CROSS-LAYER OPTIMIZATION FOR UWB-BASED AD HOC NETWORKS

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***Abstract*— In this paper, we consider a UWB-based ad hoc network and study how to maximize data rate utility for a group of communication sessions. We formulate the data rate utility problem into a nonlinear programming (NLP) problem through a cross-layer approach by taking into consideration of scheduling and power control at link layer and routing at network layer. The main contribution of this paper is the development of a solution procedure based on branch-and-bound framework. We employ a powerful optimization technique called reformulation linearization technique (RLT) to obtain a linear relaxation, which is a key component required in the branch-and-bound framework. Numerical results demonstrate the importance of cross-layer approach and offer some important insights.**

***Index Terms*— Nonlinear programming, optimization, cross-layer design, ultra-wideband (UWB), ad hoc networks, data rate utility, scheduling, power control, routing.**

I. INTRODUCTION

In this paper, we consider a group of source-destination communication pairs, which we call sessions, in UWB-based ad hoc networks. The objective is to maximize the total utility, where a session's utility is defined as the log function of its data rate [11]. Clearly, this optimization problem involves issues from different layers, i.e., link layer scheduling, power control, and network layer routing. The link layer scheduling component deals with how to use time slots for transmission and reception. The power control component considers how much transmission power a node should use in a particular time slot. Finally, the routing problem at the network level considers what set of paths the data from a source node should take to its destination node.

We aim to investigate the problem through formal nonlinear optimization technique. The outcome of this effort will fill in a critical theoretical gap on this problem. It will also contribute some important understanding, some of which were overlooked by prior research efforts. The main contribution of this paper is the development of a solution procedure to the nonlinear programming problem based on the *branch-and-bound* framework [8] and a powerful technique called *Reformulation-Linearization Technique* (RLT) [13]. During each iteration of the branch-and-bound procedure, it is important (from computation perspective) to select a partition variable. We propose a partition variable selection policy not only based on the relaxation error, but also based on their relative significance in our problem. It turns out that such policy offers a solution much faster than the policy that is solely based on relaxation error.

In our numerical results, we further explore the properties of this cross-layer optimization problem. First and foremost, we show that performance gap between a cross-layer formulation and a decoupled-design is huge, thereby underlining the importance of cross-layer optimization. In addition, we observe that the number of time slots does not need to be large to have near-optimal performance. This result is important in practice as fewer number of slots will lead to less computation time in solving the optimization problem.

II. NETWORK MODEL AND OPTIMIZATION SPACE

We consider an ad hoc network consisting of N nodes and L uni-directional source-destination communication sessions over a two-dimensional area. We now take a closer look at each components of this cross-layer optimization problem.

Scheduling. At the link level, the scheduling problem deals with how to coordinate transmission among the nodes in each "time slot." An important constraint that must be met is that a node cannot send and receive data within the same time slot. Given the number of time slots K, denote t_k the *normalized length* for time slot k, i.e., the length of time slot k over the total length of all different time slots. We have

$$\sum_{k=1}^{K} t_k = 1 \ .$$

Power Control. Power control problem deals with how much power a node should employ to transmit data in a particular time slot. Denote p_{ij}^k as the power that node i expends in time slot k for sending data to node j. Although a node cannot send and receive within the same time slot, a node can transmit to multiple nodes within the same time slot. The total power that a node i can expend at time slot k must satisfy the following power limit [16],

$$\sum_{j\in\mathcal{T}_i} p_{ij}^k \le P_{\max} ,$$

where $\mathcal{T}_i$ is the set of one-hop neighbors of node i. This requirement comes from the power density limitation of UWB, i.e., $g_{\text{nom}} \sum_{j\in\mathcal{T}_i} p_{ij}^k / W \le Q_{\max}$, where $P_{\max} = \frac{Q_{\max} W}{g_{\text{nom}}}$, $Q_{\max}$ is the maximum allowed transmission power spectral density, g_{nom} is the gain at some fixed nominal distance [12], and $W = 7.5$ GHz is the spectrum for the UWB network.

A widely-used model for power gain is

$$g_{ij} = d_{ij}^{-\alpha} , \tag{1}$$

where d_{ij} is the distance between nodes i and j and α is the path loss index. Denote $\mathcal{I}_i$ as the set of nodes that can make interference at node i and η as the ambient Gaussian noise density. Then the achievable rate from node i to node j within time slot k is

$$c_{ij}^k = t_k W \log_2 \left(1 + \frac{g_{ij} p_{ij}^k}{\eta W + \sum_{m\in\mathcal{I}_j, q\in\mathcal{T}_m}^{(m,q)\ne(i,j)} g_{mj} p_{mq}^k}\right) . \tag{2}$$

Routing. The routing problem at the network level considers, for a session l, $1 \le l \le L$, how to relay a rate of $r(l)$ from source node $s(l)$ to the destination node $d(l)$. To take advantage of the multi-path availability within an ad hoc network (i.e., network diversity), we allow a source node to split its data into sub-flows and take different paths to the destination node. Denote $f_{ij}(l)$ the data rate that is attributed to the l-th session on link (i, j). If node i is the source node $s(l)$, then

$$\sum_{j\in\mathcal{T}_i} f_{ij}(l) = r(l) . \tag{3}$$

If node i is an intermediate relay node, i.e., $i \ne s(l)$ and $i \ne d(l)$, then we have the following flow balance:

$$\sum_{j\in\mathcal{T}_i}^{j\ne s(l)} f_{ij}(l) - \sum_{m\in\mathcal{T}_i}^{m\ne d(l)} f_{mi}(l) = 0 . \tag{4}$$

If node i is the destination node $d(l)$, then

$$\sum_{m\in\mathcal{T}_i} f_{mi}(l) = r(l) . \tag{5}$$

It can be easily verified that if (3) and (4) are satisfied, (5) must be satisfied. As a result, there is no need to list (5) in the formulation once we have (3) and (4).

Since the sum of data rates on link (i, j) cannot be greater than the link capacity, we have

$$\sum_{1\le l\le L}^{s(l)\ne j, d(l)\ne i} f_{ij}(l) \le \sum_{k=1}^{K} c_{ij}^k .$$

III. Maximizing Data Rate Utility Problem

In this section, we formulate the maximizing data rate utility problem as an optimization problem. We use $\sum_{l=1}^{L} \ln r(l)$ as a utility metric in the network optimization problem, although our proposed solution procedure is general and can be applied to other utility functions. The motivation for this choice is that such log-based utility function can make a good compromise between *fairness* and *efficiency* [10].

To ensure that a node cannot send and receive within the same time slot, we introduce the notion of *self-interference parameter* g_{jj} [7] with the property of $g_{jj} \gg \max\{g_{ij} : i \in \mathcal{T}_j\}$ and incorporate this into the bit rate calculation in (2)

$$c_{ij}^k = t_k W \cdot \log_2 \left(1 + \frac{g_{ij} p_{ij}^k}{\eta W + \sum_{m\in\mathcal{I}_j, q\in\mathcal{T}_m}^{(m,q)\ne(i,j)} g_{mj} p_{mq}^k + \sum_{q\in\mathcal{T}_j} g_{jj} p_{jq}^k}\right) . \tag{6}$$

Thus, when any $p_{jq}^k > 0$, i.e., node j is transmitting to a node q, we have $c_{ij}^k \approx 0$ even if $p_{ij}^k > 0$. In other words, when node j is transmitting to any node q, the link capacity on link (i, j) is *effectively* shut down to 0. To write (6) in a more compact form, we re-define $\mathcal{I}_i$ to include node i. Thus, (6) is now in the same form as in (2). Denote $Y_i^k = \sum_{m\in\mathcal{I}_i, q\in\mathcal{T}_m} g_{mi} p_{mq}^k$. We have

$$\begin{aligned} c_{ij}^k &= t_k W \log_2 \left(1 + \frac{g_{ij} p_{ij}^k}{\eta W + \sum_{m\in\mathcal{I}_j, q\in\mathcal{T}_m}^{(m,t)\ne(i,j)} g_{mj} p_{mq}^k}\right) \\ &= t_k W \log_2 \left(1 + \frac{g_{ij} p_{ij}^k}{\eta W + Y_j^k - g_{ij} p_{ij}^k}\right) . \end{aligned}$$

The maximum utility problem (MUP) can now be formulated as follows.

Maximum Utility Problem (MUP):

$$\begin{aligned} \text{Max} \quad & \sum_{l=1}^{L} \ln r(l) \\ \text{s.t.} \quad & \sum_{k=1}^{K} t_k = 1 \\ & \sum_{j\in\mathcal{T}_i} p_{ij}^k \le P_{\max} \quad (1 \le i \le N, 1 \le k \le K) \\ & Y_i^k = \sum_{m\in\mathcal{I}_i, q\in\mathcal{T}_m} g_{mi} p_{mq}^k \quad (1 \le i \le N, 1 \le k \le K) \\ & c_{ij}^k = t_k W \log_2 \left(1 + \frac{g_{ij} p_{ij}^k}{\eta W + Y_j^k - g_{ij} p_{ij}^k}\right) \\ & \qquad (1 \le i \le N, j \in \mathcal{T}_i, 1 \le k \le K) \qquad (7) \\ & \sum_{k=1}^{K} c_{ij}^k - \sum_{1\le l\le L}^{s(l)\ne j, d(l)\ne i} f_{ij}(l) \ge 0 \quad (1 \le i \le N, j \in \mathcal{T}_i) \end{aligned}$$

$$\sum_{\substack{j\in\mathcal{T}_i\\ j\neq s(l)}} f_{ij}(l) - r(l) = 0 \qquad (1 \le l \le L, i = s(l))$$

$$\sum_{\substack{j\in\mathcal{T}_i}} f_{ij}(l) - \sum_{\substack{m\in\mathcal{T}_i\\ m\neq d(l)}} f_{mi}(l) = 0 \qquad (1 \le l \le L, 1 \le i \le N, i \neq s(l), d(l))$$

$$r(l), f_{ij}(l) \ge 0 \quad (1\le l\le L, 1\le i\le N, i\neq d(l), j \in \mathcal{T}_i, j \neq s(l))$$

$$x_i^k = 0 \text{ or } 1, t_k, p_{ij}^k, c_{ij}^k \ge 0 \quad (1\le i\le N, j\in\mathcal{T}_i, 1\le k\le K)$$

To remove the non-polynomial term in (7), we use the linear rate-SINR property that is unique to UWB. That is, we have a linear approximation for log function, i.e., $\ln(1 + x) \approx x$ when $0 < x \ll 1$. We have $c_{ij}^k \approx \frac{t_k W}{\ln 2}\frac{g_{ij}p_{ij}^k}{\eta W + Y_j^k - g_{ij}p_{ij}^k}$, which is equivalent to

$$\eta W c_{ij}^k + Y_j^k c_{ij}^k - g_{ij}p_{ij}^k c_{ij}^k - \frac{W g_{ij}}{\ln 2} t_k p_{ij}^k = 0\,. \qquad (8)$$

Finally, without loss of generality, we let t_k have the following property: $t_1 \le t_2 \le \cdots \le t_k$. This ordering will help speed up the computation.

With the above re-formulations, we now have a revised MUP (or R-MUP) formulation, which is a *nonlinear programming* (NLP) problem and is NP-hard in general [3]. In the next section, we develop a solution procedure based on branch-and-bound [8] and the novel *Reformulation-Linearization Technique* (RLT) [13] to solve this NLP problem.

IV. A Solution Procedure to R-MUP

We find that branch-and-bound approach is most effective in solving our problem. Under the so-called branch-and-bound approach, we aim to provide a $(1-\varepsilon)$ optimal solution, where ε is a small positive constant reflecting our desired accuracy in the final solution. Initially, branch-and-bound analyzes all variables in non-linear terms (denote these variables as partition variables) and determines the value intervals for these variables. By using some *relaxation technique*, branch-and-bound obtains a linear programming (LP) relaxation for the original NLP problem; its solution provides an upper bound (UB) to the objective function. With the relaxation solution as a starting point, branch-and-bound uses a *local search* algorithm to find a feasible solution to the original NLP problem, which provides a lower bound (LB) for the objective function. If the obtained lower and upper bounds are close to each other, i.e., $LB \ge (1 - \varepsilon)UB$, we are done.

If the relaxation errors for non-linear terms are not small, then the lower bound LB could be far away from the upper bound UB. To close this gap, we must have a tighter LP relaxation, i.e., with smaller relaxation errors. This could be achieved by further narrowing down the value intervals of partition variables. Specifically, branch-and-bound selects a partition variable and divides its value interval into two intervals by its value in the relaxation solution. Then the original problem (denoted as problem 1) is divided into two new problems (denoted as problem 2 and problem 3). Again, branch-and-bound performs relaxation and local search on these two new problems. Now we have LB_2 and UB_2 for problem 2 and LB_3 and UB_3 for problem 3. Since the relaxations in problems 2 and 3 are both tighter than that in problem 1, we have $UB_2, UB_3 \le UB_1$ and $LB_2, LB_3 \ge LB_1$. The upper bound of the original problem is updated from $UB = UB_1$ to $UB = \max\{UB_2, UB_3\}$ and the lower bound of the original problem is updated from $LB = LB_1$ to $LB = \max\{LB_2, LB_3\}$. As a result, we now have smaller gap between UB and LB. If $LB \ge (1 - \varepsilon)UB$, we are done. Otherwise, we choose a problem with the maximum upper bound and perform partition for this problem.

Note that during the iteration process for branch-and-bound, if we find a problem z with $(1 - \varepsilon)UB_z \le LB$, then we can conclude that this problem cannot provide much improvement on LB. That is, further branch on this problem will not yield much improvement and we can thus remove this problem from further consideration. Eventually, once we find $LB \ge (1 - \varepsilon)UB$ or the problem list is empty, branch-and-bound procedure terminates. It has been proved that under very general conditions, a branch-and-bound solution procedure always converges [13].

In the rest of this section, we develop important components in the branch-and-bound solution procedure.

Initial Value Intervals for Partition Variables. For R-MUP, $t_k, p_{ij}^k, c_{ij}^k, r(l)$, and Y_i^k are the variables that are in nonlinear terms whose value intervals are candidates to be partitioned. It is easy to obtain the following bounds: $0 \le t_k \le \frac{1}{K+1-k}$, $0 \le p_{ij}^k \le P_{\max}$, $0 \le c_{ij}^k \le W\log_2\frac{g_{ij}P_{\max}}{\eta W}$, and $0 \le Y_i^k \le P_{\max}\sum_{m\in\mathcal{I}_i} g_{mi}$.

We now develop an upper bound for $r(l)$. Since $r(l)$ should be no more than the maximum transmission rate from source node $s(l)$ and the maximum receiving rate to destination node $d(l)$, we analyze these two end rates individually. At source node $s(l)$, its transmission upper bound $C_{s(l)}$ can be calculated by having node $s(l)$ transmit to its nearest neighbor with peak power on all time slots. Assuming the nearest neighbor of $s(l)$ is j, we have $C_{s(l)} = W\log_2\left(1 + \frac{g_{s(l),j}P_{\max}}{\eta W}\right)$. At destination node $d(l)$, it turns out that an upper bound for receiving rate $C_{d(l)}$ is achieved when each node $m \in \mathcal{T}_{d(l)}$ transmits to $d(l)$ with peak power in all time slots [14] and we have

$$C_{d(l)} = \sum_{m \in \mathcal{T}_{d(l)}} W \log_2 \left(1 + \frac{g_{m,d(l)} P_{\max}}{\eta W + P_{\max} \sum_{i \in \mathcal{T}_{d(l)}}^{i \neq m} g_{i,d(l)}} \right) .$$

Based on the rate analysis at source node $s(l)$ and destination node $d(l)$ for the l-th session, we have $0 \leq r(l) \leq \min\{C_{s(l)}, C_{d(l)}\}$.

Linear Relaxation. During each iteration of the branch-and-bound procedure, we need a linear relaxation to obtain an upper bound of the objective function. For the polynomial term, we propose to employ *Reformulation-Linearization Technique* (RLT) [13]. For the non-polynomial term (i.e., $\log$ term), we propose to employ three tangential supports, which is a convex envelope linear relaxation.

We first show how RLT can obtain a linear relaxation for a polynomial term. Specifically, $Y_j^k c_{ij}^k$, $p_{ij}^k c_{ij}^k$, and $t_k p_{ij}^k$ in (8) are polynomial terms. RLT enables to use new variables to replace those polynomial terms and add linear constraints for these new variables, thus relaxing nonlinear constraint into linear constraints.

As an example, we introduce a new variable u_{ij}^k for $Y_j^k c_{ij}^k$, i.e., $u_{ij}^k = Y_j^k c_{ij}^k$. Assume $(Y_j^k)_L \leq Y_j^k \leq (Y_j^k)_U$ and $(c_{ij}^k)_L \leq c_{ij}^k \leq (c_{ij}^k)_U$, we have $[Y_j^k - (Y_j^k)_L] \cdot [c_{ij}^k - (c_{ij}^k)_L] \geq 0$, $[Y_j^k - (Y_j^k)_L] \cdot [(c_{ij}^k)_U - c_{ij}^k] \geq 0$, $[(Y_j^k)_U - Y_j^k] \cdot [c_{ij}^k - (c_{ij}^k)_L] \geq 0$, and $[(Y_j^k)_U - Y_j^k] \cdot [(c_{ij}^k)_U - c_{ij}^k] \geq 0$. From the above relationships, we obtain the following *linear* constraints (also called RLT constraints [13]) for u_{ij}^k.

$$(Y_j^k)_L \cdot c_{ij}^k + (c_{ij}^k)_L \cdot Y_j^k - u_{ij}^k \leq (Y_j^k)_L \cdot (c_{ij}^k)_L$$
$$(Y_j^k)_U \cdot c_{ij}^k + (c_{ij}^k)_L \cdot Y_j^k - u_{ij}^k \geq (Y_j^k)_U \cdot (c_{ij}^k)_L$$
$$(Y_j^k)_L \cdot c_{ij}^k + (c_{ij}^k)_U \cdot Y_j^k - u_{ij}^k \geq (Y_j^k)_L \cdot (c_{ij}^k)_U$$
$$(Y_j^k)_U \cdot c_{ij}^k + (c_{ij}^k)_U \cdot Y_j^k - u_{ij}^k \leq (Y_j^k)_U \cdot (c_{ij}^k)_U$$

Through this relaxation, we can replace $Y_j^k c_{ij}^k$ with u_{ij}^k in (8) and adding RLT constraints for u_{ij}^k into the R-MUP formulation. Following the same token, we can have linear relaxation for all polynomial terms.

Now we show how to obtain a linear relaxation for a non-polynomial term. We can denote $h_l = \ln r(l)$ for $\ln r(l)$. Note that the function $y = \ln x$, over suitable bounds of x, can be bounded by four segments (or a convex envelope), where segments I, II, and III are tangential supports and segment IV is the chord (see Fig. 1). In particular, three tangent segments are at $(x_L, \ln x_L)$, $(\beta, \ln \beta)$, and $(x_U, \ln x_U)$, where $\beta = \frac{x_L \cdot x_U \cdot (\ln x_U - \ln x_L)}{x_U - x_L}$ is the horizontal location for the point intersects extended tangent segments I and III; segment IV is the segment that joins points $(x_L, \ln x_L)$ and $(x_U, \ln x_U)$. The convex region defined by the four segments can be described by the following four *linear* constraints.

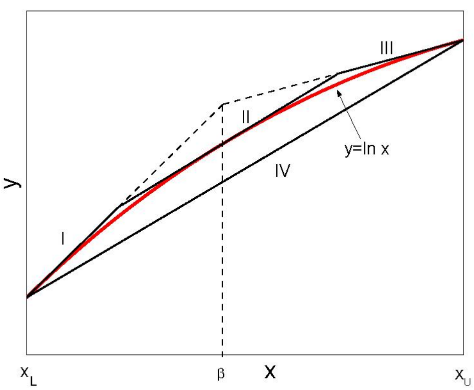


Fig. 1. A convex envelope for $y = \ln x$.

$$x_L \cdot y - x \leq x_L(\ln x_L - 1)$$
$$\beta \cdot y - x \leq \beta(\ln \beta - 1)$$
$$x_U \cdot y - x \leq x_U(\ln x_U - 1)$$
$$(x_U - x_L)y + (\ln x_L - \ln x_U)x \geq x_U \cdot \ln x_L - x_L \cdot \ln x_U$$

As a result, the non-polynomial (log) term can also be relaxed into linear objective and constraints.

Local Search Algorithm. For a problem z, the local search algorithm determines a feasible solution ψ_z based on the relaxation solution $\hat{\psi}_z$. Denote $\mathbf{t}, \mathbf{p}$, and $\mathbf{f}$ as the vectors for variables t_k, p_{ij}^k, and $f_{ij}(l)$, respectively. In our local search, we set $\mathbf{t} = \hat{\mathbf{t}}$.

Note that in R-MUP, we introduced the notion of self-interference parameter to remove the binary variables in MUP. Then in $\hat{\mathbf{p}}$, it is possible that $\hat{p}_{ij}^k > 0$ and $\hat{p}_{mi}^k > 0$ for certain node i within a time slot k. Therefore, it is necessary to find a new $\mathbf{p}$ from $\hat{\mathbf{p}}$ by changing some $\hat{p}_{ij}^k$ or $\hat{p}_{mi}^k$ to 0 such that no node can transmit and receive within the same time slot. We follow the following two guidelines when we set such transmission power to 0. First, we try to maintain the same connectivity in ψ_z as that in $\hat{\psi}_z$ wherever possible. Second, we try to split the total time slots used at a node i into two groups of equal length wherever possible, one group for transmission and the other group for receiving. More details can be found in [14].

After we obtain $\mathbf{t}$ and $\mathbf{p}$ for ψ_z, we can compute c_{ij}^k from (7). Then an optimal routing solution $\mathbf{f}$ (under $\mathbf{t}$ and $\mathbf{p}$) can be obtained by solving a concave optimization problem through standard approach [14].

Additional Details. Note that branch-and-bound chooses a partition variable in the nonlinear term with the maximum relaxation error, where the relaxation error for a nonlinear term is the difference between the value of this term and the value of its corresponding new variable

TABLE I

DESCRIPTION OF 5 UNI-DIRECTIONAL SESSIONS IN A 15-NODE NETWORK.

Session Index	$s(l) \Rightarrow d(l)$	Session Index	$s(l) \Rightarrow d(l)$
1	$9 \Rightarrow 15$	4	$2 \Rightarrow 12$
2	$4 \Rightarrow 5$	5	$1 \Rightarrow 11$
3	$10 \Rightarrow 2$		

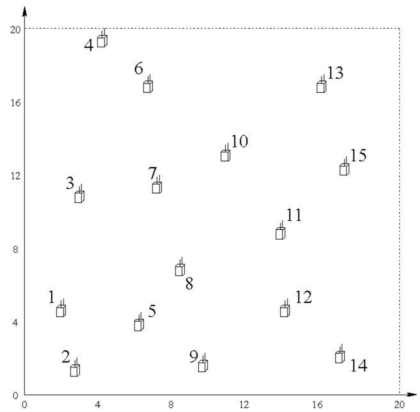


Fig. 2. A 15-node ad hoc network with 5 sessions.

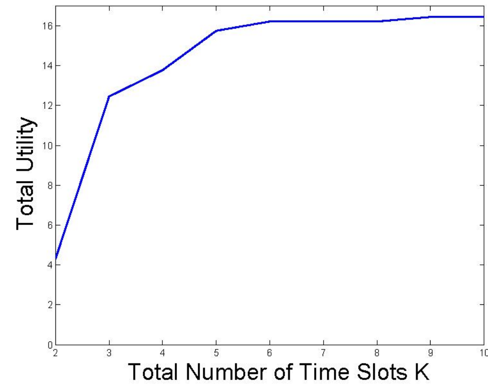


Fig. 3. The total utility as a function of total available time slots K for five sessions.

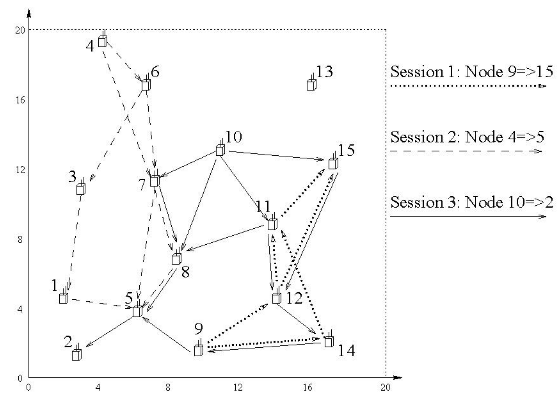


Fig. 4. Optimal routing for three sessions with $K = 6$

in the relaxation solution. If this nonlinear term has multiple variables, e.g., $Y_j^k c_{ij}^k$, then we need to choose a partition variable from Y_j^k and c_{ij}^k. Specifically, if $((Y_j^k)_U - (Y_j^k)_L) \cdot \min\{\hat{Y}_j^k - (Y_j^k)_L, (Y_j^k)_U - \hat{Y}_j^k\} \geq ((c_{ij}^k)_U - (c_{ij}^k)_L) \cdot \min\{\hat{c}_{ij}^k - (c_{ij}^k)_L, (c_{ij}^k)_U - \hat{c}_{ij}^k\}$ we partition on Y_j^k and obtain two new value intervals $[(Y_j^k)_L, \hat{Y}_j^k]$ and $[\hat{Y}_j^k, (Y_j^k)_U]$. Otherwise, we partition on c_{ij}^k.

For our specific problem, by exploiting the physical interpretation of certain variable and weighing its significance, further improvement can be made on partition variable selection policy. For example, it is clear that variable t_k directly affects the final solution. As a result, the algorithm will run much more efficiently if we give it higher priority when we choose a partition variable. This is precisely what we have done in our algorithm implementation, where we give the highest priority to t_k, the second highest priority to p_{ij}^k, then c_{ij}^k, and consider Y_i^k last when we choose a partition variable. Note that this choice will not hamper the convergence property of the algorithm [13], although it will yield different computational time.

There are two types of problems that can be eliminated before solving their LP relaxations. In the first case, if a problem is found to be infeasible, then there is no need to solve a full scale LP relaxation. For example, after we partition on p_{ij}^k, if a node must send and receive within the same time slot in a new problem, i.e., $(p_{ij}^k)_L > 0$ and $(p_{mi}^k)_L > 0$, then this new problem must be infeasible.

In the second case, if a problem cannot provide significant improvement, then there is no need to solve a full scale LP either. For example, after we partition on $r(l)$, if $(1 - \varepsilon) \sum_{l=1}^{L} \ln(r(l))_U \leq LB$, then this new problem cannot provide significant improvement and can be eliminated from problem list.

V. SIMULATION RESULTS

In this section, we present some important numerical results to offer further insights on the optimization problem. These results are important as they are not obvious from our theoretical development of the solution procedure in the last section.

We first describe the simulation settings. We consider a randomly generated network of 15 nodes deployed over a 20×20 area. There are 5 sessions (see Table I and Fig. 2). All distances are based on normalized length in (1). The path loss index is $\alpha = 2$ and the nominal gain is chosen as $g_{\text{nom}} = 0.02$. The power density limit $Q_{\max}$ is assumed to be 1% of the white noise η [12].

Scheduling. We first investigate how the total utility is affected when the total available time slots K change.

TABLE II

TOTAL UTILITY AND RATE UNDER DIFFERENT TIME SLOT LENGTH ALLOCATION POLICIES WITH $K = 6$ AND $L = 3$.

Allocation of Normalized Time Slot Length	$\sum_{l=1}^{L} \ln r(l)$	$\sum_{l=1}^{L} r(l)$
Optimal: (0.14, 0.14, 0.14, 0.15, 0.21, 0.22)	11.61	149.14
Equal: (0.16, 0.16, 0.17, 0.17, 0.17, 0.17)	9.51	78.42
Random: (0.11, 0.11, 0.13, 0.16, 0.20, 0.29)	9.15	69.04
Random: (0.14, 0.15, 0.16, 0.18, 0.18, 0.19)	9.63	84.14

TABLE III

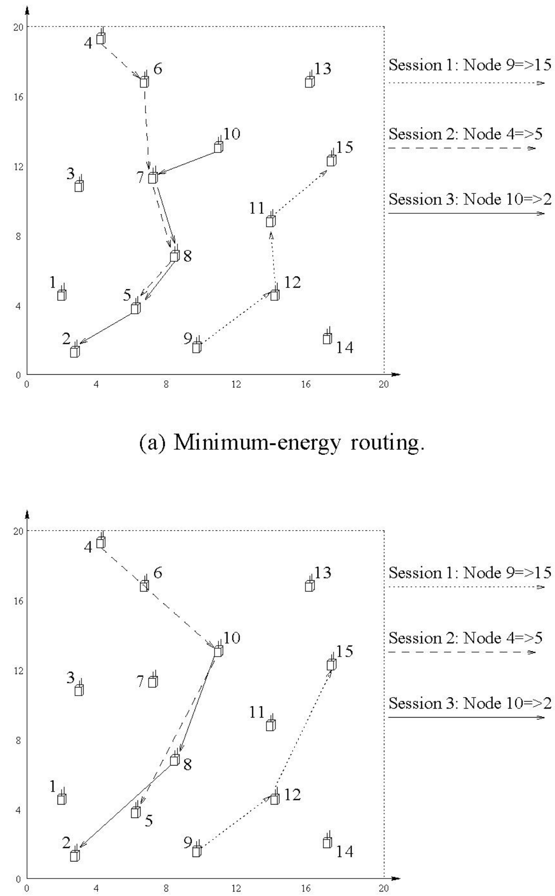
TOTAL UTILITY AND RATE UNDER DIFFERENT ROUTING STRATEGIES FOR $K = 6$ AND $L = 3$.

Routing Strategy	$\sum_{l=1}^{L} \ln r(l)$	$\sum_{l=1}^{L} r(l)$
Optimal Routing	11.61	149.14
Minimum-Energy Routing	9.39	117.25
Minimum-Hop Routing	8.63	53.99

One would expect the total utility is a non-decreasing function of K. This is because the more time slots available, the more opportunity (or larger design space) for each node to avoid interference with other nodes, and thus, this yields higher utility for the network. However, there is a price to pay for having a large K. The larger the K is, the larger the total length of all time slots in a cycle, and thus the larger delay at each node. Further, the larger the K, the larger the size of our optimization problem, which means more computational time will incur. Therefore, we wish to choose a suitably K that can provide near-optimal performance.

The total utility under different K is shown in Fig. 3. Since multi-hop is not allowed when $K = 1$ (a node cannot send and receive within the same time slot), the minimum required K is 2. As expected, the total utility is a non-decreasing function of K. But what really is interesting is that there is a "knee" point in this performance figure, i.e., when K is beyond 6, there is hardly much increase in the total utility. This suggests that for practical purpose, it is sufficient to choose $K = 6$ time slots for this particular network.

We now show how the allocation of length for each time slot affects the performance. To plot the complete routing topology (multi-path for each session) legibly on a figure, We use the first three sessions ($l = 1, 2, 3$) in Table I for this investigation. Figure 4 shows an optimal routing for these three sessions. Clearly, an optimal routing for each session is a multi-path routing. Given this routing topology, Table II shows the results under different allocation for normalized time slot length. The first policy is the optimal length allocation obtained via our solution to the NLP problem, which apparently



(a) Minimum-energy routing.

(b) Minimum-hop routing.

Fig. 5. Other routing strategies for $L = 3$ in the 15-node network.

offers the largest total utility. The second time slot length allocation policy is equal allocation. Finally, we also list the performance under two random time slot length allocations. Note that equal allocation is not a good policy and has about the same performance as a random allocation policy.

Routing. For a given optimal time slot length (obtained through our solution procedure), we now study the impact of routing on our cross-layer optimization problem. In addition to our cross-layer optimal routing, we also consider the following two routing approaches, namely, minimum-energy routing and minimum-hop routing. Under the minimum-energy routing, the energy cost is defined as d_{ij}^2 for link (i, j). Table III shows the results in this study. Clearly, the cross-layer optimal routing outperforms both minimum-energy and minimum-hop routing approaches. Both minimum-energy routing and minimum-hop routing are minimum-cost routing (with different link cost). Minimum-cost routing only uses a single-path, i.e., multi-path routing is not allowed, which may not provide good solution. Moreover, it is very likely that multiple sessions share the same path (see path $7 \to 8 \to 5$ in Fig. 5(a)). Thus, the rates for these sessions are bounded by the capacity of this path.

VI. Related Work

An overview of UWB technology is given [9]. Physical layer issues associated with UWB-based multiple access communications can be found in [4], [5], [15] and references therein. In this section, we focus our attention on related work addressing networking related problems for UWB-based networks.

In [6], Negi and Rajeswaran first showed that, in contrast to previously published results, the throughput for UWB-based ad hoc networks increases with node density. This result demonstrates the significance of physical layer properties on network layer metrics such as network capacity. In [2], Cuomo et al. studied a multiple access scheme for UWB. The impact of routing, however, was not addressed.

The most relevant research to this work are [7] and [11]. In [7], Negi and Rajeswaran studied how to proportionally maximize rates in a single-hop UWB network. In contrast, we consider a multi-hop network environment where routing is also part of the cross-layer optimization problem. As a result, our problem is considerably more difficult. In [11], Radunovic and Le Boudec explored the same problem as we studied in this paper. The authors studied some simple cases and attempted to generalize the results from these simple cases to a network with general topology. Many of these results are heuristic in nature. On the other hand, in this paper, we have developed a formal solution procedure to solve the complex cross-layer optimization problem instead of resorting to heuristics.

VII. Conclusions

In this paper, we studied the data rate utility problem in a UWB-based ad hoc network. We formulated the problem into a nonlinear programming with consideration of link layer and network layer variables. We proposed a solution procedure based on branch-and-bound framework. A powerful technique that we employed in branch-and-bound framework is the reformulation linearization technique, which is able to replace nonlinear terms in the constraints with linear ones. Numerical results demonstrate the significance of cross-layer consideration and offers insights on the impact of each component in the optimization space.

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References

[1] M.S. Bazaraa, H.D. Sherali, and C.M. Shetty, *Nonlinear Programming: Theory and Algorithms,* second edition, John Wiley & Sons, Inc., New York, NY, 1993.

[2] F. Cuomo, C. Martello, A. Baiocchi, and F. Capriotti, "Radio resource sharing for ad hoc networking with UWB," *IEEE J. on Selected Areas in Commun.*, vol. 20, no. 9, pp. 1722–1732, Dec. 2002.

[3] M.R. Garey and D.S. Johnson, *Computers and Intractability: A Guide to the Theory of NP-completeness,* pp. 245–248, W. H. Freeman and Company, New York, NY, 1979.

[4] IEEE 802.15 WPAN High Rate Alternative PHY Task Group 3a, `http://grouper.ieee.org/groups/802/15/pub/TG3a.html`.

[5] *IEEE Journal on Selected Areas in Communications – Special Issue on Ultra-Wideband Radio in Multiaccess Wireless Communications,* Guest Editors: N. Blefari-Melazzi, M.G. Di Benedettio, M. Geria, H. Luediger, M.Z. Win, and P. Withington, vol. 20, no. 9, Dec. 2002.

[6] R. Negi and A. Rajeswaran, "Capacity of power constrained ad-hoc networks," in *Proc. IEEE INFOCOM,* pp. 443–453, Hong Kong, China, March 7-11, 2004.

[7] R. Negi and A. Rajeswaran, "Scheduling and power adaptation for networks in the ultra wide band regime," in *Proc. IEEE Globecom,* pp. 139–145, Dallas, TX, Nov. 29-Dec. 3, 2004.

[8] G.L. Nemhauser and L.A. Wolsey, *Integer and Combinatorial Optimization,* pp. 354–367, John Wiley & Sons, New York, NY, 1999.

[9] R.C. Qiu, H. Liu, and X. Shen, "Ultra-wideband for multiple access communications," *IEEE Communications Magazine,* vol. 43, issue 2, pp. 80–87, Feb. 2005.

[10] B. Radunovic and J.-Y. Le Boudec, "Rate performance objectives of multi-hop wireless networks," in *Proc. IEEE INFOCOM,* pp. 1916–1927, Hong Kong, China, March 7-11, 2004.

[11] B. Radunovic and J.-Y. Le Boudec, "Optimal power control, scheduling, and routing in UWB networks," *IEEE J. Selected Areas in Commun.*, vol. 22, no. 7, pp. 1252–1270, Sept. 2004.

[12] A. Rajeswaran, G. Kim, and R. Negi, "A scheduling framwork for UWB & cellular networks," in *Proc. First International Conference on Broadband Networks (Broadnets),* pp. 386–395, San Jose, CA, Oct. 25-29, 2004.

[13] H.D. Sherali and W.P. Adams, *A Reformulation-Linearization Technique for Solving Discrete and Continuous Nonconvex Problems,* Chapter 8, Kluwer Academic Publishers, Dordrecht/Boston/London, 1999.

[14] Y. Shi, Y.T. Hou, and H.D. Sherali, "Nonlinear Optimization of Data Rate Utility Problem for UWB-based Ad Hoc Networks," *Technical Report,* the Bradley Department of Electrical and Computer Engineering, Virginia Tech, Blacksburg, VA, Oct. 2005. Available at `http://www.ece.vt.edu/thou/Research.html`.

[15] M. Win and R. Scholtz, "Ultra-wide bandwidth time-hopping spread-spectrum impulse radio for wireless multiple-access communications," *IEEE Trans. on Commun.*, vol. 48, issue 4, pp. 679–691, April 2000.

[16] H. Zhang and J. Hou, "Capacity of wireless ad-hoc networks under ultra wide band with power constraint," in *Proc. IEEE INFOCOM,* pp. 455–465, Miami, FL, March 13-17, 2005.