45}^4 = 0.0874$, $w_{4B}^4 = 0.9126$, $w_{5B}^4 = 0.0874$;
$w_{5B}^5 = 1.0000$.
Based on these w_{ik}^s and w_{iB}^s, in Step 2 of Algorithm 1, we obtain the following flow rates via (14) and (15).
$f_{12}(t) = w_{12}^1 g_1(t) = 0.0797 g_1(t)$,
$f_{13}(t) = w_{13}^1 g_1(t) = 0.5817 g_1(t)$,
$f_{14}(t) = w_{14}^1 g_1(t) = 0.3386 g_1(t)$,
$f_{2B}(t) = w_{2B}^1 g_1(t) + w_{2B}^2 g_2(t) = 0.0797 g_1(t) + g_2(t)$,
$f_{34}(t) = w_{34}^1 g_1(t) + w_{34}^3 g_3(t) = 0.5817 g_1(t) + g_3(t)$,
$f_{45}(t) = w_{45}^1 g_1(t) + w_{45}^3 g_3(t) + w_{45}^4 g_4(t) = 0.0804 g_1(t) + 0.0874 g_3(t) + 0.0874 g_4(t)$,
$f_{4B}(t) = w_{4B}^1 g_1(t) + w_{4B}^3 g_3(t) + w_{4B}^4 g_4(t) = 0.8399 g_1(t) + 0.9126 g_3(t) + 0.9126 g_4(t)$,
$f_{5B}(t) = w_{5B}^1 g_1(t) + w_{5B}^3 g_3(t) + w_{5B}^4 g_4(t) + w_{5B}^5 g_5(t) = 0.0804 g_1(t) + 0.0874 g_3(t) + 0.0874 g_4(t) + g_5(t)$.

We now calculate the lifetime for each of the five nodes. For node 1, we have

$$\begin{aligned} & \int_0^T [c_{12} f_{12}(t) + c_{13} f_{13}(t) + c_{14} f_{14}(t)] dt \\ = & \int_0^T [c_{12} \cdot 0.0797 g_1(t) + c_{13} \cdot 0.5817 g_1(t) + c_{14} \cdot 0.3386 g_1(t)] dt \\ = & \int_0^T 1.1875 * 10^{-6} \cdot g_1(t) dt = 1.1875 * 10^{-6} \cdot \bar{g}_1 T \\ = & \ 1.0331 * 10^{-2} T \le e_1 = 70 * 10^3 \ . \end{aligned}$$

Thus, the energy constraint on node 1 shows that the network lifetime is at most $\frac{70*10^3}{1.0331*10^{-2}} = 6.7757 * 10^6$ seconds, or 78.42 days. Following the same token, the energy constraints on nodes $2, 3, 4$, and 5 show that the network lifetime is at most 376.44, 86.80, 85.92, and 79.45 days, respectively. Thus, the network lifetime T is 78.42 days. Therefore, $\frac{T^* - T}{T^*} = \frac{84.24 - 78.42}{84.24} = 6.91\%$, which is much smaller than the upper bound of 22.22% calculated via Theorem 2. □

V. RELATED WORK

There has been active research on addressing issues associated with energy constraints in wireless sensor networks. Several review papers (*e.g.,* [9], [12]) have examined various issues when designing an energy-aware sensor network. In this section, we briefly summarize related efforts on power control and network lifetime.

Power control capability has been under intensive research at different layers in recent years. At the *network* layer, most work on power control can be classified into two areas. The first area is comprised of strategies to find an optimal transmitter power to control the *connectivity* properties of the network (see, e.g., [8], [13], [15]). A common theme in these strategies is to formulate power control as a network layer problem and then to adjust each node's transmission power so that different network connectivity topologies can be achieved for different objectives. The second area could be called *power-aware routing.* Most schemes use a shortest path algorithm with a power-based metric (see e.g., [16], [19]), rather than a hop-count based metric. However, energy-aware (e.g., minimum energy path) routing cannot ensure good performance of maximum network lifetime.

The notion of network lifetime has been a focus in sensor networking research in recent years. The most relevant work on network lifetime related to our research is [2]. As discussed, the LP formulation in [2] can only address the simple constant source bit rate case. The more difficult problem for time-varying source bit rate has not been explored before this paper.

VI. CONCLUSIONS

In this paper, we studied flow routing problem for an energy-constrained wireless sensor network where the source bit rate could be time-varying. Our objective is to find a flow routing algorithm such that the network lifetime can be maximized. We presented a flow routing algorithm that has the following performance guarantees: (1) When the average source rate of each node is known a priori, the flow routing solution obtained via our algorithm is optimal and gives maximum network lifetime performance; (2) When the average source rate of each node is unknown but is within a fraction (ε) of an estimated rate value, the network lifetime given by the flow routing solution using the proposed algorithm is within $\frac{2\varepsilon}{1-\varepsilon}$ from the optimum. These results constitute an important step in algorithm design for flow routing problems in energy-constrained sensor networks.

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