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Is Network Coding Always Good for Cooperative Communications?

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***Abstract*—Network coding (NC) is a promising approach to reduce time-slot overhead for cooperative communications (CC) in a multi-session environment. Most of the existing works take advantage of the benefits of NC in CC but do not fully recognize its potential adverse effect. In this paper, we show that employing NC may not always benefit CC. We substantiate this important finding in the context of analog network coding (ANC) and amplify-and-forward (AF) CC. This paper, for the first time, introduces an important concept of *network coding noise* (NC noise). Specifically, we analyze the signal aggregation at a relay node and signal extraction at a destination node. We then use the analysis to derive a closed-form expression for NC noise at each destination node in a multi-session environment. We show that NC noise can diminish the advantage of NC in CC. Our results formalizes an important concept on using NC in CC.**

I. INTRODUCTION

Spatial diversity, in the form of employing multiple transceiver antennas (i.e., MIMO), has shown to be very effective in increasing network capacity. However, equipping a wireless node with multiple antennas may not always be practical, as the footprint of multiple antennas may not fit on a wireless node (e.g. a handheld wireless device). In order to achieve spatial diversity without requiring multiple transceiver antennas on the same node, the so-called *cooperative communications* (CC) could be employed [13], [18]. Under CC, each node is equipped with only a *single* transceiver and spatial diversity is achieved by exploiting the antennas on other (cooperative) nodes in the network.

A simple form of CC can be best illustrated by a three-node example [13] shown in Fig. 1. In this figure, node s transmits to node d via one-hop, and node r acts as a cooperative relay node. Cooperative transmission from s to d is done on a frame-by-frame basis. Within a frame, there are two time slots. In the first time slot, source node s makes a transmission to destination node d. Due to the broadcast nature of wireless transmissions, transmission by node s is also overheard by relay node r. In the second time slot, node r forwards the data it overheard in the first time slot to node d.

This three-node example shows CC for a single source-destination session. In general, for multiple sessions sharing the same relay node, it will be necessary to divide a time frame into multiple mini-slots. For example, suppose there are n source nodes, n destination nodes, and one relay node. For the n source-destination pairs to take advantage of CC, it is necessary to divide a time frame into $2n$ mini-slots (see Fig. 2), with every two mini-slots assigned to a session. Note that among the $2n$ mini-slots, only n mini-slots are used for transmissions between source and destination nodes, the other n mini-slots are solely used for transmissions between relay and destination nodes to complete CC for the n sessions. Obviously, this is somewhat wasteful in terms of channel bandwidth usage.

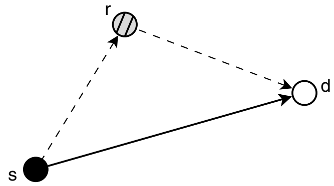

Fig. 1. A three-node schematic for cooperative communications.

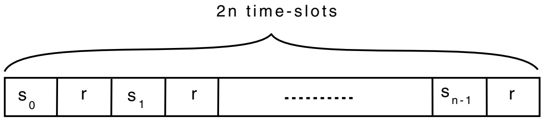

Fig. 2. A frame structure for CC when there are n source-destination pairs and one relay node.

A natural question to ask is the following: Is it possible to retain the benefits of CC while reducing its undesirable overhead (in terms of the required number of mini-slots)? If this is possible, then the benefits of CC can be substantially enhanced.

It turns out that recent advances in *network coding* (NC) [1], [2], [14], [21], [22] may offer a key to this question. Figure 3 shows a time slot structure for CC under NC. Under this scheme, the source node of each session first transmits in its respective time slot. For a given source node, its transmission is received by the corresponding destination node, and overheard by the cooperative relay node and other destination nodes (see Figs. 4(a)-4(c)). After the relay node r overhears all transmissions, it performs a linear combination of all the received signals. Then the relay node amplifies, and broadcasts the combined signal to all the destination nodes in a *single* time-slot (see last slot in Fig. 3 and Fig. 4(d)). Then each destination node extracts its desired signal by subtracting the overheard signals from the combined signal in the last time slot. In the context of n source-destination example discussed,

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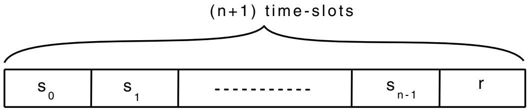


Fig. 3. Time slot structure for CC with NC.

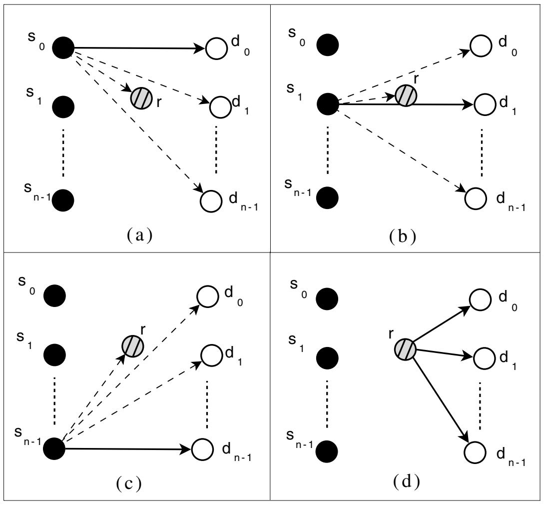


Fig. 4. Sequence of transmissions for n source nodes and one relay node.

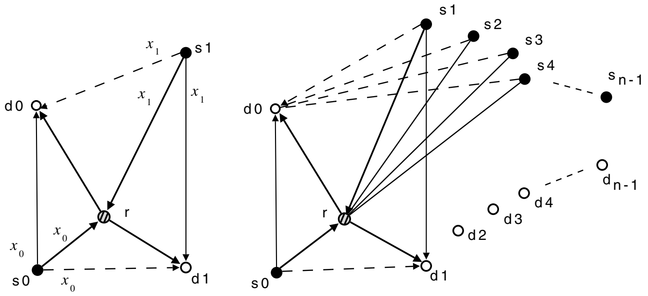
this is a reduction of $(n-1)$ time-slots! Further, due to the potential reduction of the total number of time-slots in a frame, the duration of each time-slot (for transmission) is increased (compare Fig. 2 with Fig. 3).

Ideally, after the relay node transmits the combined signal, we wish to extract the desired signal at each destination node as cleanly as possible. As we shall show in Section II, such an extraction cannot be performed perfectly. Just as one would expect, there is no "free lunch" for employing NC to conserve the number of time-slots in CC. Section III shows that the use of NC at a relay node and the signal extraction process at a destination node will inevitably bring in a non-negligible noise term. We call this noise term, introduced for the first time in this paper, as "network coding noise" (or NC noise). Due to this new NC noise at destination nodes, employing NC to perform CC may not be always beneficial. In order to substantiate this claim, we perform an in-depth analysis of network-coded cooperative communications (NCC) in the context of ANC [10], [16], [24] and AF CC [13] (denoted as A-NCC).

A. Main Contributions of This Paper

The main contributions of this paper are as follows.

- We show that employing NC may not be always beneficial to CC. In the context of A-NCC, we offer revised formulas for mutual information and achievable rate for each session.
- We offer a detailed analysis of signal aggregation at a relay node (via ANC) and the signal extraction process at a destination node. Our analysis shows the presence of NC noise due to the use of ANC. We identify this NC noise as the main adverse effect of NC.



(a) Two-session example. (b) Multiple-session example.

Fig. 5. Examples illustrating NC in CC.

- We use numerical results to demonstrate that the NC noise can outweigh the advantage of ANC. We show that in some cases, A-NCC may perform worse than AF CC and direct transmission schemes.

B. Paper Organization

The remainder of this paper is organized as follows. In Section II, we introduce NC noise. In Section III, we consider A-NCC and offer a theoretical analysis for the signal aggregation at a relay node and signal extraction at a destination node. We also derive an expression for the NC noise. In Section IV, we develop revised formulas for mutual information and achievable rate for each session under A-NCC. Section V presents numerical results and shows that NC may not be always beneficial to CC. Section VI discusses related work and Section VII concludes this paper.

II. THE PROBLEM

We show in this section, that employing NC to perform CC has its side-effects (in the form of NC noise), and may not be always beneficial to individual sessions. To illustrate the significance of NC noise, we consider an example shown in Fig. 5(a), where there are two source-destination pairs (s_0–d_0 and s_1–d_1) and one-relay (r) node. Both source-destination pairs will use the same relay node for AF CC and ANC is employed at the relay node. Based on our discussion for Fig. 3, a frame is divided into three time-slots. In the first time-slot, s_0 broadcasts signal x_0 to d_0, which is overheard by r and d_1; in the second time slot, s_1 broadcasts signal x_1 to d_1, which is overheard by r and d_0; then the relay node r performs ANC by combining the overheard signals from s_0 and s_1, and then amplifies and broadcasts the combined signal in the third time slot to d_0 and d_1. The destination node d_0 receives one copy of signal x_0 in the first time slot. It also overhears a copy of signal x_1 (denoted by $y_{s_1 d_0}$) in the second time slot. In the third time slot, destination node d_0 receives the combined signal, denoted as $y_{s_0 r d_0} + y_{s_1 r d_0}$.

Ideally, one would wish that the destination node d_0 in Fig. 5(a) can cleanly extract $y_{s_0 r d_0}$ by having the combined signal $(y_{s_0 r d_0} + y_{s_1 r d_0})$ subtract the overheard signal $y_{s_1 d_0}$. But in reality, $y_{s_1 r d_0} \neq y_{s_1 d_0}$ due to two different paths. As a result of such subtraction, a new noise term, called "ANC noise", will be introduced at d_0. The value of ANC noise will be $[y_{s_1 r d_0} - y_{s_1 d_0}]$. When the number of sessions increase (see Fig. 5(b)), this situation will worsen, as the aggregate noise

TABLE I
NOTATION

Symbol	Definition
$C_{\text{A-NCC}}(s_i, r_j, d_i)$	Achievable rate for s_i-d_i pair with relay r_j under A-NCC
$C_{\text{AF}}(s_i, r_j, d_i)$	Achievable rate for s_i-d_i pair with relay r_j under AF CC (no NC)
$C_{\text{D}}(s_i, d_i)$	Achievable rate for s_i-d_i pair under direct transmission
h_{uv}	Channel gain between nodes u and v
n	Number of source nodes in the network
r	Relay node
x_s	Signal transmitted by node s
y_{uv}	Received signal at node v (form node u)
$y_{(s_i \cup s_j)rd}$	Combination of signals from s_i and s_j received by destination node d (with ANC at relay node r)
z_v	Background noise at node v
z_v^{new}	ANC noise at node v
s_i	The i-th source node
$\mathcal{S}_r$	Set of source nodes using relay node r to perform A-NCC
SNR_{uv}	The signal noise ratio at node v when u is transmitting
W	Total bandwidth available in the network
P_u	Transmission power at node u
σ_v^2	Variance of background noise at node v
$\sigma^2_{z_v^{new}}$	Variance of ANC noise at node v
α_r	Amplifying factor at relay r

at d_0 in Fig. 5(b) now becomes $\sum_{i=1}^{n-1}[y_{s_i r d_0} - y_{s_i d_0}]$. As expected, the amount of ANC noise can grow as n increases. Therefore, it is not clear whether the advantage of employing ANC can outweigh the disadvantage of ANC noise. For a given session, the mutual information (or achievable rate) formula for A-NCC is critical in deciding whether to employ A-NCC or not. Due to ANC noise, we cannot use the mutual information formula for AF CC derived in the seminal work in [13] by Laneman et al. The actual mutual information is likely to be smaller.

III. ANALYSIS OF NC NOISE

We first consider the simple two-session example in Fig. 5(a). For this simple network, we analyze the procedure used to perform ANC at the relay node. Then we analyze the procedure employed by the destination node to extract its desired signal. This is followed by the derivation of ANC noise. Finally, we extend the result for the two-session case to the general case with n-sessions shown in Fig. 5(b). Table I shows all the notation used in this paper.

A. Two-Session Case

Fig. 5(a) shows our two-session case. Assume that the signal x_0 transmitted by source s_0 in the first time slot is for packet p_0, and the signal x_1 transmitted by source s_1 in the second time slot is for packet p_1. We assume the channel gains between all the nodes are independent to each other. Denote the channel gain between two nodes, s_0 and d_0, as $h_{s_0 d_0}$. Assume that the background noise at a node r, denoted by z_r, to be white Gaussian with zero mean and variance as σ_r^2. Denote y_{srd} as the signal received by a destination node d that is transmitted by a relay node r and originated from some source s. Denote y_{sr} as the signal received by the relay node r that is transmitted and originated at some source s. The amplifying factor used by the relay node r for AF CC as α_r.

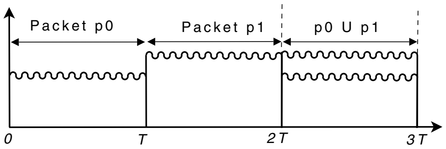


Fig. 6. Transmission behavior for A-NCC for two sessions.

Combining Signals at Relay Node Since node s_0 transmits in the first time slot, we can express the signals received by other nodes during the first time slot as

$$y_{s_0 r} = h_{s_0 r} x_0 + z_r, \tag{1}$$

$$y_{s_0 d_0} = h_{s_0 d_0} x_0 + z_{d_0},$$

$$y_{s_0 d_1} = h_{s_0 d_1} x_0 + z_{d_1}. \tag{2}$$

In the second time slot, when node s_1 transmits, the signals received by other nodes in the network can be expressed as

$$y_{s_1 r} = h_{s_1 r} x_1 + z_r,$$

$$y_{s_1 d_0} = h_{s_1 d_0} x_1 + z_{d_0},$$

$$y_{s_1 d_1} = h_{s_1 d_1} x_1 + z_{d_1}.$$

Then in the third time slot, relay node r combines x_0 and x_1, and then amplifies and broadcasts the combined signal. Figure 6 shows the transmission behavior for A-NCC for two sessions.

To understand how the destination node will separate this combined signal, we focus on one of the destination nodes d_1.

Signal Extraction at a Destination Node The destination node d_1 has received a combined signal ($x_0 \cup x_1$) from the relay node in the third time slot, and it has overheard the signal x_0 transmitted by source s_0 in the first time slot. Using these two signals, d_1 can extract a copy of signal x_1 from the combined signal as follows.

Denote the combined signal received by destination node d_1 in the third time slot as

$$y_{(s_0 \cup s_1) r d_1}(t) = \alpha_r h_{r d_1} \left[y_{s_0 r}(t-2T) + y_{s_1 r}(t-T) \right] + z_{d_1}(t), \quad \forall t \in (2T, 3T], \tag{3}$$

where the value of α_r can be specified similar to the α_r in [13], i.e.

$$\alpha_r^2 = \frac{P_r}{|h_{s_0 r}|^2 P_{s_0} + |h_{s_1 r}|^2 P_{s_1} + 2\sigma_r^2}\,, \tag{4}$$

where P_r, P_{s_0}, and P_{s_1} are the transmission powers of nodes r, s_0, and s_1, respectively.

By using (1), the combined signal in (3) can be expanded as

$$y_{(s_0 \cup s_1) r d_1}(t) = \alpha_r h_{r d_1} \left[h_{s_0 r} x_0(t-2T) + z_r(t-2T) + y_{s_1 r}(t-T) \right] + z_{d1}(t), \quad \forall t \in (2T, 3T]. \tag{5}$$

Using (2), Eq. (5) can be re-written as

$$y_{(s_0 \cup s_1) r d_1}(t) = \frac{\alpha_r h_{r d_1} h_{s_0 r}}{h_{s_0 d_1}} \left[y_{s_0 d_1}(t-2T) - z_{d_1}(t-2T) \right] + \alpha_r h_{r d_1} y_{s_1 r}(t-T) + z_{d1}(t) + \alpha_r h_{r d_1} z_r(t-2T), \quad \forall t \in (2T, 3T]. \tag{6}$$

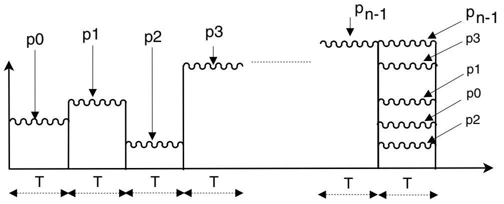


Fig. 7. Transmission behavior for A-NCC for n sessions.

Equation (6) represents the signal that the destination node d_1 will receive in the third time slot. In the first time slot, destination node d_1 had overheard the transmission of s_0, which is given by (2), and can be re-written as

$$y_{s_0 d_1}(t) = h_{s_0 d_1} x_0(t) + z_{d_1}(t), \quad \forall t \in [0, T]. \tag{7}$$

Since we assume that the channel gains and amplification factor are given, destination node d_1 can multiply (7), by a factor $\frac{\alpha_r h_{rd_1} h_{s_0 r}}{h_{s_0 d_1}}$, and subtract it from (6). We have

$$\begin{aligned} \hat{y}_{rd_1}(t) = {} & y_{(s_0 \cup s_1) r d_1}(t) - \tfrac{\alpha_r h_{rd_1} h_{s_0 r}}{h_{s_0 d_1}} y_{s_0 d_1}(t - 2T) \\ = {} & \alpha_r h_{rd_1} y_{s_1 r}(t - T) + z_{d1}(t) + \alpha_r h_{rd_1} z_r(t - 2T) \\ & - \frac{\alpha_r h_{rd_1} h_{s_0 r}}{h_{s_0 d_1}} z_{d_1}(t - 2T), \quad \forall t \in (2T, 3T] \end{aligned} \tag{8}$$

Equation (8) represents the signal for packet p_1 that destination node d_1 can construct, using the combined signal received in the third time slot and the signal overheard in the first time slot. We find that instead of z_{d_1}, we now have a *new* noise term in this constructed signal, which we denote as $z_{d_1}^{new}$, i.e.,

$$\begin{aligned} z_{d_1}^{new}(t) = {} & z_{d_1}(t) + \alpha_r h_{rd_1} z_r(t - 2T) - \\ & \tfrac{\alpha_r h_{rd_1} h_{s_0 r}}{h_{s_0 d_1}} \, z_{d_1}(t - 2T), \quad \forall t \in (2T, 3T]. \end{aligned} \tag{9}$$

We call $z_{d_1}^{new}$ as "ANC noise". Note that $z_{d_1}^{new}$ has a zero mean (because $E[z_{d_1}] = E[z_r] = 0$), and the variance of $z_{d_1}^{new}$ is

$$\sigma^2_{z_{d_1}^{new}} = \sigma^2_{d_1} + (\alpha_r h_{rd_1})^2 \ \sigma^2_r \ + \left(\frac{\alpha_r h_{s_0 r} h_{rd_1}}{h_{s_0 d_1}} \right)^2 \sigma^2_{d_1}, \tag{10}$$

which is larger than the original noise variance $\sigma^2_{d_1}$.

B. The General Multi-Session Case

We now consider the general case, where there are n source-destination pairs and one relay node in the network (see Fig. 5(b)). All the n source-destination pairs will share the same relay node by employing ANC. As mentioned earlier, signal transmission will require $(n+1)$ time slots (see Fig. 3).

The signal aggregation process in this scenario follows the same token as in the two-session case. Figure 7 shows the details in each time slot.

In this general multi-session case, a given destination node d_i overhears a copy of the signal for all the packets (including p_i) during the first n time slots. In order to construct the signal for the second copy of packet p_i, the destination node d_i will again follow the same procedure that we discussed in the two-session case. Now, instead of subtracting only one signal from the combined signal, the destination node will subtract the signals overheard from multiple sources. The expressions for the ANC noise can be obtained by generalizing (9) as follows

$$\begin{aligned} z_{d_i}^{new}(t) = {} & z_{d_i}(t) + \textstyle\sum_{s_j \in \mathcal{S}_r}^{s_j \neq s_i} \alpha_r h_{rd_i} z_r(t - |\mathcal{S}_r| T) - \\ & \textstyle\sum_{s_j \in \mathcal{S}_r}^{s_j \neq s_i} \frac{\alpha_r h_{rd_i} h_{s_j r}}{h_{s_j d_i}} z_{d_i}(t - |\mathcal{S}_r| T), \\ & \forall t \in (|\mathcal{S}_r| T, (|\mathcal{S}_r| + 1) T], \end{aligned} \tag{11}$$

where $\mathcal{S}_r$ is the set of n source nodes that are using relay r, and a general expression for the amplification factor α_r can be obtained by generalizing (4), which is

$$\alpha_r^2 = \frac{P_r}{|\mathcal{S}_r| \sigma_r^2 + \sum_{s_i \in S_r} P_{s_i} \ |h_{s_i r}|^2}. \tag{12}$$

The variance of ANC noise can be obtained by generalizing (10), which is

$$\begin{aligned} \sigma^2_{z_{d_i}^{new}} = {} & \sigma^2_{d_i} + \ (|\mathcal{S}_r| - 1) \, (\alpha_r h_{rd_i})^2 \, \sigma^2_r \, + \\ & \sigma^2_{d_i} \sum_{s_j \in \mathcal{S}_r}^{s_j \neq s_i} \left(\frac{\alpha_r h_{s_j r} h_{rd_i}}{h_{s_j d_i}} \right)^2. \end{aligned} \tag{13}$$

We can see that the variance of ANC noise at destination node d_i contains $\sigma^2_{d_i}$ and some additional new terms. These additional new terms are introduced due to the relay node employing ANC for aggregating multiple signals from sources in $\mathcal{S}_r$. The variance of ANC noise at the destination nodes increases with the increase in the number of sessions sharing the same relay node.

IV. COMPUTING ACHIEVABLE RATE

A. A-NCC

To compute the achievable rate under A-NCC, we assume that each time-slot in the frame has the same length T (see Fig. 7). Denote the time duration of the entire frame as t seconds. As a result, when all n pairs in the network are sharing a single relay node by employing A-NCC (see Fig. 3), every source node as well as the relay node will get the time-slot of $T = \frac{t}{n+1}$ seconds. Under such scenario, the achievable rate for one session, say (s_i, d_i), is given by

$$\begin{aligned} C_{\text{A-NCC}}(s_i, r, d_i) = {} & \left(\frac{\frac{t}{n+1}}{t} \right) \cdot W I_{\text{A-NCC}}(s_i, r, d_i) \\ = {} & \frac{W}{n+1} \cdot I_{\text{A-NCC}}(s_i, r, d_i), \end{aligned} \tag{14}$$

where $I_{\text{A-NCC}}(s_i, r, d_i)$ is the mutual information between s_i and d_i that are using relay r to employ A-NCC, and W is the available bandwidth in the network. In this equation, the effective bandwidth for the (s_i, d_i) pair is W divided by the total number of transmitting nodes (n source nodes plus one relay) in the network.

We now derive the mutual information, $I_{\text{A-NCC}}(s_i, r, d_i)$, between the pair (s_i, d_i) based on the ANC noise (Eq. (13)) at the destination nodes. For the signal transmitted by a source node s_i, the received signal at the relay node is

$$y_{s_i r} = h_{s_i r} x_i + z_r, \tag{15}$$

and the received signal at the corresponding destination node is

$$y_{s_i d_i} = h_{s_i d} x_i + z_{d_i}. \qquad (16)$$

For the signal transmitted by the relay node, the desired signal extracted by the destination node is

$$\hat{y}_{rd_i} = \alpha_r h_{rd_i} y_{s_i r} + z_{d_i}^{new},$$

which is

$$\hat{y}_{rd_i} = \alpha_r h_{rd_i} (h_{s_i r} x_i + z_r) + z_{d_i}^{new}, \qquad (17)$$

where $z_{d_i}^{new}$ is given in (11), and α_r is given in (12).

We can re-write (15), (16) and (17) into the following compact matrix form

$$\mathbf{Y} = \mathbf{H} x_i + \mathbf{B}\mathbf{Z}\ ,$$

where

$$\mathbf{Y} = \begin{bmatrix} y_{s_i d_i} \\ \hat{y}_{rd_i} \end{bmatrix}, \quad \mathbf{H} = \begin{bmatrix} h_{s_i d_i} \\ h_{rd_i} \alpha_r h_{s_i r} \end{bmatrix},$$

$$\mathbf{B} = \begin{bmatrix} 0 & 1 & 0 \\ \alpha_r h_{rd_i} & 0 & 1 \end{bmatrix}, \quad \text{and} \quad \mathbf{Z} = \begin{bmatrix} z_r \\ z_{d_i} \\ z_{d_i}^{new} \end{bmatrix}.$$

It was shown in [13] that we can model the above channel that combines both the direct path (s_i to d_i) and the relay path (s_i to r to d_i) as a one-input two-output complex Gaussian vector channel. The mutual information between s_i and d_i is

$$I_{\text{A-NCC}}(s_i, r, d_i) = \log\det\left(\mathbf{I} + (P_{s_i}\mathbf{H}\mathbf{H}^\dagger)\left(\mathbf{B}E\left[\ \mathbf{Z}\mathbf{Z}^\dagger\right]\mathbf{B}^\dagger\right)^{-1}\right), \qquad (18)$$

where $\mathbf{I}$ is the identity matrix, $\dagger$ represents the complex conjugate transpose, $E[\cdot]$ is the expectation function, and

$$E\left[\mathbf{Z}\mathbf{Z}^\dagger\right] = \begin{bmatrix} \sigma_r^2 & 0 & 0 \\ 0 & \sigma_{d_i}^2 & 0 \\ 0 & 0 & \sigma_{z_{d_i}^{new}}^2 \end{bmatrix}.$$

Expanding (18) gives us the value of mutual information

$$I_{\text{A-NCC}}(s_i, r, d_i) = \log_2\left(1 + \frac{|h_{s_i d_i}|^2 P_{s_i}}{\sigma_{d_i}^2} + \frac{P_{s_i}|h_{rd_i}\alpha_r h_{s_i r}|^2}{|h_{rd_i}\alpha_r|^2\sigma_r^2 + \sigma_{z_{d_i}^{new}}^2}\right),$$

which can be further rewritten as

$$I_{\text{A-NCC}}(s_i, r, d_i) = \log_2\left(1 + \text{SNR}_{s_i d_i} + \frac{\text{SNR}_{s_i r}\text{SNR}_{rd_i}}{|\mathcal{S}_r|\frac{\sigma_{z_{d_i}^{new}}^2}{\sigma_{d_i}^2} + \text{SNR}_{rd_i} + \frac{\sigma_{z_{d_i}^{new}}^2}{\sigma_{d_i}^2}\ \sum_{s_j \in \mathcal{S}_r}\ \text{SNR}_{s_j r}}\right), \qquad (19)$$

where $\text{SNR}_{s_i d_i} = \frac{P_{s_i}}{\sigma_{d_i}^2}|h_{s_i d_i}|^2$, $\text{SNR}_{s_i r} = \frac{P_{s_i}}{\sigma_r^2}|h_{s_i r}|^2$, and $\text{SNR}_{rd_i} = \frac{P_r}{\sigma_{d_i}^2}|h_{rd_i}|^2$.

For the special case of $|\mathcal{S}_r| = 1$, i.e., the one-session (three-node model) in [13], we have $i = 0$, $\sigma_{z_{d_i}^{new}}^2 = \sigma_{d_i}^2$, and (19) is reduced to

$$I_{\text{AF}}(s_i, r, d_i) = \log_2\left(1 +\ \text{SNR}_{s_i d_i} + \frac{\text{SNR}_{s_i r}\ \text{SNR}_{rd_i}}{1 + \text{SNR}_{rd_i} + \text{SNR}_{s_i r}}\ \right), \qquad (20)$$

which is exactly the result in [13].

For a given session, whether employing A-NCC is beneficial or not can be determined by comparing the achievable rate under A-NCC (i.e., Eq. (14)) with the achievable rates of the schemes where (i) the session performs AF CC without ANC, and (ii) the session employs direct transmission. We now discuss the latter two schemes.

B. AF CC (without ANC)

Under this scheme [13], the source node of the session performs AF CC with the help of relay node, and the relay node does not employ ANC for this session. Both the source node and the relay node will get the time-slot duration of $t/2n$ (see Fig. 2), and the achievable rate for a session (s_i, d_i) can be given using (20), which is

$$\begin{aligned} C_{\text{AF}}(s_i, r, d_i) = &\quad \left(\frac{\frac{t}{2n}}{t}\right) W I_{\text{AF}}(s_i, r, d_i) \\ = \frac{W}{2n}\log_2 &\left(1 + \text{SNR}_{s_i d_i} + \frac{\text{SNR}_{s_i r}\text{SNR}_{rd_i}}{1 + \text{SNR}_{rd_i} + \text{SNR}_{s_i r}}\right). \qquad (21) \end{aligned}$$

C. Direct Transmission

Direct transmission mode should be employed by a session when achievable rate under A-NCC or AF CC is worse than the achievable rate under direct transmission. Under the direct transmission mode, a source node does not perform CC and transmits directly to the destination node. The time-slot duration assigned to a source node under such scheme is t/n. For a session (s_i, d_i) , the achievable rate under direct transmission is

$$\begin{aligned} C_{\text{D}}(s_i, d_i) = &\quad \left(\frac{\frac{t}{n}}{t}\right) \cdot W\log_2(1 + \text{SNR}_{s_i d_i}) \\ = &\quad \frac{W}{n} \cdot \log_2(1 + \text{SNR}_{s_i d_i})\ . \qquad (22) \end{aligned}$$

V. NUMERICAL RESULTS

In this section, we present some numerical results to show the impact of ANC on AF CC.

A. Parameter Settings

We assume the total bandwidth in the network $W = 22$ MHz. All source nodes transmit with 1 Watt of power. The variance of Gaussian noise at every node is 10^{-10} Watts, and the path loss index is 4. For simplicity, the channel gain $|h_{uv}|^2$ between two nodes was modeled as d_{uv}^{-4}, where d_{uv} is the distance between u and v, and 4 is the path loss index.

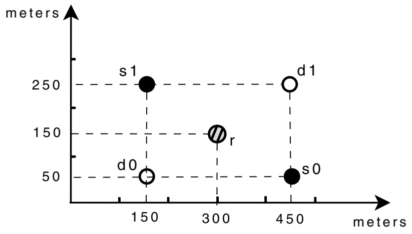


Fig. 8. Network topology for Case 1.

TABLE II
COMPARISON OF ACHIEVABLE RATE FOR EACH SESSION UNDER DIFFERENT SCHEMES FOR CASE 1.

Session	A-NCC (Mbps)		AF CC (Mbps)	Direct Tra- -nsmission (Mbps)
	Considering ANC Noise	Ignoring ANC Noise		
(s_0, d_0)	16.60	19.24	14.43	12.17
(s_1, d_1)	16.60	19.24	14.43	12.17

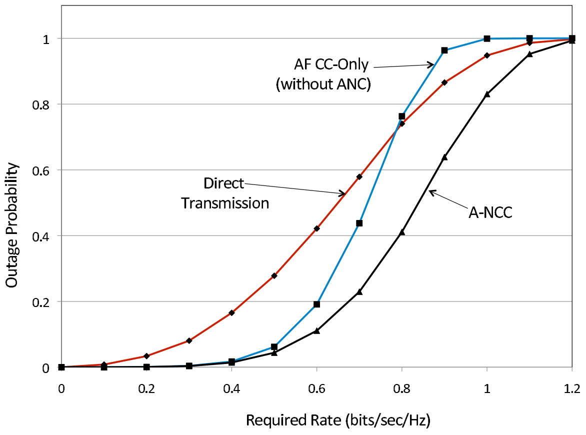


Fig. 9. Outage probability of session $s_0 \rightarrow d_0$ for Case 1.

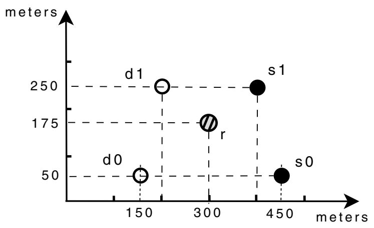


Fig. 10. Network topology for Case 2.

TABLE III
COMPARISON OF ACHIEVABLE RATE FOR EACH SESSION UNDER DIFFERENT SCHEMES FOR CASE 2.

Session	A-NCC (Mbps)		AF CC (Mbps)	Direct Tra- -nsmission (Mbps)
	Considering ANC Noise	Ignoring ANC Noise		
(s_0, d_0)	11.31	17.10	12.82	12.17
(s_1, d_1)	32.77	33.43	25.07	29.98

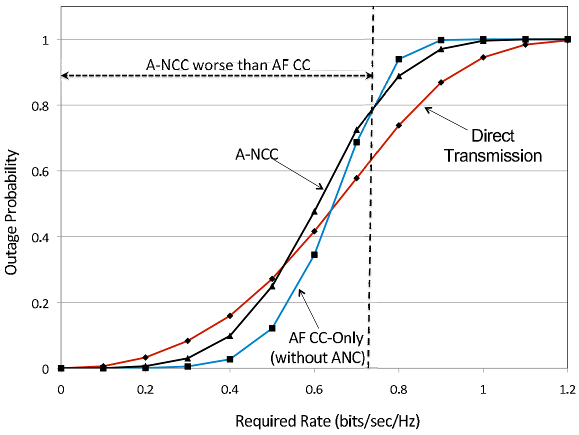


Fig. 11. Outage probability of session $s_0 \rightarrow d_0$ for Case 2.

B. The Two-Session Case

We first consider a simple two session $(s_0 \rightarrow d_0, s_1 \rightarrow d_1)$ and one relay node (r) network. To show how A-NCC performs, we consider three cases.

Case 1: A-NCC Better Than AF CC and Direct Transmission Network topology for this case is shown in Fig. 8. Table II shows the achievable rate of both sessions under different schemes. There are five columns in this table. The first column shows the source and destination nodes of each session. The second column shows the achievable rate for each session under A-NCC when ANC noise is considered (i.e. based on Eq. (14)), whereas the third column shows the achievable rate for each session under A-NCC when ANC noise is ignored, which we denote as $\bar{C}_{\text{A-NCC}}(s_i, r, d_i)$, i.e.,

$$\bar{C}_{\text{A-NCC}}(s_i, r, d_i) = \frac{W}{|\mathcal{S}_r| + 1} \cdot \log_2 \left(1 + \text{SNR}_{s_i d_i} + \frac{\text{SNR}_{s_i r}\ \text{SNR}_{r d_i}}{1 + \text{SNR}_{r d_i} + \text{SNR}_{s_i r}} \right), \quad (23)$$

where $|\mathcal{S}_r| = 2$ in this case. The fourth column shows the achievable rate for each session when the session performs AF CC without the use of ANC (i.e., based on Eq. (21) and the case shown in Fig. 2). The fifth column shows the achievable rate of each session under direct transmission (i.e. based on Eq. (22)).

We can see that in this topology, achievable rate under A-NCC (column 2) is better than the achievable rate under AF CC without ANC (column 4) and direct transmission (column 5) schemes. We also show the plots for outage probability of session $s_0 \rightarrow d_0$ under all three schemes in Fig. 9. For each scheme, the outage probability is computed by counting the number of outages under $10,000$ channel realizations. We assume that all the channels in the network are Rayleigh faded. We can see that in this case, the outage probability of session $s_0 \rightarrow d_0$ is lowest under the A-NCC scheme.

However, the next two cases show that the performance of A-NCC is not always better, and in some scenarios A-NCC may perform worse than the other two schemes.

Case 2: A-NCC Worse Than AF CC The network topology for this case is shown in Fig. 10. Similar to Table II, the achievable rates for both the sessions in this case are shown in Table III. We can see that for session $s_0 \rightarrow d_0$, the achievable rate under A-NCC (column 2) is worse than the achievable

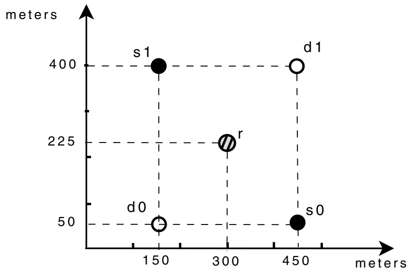


Fig. 12. Network topology for Case 3.

TABLE IV
COMPARISON OF ACHIEVABLE RATE FOR EACH SESSION UNDER DIFFERENT SCHEMES FOR CASE 3.

Session	A-NCC (Mbps)		AF CC (Mbps)	Direct Tra--nsmission (Mbps)
	Considering ANC Noise	Ignoring ANC Noise		
(s_0, d_0)	11.82	13.43	10.08	12.17
(s_1, d_1)	11.82	13.43	10.08	12.17

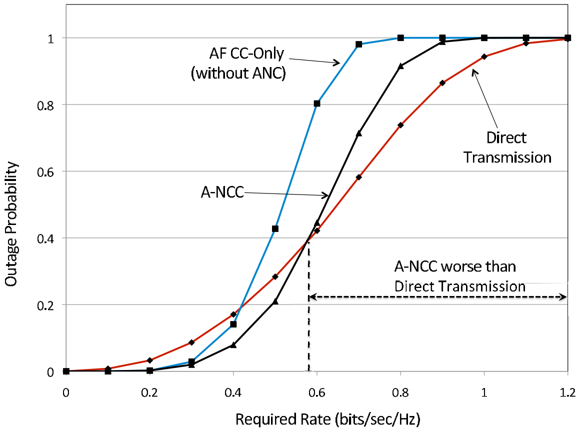


Fig. 13. Outage probability of session $s_0 \rightarrow d_0$ for Case 3.

rate under the other two schemes, i.e. column 4 and column 5. But for session $s_1 \rightarrow d_1$, the achievable rate under A-NCC (column 2) is better than the other two schemes.

For this case, Fig. 11 shows the outage probability of session $s_0 \rightarrow d_0$ under all three schemes. We find that for this case, there is a region in which the outage probability under A-NCC is worse than the scheme employing AF CC without ANC.

Case 3: A-NCC Worse Than Direct Transmission The network topology to illustrate this is shown in Fig. 12. The achievable rates for both sessions in this case are shown in Table IV. We find that the achievable rate under A-NCC (column 2) is worse than the achievable rate under direct transmission scheme (column 5), but better than the scheme in which AF CC is employed without ANC (column 4).

For this case, Fig. 13 shows the outage probability of session $s_0 \rightarrow d_0$ under all three schemes. We can see that for this case, when the required bit rate is above 0.6 bits/sec/Hz, the outage probability under A-NCC is worse than the outage probability under direct transmission scheme.

Summary. From the above three cases, we conclude that it is possible for A-NCC to perform worse than AF CC or direct transmission. The reason for this behavior is the ANC noise infused at the destination nodes. Since the amount of ANC noise depends upon the channel gains between the participating nodes, the performance of A-NCC also depends upon the channel gains of the participating sessions.

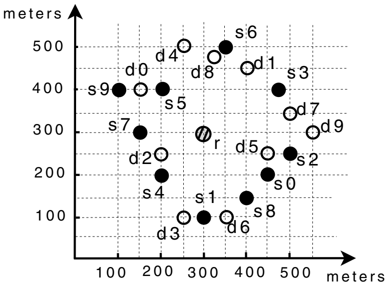


Fig. 14. A 10-session one-relay network topology.

We also find that under each topology, the achievable rate of each session in column 2 is *always* less than that in column 3. This underscores the importance of ANC noise, and shows that ignoring ANC noise will lead to overly optimistic (or inflated) results.

C. A General Multi-session Network

In this section, we consider a general network with multiple sessions. The network topology is shown in Fig. 14, where we have 10 sessions and one relay node. We show that when more sessions employ A-NCC and share the same relay node, ANC noise at destination nodes will increase. This increase in ANC noise will have a direct impact on the effective SNR and the achievable rate of individual sessions. As a result, we find that when the amount of ANC noise increases beyond a certain threshold, employing A-NCC will not be beneficial for some sessions. We chose session $s_0 \rightarrow d_0$ in our study to demonstrate this finding.

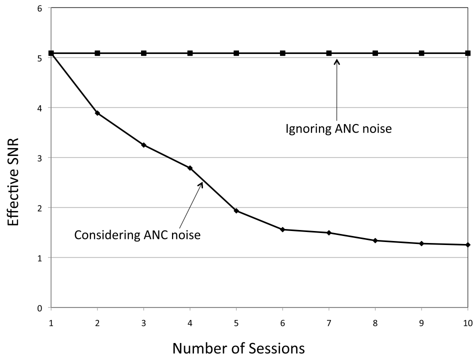
To start with, all the sessions are active, but only one session $(s_0 \rightarrow d_0)$ is using the relay node r to perform AF CC. The effective bandwidth for every session is $W/10$. We first determine the effect of adding more sessions on the effective SNR of $s_0 \rightarrow d_0$. Here, the effective SNR for a session $s_i \rightarrow d_i$ is defined based on Eq. (14), i.e.,

$$\text{SNR}_{\text{eff}}(s_i, r, d_i) = \text{SNR}_{s_i d_i} + \frac{\text{SNR}_{s_i r}\text{SNR}_{r d_i}}{|\mathcal{S}_r|\frac{\sigma^2_{z^{new}_{d_i}}}{\sigma^2_{d_i}} + \text{SNR}_{r d_i} + \frac{\sigma^2_{z^{new}_{d_i}}}{\sigma^2_{d_i}}\sum_{s_j \in \mathcal{S}_r} \text{SNR}_{s_j r}}, \quad (24)$$

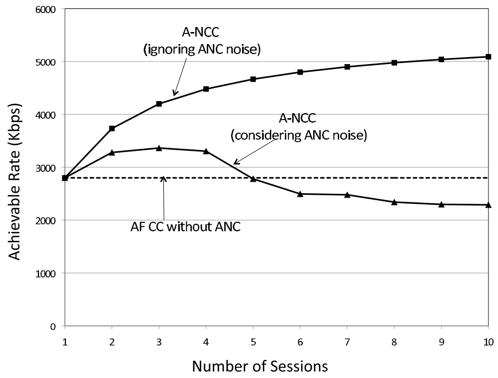
where $\mathcal{S}_r$ is the set for those source nodes using the relay node r.

Since only one session (i.e., $s_0 \rightarrow d_0$) is using the relay node initially, the value of $n(= |\mathcal{S}_r|)$ in Eq. (24) will be one for session $s_0 \rightarrow d_0$, and $\mathcal{S}_r$ will contain only s_0. Then we let session $s_1 \rightarrow d_1$ also use the relay node for performing AF CC. Now, both $s_0 \rightarrow d_0$ and $s_1 \rightarrow d_1$ are employing A-NCC, and the other eight sessions are using direct transmission. Note that now, the value of $n(= |\mathcal{S}_r|)$ in Eq. (24) for these two sessions will be two, and $\mathcal{S}_r$ will contain $\{s_0, s_1\}$. The process continues until all 10 sessions start employing A-NCC by sharing the same relay node.

We plot the effect of adding more sessions on the effective SNR of session $s_0 \rightarrow d_0$ in Fig. 15(a). Horizontal axis of



(a) Impact on effective SNR for session $s_0 \rightarrow d_0$.



(b) Impact on achievable rate for session $s_0 \rightarrow d_0$.

Fig. 15. Example illustrating the negative effect of ANC on session $s_0 \rightarrow d_0$.

Fig. 15(a) shows the number of active sessions sharing the same relay node, and the vertical axis shows the value of effective SNR for session $s_0 \rightarrow d_0$. Figure 15(a) contains two curves, one for the effective SNR when ANC noise is ignored, and the other is for the effective SNR when ANC noise is considered (i.e., using Eq. (24)). We can see that as more sessions start using the relay node, the effective SNR of session $s_0 \rightarrow d_0$ (when ANC noise is considered) decreases. This is due to the increase in ANC noise at the destination node for session $s_0 \rightarrow d_0$.

This reduction in the effective SNR of session $s_0 \rightarrow d_0$ directly impacts the achievable rate of this session. This impact is illustrated in Fig. 15(b). Horizontal axis of Fig. 15(b) shows the number of sessions in the network that are sharing the same relay node, and the vertical axis shows the achievable rate of session $s_0 \rightarrow d_0$. We plot three curves in Fig. 15(b). The top curve shows the achievable rate for session $s_0 \rightarrow d_0$, which is calculated by ignoring the impact of ANC noise (i.e., Eq. (23)). The middle curve shows the correct achievable rate of session $s_0 \rightarrow d_0$, which is calculated by taking ANC noise into account (i.e., Eq. (14). The straight line shows the achievable rate of session $s_0 \rightarrow d_0$ when ANC is not used to perform AF CC (i.e., Eq. (21)).

In Fig. 15(b), we find that employing A-NCC was beneficial for session $s_0 \rightarrow d_0$ initially due to the time-slot benefit. But as more sessions start sharing the same relay node, the adverse effect of ANC noise at d_0 starts to increase and the achievable rate of session $s_0 \rightarrow d_0$ starts to decrease. Finally, the ANC noise has reached the point where the time-slot benefit of A-NCC is no longer adequate to counter the ANC noise. In Fig. 15(a), we can also see that ignoring the ANC noise will result in inflated values of effective SNR, which are incorrect. Similarly, Fig. 15(b) shows that the achievable rate calculated by ignoring ANC noise is also inflated and incorrect.

TABLE V
COMPARISON OF VARIOUS DATA TRANSFER SCHEMES FOR A 10-SESSION NETWORK.

Session	A-NCC (Mbps)		AF CC (Mbps)	Direct Tra--nsmission (Mbps)
	Considering ANC Noise	Ignoring ANC Noise		
$(s0, d0)$	2.24	4.97	2.73	1.41
$(s1, d1)$	1.92	4.49	2.47	1.36
$(s2, d2)$	3.88	5.45	3.00	2.43
$(s3, d3)$	1.56	3.93	2.16	1.24
$(s4, d4)$	3.22	5.17	2.85	2.34
$(s5, d5)$	4.54	6.82	3.75	2.63
$(s6, d6)$	1.33	3.77	2.07	1.00
$(s7, d7)$	2.49	4.84	2.66	1.50
$(s8, d8)$	2.54	5.15	2.83	1.79
$(s9, d9)$	0.72	2.60	1.43	0.61

We now consider the final instance when all the sessions in the network are using the same relay node to perform A-NCC. Table V shows the achievable rates of all the sessions in the network under different transmission schemes (similar to Tables II, III, and IV). Comparing columns 2, 4 and 5, we can see that employing A-NCC (column 2) is not beneficial for most of the sessions (i.e. sessions 0, 1, 3, 6, 7, 8, 9). We also find that for some sessions (i.e. sessions 2, 4, 5), A-NCC (column 2) does offer better results than AF CC without ANC (column 4) and the direct transmission (column 5). We can see that the achievable rate for each session in column 2 is always less than that in column 3, which is consistent with the results we obtained in the simple two-session network.

As a result, we find that employing NC to perform CC can improve the achievable rate of individual sessions only under some (and not all) scenarios. The channel gains between the relay node, and the source/destination nodes of the participating sessions play important roles in the overall performance of A-NCC.

VI. RELATED WORK

In this section, we briefly review related work in CC and NC. Then we describe several recent efforts on employing NC to perform CC.

The concept of CC can be traced back to the three-terminal communication channel (or a relay channel) in [20] by Van Der Meulen. Shortly after, Cover and El Gamal studied the general relay channel and established an achievable lower bound for data transmission [3]. These two seminal works laid the foundation for the present-day research on CC. Recent research on CC aims to exploit distributed antennas on other nodes in the network, and has resulted in various protocols at physical layer [4], [5], [6], [8], [13], [15], [18], [19] and at the network layer [11], [17], [23]. The AF CC model we used in this paper is based on [13].

The concept of NC was first introduced by Ahlswede et al. in their seminal work [1], where they showed how NC can save bandwidth for multicast flows in a wired network. In the context of wireless networks, NC can be described as either digital network coding (DNC) (see e.g. [9]), or ANC (see e.g. [10], [24]). For readers interested in NC, we refer them to the NC bibliography in [7].

The research most relevant to our work is [2], [14], [21], [22]. Bao et al. [2] were the first ones to employ the technique of performing CC with the help of NC in a multi-source single-destination network. They showed that the achievable rate and outage probability of a network can be improved if CC is performed with the help of NC. Shortly after, Peng et al. [14] considered a network with a single relay node and multiple source-destination pairs. They again showed that performing CC with the help of NC can reduce the outage probability of the entire network. More recently, Xiao et al. [21] considered a two-source single-destination network and showed that performing CC with the help of NC can reduce packet error rates. In [22], Xu and Li considered a cellular network with bi-directional traffic, and showed the improvement in network throughput when NC is employed to perform CC. The positive results in these works show the benefits of applying NC to CC in scenarios where NC benefits CC. However, as we have shown in this paper, NC could introduce non-negligible noise at destination nodes, which can potentially undermine its advantage.

We believe that the concept of NC noise have wider implications than explored in this paper. The concept of NC noise can extend beyond the A-NCC model discussed in this paper, such as multiple relay nodes in CC, or using DNC with decode-and-forward CC, among others [12].

VII. CONCLUSION

In this paper, we investigated the important problem of how NC will affect the performance of CC. We studied this important question in context of ANC and AF CC. We showed that NC may not always be beneficial to CC. For the first time in this paper, we formalized the concept of NC noise. We derived a closed-form expression for ANC noise at each destination node in a multi-session environment. Based on this result, we further developed mutual information equation and achievable rate calculation. Using numerical results, we demonstrated the impact of ANC on the achievable rate of AF CC. Our results offer new understanding of NC noise and provide correct guidelines when applying NC in CC.

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