How to Correctly Use the Protocol Interference Model for Multi-hop Wireless Networks

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ABSTRACT

This paper tries to reconcile the tension between physical model and protocol model that have been used to characterize interference relationship in a multi-hop wireless network. The physical model (a.k.a. SINR model) is widely considered as a reference model for physical layer behavior but its application in multi-hop wireless networks is limited by its complexity. On the other hand, the protocol model (a.k.a. unified disk graph model) is simple but there have been doubts on its validity. This paper explores the following fundamental question: How to correctly use the protocol interference model? We show that in general, solutions obtained under the protocol model may be *infeasible* in practice and thus, results based on blind use of protocol model can be misleading. We propose a novel concept called "reality check" and present a method of using protocol model with reality check for wireless networks. Subsequently, we show that by appropriate setting of the interference range in the protocol model, it is possible to narrow the solution gap between the two models. Our simulation results confirm that this gap is indeed small (or even negligible). Thus, our methodology of joint reality check and interference range setting retains the protocol model as a viable approach to analyze multi-hop wireless networks.

Categories and Subject Descriptors

C.2.1 [**Computer-Communication Networks**]: Network Architecture and Design — *Wireless communication*; I.6.4 [**Computing Methodologies**]: Simulation and Modeling—*Model validation and analysis*

General Terms

Performance, Theory

Keywords

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1. INTRODUCTION

There are two widely used models to characterize interference relationship in a wireless network, namely, the physical model and the protocol model. The physical model, also known as the SINR model, is based on practical transceiver designs of communication systems that treat interference as noise. Under the physical model, a transmission is successful if and only if signal-to-interference-and-noise-ratio (SINR) at the intended receiver exceeds a threshold so that the transmitted signal can be decoded with an acceptable bit error probability. Further, capacity calculation is based on SINR (via Shannon's formula), which takes into account interference due to simultaneous transmissions by other nodes. In wireless communications, such interference model is considered as a reference model since there exist practical coding schemes to approach its solution in real systems.

However, the difficulty associated with the physical model is its computational complexity in obtaining a solution, particularly when it involves cross-layer optimization in a multi-hop network environment. This is because SINR calculation is a non-convex function with respect to the transmission powers. As a result, a solution to cross-layer optimization using the physical model is difficult to develop and its computational complexity is likely very high for large-sized networks. Consequently, most of the current approaches to cross-layer optimization employing the physical layer model follow a simplified layer-by-layer (or "layer-decoupled") approach and thus yield sub-optimal solutions (e.g., [5, 8, 10]) or instead, focus on providing asymptotic lower and upper bounds (e.g., [13, 14, 18]).

To circumvent the complexity issue associated with physical model, the so-called protocol model [13], also known as unified disk graph model, has been widely used by researchers in wireless networking community as a way to simplify the mathematical characterization of physical layer. Under the protocol model, a successful transmission occurs when a node falls inside the transmission range of its intended transmitter and falls outside the interference ranges of other non-intended transmitters. The setting of transmission range is based on a signal-to-noise-ratio (SNR) threshold. The setting of interference range is rather heuristic and remains an open problem. Under the protocol model, the impact of interference from a transmitting node is binary and is solely determined by whether or not a receiver falls within the interference range of this transmitting node. That is, if a node falls in the interference range of a non-intended transmitter, then this node is considered to be interfered and thus cannot receive correctly from its intended transmitter; otherwise, the interference is assumed to be negligible. Due to such simplification, the protocol model has been widely used in developing algorithms and protocols in wireless networks (e.g. [2, 4, 16, 17, 20, 23, 24, 25]) and can be easily applied to analyze large-sized wireless networks.

The controversy surrounding (or arguments against) the protocol model is that a binary decision of whether or not interference exists (based on interference range) does not accurately capture physical layer characteristics. For the case when a node falls in the interference range of a non-intended transmitter, the protocol model assumes that this node cannot receive correctly from its intended transmitter (due to interference). But this is overly conservative, based on capacity formula, as there could still be some capacity even with interference. On the other hand, for the case when a node falls outside the interference range of each non-intended transmitter, protocol model assumes that there is no interference. But this is somewhat optimistic as small interference from different transmitters can aggregate and may not be negligible in capacity calculation. As a result, there have been some serious doubts in the research community on the correctness of protocol interference model for wireless networks.

The goal of this paper is to reconcile the tension between physical model and protocol model by answering the following fundamental question: How to correctly use the protocol interference model? The answer to this question is important for current and future investigations on multi-hop wireless networks.

It is worth pointing out that in the physical model, interference is treated as noise. Information-theoretic study has shown that if the interference information is exploited wisely (e.g., successive decoding [1], superposition coding [3, 7], dirty paper coding [6]), a larger capacity region can be achieved. However, practical implementations of these techniques for multi-hop wireless networks remain to be developed due to the following issues. (1) These techniques, although theoretically attractive, are hard to implement for real systems due to extremely high hardware/software requirements and computational complexity. (2) In a multi-hop ad hoc network, there is no centralized infrastructure. As a result, exploiting interference information in such setting is extremely difficult. Thus, these advanced physical layer techniques will not be considered in this paper.

1.1 Main Contributions

The main contributions of this paper are the following.

- We show that in general, solutions obtained under the protocol model may not be feasible in practice. Thus, solutions based on blind use of the protocol model may offer incorrect results as there is no feasibility checking mechanism in place after a solution is obtained. Due to this oversight, the doubt on blind use of protocol model is legitimate.

- To obtain a feasible solution for the protocol model, we propose a novel concept called "reality check" and a new methodology on how to use it with protocol model to obtain a feasible solution.

- We further show that by combining reality check with appropriate setting of the interference range, it is possible to have the protocol model offer comparable results as those under the physical model. This offers us the correct approach of using the protocol model.

1.2 Paper Organization

The rest of this paper is organized as follows. In Section 2, we present a general cross-layer optimization problem under both physical model and protocol model. We briefly discuss the approaches to solve both problems and their complexities. Section 3 identifies potential infeasibility issue associated with the protocol model solution. We introduce a reality check mechanism and show how it can be used to obtain a revised solution that is feasible. In Section 4, we show the correct methodology of using the protocol interference model. That is, by combining reality check mechanism and appropriate setting of the interference range in the protocol model, it is possible to obtain comparable results under both models. Section 5 concludes this paper.

2. MATHEMATICAL MODELS, PROBLEM FORMULATION, AND SOLUTION APPROACH

For the sake of generality in this investigation, we consider a multi-hop cognitive radio network (CRN), which not only encompasses all the features in existing multi-channel multi-radio [2, 9, 17, 18, 20, 21] (including the 802.11-based radio platform) but also is positioned to be the primary radio platform in the coming decades. Thus, algorithmic and optimization results for CRNs are not only important for future wireless networks, but are also generalizations of traditional wireless networks.

2.1 Models at Multiple Layers

We consider a CRN consisting of a set of $\mathcal{N}$ nodes. In a CRN, the available frequency bands at each node depends on its location and may not be the same. Denote $\mathcal{M}_i$ the set of available frequency bands at node i and assume the bandwidth of each frequency band is W. Denote $\mathcal{M}$ the set of all frequency bands present in the network, i.e., $\mathcal{M} = \bigcup_{i \in \mathcal{N}} \mathcal{M}_i$. Denote $\mathcal{M}_{ij} = \mathcal{M}_i \bigcap \mathcal{M}_j$, which is the set of common available bands on nodes i and j and thus can be used for transmission between these two nodes.

Scheduling and Power Control (For Both Physical and Protocol Models). Scheduling can be done either in time domain or frequency domain. In this study, we consider scheduling in the frequency domain in the form of assigning frequency band. Denote

$$x_{ij}^m = \begin{cases} 1 & \text{If node } i \text{ transmits to node } j \text{ on band } m, \\ 0 & \text{otherwise.} \end{cases} \quad (1)$$

Then for a band $m \in \mathcal{M}_i$, node i cannot use it for transmission to multiple nodes or for reception from multiple nodes. Further, due to self-interference, node i cannot use it for both transmission and reception. Putting these constraints together, we have

$$\sum_{i \in \mathcal{T}_k^m} x_{ki}^m + \sum_{j \in \mathcal{T}_i^m} x_{ij}^m \le 1 \quad (i \in \mathcal{N}, m \in \mathcal{M}_i) \,, \quad (2)$$

where $\mathcal{T}_i^m$ is the set of nodes that are within the maximum transmission range from node i (under peak power $P_{\max}$) on band m.

Denote p_{ij}^m as the transmission power at node i, when node i transmits data to node j on band m. Clearly, when node i does not transmit data to node j on band m, p_{ij}^m should be 0. Under the maximum allowed transmission power limit $P_{\max}$, we have

$$p_{ij}^m \le P_{\max} x_{ij}^m \quad (i \in \mathcal{N}, m \in \mathcal{M}_i, j \in \mathcal{T}_i^m) . \qquad (3)$$

Scheduling Feasibility Constraints (For Physical Model). Under the physical model, a transmission is successful if and only if the SINR at the receiving node exceeds a certain threshold, say α. We now formulate this constraint. For a transmission from node i to node j on band m, the SINR at node j is

$$s_{ij}^m = \frac{g_{ij} p_{ij}^m}{\eta W + \sum_{k \in \mathcal{N}}^{k \neq i,j} \sum_{h \in \mathcal{T}_k^m}^{h \neq i,j} g_{kj} p_{kh}^m} ,$$

where η is the ambient Gaussian noise density, g_{ij} is the propagation gain from node i to node j, $\mathcal{T}_k^m$ is the set of nodes to which node k can transmit on band m.

Since there is a transmission from node i to node j on band m, neither i nor j can receive from other nodes on band m, i.e., $p_{ki}^m = 0$ and $p_{kj}^m = 0$. We have $\sum_{h \in \mathcal{T}_k^m} g_{kj} p_{kh}^m = \sum_{h \in \mathcal{T}_k^m}^{h \neq i,j} g_{kj} p_{kh}^m$. Denote

$$t_k^m = \sum_{h \in \mathcal{T}_k^m} p_{kh}^m = \sum_{h \in \mathcal{T}_k^m}^{h \neq i,j} p_{kh}^m \quad (k \in \mathcal{N}, m \in \mathcal{M}_k) . \qquad (4)$$

We have $s_{ij}^m = \frac{g_{ij} p_{ij}^m}{\eta W + \sum_{k \in \mathcal{N}}^{k \neq i,j} g_{kj} t_k^m}$, i.e.,

$$\eta W s_{ij}^m + \sum_{k \in \mathcal{N}}^{k \neq i,j} g_{kj} t_k^m s_{ij}^m - g_{ij} p_{ij}^m = 0$$
$$(i \in \mathcal{N}, m \in \mathcal{M}_i, j \in \mathcal{T}_i^m) . \qquad (5)$$

Note that this SINR computation also holds when $p_{ij}^m = 0$, i.e., when there is no transmission from node i to node j on band m.

Recall that under physical model, a transmission from node i to node j on band m is successful if and only if SINR at node j exceeds a threshold α, i.e., $s_{ij}^m \ge \alpha$. Then by (1), we have

$$s_{ij}^m \ge \alpha x_{ij}^m \quad (i \in \mathcal{N}, m \in \mathcal{M}_i, j \in \mathcal{T}_i^m) , \qquad (6)$$

which is the necessary and sufficient condition for successful transmission under the physical model.

For a successful transmission (i.e., if the above constraints are satisfied), the achieved capacity by this s_{ij}^m is

$$c_{ij}^m = W \log_2(1 + s_{ij}^m) \quad (i \in \mathcal{N}, m \in \mathcal{M}_i, j \in \mathcal{T}_i^m) . \qquad (7)$$

Of course, the actual data rate is dependent on a number of other parameters, such as modulation, coding schemes, BER constraints, detector schemes, etc., and will be lower than that obtained by the Shannon capacity formula.

Scheduling Feasibility Constraints (For Protocol Model). Under protocol model, a transmission is successful if and only if the receiving node is within the transmission range of the intended transmitting node and is outside the interference range of each non-intended transmitting node. When power control is employed at each transmitting node, the transmission range and interference range can be varied and may be different from the others. As a result, the interference relationship among nodes becomes more complicated. In [22], Shi and Hou showed that the conditions for successful transmission from node i to node j with an interfering transmission from node k to node h can be formulated as follows:

$$p_{ij}^m \in \left[\left(\frac{d_{ij}}{R_T^{\max}} \right)^n P_{\max} x_{ij}^m ,\ P_{\max} x_{ij}^m \right]$$
$$(i \in \mathcal{N}, m \in \mathcal{M}_i, j \in \mathcal{T}_i^m) ,$$

$$p_{kh}^m \le P_{\max} - \left[1 - \left(\frac{d_{kj}}{R_I^{\max}} \right)^n \right] P_{\max} x_{ij}^m$$
$$(i \in \mathcal{N}, m \in \mathcal{M}_i, j \in \mathcal{T}_i^m, k \in \mathcal{I}_j^m, k \neq i, h \in \mathcal{T}_k^m) ,$$

where d_{ij} is the physical distance between nodes i and j, $R_T^{\max}$ and $R_I^{\max}$ are the maximum transmission and interference ranges (under peak transmission power $P_{\max}$), respectively, and $\mathcal{I}_j^m$ is the set of nodes that may contribute towards non-negligible interference at node j. These constraints are based on the uniform propagation gain $g_{ij} = d_{ij}^{-n}$, where n is the path loss index.

To better understand the physical meaning of these two constraints, we consider a general propagation gain function $g_{ij} = g(d_{ij})$. Further, denote

$$P_{ij}^T = \frac{g(R_T^{\max})}{g(d_{ij})} P_{\max} \qquad (8)$$

and $P_{kj}^I = \frac{g(R_I^{\max})}{g(d_{kj})} P_{\max}$, which are the minimum required power for transmission from node i to node j and the maximum allowed transmission power at node k when node j is receiving, respectively. We have the following constraints for successful transmission from node i to node j (with a concurrent transmission from node k to node h) under the protocol model.

$$p_{ij}^m \in [P_{ij}^T x_{ij}^m ,\ P_{\max} x_{ij}^m] \qquad (i \in \mathcal{N}, m \in \mathcal{M}_i, j \in \mathcal{T}_i^m) , \ (9)$$

$$p_{kh}^m \le P_{\max} - (P_{\max} - P_{kj}^I) x_{ij}^m$$
$$(i \in \mathcal{N}, m \in \mathcal{M}_i, j \in \mathcal{T}_i^m, k \in \mathcal{I}_j^m, k \neq i, h \in \mathcal{T}_k^m) . (10)$$

For a successful transmission (i.e., the above two constraints are satisfied), the interference from any other transmitter is considered "negligible" under protocol model and the achieved capacity is

$$c_{ij}^m = W \log_2 \left(1 + \frac{g_{ij} p_{ij}^m}{\eta W} \right) \ (i \in \mathcal{N}, m \in \mathcal{M}_i, j \in \mathcal{T}_i^m). \qquad (11)$$

Note that this capacity computation also holds for $p_{ij}^m = 0$, i.e., when there is no transmission from node i to node j on band m.

In our problem, power control can be performed at each node. When the peak power $P_{\max}$ is used at a node i, this node has the maximum transmission range $R_T^{\max}$, which can be computed based on minimum required receiving power $(g(R_T^{\max}) \cdot P_{\max})$ at a receiving node j. When the transmission power p is less than $P_{\max}$, the same minimum required receiving power should be met. If node i can transmit to node j, then we have $g(d_{ij}) \cdot p \ge g(R_T^{\max}) \cdot P_{\max}$. Thus, the transmission range is

$$R_T(p) = g^{-1} \left(\frac{g(R_T^{\max}) \cdot P_{\max}}{p} \right) , \qquad (12)$$

which is a function of transmission power p. Similarly, the interference range is

$$R_I(p) = g^{-1}\left(\frac{g(R_I^{\max}) \cdot P_{\max}}{p}\right) . \quad (13)$$

Routing (For Both Physical and Protocol Models). Among the set of $\mathcal{N}$ nodes in the ad hoc network, we assume there is a set of $\mathcal{L}$ active user communication (unicast) sessions. Denote $s(l)$ and $d(l)$ the source and destination nodes of session $l \in \mathcal{L}$ and $r(l)$ the minimum rate requirement (in b/s) of session l. Suppose that our objective is to maximize a scaling factor K for all sessions' requirements. That is, for each each session $l \in \mathcal{L}$, $Kr(l)$ amount of data rate is to be transmitted from $s(l)$ to $d(l)$. To route each of these flows from its respective source node to destination node, it is necessary to employ multi-hop due to the limited transmission range of a node. Further, to maximize the scaling factor, it is necessary to employ multi-path.

Mathematically, this can be modeled as follows. Denote $f_{ij}(l)$ the data rate from node i to node j that is attributed to session l, where $i \in \mathcal{N}, j \in \mathcal{T}_i = \bigcup_{m \in \mathcal{M}_i} \mathcal{T}_i^m$. If node i is the source node of session l, i.e., $i = s(l)$, then

$$\sum_{j \in \mathcal{T}_i} f_{ij}(l) = Kr(l) \quad (l \in \mathcal{L}, i = s(l)) . \quad (14)$$

If node i is an intermediate relay node for session l, i.e., $i \neq s(l)$ and $i \neq d(l)$, then

$$\sum_{j \in \mathcal{T}_i}^{j \neq s(l)} f_{ij}(l) = \sum_{k \in \mathcal{T}_i}^{k \neq d(l)} f_{ki}(l) \ (l \in \mathcal{L}, i \in \mathcal{N}, i \neq s(l), d(l)). \quad (15)$$

If node i is the destination node of session l, i.e., $i = d(l)$, then

$$\sum_{k \in \mathcal{T}_i} f_{ki}(l) = Kr(l) \quad (l \in \mathcal{L}, i = d(l)) . \quad (16)$$

It can be easily verified that once (14) and (15) are satisfied, (16) must also be satisfied. As a result, it is sufficient to have (14) and (15) in the formulation.

In addition to the above flow balance equations at each node $i \in \mathcal{N}$ for session $l \in \mathcal{L}$, the aggregated flow rates on each radio link cannot exceed this link's capacity. Therefore, for a link $i \to j$, we have

$$\sum_{l \in \mathcal{L}}^{s(l) \neq j, d(l) \neq i} f_{ij}(l) \leq \sum_{m \in \mathcal{M}_{ij}} c_{ij}^m \quad (i \in \mathcal{N}, j \in \mathcal{T}_i) , \quad (17)$$

where c_{ij}^m is computed by (7) under the physical model or by (11) under the protocol model.

2.2 Problem Formulation and Solution Approach

Objective Function. In our problem formulation, we consider to maximize a capacity related objective function. Specifically, we choose to maximize the scaling factor K for all sessions' rate requirements. There are many other objectives that can also be used in this investigation, e.g., the sum of all sessions' rates, the sum of log utility of session rates, etc. In general, we could consider an objective function in the form of the total utility of session rates, with the utility of a session being a concave function of its rate.

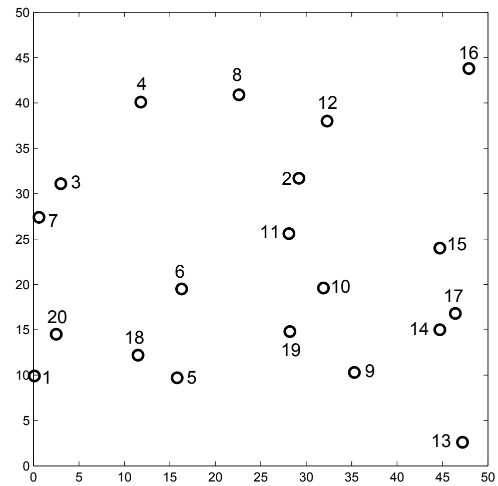


Figure 1: A 20-node 5-session network topology.

We emphasize that the same methodology that we will develop regarding how to correctly use the protocol model is applicable to all these objective settings.

Problem Formulation. Under the physical model, putting together all the constraints for scheduling, power control, and flow routing, we have: Maximize K subject to constraints (2), (3), (4), (5), (6), (7), (14), (15), and (17). While under the protocol model, the constraints are (2), (9), (10), (11), (14), (15), and (17).

Solution Approach. Both cross-layer optimization problems are in the form of *mixed-integer non-linear programming* (MINLP) problem, which is NP-hard in general [11]. A solution procedure based on the branch-and-bound (similar to [22]) can be developed to solve these MINLP optimization problems.

An important step in branch-and-bound approach is constructing and solving a linear relaxation for the original optimization problem at each iteration. Under the physical model, the linear relaxation has $O(N^3M)$ variables, while under the protocol model, the linear relaxation has $O(N^2M)$ variables. Since the number of variables directly impacts complexity, the complexity of solving a protocol model problem is much lower than a physical model problem.

3. A REALITY CHECK MECHANISM FOR PROTOCOL MODEL SOLUTION

In this section, we identify the potential infeasibility problem associated with the protocol model solution. Then we introduce a reality check mechanism and show how it can be used to obtain a revised solution that is feasible.

3.1 Infeasibility in Protocol Model Solution

Under the protocol model, the impact of interference from neighboring nodes is binary and is solely determined by whether or not the node falls within the interference range of non-intended transmitters. However, as the following example shows, solutions obtained under the protocol model may not be feasible in practice.

Example 1. *Consider a* 20*-node* 5*-session network in Figure 1. The location and available bands at each node are*

Table 1: Location and available frequency bands at each node for a 20-node 5-session network

Node	Location	Available Bands	Node	Location	Available Bands	Node	Location	Available Bands
1	$(0.1, 9.9)$	2, 5, 6, 7, 9, 10	8	$(22.6, 40.9)$	2, 5, 6, 7, 8, 9	15	$(44.7, 24)$	2, 5, 6, 8
2	$(29.2, 31.7)$	1, 3, 4	9	$(35.3, 10.3)$	2, 3, 4, 5, 6, 7, 8, 9, 10	16	$(47.9, 43.8)$	2, 5, 6, 8, 9, 10
3	$(3, 31.1)$	2, 6, 7, 8, 9, 10	10	$(31.9, 19.6)$	2, 6, 7, 8, 9, 10	17	$(46.4, 16.8)$	2, 5, 6, 7, 8
4	$(11.8, 40.1)$	2, 8	11	$(28.1, 25.6)$	2, 5, 6, 8, 9, 10	18	$(11.5, 12.2)$	1, 2, 5, 6, 7, 8, 9, 10
5	$(15.8, 9.7)$	7, 8, 9, 10	12	$(32.3, 38)$	2, 5, 6, 7, 8, 10	19	$(28.2, 14.8)$	3, 4
6	$(16.3, 19.5)$	2, 6, 10	13	$(47.2, 2.6)$	2, 6	20	$(2.5, 14.5)$	5, 6, 10
7	$(0.6, 27.4)$	1, 3	14	$(44.7, 15)$	5, 6, 7, 8, 9, 10			

shown in Table 1. The source node, destination node, and minimum rate requirement of each session are shown in Table 2. The setting of parameters are $W = 50$, $\alpha = 3$, $R_T^{max} = 20$ *and* $P_{max} = 4.8 \cdot 10^5 \eta W$. *All units are normalized appropriately. When the maximum interference range is* $R_I^{max} = 35$, *we have the following power control and scheduling solution:*

$x^2_{8,11} = 1, \quad p^2_{8,11} = 0.6 \cdot P_{max}$;
$x^7_{9,17} = 1, \quad p^7_{9,17} = 0.2 \cdot P_{max}$;
$x^9_{11,10} = 1, \quad p^9_{11,10} = 0.1 \cdot P_{max}$;
$x^7_{12,8} = 1, \quad p^7_{12,8} = 0.1 \cdot P_{max}$;
$x^8_{12,11} = 1, \quad p^8_{12,11} = 0.4 \cdot P_{max}$;
$x^2_{13,9} = 1, \quad p^2_{13,9} = 0.4 \cdot P_{max}$;
$x^5_{15,14} = 1, \quad p^5_{15,14} = 0.1 \cdot P_{max}$;
$x^6_{16,12} = 1, \quad p^6_{16,12} = P_{max}, \quad x^{10}_{16,12} = 1, \quad p^{10}_{16,12} = 0.5 \cdot P_{max}$;
$x^5_{18,1} = 1, \quad p^5_{18,1} = 0.2 \cdot P_{max}$.

There are two transmissions on band 2, *i.e., from node* 8 *to node* 11 *and from node* 13 *to node* 9 *(see the location of each node in Table 1). The transmission power at node* 8 *is* $0.6 \cdot P_{max}$. *Using (12), the transmission range at node* 8 *is* 17.60, *which is larger than the distance* 16.26 *between nodes* 8 *and* 11. *Thus, node* 11 *is in the transmission range of node* 8. *The transmission power is* $0.4 \cdot P_{max}$ *at node* 13. *Using (13), the interference range is* 27.83, *which is smaller than the distance* 29.90 *between nodes* 13 *and* 11. *Thus, node* 11 *is not in the interference range of node* 13. *Under the protocol model, the transmission from node* 8 *to node* 11 *is successful. Further, since node* 11 *is not in the interference range of node* 13, *it is assumed that there is no interference from node* 13. *We have that SNR at node* 11 *is* $(g_{8,11} \cdot p^2_{8,11})/(\eta W) = 4.1216$. *By (11), the capacity on link* $8 \to 11$ *is* 117.84. *We note that in the protocol model solution, the flow rate from node* 8 *to node* 11 *is* 117.84, *i.e., the computed capacity is* 100% *utilized.*

However, the interference from node 13 *at node* 11 *is* $g_{13,11} \cdot p^2_{13,9} = 0.2403 \cdot \eta W$, *which is not zero. The SINR at node* 11 *is* 3.323. *By (7), the actual capacity from node* 8 *to node* 11 *is* 105.61. *But in the protocol model solution, the flow rate on this link is* 117.84, *which is larger than this link's capacity. Therefore, the solution obtained under the protocol model is infeasible.* □

Due to such potential infeasibility in the protocol model solution, results based on blind use of protocol model may be incorrect.

3.2 The Reality Check Mechanism

As we discussed in previous section, capacity calculation under the protocol model is not accurate. To find out what result can really be achieved under the protocol model solution, it is necessary to go through a validation process. In this section, we introduce a new "reality check" mechanism for a protocol model solution. The goal of reality check is to find the achievable result under a given protocol model solution. Reality check result can also be viewed as a corrected/revised result based on the protocol model solution.

Table 2: Source node, destination node, and minimum rate requirement of each session in the 20-node 5-session network

Session l	Source Node $s(l)$	Destination Node $d(l)$	Minimum Rate Requirement $r(l)$
1	16	10	9
2	18	1	1
3	12	11	4
4	13	17	3
5	15	14	2

Specifically, for a given protocol model solution, we have the knowledge of routing, scheduling, and power control for each node in the network. Under reality check, instead of using the capacity computed by (11) (which neglects the impact of interference), we use (7) to *re-compute* actual achievable capacity between the nodes under the given scheduling and power control solution (as illustrated in Example 1). Using this accurate capacity calculation among the nodes, we can re-compute the achievable result (i.e., objective function) by a mathematical program to obtain a feasible routing solution. This new routing, along with the original scheduling and power control, offer a feasible solution. We call this result the reality check result, which is formally defined as follows.

Definition 1. *Reality check result is defined as the achievable result for a given protocol model solution.*

In reality check, we only need to re-compute capacities and adjust flow rates. Although we use the same capacity formula as that in physical model, this formula is only used for simple calculations instead of being part of a complex optimization problem as under the physical model. Therefore, the complexity of this reality check mechanism is very small.

Example 2. *We again consider the* 20*-node* 5*-session network example discussed in Example 1. The objective function (i.e., the maximum scaling factor) under the protocol model solution is* 20.47 *(without reality check).*

We have examined the capacity from node 8 *to node* 11 *in Example 1. In the protocol model solution, the computed capacity is* 117.84 *and the flow rate on this link is also* 117.84. *However, we find that the actual capacity is* 105.61. *As a result, the achievable objective value will be smaller than* 20.47.

To obtain the achievable objective function, we re-compute capacity for all links following the same token in Example 1. We list the actual capacity and the capacity computed in protocol model in Table 3. Using revised capacities (from reality check), we can re-compute a feasible solution using a linear program. The new achieved objective value is 18.34 *(vs.* 20.47 *in the blind use of protocol model solution).* □

Table 3: Actual capacity and the capacity computed in protocol model

Link	Actual Capacity	Capacity Computed in Protocol Model
$8 \to 11$	105.61	117.84
$9 \to 17$	102.67	108.67
$11 \to 10$	328.78	328.78
$12 \to 8$	119.47	123.98
$12 \to 11$	145.70	145.70
$13 \to 9$	113.75	126.28
$15 \to 14$	148.07	152.81
$16 \to 12$	243.25	243.25
$18 \to 1$	131.58	132.18

The following statement summarizes our discussion on the reality check result.

PRINCIPLE 1. *The reality check result offers a correct measure of achievable result by a protocol model solution.*

The above principle offers a meaningful performance measurement criteria for the protocol model. The efficacy of protocol model depends on the performance gap between its reality check result and the result obtained under the physical model. If this performance gap is small, then the protocol model is a good approximation and can be used as an effective tool for analyzing wireless networks. On the other hand, if this performance gap is large, then the protocol model may not be very useful.

The following lemma shows the relationship between the reality check result for the protocol model and the optimal result obtained under the physical model.

LEMMA 1. *Reality check result cannot exceed the optimal result under the physical model.*

PROOF. Note that both solutions employ the same accurate link capacity computation (7) and the final results (i.e., objective function values) are both feasible. Since the optimal physical model solution has the largest scaling factor among all feasible solutions, while reality check result is only one possible feasible solution, then reality check result cannot exceed the the optimal result under the physical model. □

4. INTERFERENCE RANGE SETTING FOR THE PROTOCOL MODEL

In this section, we investigate the performance gap of the protocol model in relation to the physical model.

4.1 Setting of Interference Range

To perform a meaningful comparison, both physical model and protocol model should use the same underlying physical layer mechanism. For the protocol model, we have two parameters, i.e., the maximum transmission and interference ranges $R_T^{\max}, R_I^{\max}$. Since the underlying physical layer mechanism is the same, the parameter $R_T^{\max}$ should be consistent with the α parameter in the physical model. Under the ideal scenario, when there is no concurrent transmission in the same band, two nodes with distance $R_T^{\max}$ should be able to communicate with each other under the maximum transmission power $P_{\max}$ and the SINR should

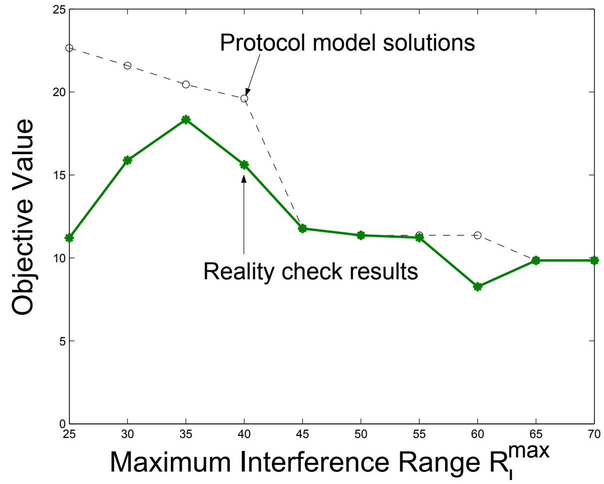


Figure 2: Protocol model solutions and corresponding reality check results for the 20-node 5-session network.

be α (same as that under physical model). Thus, we have $\frac{g(R_T^{\max})\cdot P_{\max}}{\eta W} = \alpha$. As a result,

$$R_T^{\max} = g^{-1}\left(\frac{P_{\max}}{\alpha\eta W}\right) . \quad (18)$$

Note that the maximum interference range, $R_I^{\max}$, is a parameter introduced by the protocol model and there is no corresponding parameter in the physical model. The only requirement on $R_I^{\max}$ is $R_I^{\max} > R_T^{\max}$, i.e., a lower bound for $R_I^{\max}$ is $R_T^{\max}$. To find an upper bound for $R_I^{\max}$, we want to determine a constant $(R_I)^U$ such that if $R_I^{\max} \geq (R_I)^U$, then when any link is active, all other links on the same band in the network cannot be active. Consider a link $k \to h$ on a band m. Using (8) and (13), its interference range is at least $R_I(P_{kh}^T) = g^{-1}(\frac{g(R_I^{\max})\cdot P_{\max}}{P_{kh}^T}) = g^{-1}(\frac{g(R_I^{\max})\cdot g(d_{kh})}{g(R_T^{\max})})$. For another node j with $d_{jk} \leq R_I(P_{kh}^T)$ (or equivalently, $R_I^{\max} \geq g^{-1}(\frac{g(R_T^{\max})\cdot g(d_{jk})}{g(d_{kh})})$), it cannot receive on band m when node i is transmitting to node j on band m. Thus, we can set

$$\begin{aligned}(R_I)^U &= \max\{g^{-1}(\frac{g(R_T^{\max})\cdot g(d_{jk})}{g(d_{kh})}) : j \in \mathcal{N}, m \in \mathcal{M}_j, \\ &\quad k \in \mathcal{I}_j^m, h \in \mathcal{T}_k^m\} \\ &= \max\{g^{-1}(\frac{g(R_T^{\max})\cdot g(\max\{d_{jk} : k \in \mathcal{I}_j^m\})}{\min\{g(d_{kh}) : h \in \mathcal{T}_k^m\}}) \\ &\quad : k \in \mathcal{N}, m \in \mathcal{M}_j\} .\end{aligned}$$

Any $R_T^{\max} \geq (R_I)^U$ will lead to the same interference relationship in the network, which in turn yields the same protocol model solution and the same reality check result. Thus, the range for $R_I^{\max}$ is within $[R_T^{\max}, (R_I)^U]$.

As one would expect, the setting of $R_I^{\max}$ directly affects the performance gap between the two models. We now investigate the impact of the maximum interference range ($R_I^{\max}$) setting. We consider 20, 30, 40, 50-node ad hoc networks with each node randomly located in a 50x50 area.[1] For

[1]The results for the case where the node density is kept constant while the area of network coverage increases will be discussed at the end of this section.

Table 4: Location and available frequency bands at each node for a 30-node 5-session network

Node	Location	Available Bands	Node	Location	Available Bands
1	$(7, 0.7)$	1, 2, 6, 7, 16, 17, 19, 20	16	$(30.3, 28.1)$	7, 8, 11, 16, 17, 19, 20
2	$(5, 4)$	3, 5, 9, 12, 14, 15	17	$(32, 41.1)$	7, 11, 16, 17, 19, 20
3	$(6.8, 14)$	1, 2, 6, 7, 8, 11, 16, 17, 19, 20	18	$(14.1, 33.7)$	3, 4, 5
4	$(15.7, 3.3)$	1, 2, 7, 16, 20	19	$(23, 46.4)$	3, 12, 15
5	$(9.5, 17)$	3, 4, 5, 9, 12	20	$(30.3, 9.3)$	5, 9
6	$(19.4, 17.1)$	1, 2, 6, 7, 8, 16, 19, 20	21	$(17.6, 29.2)$	1, 2, 6, 7, 8, 11, 16, 17, 19, 20
7	$(34.7, 14.6)$	3, 4, 5, 9, 12, 14	22	$(27.1, 27.8)$	9, 12, 14, 15
8	$(4.9, 25.9)$	3, 4, 12	23	$(26.9, 45.9)$	3, 4, 5, 9, 10, 12, 13, 14, 15, 17
9	$(46.6, 42.1)$	10, 18	24	$(43.3, 32.4)$	1, 2, 11, 16, 17, 20
10	$(8.3, 38.3)$	3, 4, 5, 9, 14	25	$(45.4, 8.2)$	3, 4, 5, 9, 12, 14
11	$(26.7, 11.1)$	1, 6, 7, 8, 11, 16, 17, 19, 20	26	$(43.4, 35)$	3, 5, 9, 15
12	$(36.4, 47.3)$	10, 13, 18	27	$(41.3, 45.1)$	1, 16, 20
13	$(24.3, 21.2)$	1, 2, 6, 8, 11, 19	28	$(14.4, 30.3)$	1, 2, 6, 7, 8, 11, 16, 17, 20
14	$(23.1, 0.8)$	3, 5, 9, 14	29	$(41.6, 41.7)$	3, 4, 5, 9, 10, 12, 14, 15, 18
15	$(21.4, 19.2)$	4, 9, 12, 14	30	$(25.9, 12)$	1, 2, 6, 7, 8, 11, 16, 17, 19, 20

ease of exposition, we normalize all units for distance, bandwidth, rate, and power based on (5) and (7) with appropriate dimensions. At each node, there are up to 10 available frequency bands and each band has a bandwidth of $W = 50$. The set of available bands at different nodes can be different. The number of active user communication sessions is 5 for 20 and 30-node networks. For 40 and 50-node networks, the number of active user communication sessions is either 5 or 10. The source node and destination node of each session are randomly selected. The minimum rate requirement for each session is randomly generated within $[1, 10]$.

We assume that the SINR threshold α for the physical model is $\alpha = 3$ [12]. For the protocol model, the maximum transmission range $R_T^{\max}$ and maximum interference range $R_I^{\max}$ under peak power $P_{\max}$ are two constant parameters. Note that the transmission range $R_T(p)$ and interference range $R_I(p)$ are variables that depend on transmission power p (see (12) and (13)). We assume peak transmission power $P_{\max} = 4.8 \cdot 10^5 \eta W$, with the corresponding maximum transmission range $R_T^{\max} = 20$ (by (18)). Since the maximum interference range $R_I^{\max}$ (under peak power $P_{\max}$) is a parameter for the protocol model, we investigate the impact of this parameter setting in our study. We apply the reality check mechanism for each protocol model solution.

The first set of results is for the 20-node 5-session network discussed in Example 1. The solution under the physical model has an objective value of 18.89. The results under the protocol model are shown in Fig. 2. We can see that, the reality check result for protocol model solution is different under different maximum interference range setting. The largest objective value among these reality check results is 18.34 (with $R_I^{\max} = 35$), which is very close to the physical model solution.

By Principle 1, the reality check result offers a measure of achievable result by a protocol model solution. Thus, for this network, the best maximum interference range value should be 35. In general, we have the following rule on how to set the maximum interference range for protocol model.

RULE 1. *The maximum interference range in the protocol model should be set to the value corresponding to the maximum reality check result.*

We emphasize that to set this range optimally, it is *not* necessary to solve the problem under the physical model, which involves much higher complexity.

Table 5: Source node, destination node, and minimum rate requirement of each session in the 30-node 5-session network

Session l	Source Node $s(l)$	Destination Node $d(l)$	Minimum Rate Requirement $r(l)$
1	16	28	4
2	24	11	7
3	13	1	1
4	19	29	8
5	26	15	1

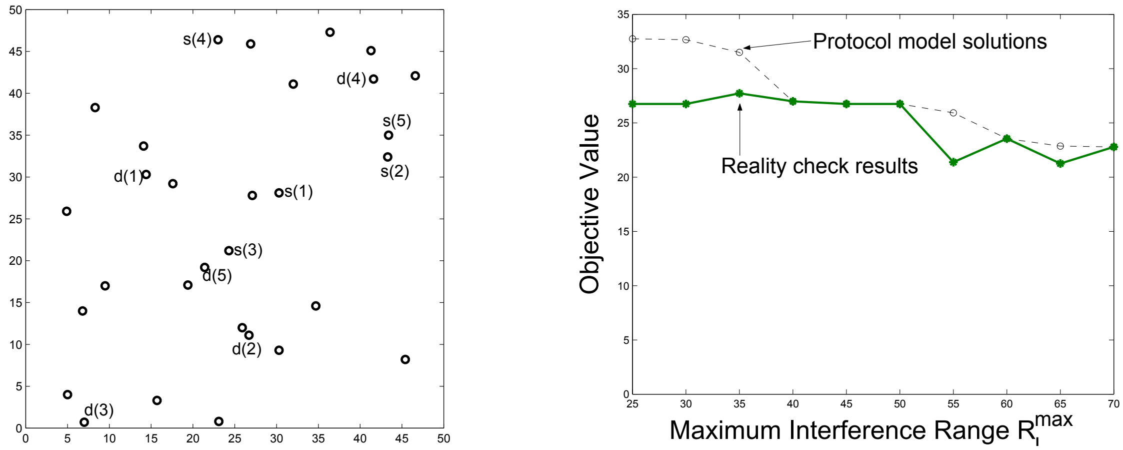
The network topology of a 30-node 5-session network is shown in Fig. 3(a). The location and available bands at each node are shown in Table 4. The source node, destination node, and minimum rate requirement of each session are shown in Table 5. The solution under physical model has an objective value of 31.18. The results under protocol model are shown in Fig. 3(b). We find that the best reality check result has an objective value of 27.72, which is within 11% of the optimum (i.e., 31.18) and the maximum interference range should be set to 35 by Rule 1.

The network topology of a 40-node 5-session network is shown in Fig. 4(a). The location and available bands at each node, the source node, destination node, and minimum rate requirement of each session are omitted due to space limitation. The solution under physical model has an objective value of 29.42. The results under protocol model are shown in Fig. 4(b), with maximum reality check result also being 29.42. By Rule 1, we find that the maximum interference range in the protocol model should be set to 35.

The network topology of a 40-node 10-session network is shown in Fig. 5(a). The solution under physical model has a maximum objective value of 16.43. The results under protocol model are shown in Fig. 5(b) with the same maximum reality check result being 16.43. By Rule 1, we find that the maximum interference range in the protocol model should be set to 30.

For a 50-node 5-session network in Fig. 6(a), the solution under physical model has an objective value of 25.27. The results under protocol model are shown in Fig. 6(b). By Rule 1, we find that the maximum interference range in the protocol model should be set to 35 or 40. Both values can achieve the same objective value of 25.02, which is very close to the optimum 25.27.

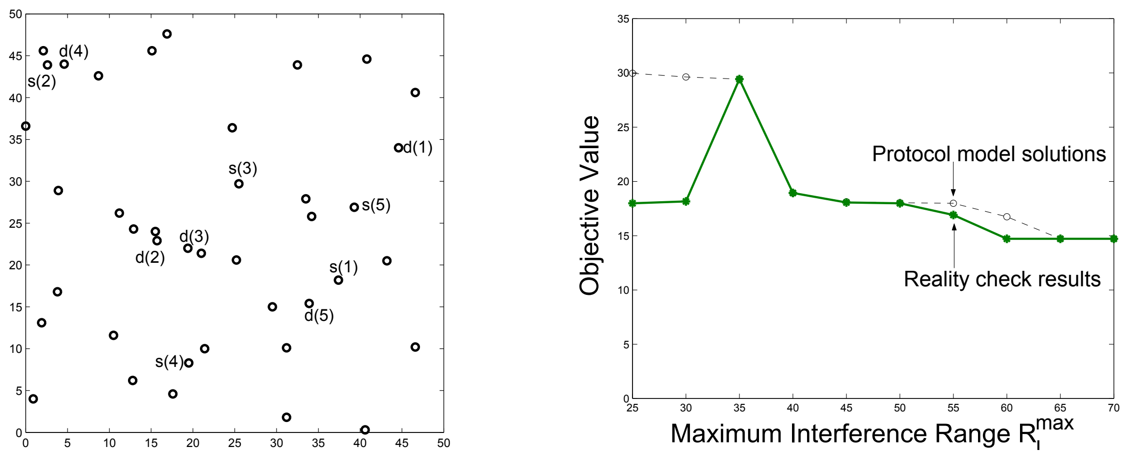
Finally, we have the results for a 50-node 10-session network in Fig. 7. The solution under physical model has an



(a) Network topology.

(b) Protocol model solutions and corresponding reality check results.

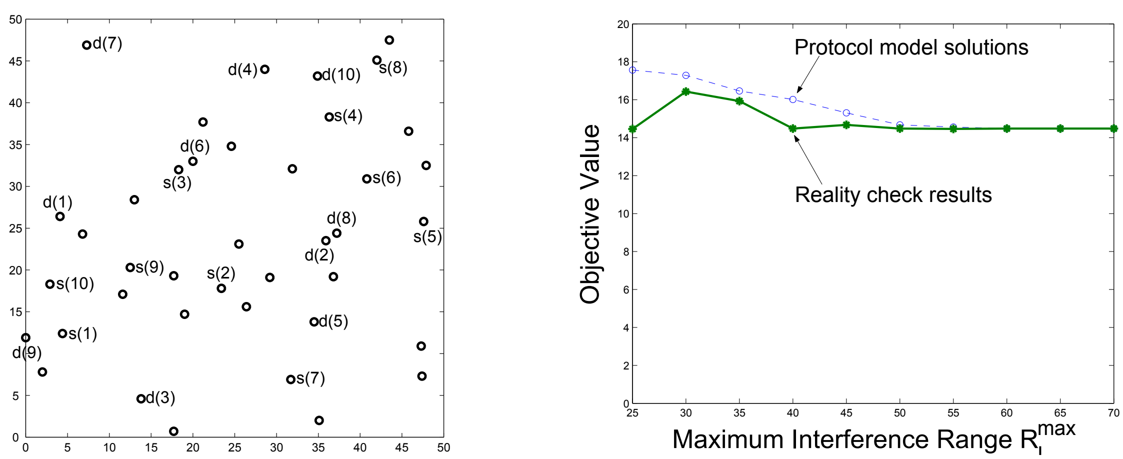
Figure 3: A 30-node 5-session network.



(a) Network topology.

(b) Protocol model solutions and corresponding reality check results.

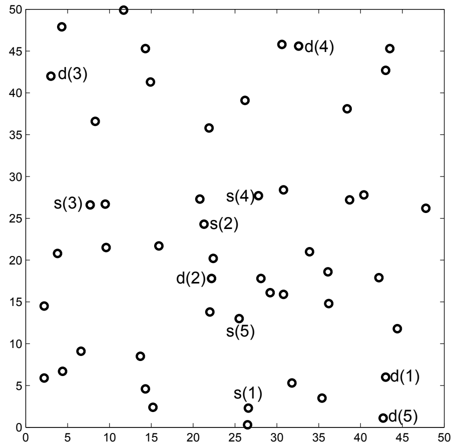
Figure 4: A 40-node 5-session network.



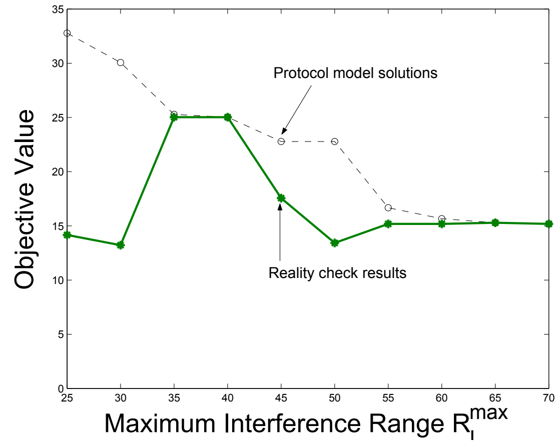
(a) Network topology.

(b) Protocol model solutions and corresponding reality check results.

Figure 5: A 40-node 10-session network.

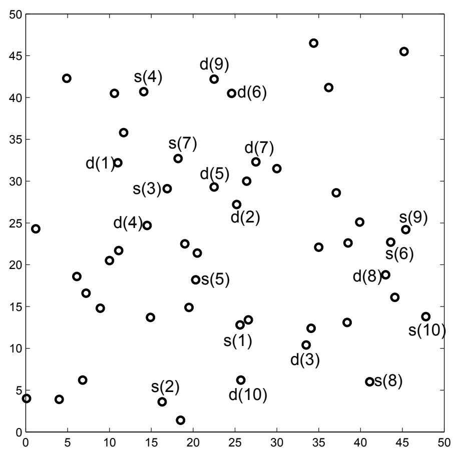


(a) Network topology.

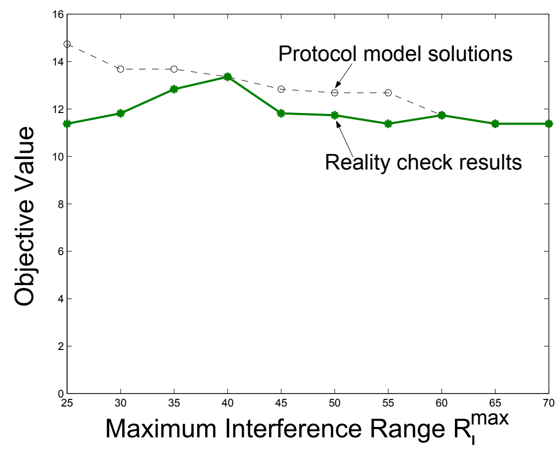


(b) Protocol model solutions and corresponding reality check results.

Figure 6: A 50-node 5-session network.



(a) Network topology.



(b) Protocol model solutions and corresponding reality check results.

Figure 7: A 50-node 10-session network.

objective value of 13.36. By Rule 1, we find that the maximum interference range in the protocol model should be set to 40. The corresponding objective value is the same as that under physical model, i.e., 13.36.

In summary, we have shown that the setting of maximum interference range in the protocol model has a direct impact on the performance result. Although the search interval for the maximum interference range can be specified, an efficient search algorithm for finding the optimal setting remains an open problem and will be investigated in our future research.

4.2 The Gap

Under protocol model solutions, we can see that (in Figs. 2, 3(b), 4(b), 5(b), 6(b), and 7(b)) the actual objective value in reality check result is no more than that in protocol model solution.

When the maximum interference range is set too small (e.g., 25), the protocol model provides an incorrect solution (even larger than that under the physical model). After we perform reality check on protocol model solution, the achieved result could be much lower than that under the physical model. This is because, under a very small interference range, the "negligible" interference from neighboring nodes could be large (non-negligible). As a result, the accurate capacity calculated via (7) could be much smaller than that computed in the protocol model by (11).

On the other hand, when the maximum interference range is set too large, the gap between protocol model solution and its reality check result can be small. This is because, with a very large interference range, spectrum may not be re-used at different nodes. As a result, there is no interference from other nodes. In this case, the link capacity computation in (11) is the same as that in (7) and thus, there is no performance degradation after reality check. However, as we have seen in all the results, setting the maximum interference range too large will lead to very conservative results, i.e., much smaller than those under the physical model.

All results presented so far are based on networks in a fixed area with varying node density. We also performed simulations on networks with a fixed node density while the area of network was changed. Results for this set of simulations are consistent with observations in this section. To conserve space, we will not include these simulation results.

Based on our simulation results, we draw the following observation.

Observation 1. *Under the optimal setting of the maximum interference range, the reality check result for protocol model solution is close to (or the same as) the physical model solution.*

The significance of the above observation is that it enables us to use the protocol model (with reality check) as a good simplification for the physical model as long as maximum interference range is set correctly.

5. CONCLUSION

This paper aimed to reconcile the tension between physical model and protocol model. We showed that in general, solutions obtained directly under the protocol model are likely to be infeasible in practice and thus blind use of protocol model is likely to offer incorrect results. To address this problem, we proposed a new mechanism called "reality check" and showed how it can be used to obtain a feasible protocol model solution. Subsequently, we showed that by appropriate setting of the interference range in the protocol model, it is possible to narrow the solution gap between protocol model and physical model. Our simulation results confirmed that this gap is indeed small, thereby suggesting that our method of joint reality check and interference range setting can make the protocol model a viable approach to analyze multi-hop wireless networks.

6. ACKNOWLEDGMENTS

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