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On Node Lifetime Problem for Energy-Constrained Wireless Sensor Networks

Y. THOMAS HOU * and YI SHI
The Bradley Department of Electrical and Computer Engineering, Virginia Tech, Blacksburg, VA 24061, USA

HANIF D. SHERALI
The Grado Department of Industrial and Systems Engineering, Virginia Tech, Blacksburg, VA 24061, USA

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A fundamental problem in wireless sensor networks is to maximize network lifetime under given energy constraints. In this paper, we study the network lifetime problem by considering not only maximizing the time until the first node fails, but also maximizing the lifetimes for all the nodes in the network, which we define as the *Lexicographic Max-Min (LMM) node lifetime* problem. The main contributions of this paper are two-fold. First, we develop a polynomial-time algorithm to derive the LMM-optimal node lifetime vector, which effectively circumvents the computational complexity problem associated with an existing state-of-the-art approach, which is exponential. The main ideas in our approach include: (1) a link-based problem formulation, which significantly reduces the problem size in comparison with a flow-based formulation, and (2) an intelligent exploitation of *parametric analysis* technique, which in most cases determines the minimum set of nodes that use up their energy at each stage using very simple computations. Second, we present a simple (also polynomial-time) algorithm to calculate the flow routing schedule such that the LMM-optimal node lifetime vector can be achieved. Our results in this paper advance the state-of-the-art algorithmic design for network-wide node lifetime problem and facilitate future studies of the network lifetime problem in energy-constrained wireless sensor networks.

Keywords: energy constraint, node lifetime, lexicographic max-min, flow routing, power control, wireless sensor networks

1. Introduction

Wireless sensor networks consist of battery-powered nodes that are endowed with a multitude of sensing modalities including multimedia (e.g., video, audio) and scalar data (e.g., temperature, pressure, light, magnetometer, infrared). The demand for these networks is spurred by numerous applications that require in-situ, unattended, high-precision, and real-time observations over a vast area. Although there have been significant improvements in processor design and computing, advances in battery technology still lag behind, making energy resource the fundamental challenge in wireless sensor networking. As a consequence of the energy constraint, a new performance metric, namely, the *network lifetime*, has become a vitally important benchmark for wireless sensor networks. There have been active research efforts recently at the networking layer on devising flow routing algorithms to maximize network lifetime [4–6,8–10,16,29]. However, the network lifetime objective in most of these efforts has been centered around maximizing the time until the first node fails. Although the time until the first node fails is an important measure from the complete network coverage point of view, this performance metric alone cannot measure the lifetime performance behavior for all nodes in the network. For wireless sensor networks that are primarily designed for environmental monitoring or surveillance, the loss of a single node will only affect the coverage of one particular area and will not affect the monitoring or surveillance capabilities of the remaining nodes in the network. This is because the remaining nodes in the network can adjust their transmission power (via power control) and reconfigure themselves into a new network routing (relay) topology so that information collected at the remaining nodes can still be delivered successfully to the base-station. Consequently, it is important to investigate how to maximize the lifetime for, not only the first node, but also *all* the other nodes in the network. We call this the *Lexicographic Max-Min (LMM) node lifetime* problem, which will be formally defined in Section 2.3.

Recently, Brown et al. [7] studied this problem under the so-called "maximum node lifetime curve" problem, which is equivalent to the LMM node lifetime problem. Informally, the maximum node life curve attempts to *maximize* the time until a set of nodes drain up their energy (which we call the *drop point*) while *minimizing* the number of nodes that drain up their energy at each drop point. The main contribution by Brown et al. [7] is the development of a procedure to solve the maximum node lifetime curve problem. A key step in their procedure is to use *multiple* independent linear programming (LP) calculations to determine the minimum set of nodes at each drop point, which we call "Serial LP with Slack Variable

*Corresponding author.

analysis" (SLP-SV). Although this approach can solve the LMM node lifetime problem, its computational complexity is shown to be exponential, which could be a potential problem for large-scale networks.

Inspired by Brown et al.'s work on the LMM node lifetime problem, in this paper, we develop a polynomial time algorithm to derive the LMM-optimal node lifetime vector. In addition, we demonstrate that, for any given network configuration and initial condition, our approach is always significantly computationally more efficient than the *Slack Variable* (SV) based approach in [7]. Consequently, this leads to an even stronger performance guarantee than the commonly-used *average case complexity* criteria. The computational effectiveness of our approach accrues from two important techniques. First, we employ a *link-based* problem formulation, which significantly reduces the problem size in comparison with a flow-based formulation used in [7]. Second, which is also the most significant contribution in this paper, we exploit the so-called *parametric analysis* technique at each drop point to determine the minimum set of nodes that use up their energy. When the problem is non-degenerate, we show that this technique is a powerful tool in determining the minimum node set for each drop point. It is also extremely simple and has a linear time complexity per node in contrast with the SV-based approach proposed in [7], which requires solving multiple additional LPs at each drop point. Even for the rare case, when the problem is degenerate, using the parametric analysis technique still is more efficient than the SV-based approach as it decreases the number of additional LPs that need to be solved at each drop point.

In addition to providing an efficient polynomial time algorithm for the LMM-optimal node lifetime vector computation, we also develop a simple polynomial time algorithm that provides a corresponding flow routing schedule among the remaining alive nodes at each stage such that the LMM-optimal node lifetime vector can indeed be achieved. A nice property about this algorithm is that it can be executed in parallel (instead of in serial) for all the stages.

The remainder of this paper is organized as follows. In Section 2, we describe the system model and problem statement for this research, including the reference network architecture, nodal power dissipation behavior, and the LMM node lifetime problem description. We also describe a naive approach to address this problem and discuss why it usually gives an incorrect solution. Section 3 presents the link-based LMM problem formulation and our efficient Serial LP algorithm based on Parametric Analysis, which we call SLP-PA. In Section 4, we present a simple algorithm to calculate the flow routing schedule at each stage such that the LMM-optimal node lifetime vector can indeed be achieved. Section 5 analyzes the complexity of our algorithm and compares it with that in [7]. Numerical results using the SLP-PA approach and the corresponding flow routing schedule are given in Section 6. Section 7 reviews related work and Section 8 concludes this paper.

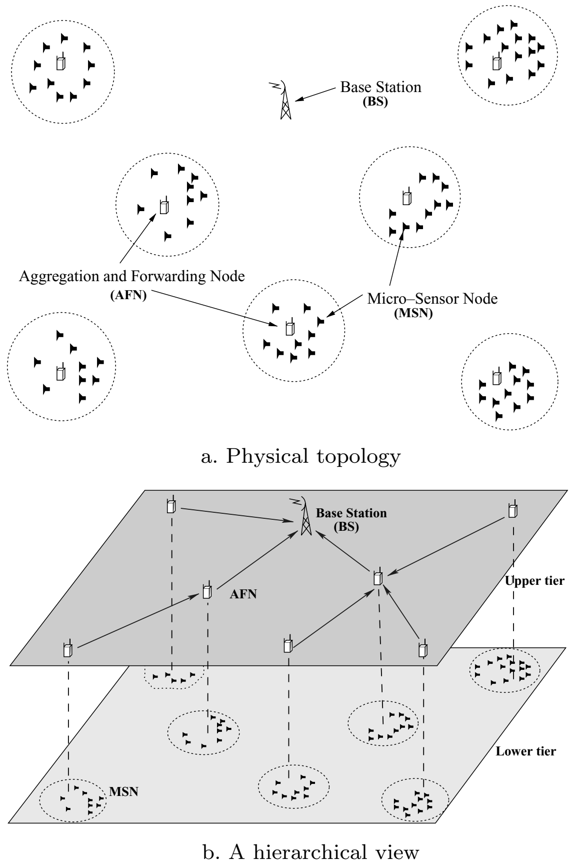


Figure 1. Reference architecture for a two-tier wireless sensor network.

2. System modeling and problem formulation

2.1. Reference network architecture

We focus on a two-tier architecture for wireless sensor networks. The two-tier network architecture is motivated by recent advances in *distributed source coding* (DSC) [11, 20, 23]. Figures 1(a) and (b) show the *physical* and *hierarchical* network topology for such a network, respectively. Here, we have three types of nodes in the network: *micro-sensor nodes* (MSNs), *aggregation and forwarding nodes* (AFNs), and a *base-station* (BS). The MSNs can be application-specific sensor nodes (e.g., temperature sensor nodes (TSNs), pressure sensor nodes (PSNs), and video sensor nodes (VSNs)) and they constitute the lower tier of the network. They are deployed in groups (or clusters) at a strategic location for surveillance or monitoring applications. The MSNs are small and low-cost; they are densely deployed within a small geographical area. The objective of an MSN is very simple: Once triggered by an event (e.g., the detection of motion or biological/chemical agents), it starts to capture live information (e.g., video), which it sends directly to the local AFN in one hop. It is worth pointing out that multi-hop routing among the MSNs may not be necessary due to the small distance between an MSN and its AFN. By deploying these inexpensive MSNs in clusters, and within proximity of a strategic location, it is possible to obtain a comprehensive view of the area situation by exploring the *correlation* among the scenes collected at each MSN [11]. Furthermore, the reliability of area surveillance capability can also be improved through redundancy among the MSNs in the same cluster.

For each cluster of MSNs, there is one AFN, which is different from an MSN in terms of both its physical properties and functions. The primary functions of an AFN are: (1) *data aggregation* (or "fusion") for information flows coming from the local cluster of MSNs, and (2) *forwarding* (or relaying) the aggregated information to the next hop AFN toward the base-station. For data fusion, an AFN analyzes the content of each data stream (e.g., video) it receives, from which it composes a complete scene by exploiting the correlation among each individual data stream from the MSNs. An AFN also serves as a relay node for other AFNs to carry traffic toward the base-station. Although an AFN is expected to be provisioned with much more energy than an MSN, it also consumes energy at a substantially higher rate (due to wireless communication over large distances). Consequently, an AFN has limited lifetime. Upon the depletion of energy at an AFN, we expect that the *coverage* for the particular area under surveillance will be lost, despite the fact that some of the MSNs within the cluster may still have remaining energy.[1] Therefore, it is essential to maximize the lifetime of each AFN, which is the main focus of this paper.

The third component in the two-tier architecture is the base-station. The base-station is, essentially, the *sink* node for all the AFNs in the network. A base-station may be assumed to have a sufficient battery resource provision, or its battery may be re-provisioned during its course of operation. Therefore, its power dissipation is not a concern in our investigation.

In summary, the main functions of the lower tier MSNs are data acquisition and compression while the upper-tier AFNs are used for data fusion and relaying the information to the base-station. The routing topology can be controlled by the power level of a node's transmitter [13,22,25,28], which in turn controls the distance coverage of an AFN. Consequently, by adjusting the power level of an AFN's transmitter, we can form different network routing topologies.

2.2. Power dissipation

For the ease of exposition, we assume that the rate of data stream generated at each AFN (after data aggregation) is at a constant bit rate. For an AFN, the power consumption by data communication (i.e., receiving and transmitting) is the dominant factor [1]. The power dissipation at the transmitter can be modeled as:

$$p_t(i,\ k) = c_{ik} \cdot r_{ik}, \qquad (1)$$

[1] We assume that each MSN can only forward information to its local AFN for processing (e.g., video fusion).

where $p_t(i,\ k)$ is the power dissipated at node i when it is transmitting to node k, r_{ik} is the bit rate transmitted from node i to node k, and c_{ik} is the power consumption cost of radio link (i, k) and is given by

$$c_{ik} = \alpha_{t1} + \alpha_{t2} \cdot d_{ik}^{m}\ , \qquad (2)$$

where α_{t1} is a *distance-independent* constant term, α_{t2} is a coefficient term associated with the *distance-dependent* term, d_{ik} is the distance between these two nodes, and m is the path loss index, with $2 \le m \le 4$ [24]. Typical values for these parameters are $\alpha_{t1} = 50$ nJ/b and $\alpha_{t2} = 0.0013$ pJ/b/m^4 (for $m = 4$) [14].[2] The power dissipation at a receiver can be modeled as [24]:

$$p_r(i) = \alpha_r r_i, \qquad (3)$$

where r_i (in b/s) is the rate of the received data stream. A typical value for the parameter α_r is 50 nJ/b [14].

2.3. The lexicographic max-min node lifetime problem

For a network having N AFNs, suppose that AFN i generates data stream at a rate g_i, and that the initial energy at this node is given by $e_i (1 \le i \le N)$. Then, it is straightforward to use a linear programming (LP) approach to find an optimal flow routing schedule such that the time until any AFN runs out of energy is maximized [8,9]. That is, we can solve:

$$\begin{aligned}
\text{Max} \quad & T \\
\text{s.t.} \quad & f_{iB} + \sum_{k \ne i} f_{ik} - \sum_{m \ne i} f_{mi} = g_i \quad (1 \le i \le N) \qquad (4)\\
& \sum_{m \ne i} \alpha_r f_{mi} T + \sum_{k \ne i} c_{ik} f_{ik} T + c_{iB} f_{iB} T \le e_i \quad (1 \le i \le N)\\
& T,\ f_{ik},\ f_{iB} \ge 0 \quad (1 \le i \ne k \le N) \qquad (5)
\end{aligned}$$

where f_{ik} and f_{iB} are data rates transmitted from AFN i to AFN k and from AFN i to the base-station B, respectively. Equations (4) state that, the total data transmitted from AFN i is equal to the total data received from other AFNs, plus the data generated locally by AFN i. Equations (5) state that the energy required to receive and transmit all these data cannot exceed its initial energy.

Note that since f_{mi}, f_{ik}, f_{iB}, and T are all variables, the above optimization problem is not linear (due to the product terms $f_{mi}\,T$, etc.). To transform it into an LP, we denote $V_{ik} = f_{ik} T$ and $V_{iB} = f_{iB} T$. Then, we may equivalently solve:

$$\begin{aligned}
\text{Max} \quad & T \\
\text{s.t.} \quad & \\
& V_{iB} + \sum_{k \ne i} V_{ik} - \sum_{m \ne i} V_{mi} - g_i T = 0 \quad (1 \le i \le N) \qquad (6)\\
& \sum_{m \ne i} \alpha_r V_{mi} + \sum_{k \ne i} c_{ik} V_{ik} + c_{iB} V_{iB} \le e_i \quad (1 \le i \le N) \qquad (7)\\
& T,\ V_{ik},\ V_{iB} \ge 0 \quad (1 \le i \ne k \le N) \qquad (8)
\end{aligned}$$

[2] In this paper, we use $m = 4$ in all of our numerical results.

where equations (6) follows by multiplying equations (4) by T. We now have an LP problem and the optimal solution for T represents the maximum time until the first node fails.

Although it is important to maximize the time until first AFN runs out of energy (also know as the network lifetime in [4,8]), it is even more important to concurrently maximize also the time that the second, third, and all subsequent AFNs run of energy. That is, it is important to find a flow routing schedule among the AFNs such that the lifetimes of all AFNs in the network can achieve the optimal *Lexicographic Max-Min* (LMM) vector. A formal definition for the LMM-optimal node lifetime vector is hereby given as follows. Numerical examples for the LMM-optimal node lifetime vector can be found in Section 6.

Definition 1. A sorted network node lifetime vector $[\tau_1, \tau_2, \ldots, \tau_N]$ with $\tau_1 \le \tau_2 \le \cdots \le \tau_N$ is LMM-optimal if and only if for any other sorted node lifetime vector $[\hat{\tau}_1, \hat{\tau}_2, \ldots, \hat{\tau}_N]$ with $\hat{\tau}_1 \le \hat{\tau}_2 \le \cdots \le \hat{\tau}_N$, there exists a k, $1 \le k \le N$, such that for $i = 1, 2, \ldots, k-1$, $\tau_i = \hat{\tau}_i$ but $\tau_k > \hat{\tau}_k$.

A naive approach to the LMM node lifetime problem would be to apply a *max-min* like iterative procedure to find the sequence of node lifetime for all AFNs in the network by considering the energy of each AFN as the bottleneck resource. Under this approach, an iterative LP for alive nodes in the form of (6) to (8) could be employed to find the maximum time until the next node fails. By calculating the remaining energy at each node at the end of the iteration, we can move on to the next iteration, until all the nodes drain their energy. Although this approach seems appealing and intuitive, we now show that it usually gives an incorrect solution.

We first must realize that there is a fundamental difference between the LMM node lifetime problem here and the classical max-min rate allocation problem described in [3,15]. That is, the LMM node lifetime problem implicitly embeds (or couples) a flow routing problem within the LMM node lifetime problem, while under the classical max-min rate allocation, there is no routing problem involved since the routes for all flows are fixed.

Due to this coupling of flow routing and LMM node lifetime optimization, we find that *any iterative LMM node lifetime algorithm requiring energy reservation among the nodes during each iteration is incorrect*. This is because, unlike max-min (which addresses only the rate allocation problem under fixed routes), starting from the first iteration, there usually exist *non-unique* flow routing solutions corresponding to the same drop point. Consequently, each of these flow routing schedules, once chosen, will yield *different* remaining energy at the AFNs for future iterations and so forth, leading to a different node lifetime vector, which may not be the same as the LMM-optimal node lifetime vector. Numerical results demonstrating the incorrectness of this naive approach (which we call *Serial LP with Energy Reservation* (SLP-ER)) will be given in Section 6.

Recently, Brown et al. [7] studied the LMM node lifetime problem under the notion of a "node lifetime curve". In their approach, they first identified the uniqueness of the LMM-optimal node lifetime vector. Based on this property, they developed an iterative procedure to solve the LMM node lifetime problem. In particular, they developed a revised simplex method to calculate the maximum node lifetime curve (equivalent to the LMM-optimal node lifetime vector). A key step in their procedure is to use multiple independent LPs to maximize the sum of slack variables in order to determine the minimum node set at each drop point. During each iteration, only the drop point and the corresponding minimum set of nodes are determined, and there is no resource reservation process among the nodes. Although their proposed approach solves the LMM node lifetime problem, there still remain significant issues to be addressed. Among others, the problem formulation proposed in [7] is shown to be of *exponential* computational complexity, which could become problematic when the scale of the network becomes large.

3. An efficient serial LP algorithm based on parametric analysis

In this section, we present an efficient algorithm for the LMM node lifetime problem. Unlike the Serial LP with Slack Variable analysis (SLP-SV) approach in [7], our approach results in a polynomial running time. Moreover, for *any* given network configuration and initial condition, our approach is much simpler than the Slack Variable based (SV-based) approach in [7]. The computational effectiveness of our approach hinges upon two important techniques. First, we employ a link-based problem formulation that significantly reduces the problem size in comparison with a flow-based formulation adopted in [7]. Second, we invoke a *parametric analysis* procedure at each stage to determine the minimum node set at each drop point. For non-degenerate problems, this parametric analysis results in only a linear time computational complexity per node, while the SV-based approach in [7] requires the solution of multiple independent LPs to determine the minimum set of nodes at each drop point. Even for the rare case when the problem is degenerate, using our parametric analysis technique is still more efficient than the SV-based approach because it decreases the number of additional LPs that needs to be solved at each drop point. In the remainder of this section, we elaborate on the details of our Serial LP algorithm based on Parametric Analysis (SLP-PA). Table 1 shows the notation used in this paper.

3.1. Link-based formulation

Suppose that $[\tau_1, \tau_2, \ldots, \tau_N]$ with $\tau_1 \le \tau_2 \le \cdots \le \tau_N$ is LMM-optimal. If $\tau_k = \tau_{k+1}$, then the respective nodes corresponding to τ_k and τ_{k+1} have the same node lifetime, i.e., the corresponding nodes for τ_k and τ_{k+1} use up their energy at the same time. To keep a track of *distinct* node lifetimes, we remove all repetitive elements in the vector and rewrite it as $[a_1, a_2, \ldots, a_n]$ such that $a_1 < a_2 < \cdots < a_n$, where

Table 1
Notation.

Symbol	Definition
N	The number of AFNs in the network
e_i	The initial energy at AFN i
g_i	The data rate generated at AFN i
α_r	Power consumption coefficient for receiving data
c_{ik} (or c_{iB})	Power consumption coefficient for transmitting data from AFN i to AFN k (or the base-station B)
T_i	AFN i's lifetime under the LMM-optimal node lifetime vector
τ_i	The i-th node lifetime in the sorted LMM-optimal node lifetime vector, i.e. $\tau_1 \le \tau_2 \le \cdots \le \tau_N$
n	The number of distinct lifetimes in the sorted LMM-optimal node lifetime vector
a_i	The i-th distinct lifetime in the sorted LMM-optimal node lifetime vector, i.e. $a_1(=\tau_1) < a_2 < \cdots < a_n(=\tau_N)$
δ_i	$= a_i - a_{i-1}$
S_i	The minimum set of nodes that uses up energy at time a_i
$\hat{S}_i$	The set of all possible AFNs which may use up energy at time a_i, $S_i \subseteq \hat{S}_i$
V_{ik}(or V_{iB})	The total bit volume from AFN i to AFN k (or the base-station B)
f_{ik} (or f_{iB})	The flow rate from AFN i to AFN k (or the base-station B)
x	The optimal solution for LP-LMM
w	The optimal solution for the dual problem of LP-LMM
b	Right-hand-side (RHS) of LP-LMM
I_i	A vector having a single 1 corresponding to the index i of Eq. (9) and 0 elsewhere
$\mathcal{B}$	The columns corresponding to the basic variables of LP-LMM
$\mathcal{Z}$	The columns corresponding to the non-basic variables of LP-LMM
$c_{\mathcal{B}}$	The parameters in the objective function corresponding to the basic variables of LP-LMM
$c_{\mathcal{Z}}$	The parameters in the objective function corresponding to the non-basic variables of LP-LMM
$x_{\mathcal{B}}$	Part of optimal solution corresponding to the basic variables of LP-LMM
$x_{\mathcal{Z}}$	Part of optimal solution corresponding to the non-basic variables of LP-LMM

$a_1 = \tau_1$, $a_n = \tau_N$, and $n \le N$. Corresponding to these drop points, denote $S_1, S_2, \ldots, S_n$ as the sets of nodes that drain their energy at the drop points $a_1, a_2, \ldots, a_n$, respectively. Clearly, $|S_1| + |S_2| + \ldots + |S_n| = |S| = N$ where S denotes the set of all N AFNs in the network. The problem is to find the LMM-optimal values of $a_1, a_2, \ldots, a_n$ and the corresponding sets $S_1, S_2, \ldots, S_n$.

To formulate this problem into an iterative form, we define $a_0 = 0$ and $S_0 = \emptyset$. Furthermore, denote $\delta_l = a_l - a_{l-1}$. Then, the iterative optimization problem (starting with $l = 1$) for the LMM node lifetime problem becomes,

LP-LMM: Max δ_l
s.t.

$$V_{iB} + \sum_{k \ne i} V_{ik} - \sum_{m \ne i} V_{mi} - \delta_l g_i = a_{l-1} g_i, \quad \left(i \notin \bigcup_{j=0}^{l-1} S_j \right) \tag{9}$$

$$V_{iB} + \sum_{k \ne i} V_{ik} - \sum_{m \ne i} V_{mi} = a_h g_i, \quad (i \in S_h, h < l) \tag{10}$$

$$\sum_{m \ne i} \alpha_r V_{mi} + \sum_{k \ne i} c_{ik} V_{ik} + c_{iB} V_{iB} \le e_i, \quad \left(i \notin \bigcup_{j=0}^{l-1} S_j \right) \tag{11}$$

$$\sum_{m \ne i} \alpha_r V_{mi} + \sum_{k \ne i} c_{ik} V_{ik} + c_{iB} V_{iB} = e_i, \quad (i \in S_h, h < l)$$

$$V_{ik}, V_{iB}, \delta_l \ge 0, \quad (1 \le i \ne k \le N). \tag{12}$$

The set of constraints in (9) state that the total in-coming and local data bit volumes are equal to the total out-going data bit volumes for each node that still has remaining energy at time a_{l-1}. The set of constraints in (10) say that the total in-coming and local data volumes are equal to the out-going data bit volume for each node that no longer has any remaining energy at time a_{l-1}. The set of constrains in (11) state that the total energy consumed for receiving and transmitting data bit volumes is no more than the initial energy for each node that has remaining energy at time a_{l-1}. The set of constraints in (12) say that the total energy consumed for receiving and transmitting data is equal to the initial energy for each node that no longer has any remaining energy at time a_{l-1}.

The above LP formulation can be rewritten in the form: **Max** cx, **s.t.** $Ax = b$ and $x \ge 0$, the dual problem for which is given by: **Min** wb, **s.t.** $wA \ge c$ and w unrestricted [2]. Both can be solved simultaneously by standard LP techniques (e.g., [2]) in polynomial-time. Although solving LP-LMM gives the optimal value for δ_l, we need yet to determine the *minimum* set of nodes corresponding to this δ_l, which is the main task in this investigation. In the remainder of this section, we exploit post-LP parametric analysis techniques [2] to determine the minimum node set for each drop point.

3.2. Minimum node set determination with parametric analysis

Denote $\hat{S}_l \neq \emptyset$ to be the set of nodes for which the constraints (11) are binding at optimality for LP-LMM, i.e., the set of nodes that achieve *equality* in (11). Although at least one of the nodes in $\hat{S}_l$ must belong to S_l (the minimum node set at a_l), some of the nodes in $\hat{S}_l$ may still be further "stretched" to live longer under alternative flow routing schedules. In the special case, if $|\hat{S}_l| = 1$, then $S_l = \hat{S}_l$; otherwise, we need to determine the minimum set of $S_l (S_l \subseteq \hat{S}_l)$ that achieves the LMM-optimal solution.

We find the so-called *parametric analysis* (PA) technique [2] is most effective in addressing this type of problems. The main idea of parameter analysis is to find how a small perturbation of some component in the LP-LMM will affect the solution. In particular, consider a small increase in the right-hand-side (RHS) of (9), i.e., changing b_i to $b_i + \epsilon_i$, where $\epsilon_i > 0$. Then this node i belongs to S_l *if and only if* $\frac{\partial^+ \delta_l}{\partial \epsilon_i}(0) < 0$, i.e., a small increase in node i's lifetime (in terms of total bit volume generated at node i) leads to a decrease in the next drop point.

To compare $\frac{\partial^+ \delta_l}{\partial \epsilon_i}(0)$ with 0, we resort to an important duality relationship in LP theory. If x and w are the respective optimal solutions to the primal and dual problems, then based on the parametric duality property [2], we have

$$\frac{\partial^+ \delta_l}{\partial \epsilon_i}(0) = \frac{\partial^+ (cx)}{\partial b_i}(b_i) \leq w_i \ . \tag{13}$$

Note that by the nature of the problem, we have $w_i \leq 0$ for an optimal dual solution. Recall that these w_i can be easily obtained at the same time when we solve the primal LP problem. Therefore, if $w_i < 0$, then we can determine immediately that $i \in S_l$. On the other hand, if we find that $w_i = 0$, it is not clear whether $\frac{\partial^+ \delta_l}{\partial \epsilon_i}(0)$ is strictly negative or 0 and further analysis is thus needed.

For each node i with $w_i = 0$, we must perform a complete PA to see whether this RHS can be further increased without changing the objective value of LP-LMM. If there is no change, then we can determine that node $i \notin S_l$; otherwise, $i \in S_l$.

Assume that the optimal solution is $(x_{\mathcal{B}}, x_{\mathcal{Z}})$, where $x_{\mathcal{B}}$ and $x_{\mathcal{Z}}$ denote the set of basic and non-basic variables; $\mathcal{B}$ and $\mathcal{Z}$ denote the columns corresponding to the basic and non-basic variables; $c_{\mathcal{B}}$ and $c_{\mathcal{Z}}$ denote the objective function coefficient vectors for the basic and non-basic variables; and q denotes the objective value. Then the corresponding canonical equations yield [2]

$$q + \left(c^t_{\mathcal{B}} \mathcal{B}^{-1} \mathcal{Z} - c^t_{\mathcal{Z}}\right) x_{\mathcal{Z}} = c^t_{\mathcal{B}} \mathcal{B}^{-1} b,$$
$$x_{\mathcal{B}} + \mathcal{B}^{-1} \mathcal{Z} x_{\mathcal{Z}} = \mathcal{B}^{-1} b.$$

If b is replaced by $b + \epsilon_i I_i$, where the column vector I_i has a single 1 corresponding to node i in the set of constraints (9) and has 0 elements otherwise, then the only change in the constraints due to this perturbation is that $\mathcal{B}^{-1} b$ will be replaced by $\mathcal{B}^{-1}(b + \epsilon_i I_i)$. Consequently, the objective value for the current basis becomes $c^t_{\mathcal{B}} \mathcal{B}^{-1}(b + \epsilon_i I_i)$. Furthermore, as long as $\mathcal{B}^{-1}(b + \epsilon_i I_i)$ is nonnegative, the current basis remains optimal. Denote $\bar{b} = \mathcal{B}^{-1} b$ and $\mathcal{B}^{-1}_i = \mathcal{B}^{-1} I_i$ and let $\hat{\epsilon}_i$ be an upper bound for ϵ_i such that the current basis remains optimal, we have

$$\hat{\epsilon}_i = \min_j \left\{ \frac{\bar{b}_j}{-\mathcal{B}^{-1}_{ij}} : \mathcal{B}^{-1}_{ij} < 0 \right\} \tag{14}$$

If $\hat{\epsilon}_i > 0$, the optimal objective value varies according to $c^t_{\mathcal{B}} \mathcal{B}^{-1}(b + \epsilon_i I_i)$ for $0 < \epsilon_i \leq \hat{\epsilon}_i$. Since $w = c^t_{\mathcal{B}} \mathcal{B}^{-1}$ and $w_i = 0$, we have $c^t_{\mathcal{B}} \mathcal{B}^{-1} I_i = w_i = 0$. Thus, the objective value will *not* change for $\epsilon_i \in (0, \hat{\epsilon}_i]$, and consequently, the lifetime for node i can be "stretched" to last longer beyond current drop point a_l. That is, node i does not belong to the minimum node set S_l.

For most practical problems, this directly yields whether $i \in S_l$ or $i \notin S_l$ (for all $i \in \hat{S}_l$). But in the rare event where $\hat{\epsilon}_i = 0$, the problem is degenerate. To develop a polynomial-time algorithm, denote W_l as the set of all nodes with $w_i < 0$ and U_l the set of all nodes with $w_i = 0$ and $\hat{\epsilon}_i = 0$. Then we solve the following LP to maximize the slack variables (SV) for nodes in U_l.

MSV: Max $\sum_{i \in U_l} \epsilon_i$
s.t.

$$V_{iB} + \sum_{k \neq i} V_{ik} - \sum_{m \neq i} V_{mi} - \epsilon_i g_i = a_l g_i, \quad (i \in U_l)$$
$$V_{iB} + \sum_{k \neq i} V_{ik} - \sum_{m \neq i} V_{mi} = a_h g_i, \ (i \in S_h, 1 \leq h < l)$$
$$V_{iB} + \sum_{k \neq i} V_{ik} - \sum_{m \neq i} V_{mi} = a_l g_i, \quad \left(i \notin U_l \bigcup_{h=1}^{l-1} S_h\right)$$
$$\sum_{m \neq i} \alpha_r V_{mi} + \sum_{k \neq i} c_{ik} V_{ik} + c_{iB} V_{iB} = e_i, \quad \left(i \in U_l \bigcup W_l \bigcup_{h=1}^{l-1} S_h\right)$$
$$\sum_{m \neq i} \alpha_r V_{mi} + \sum_{k \neq i} c_{ik} V_{ik} + c_{iB} V_{iB} \leq e_i, \quad \left(i \notin U_l \bigcup W_l \bigcup_{h=1}^{l-1} S_h\right)$$
$$V_{ik}, V_{iB}, \epsilon_i \geq 0 \ , (1 \leq i \neq k \leq N).$$

If the optimal objective value is 0, then no node in U_l can have a positive ϵ_i, i.e., these nodes should all belong to S_l. That is, these nodes should all belong to S_l and we have $S_l = W_l + U_l$. On the other hand, if the optimal objective value is positive, then some nodes in U_l must have positive ϵ_i, i.e., these nodes should not belong to S_l. Consequently, we remove these nodes from U_l and if $U_l \neq \emptyset$, we solve another MSV. This procedure will terminate when the optimal objective value is 0 or $U_l = \emptyset$.

In a nutshell, the complete PA procedure for the determination of whether or not a node $i \, (\in \hat{S}_l)$ belongs to the minimum node set S_l can be summarized as follows.

Algorithm 1 (Minimum node set determination with PA)

1. Initialize $W_l = \emptyset$ *and* $U_l = \emptyset$.
2. For each node $i \in \hat{S}_l$,
   a) if $w_i < 0$, add node i to W_l;
   b) otherwise, using $\mathcal{B}^{-1}$ (which is readily available after solving an LP-LMM), compute $\bar{b} = \mathcal{B}^{-1}b$, $\mathcal{B}_i^{-1} = \mathcal{B}^{-1}I_i$, and $\hat{\epsilon}_i$ according to (14). If $\hat{\epsilon}_i = 0$, add i to U_l
3. If $U_l = \emptyset$, let $S_l = W_l$ and stop, else build and solve an MSV.
4. If the optimal objective value is 0, let $S_l = W_l + U_l$ and stop. Otherwise, remove all nodes with $\epsilon_i > 0$ from U_l and go to Step 3.

The following lemma establishes an important property for the minimum node set obtained at each drop point. The proof is given in the appendix.

Lemma 1. The set of physical nodes corresponding to the minimum node set at each drop point under the *LMM-optimal solution* is unique.

4. LMM-optimal flow routing schedule

4.1. Non-uniqueness of flow routing schedule

The solution to the LMM node lifetime problem would not be complete without a corresponding flow routing schedule. The first question to ask is whether such a flow routing schedule is unique. We show that, although the set of physical nodes corresponding to the minimum node set at each stage is unique (Lemma 1), the flow routing schedule is *non-unique*. This is because upon the completion of the last LP-LMM in SLP-PA, there usually exist non-unique bit volume solutions (V_{ik} and V_{iB} values along each radio link), all of which can achieve the same unique objective values (δ_i values). As the bit volumes (V_{ik} and V_{iB} values) along each radio link are non-unique, the corresponding flow routing schedule is, as a result, also non-unique. This observation is formally stated in the following lemma.

Lemma 2. The flow routing schedule corresponding to the LMM-optimal node lifetime vector can be non-unique.

Incidentally, this result corrects an error in [7] (Lemma 3.2), which incorrectly states that such a flow routing schedule is unique.

4.2. An optimal flow routing schedule

Given that the optimal flow routing schedule is non-unique, there are potentially many possible flow routing schedules that achieve the LMM-optimal node lifetime vector. In this section, we present a simple polynomial-time algorithm that provides an LMM-optimal flow routing schedule.

The main task in this algorithm is to define flows from the bit volumes (V_{ik} and V_{iB} values), which are obtained upon the completion of the last LP-LMM in our SLP-PA approach. Note that the bit volumes obtained here represent the *total* amount of bit volume being transported between the nodes during $[0,\ a_n]$, where $a_n = \tau_N$ is the time that the last set of nodes drain their energy. The main result here is that if we let the total amount of out-going flow at a node be distributed *proportionally to the bit volumes* on each outgoing link for all the remaining alive nodes at each stage, then we can achieve the drop points $a_1,\ a_2,\ \ldots, a_n$ as well as the corresponding minimum node sets $S_1,\ S_2,\ \ldots,\ S_n$. The algorithm is formally described as follows.

Algorithm 2 (An optimal flow routing schedule). Upon the completion of the SLP-PA algorithm for the LMM node lifetime vector, we have the drop points (in strictly increasing order) $a_1, a_2, \ldots, a_n$, the corresponding minimum physical node sets $S_1, S_2, \ldots,\ S_n$, and the total amount of bit volume on each radio link (i.e., V_{ik} and V_{iB}). The following algorithm gives an LMM-optimal flow routing schedule for the corresponding time interval $(a_{l-1}, a_l]$, where $a_0 = 0$ and $l = 1,\ 2, \ldots,\ n$.

1. Denote $U_l = S - \bigcup_{j=0}^{l-1} S_j$, with $S_0 = \emptyset$. Initialize all flows to zero, i.e., $f_{ik}^{(l)} = 0$, $f_{iB}^{(l)} = 0$ for $1 \le i \ne k \le N$.
2. If $U_l = \emptyset$, then stop, else choose a node i from U_l such that[3]:
   - either node i does not receive data from any other node, or
   - all nodes from which node i receives data are not in U_l
3. The flow routing at node i during $(a_{l-1}, a_l]$ is then defined as
$$f_{ik}^{(l)} = \frac{V_{ik}}{V_{iB} + \sum_{k \ne i} V_{ik}} \left(\sum_{m \ne i} f_{mi}^{(l)} + g_i \right) \qquad (\forall k \ne i),$$
$$f_{iB}^{(l)} = \frac{V_{iB}}{V_{iB} + \sum_{k \ne i} V_{ik}} \left(\sum_{m \ne i} f_{mi}^{(l)} + g_i \right),$$
   where the $f_{mi}^{(l)}$ values, if not zero, have all been defined before calculating the flow routing for node i.
4. Let $U_l = U_l - \{i\}$ and go to Step 2.

As shown in this algorithm, for each time interval $(a_{l-1}, a_l]$, $l = 1,\ 2, \ldots, n$, we initialize U_l as the set of remaining alive nodes at this stage, which is represented by $U_l = S - \cup_{j=0}^{l-1} S_j$. For these nodes, we compute a flow routing by starting with the "boundary" nodes and then move to the "interior" nodes. More precisely, we will calculate the flow routing for a node i

[3] Such an i must exist when $U_l \ne \emptyset$. For an LMM-optimal solution, there is no cycle, i.e., we do not have $V_{i_1,i_2}, \ldots, V_{i_{k-1},i_k}, V_{i_k,i_1} > 0$. Otherwise, by reducing these volumes a little further, we can increase the corresponding nodes' lifetimes.

if and only if we have calculated the flow routing for each node m that has traffic coming into node i. The out-going flow at node i is calculated by distributing the aggregated in-coming flow *proportionally* according to the overall bit volume along its out-going radio links. As an example, suppose that during $(a_4, a_5]$, node 2 receives an aggregated flow of rate 2 kb/s and generates 0.4 kb/s amount of data locally. Assume that $V_{24} = 100$ kb, $V_{25} = 200$ kb, and $V_{2B} = 300$ kb over $[0, a_n]$. Then the out-going flow at node 2 is routed as follows: $f_{24}^{(5)} = 0.4$ kb/s, $f_{25}^{(5)} = 0.8$ kb/s, and $f_{2B}^{(5)} = 1.2$ kb/s.

We now give a formal proof that the flow routing schedule defined by Algorithm 2 will indeed give the LMM-optimal node lifetime vector.

Proof: For $t \in (a_{l-1}, a_l]$, denote $f_{ik}(t) = f_{ik}^{(l)}$ and $f_{iB}(t) = f_{iB}^{(l)}$, $l = 1, 2, \ldots, n$; $G_i(t) = g_i$ for $t \le T_i$ and $G_i(t) = 0$ for $t > T_i$. To show that the flow routing schedule defined in Algorithm 2 indeed gives the LMM-optimal node lifetime vector, it is sufficient to show that each physical node i ($i = 1, 2, \ldots, N$) has lifetime $\mathrm{T_i}$ under this flow routing schedule, i.e.,

$$\int_{t=0}^{T_i} \left[\sum_{m \neq i} \alpha_r f_{mi}(t) + \sum_{k \neq i} c_{ik} f_{ik}(t) + c_{iB} f_{iB}(t) \right] dt = e_i. \tag{15}$$

To show that (15) is true, it is sufficient to show that, $\int_{t=0}^{T_i} f_{mi}(t)dt = V_{mi}$, $\int_{t=0}^{T_i} f_{ik}(t)dt = V_{ik}$, and $\int_{t=0}^{T_i} f_{iB}(t)dt = V_{iB}$ hold. This is equivalent to showing that

$$\int_{t=0}^{T} f_{mi}(t)dt = V_{mi}\ , \tag{16}$$

$$\int_{t=0}^{T} f_{ik}(t)dt = V_{ik}\ , \tag{17}$$

$$\int_{t=0}^{T} f_{iB}(t)dt = V_{iB}\ , \tag{18}$$

and

$$f_{mi}(t) = 0 \quad \text{for } t > T_i, \tag{19}$$

$$f_{ik}(t) = 0 \quad \text{for } t > T_i, \tag{20}$$

$$f_{iB}(t) = 0 \quad \text{for } t > T_i. \tag{21}$$

To show that (16) holds, it is sufficient to show that (17) holds for $1 \le i \neq k \le N$. We now show that this is true.

1. Suppose that node i is a "boundary" node that does not receive any flow from other nodes. Hence, we have $V_{mi} = 0$, and so, $f_{mi}(t) = 0$. Consequently,

$$\begin{aligned} &\int_{t=0}^{T} f_{ik}(t)dt \\ &= \int_{t=0}^{T} \frac{V_{ik}}{V_{iB} + \sum_{k \neq i} V_{ik}} \cdot G_i(t)dt \\ &= \frac{V_{ik}}{V_{iB} + \sum_{k \neq i} V_{ik}} \cdot g_i T_i \\ &= \frac{V_{ik}}{V_{iB} + \sum_{k \neq i} V_{ik}} \left(V_{iB} + \sum_{k \neq i} V_{ik} \right) = V_{ik}\ . \end{aligned}$$

The second equality holds since $G_i(t) = g_i$ for $t \in [0,\ T_i]$ and $G_i(t) = 0$ otherwise. The third equality follows since the bit volumes V_{ik} and V_{iB} must meet the volume balance property at node i.

2. Now, let us suppose that node i is not a "boundary" node and thus will receive flow from some nodes m. Based on the selection of node i, node m's out-going flows have already been defined. Moreover, it is supposed that node m has already met the criteria in (16), particularly, $\int_{t=0}^{T} f_{mi}(t)dt = V_{mi}$. Based on the definition for flow routing in Algorithm 2, we have

$$\begin{aligned} &\int_{t=0}^{T} f_{ik}(t)dt \\ &= \int_{t=0}^{T} \frac{V_{ik}}{V_{iB} + \sum_{k \neq i} V_{ik}} \left(\sum_{m \neq i} f_{mi}(t) + G_i(t) \right) dt \\ &= \frac{V_{ik}}{V_{iB} + \sum_{k \neq i} V_{ik}} \left(\sum_{m \neq i} V_{mi} + g_i T_i \right) \\ &= \frac{V_{ik}}{V_{iB} + \sum_{k \neq i} V_{ik}} \left(V_{iB} + \sum_{k \neq i} V_{ik} \right) = V_{ik}. \end{aligned}$$

The second equality holds since $\int_{t=0}^{T} f_{mi}(t)dt = V_{mi}$ (which we have proved) and $\int_{t=0}^{T} G_i(t)dt = g_i T_i$. The third equation holds since the bit volumes V_{ik} and V_{iB} must meet the volume balance property at node i.

Combining (i) and (ii), we have proved that (17) holds for $1 \le i \neq k \le N$. Following the same argument, we can prove that (18) also holds for $1 \le i \le N$.

Next, we prove (19), (20) and (21) also hold. For $t > T_i$, suppose that $t \in (a_{j-1}, a_j]$. Then we have $a_{j-1} \ge T_i$. Since under Algorithm 2, positive flow routing for $f_{ik}^{(l)}$ and $f_{iB}^{(l)}$ are only defined for $T_i > a_{l-1}$, we have $f_{ik}^{(j)} = 0$ and for $t > T_i$, i.e., (20) and (21) both hold. Now we show that (19) also holds. If $V_{mi} = 0$, then $f_{mi}(t) = 0$ for $t > T_i$ holds trivially. If $V_{mi} > 0$, we must have $T_i \ge T_m$ under the LMM-optimal solution. Otherwise (i.e., $T_i < T_m$), we can further increase node i's lifetime by decreasing V_{mi} while increasing V_{mB}, but this contradicts the assumption that T_i is the optimal node lifetime for node i under the LMM-optimal solution. Since we have $a_{j-1} \geq T_i$, then $a_{j-1} \geq T_m$. Consequently, $f_{mi}(t) = 0$ for $t > T_i$ by the flow routing construction in Algorithm 2.

An important property for Algorithm 2 is that the flow routing schedule during each interval $(a_{l-1}, a_l]$, $l = 1, 2, \ldots, n$, can be calculated *independently* by referencing the LMM-optimal solution. Consequently, this property enables a *parallel* computation of the flow routing schedule, i.e., for computing the flow routing for all n intervals at the same time.

5. Computational complexity analysis

Complexity of SLP-PA. We now analyze the complexity of our SLP-PA approach to solve the LMM node lifetime problem. First we consider the complexity of finding each node's lifetime and the total bit volume transmitted along each link. At each stage, we solve an LP problem, both its primal and dual have a complexity of $O({n_A}^3 L)$, where n_A is the number of constraints or variables in the problem, whichever is larger, and L is the number of binary bits required to store the data [2]. Since the number of variables is $O(N^2)$ and is larger than the number of constraints, which is $O(N)$, the complexity of solving the LP is $O(N^6 L)$. After solving an LP at each stage, we need to determine whether or not a node that just reached its energy binding constraint belongs to the minimum node set for this stage. Note that w and $\hat{b} = \mathcal{B}^{-1} b$ can be readily obtained when we solve the primal LP problem. To determine whether a node, say i belongs to the minimum node set, we examine w_i. If $w_i < 0$, then node i belongs to the minimum node set and the complexity is $O(1)$. On the other hand, if $w_i = 0$, we need to further examine whether $\hat{\epsilon}_i > 0$ or not. Based (14), the computation for $\hat{\epsilon}_i$ is $O(N)$. So at each stage, the complexity in PA for each node is $O(N)$. The total complexity of PA at each stage for the node set is thus $|\hat{S}_l| \cdot O(N)$ or $O(N \cdot N) = O(N^2)$. Thus, the complexity at each stage is $O(N^6 L) + O(N^2) = O(N^6 L)$. As there are at most N stages, the overall complexity is $O(N^7 L)$.

We now analyze the complexity for the degenerate case. Upon the completion of Step 2 in Algorithm 1, we denote $U_l^{(0)} = U_l$. Since we need to solve at most $|U_l^{(0)} - S_l|$ LPs, the complexity is $|U_l^{(0)} - S_l| \cdot O(N^6 L)$ or $O(N \cdot N^6 L) = O(N^7 L)$. Hence, the complexity at each stage is $O(N^6 L) + O(N^2) + O(N^7 L) = O(N^7 L)$. Since there are at most N stages, the overall complexity is $O(N^8 L)$.

We now analyze the complexity of Algorithm 2 to find the flow routing schedule. At each stage, we need to define the transmission rates for the remaining alive nodes. Since the complexity of defining each node's flow rates is $O(N)$, the complexity of calculating the flow routing schedule at each stage is thus $O(N^2)$. Since there are at most N stages, the overall complexity of Algorithm 2 is $O(N^3)$. Now combining the complexity of both parts in our approach, the overall complexity is $O(N^7 L) + O(N^3) = O(N^7 L)$ for the non-degenerate case and $O(N^8 L) + O(N^3) = O(N^8 L)$ for the degenerate case. Both are polynomial.

Comparison with SLP-SV. We now compare the complexity of our approach with the SLP-SV approach in [7]. First of all, SLP-SV needs to keep track of each *sub-flow* along its route from the source node toward the base-station. Such a flow-based (or more precisely, sub-flow based) approach usually makes the size of the LP coefficient matrix exponential, which leads to an exponential-time algorithm even with the most efficient LP technique (e.g., [2]).[4] Second, even if a link-based LP formulation such as ours is adopted in [7], the computational efficiency of Slack Variable based (SV-based) approach would be still worse than SLP-PA. This is because that at each stage, the SV-based approach in [7] solves several *additional* LPs (up to $|\hat{S}_l - S_l|$) to determine S_l, in contrast with the simpler parametric analysis for the SLP-PA approach, which only involves $O(N^2)$ effort for the non-degenerate case. Even for the degenerate case, the number of additional LPs are up to $|U_l^{(0)} - S_l| (\leq |\hat{S}_l - S_l|)$. Consequently, for any problem, our approach is computationally more efficient than the SLP-SV approach in [7].

Finally, we discuss a hybrid link-flow approach mentioned in [7]. This approach requires a sub-flow accounting on each link and results in an order of magnitude more constraints than the link-based approach proposed in this paper. Although this approach can solve the LMM node lifetime problem in polynomial-time (e.g., using interior point methods [2]), the overall complexity is still orders of magnitude higher than that for our proposed SLP-PA approach. Furthermore, there remains the additional burden associated with the SLP-SV approach for solving the additional LPs even using the hybrid link-flow based approach.

6. Numerical Investigation

In this section, we use numerical results to illustrate the solution of LMM node lifetime problems and compare our SLP-PA to some other approaches. In particular, we will compare SLP-PA with the naive approach (see Section 2.3) that uses a serial LP "blindly" to solve the LMM node lifetime problem. We call this naive approach *Serial LP with energy reservation* (SLP-ER). As discussed in Section 2.3, the naive SLP-ER approach requires energy reservation at each stage and cannot give the correct LMM-optimal solution. We also compare our SLP-PA approach with the *Minimum-Power Routing* (MPR) approach that has been considered in the literature (see, e.g. [12,13,17–19,21,26,27]) and is used to achieve energy efficiency. Under the MPR approach, an AFN always chooses the path that consumes the minimum amount of power toward the base-station. As discussed earlier, although energy-efficient from a per-bit delivery perspective, the MPR approach cannot achieve the LMM-optimal objective.

[4] Incidentally, the revised simplex method proposed in [7] is not as efficient as that in [2] and is itself exponential.

Table 2
Locations (in meters) for each AFN in a *10*-node network.

AFN	Location (x_i, y_i)	AFN	Location (x_i, y_i)
1	(400, −320)	6	(−500, 100)
2	(300, 440)	7	(−400, 0)
3	(−300, −420)	8	(420, 120)
4	(320, −100)	9	(200, 140)
5	(340, −120)	10	(220, −340)

Table 3
Locations (in meters) for each AFN in a 20-node network.

AFN	Location (x_i, y_i)	AFN	Location (x_i, y_i)
1	(200, 130)	11	(110, −230)
2	(−400, −380)	12	(-210, 0)
3	(−100, 420)	13	(210, 320)
4	(0, 430)	14	(300, −480)
5	(−410, 440)	15	(−420, −420)
6	(−200, 230)	16	(120, −240)
7	(400, −490)	17	(220, −440)
8	(410, −300)	18	(−220, 240)
9	(100, 310)	19	(−500, −110)
10	(100, 140)	20	(0, −330)

6.1. Network configurations and parameter settings

We consider two network topologies. The first network consists of 10 AFNs while the second network consists of 20 AFNs. Under each network, the base-station B is located at the origin (0, 0) (in meters). The locations for these 10 and 20 AFNs are generated at random and are shown in Tables 2 and 3, respectively.

6.2. Results

10-AFN network. We assume that the initial energy at each AFN is 50 kJ and local data generated by each AFN is 0.2 kb/s. The power dissipation behaviors for transmission and reception are defined in (1) and (3), respectively.

Table 4 gives each AFN's lifetime (days) under each approach.[5] The "sorted" index column represents the node index, in which the AFNs are sorted by their node lifetimes in nondecreasing order. Clearly, the node lifetime vector under SLP-PA dominates that under the SLP-ER and MPR approaches with respect to the LMM-optimal node lifetime vector definition (see Definition 1). For example, comparing the node lifetime vector under SLP-PA and SLP-ER, we find that $\tau_1^{\text{SLP-PA}} = \tau_1^{\text{SLP-ER}}$, $\tau_2^{\text{SLP-PA}} = \tau_2^{\text{SLP-ER}}$, $\tau_3^{\text{SLP-PA}} = \tau_3^{\text{SLP-ER}}$, and $\tau_4^{\text{SLP-PA}} > \tau_4^{\text{SLP-ER}}$. Similarly, comparing the node lifetime vector under SLP-PA and MPR, we have $\tau_1^{\text{SLP-PA}} > \tau_1^{\text{MPR}}$. In general, τ_1^{MPR} (28.91 days) is the smallest among the three approaches (45.71 days under both SLP-PA and SLP-ER) since minimum power routing does not

[5]The results for the SLP-SV are not shown since they are the same as those under SLP-PA. The difference is in the computational complexity.

Table 4
Node Lifetime performance(in days) under the three approaches for the 10-AFN Network.

Sorted	SLP-PA		SOP-ER		MPR	
index	τ_i	AFN	τ_i	AFN	τ_i	AFN
1	45.71	3	45.71	1	28.91	7
2	45.71	6	45.71	2	46.09	3
3	45.71	7	45.71	3	61.63	6
4	146.08	1	45.71	5	87.75	9
5	146.08	2	45.71	6	92.77	4
6	146.08	4	45.71	7	118.79	5
7	146.08	5	45.71	10	142.96	8
8	146.08	8	303.70	4	150.29	2
9	146.08	9	303.70	8	157.62	10
10	146.08	10	303.70	9	182.55	1

guarantee a good performance with respect to node lifetime performance. Although SLP-ER and SLP-PA have the same node lifetime (45.71 days) at the first stage, SLP-PA gives a smaller AFN set ($|S_1|^{\text{SLP-PA}} = 3$) at this drop point than SLP-ER ($|S_1|^{\text{SLP-ER}} = 7$), which shows that the naive SLP-ER approach cannot offer the correct solution to the LMM node lifetime problem.

Another way to visualize the LMM node lifetime performance in Table 4 is to plot the total number of remaining "alive" AFNs over time, which is given in Figure 2. Viewing Figure 2(a), in the beginning, all 10 AFNs are alive. As time goes on, one AFN under MPR drains its energy (at 28.91 days) and the remaining alive AFNs drop to 9. Under both the SLP-PA and SLP-ER approaches, the first drop point takes place at 45.71 days, during which 3 AFNs drain their energy under SLP-PA while 7 AFNs drain energy under SLP-ER. Among the three approaches, only the SLP-PA provides a node lifetime solution that meets the LMM-optimal definition (see Definition 1).

20-AFN network. For the 20-AFN network (Table 3), we assume that the initial energy at each AFN is 50 kJ and that the local data generated by each AFN is 0.5 kb/s. Table 5 shows the sorted node lifetime performance under the three approaches. Plots for the node lifetime curve are given in figure 2(b). Again, we have similar observations as those for the 10-AFN network.

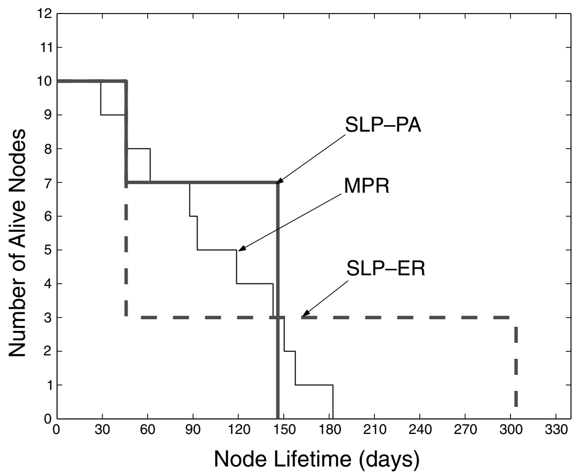
Flow routing schedule. We now show how to use Algorithm 2 to calculate a flow routing schedule that achieves the LMM-optimal node lifetime vector for the 10-AFN network. Under the SLP-PA approach, we have $a_1 = 45.71$ days with $S_1 = \{3, 6, 7\}$ and $a_2 = 146.08$ days with $S_2 = \{1, 2, 4, 5, 8, 9, 10\}$.

Also, we obtain the following bit volumes (all in 10^4 kb) among the nodes from the last LP-LMM solution:

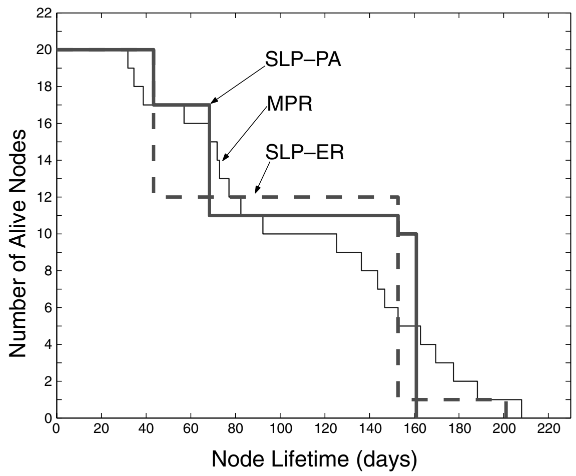
$$V_{1,5} = 320.0419, \quad V_{1,B} = 46.7550;$$

$$V_{2,9} = 233.8006, \quad V_{2,B} = 18.6306;$$

$$V_{3,7} = 48.6548, \quad V_{3,B} = 30.3317;$$



a. 10-AFN network



b. 20-AFN network

Figure 2. Node lifetime curves under the three approaches for the 10-AFN and 20-AFN networks.

Table 5
Lifetime (days) for the 20-AFN network.

Sorted	SLP-PA		SOP-ER		MPR	
index	τ_i	AFN	τ_i	AFN	τ_i	AFN
1	43.35	2	43.35	2	31.85	19
2	43.35	15	43.35	7	34.54	11
3	43.35	19	43.35	8	38.72	2
4	68.32	7	43.35	14	56.99	15
5	68.32	8	43.35	15	67.98	16
6	68.32	11	43.35	16	71.79	8
7	68.32	14	43.35	17	72.88	17
8	68.32	16	43.35	19	77.08	14
9	68.32	17	152.72	1	82.40	7
10	152.72	5	152.72	3	92.27	10
11	160.91	1	152.72	4	125.25	6
12	160.91	3	152.72	5	136.33	1
13	160.91	4	152.72	6	143.59	12
14	160.91	6	152.72	9	146.77	9
15	160.91	9	152.72	10	152.72	5
16	160.91	10	152.72	12	162.77	20
17	160.91	12	152.72	13	169.59	18
18	160.91	13	152.72	18	177.54	13
19	160.91	18	152.72	20	188.26	4
20	160.91	20	201.09	11	208.04	3

$$V_{4,B} = 303.3560;$$
$$V_{5,4} = 50.9249, \quad V_{5,8} = 390.6881, \quad V_{5,B} = 130.8601;$$
$$V_{6,7} = 22.2673, \quad V_{6,B} = 56.7191;$$
$$V_{7,B} = 149.9086;$$
$$V_{8,9} = 576.2578, \quad V_{8,B} = 66.8615;$$
$$V_{9,B} = 1062.4895;$$
$$V_{10,1} = 114.3658, \quad V_{10,B} = 138.0654$$

We now find the flow routing schedule for each interval, i.e., $[0,\ a_1]$ and $(a_1,\ a_2]$, respectively. For time interval $[0, a_1]$, we get the following.

- Nodes 2, 3, 6, and 10 do not receive any data. Using Algorithm 2, node 2 sends 0.185 kb/s to node 9 and 0.015 kb/s to the base-station B. Similarly, node 3 sends 0.123 kb/s to node 7 and 0.077 kb/s to the base-station B; node 6 sends 0.057 kb/s to node 7 and 0.143 kb/s to the base-station B; node 10 sends 0.091 kb/s to node 1 and 0.109 kb/s to the base-station B.
- Now, since the in-coming flow to nodes 1 and 7 are defined, we can calculate their out-going flow rates. Using Algorithm 2, node 1 sends 0.254 kb/s to node 5 and 0.037 kb/s to the base-station B; node 7 sends 0.380 kb/s to the base-station B.
- Next, we consider node 5. After calculation, we find that node 5 should send 0.040 kb/s to node 4, 0.310 kb/s to node 8, and 0.104 kb/s to the base-station B.
- Following this, we consider nodes 4 and 8. We find that node 4 sends 0.240 kb/s to the base-station B; node 8 sends 0.457 kb/s to node 9 and 0.053 kb/s to the base-station B.
- Finally, we consider node 9. Using Algorithm 2, we find that node 9 sends 0.842 kb/s to the base-station B.

In summary, during $[0,\ a_1] = [0, 45.71]$, we have the following flow rates (all in kb/s):

$$f_{1,5} = 0.254, \quad f_{1,B} = 0.037;$$
$$f_{2,9} = 0.185, \quad f_{2,B} = 0.015;$$
$$f_{3,7} = 0.123, \quad f_{3,B} = 0.077;$$
$$f_{4,B} = 0.240;$$
$$f_{5,4} = 0.040, \quad f_{5,8} = 0.310, \quad f_{5,B} = 0.104;$$
$$f_{6,7} = 0.057, \quad f_{6,B} = 0.143;$$
$$f_{7,B} = 0.380;$$
$$f_{8,9} = 0.457, \quad f_{8,B} = 0.053;$$

$$f_{9,B} = 0.842;$$

$$f_{10,1} = 0.091, \quad f_{10,B} = 0.109.$$

Likewise, applying Algorithm 2 during $(a_1, a_2] = (45.71, 146.08]$, we obtain the following flow routing rates (all in kb/s):

$$f_{1,5} = 0.254, \quad f_{1,B} = 0.037;$$

$$f_{2,9} = 0.185, \quad f_{2,B} = 0.015;$$

$$f_{4,B} = 0.240;$$

$$f_{5,4} = 0.040, \quad f_{5,8} = 0.310, \quad f_{5,B} = 0.104;$$

$$f_{8,9} = 0.457, \quad f_{8,B} = 0.053;$$

$$f_{9,B} = 0.842;$$

$$f_{10,1} = 0.091, \quad f_{10,B} = 0.109.$$

It is easy to verify that above flow routing schedule will indeed obtain the LMM-optimal node lifetime vector.

The flow routing schedule that achieves the LMM-optimal node lifetime vector for the 20-AFN network can be obtained in a similar manner (by using Algorithm 2). This is omitted for the sake of brevity.

7. Related work

The closest work related to ours is that in [7], which has been discussed in detail in the paper. In this section, we briefly review relevant work that contributed to the background for our investigation.

There have been many recent efforts in the area of *power-aware routing* (see e.g., [12,13,17–19,21,26,27]). Most schemes under power-aware routing use a shortest path algorithm with a power-based metric, rather than a hop-count based metric. However, as we have shown in the numerical results section, energy-aware (e.g., minimum-power path) routing may not ensure good performance in maximizing network lifetime. For example, using the most energy-efficient route may still result in a premature depletion of energy at certain nodes, which is not optimal from the network lifetime perspective.

The notion of network lifetime for wireless sensor networks has been studied in [4–6,8–10,16,29]. The notion of network lifetime discussed in these work focuses on the time until the first node fails without further consideration of the remaining nodes in the network. As wireless sensor networks will typically remain useful even if some nodes run out of energy, it is essential to further investigate how to maximize the lifetime for all the remaining nodes in the network, which is the focus of this paper.

8. Conclusions

In this paper, we considered the problem of how to maximize the lifetime for all the nodes in a wireless sensor network. We formally defined this optimization problem as the Lexicographic Max-Min (LMM) node lifetime problem and investigated approaches to solve it. The main contributions in this paper are two-fold. First, we developed a polynomial-time algorithm to obtain the LMM-optimal node lifetime vector, which improves upon the computational complexity associated with a state-of-the-art algorithm. Second, we presented a simple (also polynomial-time) algorithm to calculate the flow routing schedule among the AFNs such that the LMM-optimal node lifetime vector can be achieved. The results in this paper help lay the essential algorithmic foundation for studying network lifetime problems in energy-constrained wireless sensor networks.

Appendix: Proof of Lemma 1

By the definition of LMM-optimal node lifetime vector (see Definition 1), the optimal node lifetimes (λ_l values) are unique and the corresponding number of nodes in the minimum node set ($|S_l|$ values) are also unique. To show that the group of *physical* nodes in each S_l is also unique, we employ the parametric simplex approach to determine the minimum node set as follows.

In essence, the parametric simplex approach solely relies on PA technique without resorting the SV approach even when the problem is degenerate. That is, when the problem is degenerate, i.e., for some node $i \in \hat{S}_l$, we have $w_i = 0$ and $\hat{\epsilon}_i = 0$, then the basis can change while the optimal objective value remains unchanged. We can analyze W_i and ϵ_i under the new basis to determine whether or not node i belongs to the minimum node set S_l. If we still have $w_i = 0$ and $\hat{\epsilon}_i = 0$, the basis can change again with the same optimal objective value. To prevent cycling back to a previous basis, we can use a de-cycling rule [2]. Thus, this procedure is guaranteed to terminate within a finite number of steps and we can determine whether or not node i indeed belongs to the minimum node set S_l.

Note that in the above parametric simplex approach, the set of physical nodes corresponding to S_l is *uniquely* determined since the analysis is conducted independently for each node. Therefore, upon the completion of all stages, the group of physical nodes in each minimum node set is unique.

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Y. Thomas Hou obtained his B.E. degree from the City College of New York in 1991, the M.S. degree from Columbia University in 1993, and the Ph.D. degree from Polytechnic University, Brooklyn, New York, in 1998, all in Electrical Engineering. From 1997 to 2002, Dr. Hou was a research scientist and project leader at Fujitsu Laboratories of America, IP Networking Research Department, Sunnyvale, California (Silicon Valley). Since Fall 2002, he has been an Assistant Professor at Virginia Tech, the Bradley Department of Electrical and Computer Engineering, Blacksburg, Virginia. Dr. Hou's research interests are in the algorithmic design and optimization for network systems. His current research focuses on wireless sensor networks and multimedia over wireless ad hoc networks. In recent years, he has worked on scalable architectures, protocols, and implementations for differentiated services Internet; service overlay networking; multimedia streaming over the Internet; and network bandwidth allocation policies and distributed flow control algorithms. He has published extensively in the above areas and is a co-recipient of the 2002 *IEEE International Conference on Network Protocols* (ICNP) Best Paper Award and the 2001 *IEEE Transactions on Circuits and Systems for Video Technology* (CSVT) Best Paper Award. He is a member of ACM and a senior member of IEEE.

E-mail: thou@vt.edu

Yi Shi received his B.S. degree from University of Science and Technology of China, Hefei, China, in 1998, a M.S. degree from Institute of Software, Chinese Academy of Science, Beijing, China, in 2001, and a second M.S. degree from Virginia Tech, Blacksburg, VA, in 2003, all in computer science. He is currently working toward his Ph.D. degree in electrical and computer engineering at Virginia Tech. While in undergraduate, he was a recipient of Meritorious Award in International Mathematical Contest in Modeling and 1997 and 1998, respectively. Yi's current research focuses on algorithms and optimization for wireless sensor networks and wireless ad hoc networks. His work has appeared in highly selective international conferences (e.g., *ACM MobiCom and MobiHoc*).
E-mail: yshi@vt.edu

Hanif D. Sherali is the W. Thomas Rice Endowed Chaired Professor of Engineering in the Industrial and Systems Engineering Department at Virginia Polytechnic Institute and State University. His area of research interest is in discrete and continuous optimization, with applications to location, transportation, and engineering design problems. He has published about 200 papers in Operations Research journals, has co-authored four books in this area, and serves on the editorial board of eight journals. He is a member of the National Academy of Engineering.
E-mail: hanifs@vt.edu