Maximizing the Lifetime of Wireless Sensor Networks through Optimal Single-Session Flow Routing

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Abstract—Wireless sensor networks are becoming increasingly important in recent years due to their ability to detect and convey real-time, in-situ information for many civilian and military applications. A fundamental challenge for such networks lies in energy constraint, which poses a performance limit on the achievable network lifetime. We consider a two-tier wireless sensor network and address the network lifetime problem for upper-tier *aggregation and forwarding nodes* (AFNs). Existing flow routing solutions proposed for maximizing network lifetime require AFNs to split flows to different paths during transmission, which we call *multisession* flow routing solutions. If an AFN is equipped with a single transmitter/receiver pair, a multisession flow routing solution requires a packet-level power control at the AFN so as to conserve energy, which calls for considerable overhead in synchronization among the AFNs. In this paper, we show that it is possible to achieve the same optimal network lifetime by power control on a much larger timescale with the so-called *single-session* flow routing solutions, under which the packet-level power control and, thus, strict requirement on synchronization are not necessary. We also show how to perform optimal single-session flow routing when the bit-rate of composite flows generated by AFNs is time-varying, as long as the average bit-rate can be estimated.

Index Terms—Sensor networks, network lifetime, energy constraint, power control, flow routing.

1 Introduction

Wireless sensor networks have recently found many new applications that will have significant impact throughout our society. In this paper, we consider a two-tier wireless sensor network that can be deployed for various sensing applications. This type of sensor network consists of a number of *sensor clusters* and a *base-station*. Each cluster is deployed around a strategic location and consists of a number of wireless *microsensor nodes* (MSNs) and one *aggregation and forwarding node* (AFN). Each MSN is able to capture and transmit data to an AFN that performs in-network information processing by aggregating all correlated data received in the same cluster (also known as *data fusion*). The AFN then sends the composite flow to the base-station through single or multihop data transmission.

One of the most important performance measures for wireless sensor networks is *network lifetime.* For a two-tier wireless sensor network considered in this paper, whenever an AFN runs out of energy, the sensing capability for that cluster is completely lost from the viewpoint of the base-station. Therefore, the most stringent definition of network lifetime would be the time until any AFN fails due to depletion of energy. Since the lifetime of each individual AFN heavily depends on its energy consumption behavior and the majority of power consumption at an AFN is due to its radio communication, it is essential to devise strategies that can minimize radio-related power consumption at AFNs. One promising approach to maximizing network lifetime is to control the output power level of radio transmitters. Since the output power level of a radio transmitter directly affects its coverage, it is important to utilize the relay capability among AFNs to forward composite flows. This offers an opportunity to dynamically control the output power level of AFNs, so that different network routing topologies can be formed and network lifetime can be extended.

This paper investigates optimal network flow routing among upper-tier AFNs with dynamic power control at AFNs, so that network lifetime can be maximized. Existing solutions to this problem, obtained under linear programming (LP) (see, e.g., [8]), require each AFN to split data flows to multiple paths during transmission, which we call *multisession flow routing solutions*. With this approach, when an AFN is equipped with a single transmitter/receiver pair, it is necessary for the AFN to perform power control at *packet-level* to conserve energy, which calls for stringent requirement in synchronization among the AFNs. Although synchronization techniques are available (see, e.g., [13], [14], [22], [37]), all these methods rely on exchanging time-stamped messages between two (or more) nodes to compute the relative drift and offset between the node clocks. To guarantee packet-level power control between a transmitter and a receiver, the synchronization requirement is stringent and will bring in considerable overhead.

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A naive alternative is to have each AFN be equipped with multiple transmitters, each of them corresponding to an outgoing flow. Since the number of concurrent flows from an AFN is in the order of $O(N)$, where N is the number of total AFNs, this approach is clearly not scalable.

In this paper, we explore a completely different approach, which we call *single-session* flow routing solutions, where no flow splitting is allowed. We are interested in achieving the same optimal network lifetime by having each AFN perform power control and topology change on a much larger time scale than the per-packet level, as in existing solutions. As a result, the synchronization requirement is extremely low and its overhead is negligible when compared to packet-level multisession flow routing solutions.

In addition to reducing the synchronization requirement, the single-session flow routing solution developed in this paper suits perfectly well when directional antennas are employed by AFNs. Directional antennas have significant advantages over omni-directional antennas in terms of minimizing communication interference and reducing power consumption. In this paper, we lay the theoretical foundation that, under omni-directional antennas, a single-session flow routing solution can achieve the same maximum network lifetime as that with a multisession flow routing solution. Consequently, this result implies that, under directional antennas (where single-session flow routing solution is necessary in many cases), many folds of network lifetime improvement can be achieved.

The goal of this investigation is to develop single-session flow routing solutions, where routing topologies are relatively static and are adjusted (via power control) on a large timescale, so that the network lifetime can be maximized. To achieve this objective, we first show that an optimal multisession solution obtained through the LP approach (e.g., [8]) can be transformed into an *equivalent* single-session flow routing solution. By *equivalent*, we mean that the maximum network lifetimes under both approaches are identical. Furthermore, the consumed energy at each AFN are identical at the end of network lifetime under both approaches. In the second part of this paper, we move on to investigate single-session flow routing solutions when the bit-rate from each AFN is time-varying, as long as the average bit-rate can be estimated beforehand. We present an equivalence theorem that shows that an optimal single-session flow routing solution for a sensor network of variable bit-rate AFNs can be obtained from an *auxiliary* network of constant bit-rate AFNs. We also show that, as long as the estimated average bit-rate is close to the actual value, the network lifetime achieved by single-session flow routing solutions is indeed approaching to the optimum.

The remainder of this paper is organized as follows: In Section 2, we present a reference model for a two-tier wireless sensor network and discuss power consumption behavior of upper-tier AFNs. In Section 3, we show how an optimal multisession flow routing solution can be transformed into an equivalent single-session flow routing solution. Section 4 studies the optimal single-session flow routing problem when the bit-rate from each AFN is time-varying. Section 5 reviews related work, and Section 6 concludes this paper.

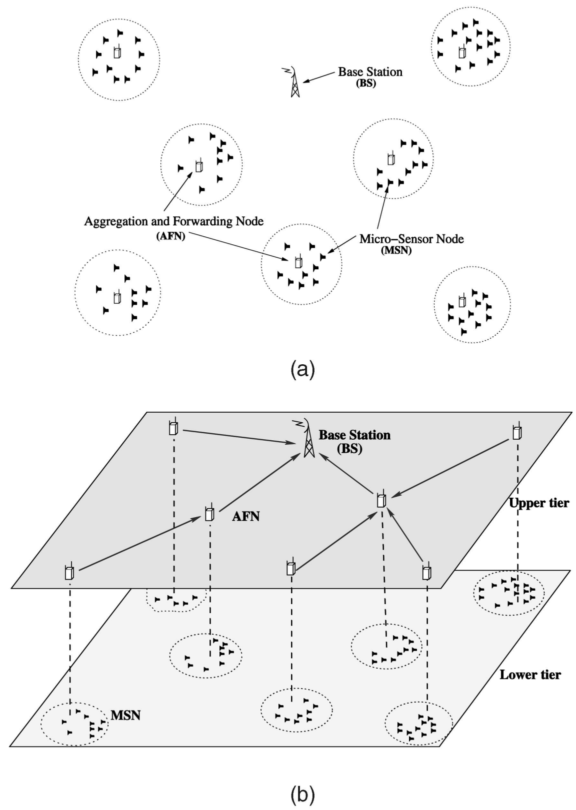

Fig. 1. Reference architecture for a two-tier wireless sensor network. (a) Physical topology. (b) A logical topology.

2 Network Reference Model

2.1 A Two-Tier Architecture for Sensor Networks

We focus on a two-tier architecture for wireless sensor networks, which was motivated by recent advances in distributed source coding (DSC) for sensor networks [9], [32]. Figs. 1a and 1b show the *physical* topology and a snapshot of the *logical* routing topology of such network, respectively. As shown in these figures, we have three types of nodes in the network: *microsensor nodes* (MSNs), *aggregation and forwarding nodes* (AFNs), and a *base-station* (BS). MSNs constitute the *lower-tier* of the network and are deployed in groups (or clusters) around strategic locations for various sensing applications. Each MSN is small and low-cost and can be densely deployed within a small geographical area. The objective of an MSN is very simple: once triggered by an event (e.g., detection of motion or biological/chemical agents), the MSN starts to capture live data (video, audio, or scalar measurement), which it sends directly to the local AFN in one hop. It is worth pointing out that multihop routing among MSNs is not necessary due to the small distance between an MSN and the local AFN. For such a small distance (e.g., less than 50 meters), the distance-dependent power consumption term is negligible (also see (1)). Therefore, the radio-related power consumption for an MSN is only determined by the distance-independent term. In this case, there is no advantage of using multihop routing among MSNs to forward data streams to the AFN. By deploying these inexpensive MSNs densely in clusters and within proximity of a strategic location, it is possible to obtain a comprehensive view of the area by exploring the correlation among the data collected by each MSN. Furthermore, the reliability of surveillance capability can be improved through redundant data collected by the MSNs in the same cluster.

Within each cluster of MSNs, there is one AFN, which is different from an MSN in terms of its physical structure and logical functions. The primary functions of an AFN include: 1) *data aggregation* (or *fusion*) for data received from the local MSNs and 2) *forwarding* (or *relaying*) the aggregated composite flows (including flows from other AFNs) to the next-hop AFN toward the base-station. For data fusion, the AFN analyzes the content of each data stream received from MSNs and then aggregates all the information through DSC [9], [32]. Here, we assume the positions of MSNs and AFNs are static after being deployed.

In addition to receiving data streams from MSNs within the local cluster and performing information fusion among the received data, an AFN has an important networking function for the upper-tier AFNs: It serves as a *relay node* for other AFNs to forward their data toward the base-station. Although an AFN is expected to be provisioned with much more energy than an MSN, it also consumes energy at a substantially higher rate (due to wireless communication over greater distances). Consequently, an AFN has a limited lifetime. Upon the depletion of energy at an AFN, the *coverage* for that particular area is lost.

The last component within the two-tier architecture is the base-station, which is the *sink* node for flows generated by all AFNs in the network. We assume that the base-station has sufficient energy provisioning (e.g., direct power supply) or its energy may be reprovisioned over time. Therefore, the base-station is not subject to the energy constraint. Further, we assume that the base station has complete knowledge of each AFN's location in the network, its initial energy, and bit generation rate. Such information can be obtained with many proposed localization techniques in the literature (see, e.g., [2], [6], [16], [20], [27]) and its discussion is beyond the scope of this paper.

In summary, the main function of the lower-tier MSNs is data acquisition, while the upper-tier AFNs are used for data fusion and forwarding the aggregated flows toward the base-station. Although the physical topology is static, there is a great degree of flexibility in terms of how the network routing topology can be formed to forward data flows from an AFN to the base-station. Power control at the transmitter of an AFN determines the radio coverage of an AFN, which, in turn, affects the network routing topology [15], [31], [34], [40]. In the remainder of this paper, we will explore how to perform single-session flow routing among the AFNs (with power control on a large time scale) so that network lifetime can be maximized.

2.2 Power Consumption Model

A detailed power consumption model for each component in a wireless sensor node can be found in [17]. For an AFN, the radio-related power consumption (i.e., in transmitter and receiver) is the dominant factor [1]. When AFN i transmits data to AFN k, the power consumption at the transmitter can be modeled as

$$p_{ik}^{t} = c_{ik} \cdot f_{ik}, \tag{1}$$

where f_{ik} (in b/s) is the bit-rate of the flow sent by AFN i to AFN k. Here, c_{ik} is the power consumption cost of link (i, k), and

$$c_{ik} = \alpha + \beta \cdot d_{ik}^{n}, \tag{2}$$

where α is a *distance-independent* term, β is a coefficient associated with the *distance-dependent* term, d_{ik} is the distance between these two nodes, n is the path loss exponent, and $2 \le n \le 4$ [33]. Typical values of these parameters are $\alpha = 50$ nJ/b and $\beta = 0.0013$ nJ/b/m^4 when $n = 4$ [17]. In this paper, we adopt $n = 4$ for all of our numerical results.

The power consumption at the receiver of AFN j can be modeled as [33]:

$$p_{j}^{r} = \rho \cdot \sum_{k \neq j} f_{kj}, \tag{3}$$

where f_{kj} (also in b/s) is the incoming bit-rate of the composite flow received by AFN j from AFN k. A typical value of ρ is 50 nJ/b [17].

3 Optimal Single-Session Flow Routing Solution

In this section, we show that a multisession flow routing solution can be transformed into an equivalent single-session flow routing solution.

3.1 State-of-the-Art and Its Limitations

We discuss the existing multisession flow routing solution by following closely to the work of Chang and Tassiulas in [8]. Suppose that the data flow's bit-rate generated by AFN i is g_i and the initial energy at AFN i is e_i. Denote T as the network lifetime for AFNs, i.e., the time duration from network initialization until any AFN drains out of energy. We then have the following incoming/outgoing flow balance equations and energy constraints for each AFN i ($i = 1, 2, \cdots, N$):

$$g_i + \sum_{m \neq i} f_{mi} = \sum_{k \neq i} f_{ik} + f_{iB}, \tag{4}$$

$$T \cdot \rho \sum_{m \neq i} f_{mi} + T \cdot \sum_{k \neq i} c_{ik} f_{ik} + T \cdot c_{iB} f_{iB} \le e_i, \tag{5}$$

where f_{ik} and f_{iB} denote the flow rates from AFN i to AFN k and to base-station B, respectively. The first N equations in (4) state that, at each AFN i, the bit-rate of the flow g_i generated by i, plus the total bit-rate of incoming flows received by i from other AFNs, is equal to the total bit-rate of outgoing flows transmitted from i. The next N inequalities in (5) state that the energy required to receive and transmit all these flows at each AFN i, at the end of network lifetime T, cannot exceed its energy constraint. Our objective is to maximize T while both (4) and (5) are satisfied.

To formulate an optimization problem for network flow routing, let $V_{ik} = f_{ik} T$ and $V_{iB} = f_{iB} T$, where V_{ik} and V_{iB} are the bit-volumes being sent from AFN i to k and B for T, respectively. We obtain the following linear programming (LP) formulation:

Max T

s.t.

$$g_i T + \sum_{m \neq i} V_{mi} - \sum_{k \neq i} V_{ik} - V_{iB} = 0 \quad (1 \leq i \leq N), \tag{6}$$

$$\sum_{m \neq i} \rho V_{mi} + \sum_{k \neq i} c_{ik} V_{ik} + c_{iB} V_{iB} \leq e_i \quad (1 \leq i \leq N), \tag{7}$$

where (6) is from the balance equations in (4) and (7) is from the energy constraints in (5). Note that T, V_{mi}, V_{ik}, and V_{iB} are variables and that g_i, ρ, c_{ik}, c_{iB}, and e_i are all constants.

We now have a standard LP formulation, i.e., **Max** cx, **s.t.** $Ax \leq b$ and $x \geq 0$. For each AFN i, we denote set Q_i containing all the AFNs k satisfying $d_{ik} < d_{iB}$, i.e., AFNs in Q_i are within the radius from AFN i to the base-station B. Then, our variable space for the LP formulation can be further reduced if we take into consideration that, for AFN i, only AFNs in Q_i may be chosen as relay nodes; that is, we can remove variable f_{ik} when $k \notin Q_i$.

Clearly, such an LP approach will yield a *multisession* flow routing solution, which requires a flow from a node to be split into multiple paths during transmission. When an AFN is equipped with a single transmitter/receiver pair, the AFN is required to perform a packet-level power control to implement the multisession flow routing solution so as to reach different next-hop nodes. As discussed in Section 1, such packet-level transmission/reception calls for stringent requirement in synchronization among the AFNs, which bring in considerable overhead.

3.2 Our Approach: Single-Session Flow Routing

In this section, we propose a completely different approach for flow routing, which we call *single-session* flow routing. Under this approach, power control and topology change are only done on a much larger time scale instead of on the per-packet basis. As a result, the synchronization requirement is extremely low and its overhead is negligible when compared to packet-level multisession flow routing solutions.

Our main contribution is to show that a multisession flow routing solution can be transformed into an equivalent single-session flow routing solution. By *equivalent*, we mean that both flow routing solutions have the same network lifetime. Besides preserving their flow balance, we also require that the per-node energy consumption at the end of network lifetime are identical under both solutions.

Theorem 1. *Suppose that we have a multisession flow routing solution ψ with maximum network lifetime T for a sensor network. Then, there exists an equivalent single-session flow routing solution $\hat{\psi}$ for the same network.*

Theorem 1 can be proved by constructing a single-session flow routing solution (denoted as $\hat{\psi}$) for a given multisession flow routing solution ψ, and showing that $\hat{\psi}$ is equivalent to ψ according to our criteria. In the following, we will describe such an algorithm. Before we perform the transformation, it is important to remove all flow cycles in ψ. This is necessary to ensure that upon the termination of the algorithm, the flow routing of each AFN will be in single-session mode. Here, a flow cycle in ψ refers to a directed cycle composed of directed links each carrying a positive flow. Cycle detection and removal procedures can use depth-first search and mark algorithms, which are discussed in the literature (see, e.g., [10]). Therefore, we will not discuss them further in this paper. It is worth pointing out that, after a cycle detection and removal procedure, the network lifetime will be identical to that obtained by solving the LP formulation.

After performing cycle detection and removal procedures, we obtain a cycle-free multisession flow routing solution ψ with maximum network lifetime T. We are now ready to perform multisession to single-session transformation. The transformation algorithm follows an *exterior-to-interior* order, i.e., we begin with nonrelay AFNs first and perform the transformation gradually on relay AFNs toward the base-station. This procedure will ensure that, by the time we perform transformation for AFN s, all the AFNs from which AFN s receives flows have already been transformed into single-session mode, and that all incoming flows to AFN s are already determined by earlier transformations on other AFNs.

The key idea of transformation is as follows: For each AFN s, its relay nodes under a single-session flow routing solution will be the same set of relay nodes under an equivalent multisession solution. However, for single-session solution, we partition network lifetime T into several durations. For each duration segment, AFN s will solely transmit its data to one particular relay node. The length of these time durations during which AFN s will transmit its outgoing flow exclusively to this respective relay node can be determined by the total bit-volume sent to this node under the multisession flow routing solution.

Under $\hat{\psi}$, denote $\hat{f}_{ik}(t)$ and $\hat{f}_{iB}(t)$ the bit-rates at time t ($0 \leq t \leq T$) from AFN i to AFN k and the base-station B, respectively. Due to the nature of single-session flow routing, at any time $t \in [0, T]$, there is only one flow in the set of $\hat{f}_{ik}(t)$ and $\hat{f}_{iB}(t)$ that has a nonzero bit-rate.

Algorithm 1. For a cycle-free multisession flow routing solution ψ with maximum network lifetime T, the following iterative algorithm obtains an equivalent single-session flow routing solution $\hat{\psi}$:

1. Identify a multisession AFN s such that
   a) either s is not receiving flows from any other AFN (i.e., a nonrelay AFN) or
   b) all AFNs from which AFN s receives flows are already in single-session mode.

   If there does not exist such a multisession AFN, we already have an equivalent single-session flow routing solution $\hat{\psi}$; otherwise, perform the following transformation for AFN s.
2. For AFN s, denote $R_s = r_1, r_2, \cdots, r_{|R_S|}$ as the set of relay nodes for AFN s under multisession solution ψ. If s has a direct flow to the base-station B under ψ, B is also included in R_s. Let $|R_s|$ denote the number of nodes in R_s. We define $|R_s|$ as the number of time duration segments for the single-session solution, i.e., $T_{s,r_1} = [0, t_1)$, $T_{s,r_2} = [t_1, t_2)$, $\cdots$, $T_{s,r_k} = [t_{k-1}, t_k)$, $\cdots$, $T_{s,r_{|R_s|}} = [t_{|R_s|-1}, t_{|R_s|}]$, with $t_{|R_s|} = T$. We will show

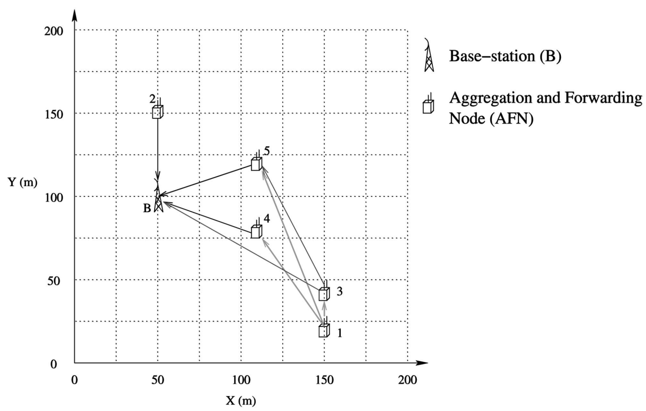

Fig. 2. A multisession flow routing solution for the sample sensor network.

$t_{|R_s|} = T$ in the correctness proof for this algorithm. T_{s,r_k} $(k = 1, 2, \cdots, |R_s|)$ are defined as follows:

$$\int_{T_{s,r_k}} \left[g_s + \sum_{m \neq s} \hat{f}_{ms}(t) \right] dt = f_{s,r_k} T \; . \tag{8}$$

Then, we have a single-session flow routing schedule for AFN s as follows:

$$\hat{f}_{s,r_k}(t) = \begin{cases} g_s + \sum_{m \neq s} \hat{f}_{ms}(t) & t \in T_{s,r_k}, \\ 0 & \text{otherwise}, \end{cases} \tag{9}$$

i.e., during T_{s,r_k}, AFN s will solely transmit to node r_k, where $k = 1,\ 2,\ \cdots,\ |R_s|$.

3. Go to Step 1.

To show that Algorithm 1 is correct, it is sufficient to show that the following two criteria are satisfied: 1) For each AFN, the rate of incoming (including self-generated) flows is equal to the rate of outgoing flow (i.e., flow balance) at any time and 2) at time T, the energy consumption at each AFN under $\hat{\psi}$ is the same as that under ψ. A complete proof is given in the Appendix.

We now examine the complexity of Algorithm 1. It is easy to see that there are at most N iterations in Algorithm 1. During each iteration, we need to determine which node s we will perform transformation ($O(N)$ complexity) and compute $|R_s| = O(N)$ time intervals for this node. Thus, the total complexity is $O(N^2)$.

It is worth pointing out that, for a specific AFN source node, since every link used by the multisession solution will also be used by the single-session solution during some time interval, the maximum number of hops for an AFN source node under both solutions is therefore identical.

So far we have not addressed any issues related to MAC layer, as such issues are not unique to our problem and can be addressed separately [8]. Nevertheless, we will offer a brief discussion here. It turns out that the constant bit rate traffic pattern (with rate change only on a very large time scale) under the single-session flow routing solution can greatly simplify the design at the MAC layer. This is because, for such nonbursty regular traffic pattern, a MAC protocol based on dedicated assignment, wherein bandwidth is shared using a predetermined allocation, offers the best performance [1]. Furthermore, the presence of a base-station can greatly simplify the coordination of media access and bandwidth allocation among the AFNs. Note that a random access or contention-based MAC scheme is more suitable for bursty traffic and is not a good choice here [1].

TABLE 1
AFN Coordinates (m), Local Flow Rate (kb/s), and Initial Energy (kJ) of the Sample Sensor Network

AFN i	Coordinates $(x_i,\ y_i)$	Local rate g_i	Initial energy e_i
1	(150, 20)	9	28
2	(50, 150)	7	26
3	(150, 40)	5	38
4	(110, 80)	1	19
5	(110, 120)	3	21

TABLE 2
Internode Flow Rates in a Multisession Solution for Example 1

	f_{ik} (kb/s)					f_{iB} (kb/s)
i	$k=1$	$k=2$	$k=3$	$k=4$	$k=5$	B
1	0	0	1.1229	5.4243	2.4528	0
2	0	0	0	0	0	7.0000
3	0	0	0	0	2.4320	3.6909
4	0	0	0	0	0	6.4342
5	0	0	0	0	0	7.8848

3.3 A Numerical Example

We use a 5-node network to illustrate how a multisession flow routing solution can be transformed into an equivalent single-session flow routing solution by using Algorithm 1.

Example 1. Referring to Fig. 2, suppose that we have 5 AFNs. The coordinates, local flow rate, and initial energy for each node are listed in Table 1. The base-station (B) is located at $(50, 100)$ m.

With the LP approach, we obtain a static multisession flow routing solution (see Fig. 2) with f_{ik} and f_{iB} listed in Table 2. For the given initial energy at each AFN, the maximum network lifetime obtained by solving the corresponding LP problem (see Section 3.1) is $T = 302.88$ days.

We now use Algorithm 1 to transform the above multisession flow routing solution into a single-session flow routing solution. According to Algorithm 1, since nodes 2, 4, and 5 are already in single-session mode, there is no need to perform transformation on them (except that the flow rates of 4 and 5 need to be recomputed). We then transform AFN 1 to a single-session routing schedule. That is, since $\int_{T_{13}} g_1 dt = f_{13} T$ and only T_{13} is unknown, we obtain $T_{13} = [0,\ 37.79)$ (in days). Similarly, we have $T_{14} = [37.79,\ 220.33)$ and $T_{15} = [220.33,\ 302.88]$. That is, during $[0,\ 37.79)$ days, AFN 1 sends its outgoing flow to AFN 3; during $[37.79,\ 220.33)$ days, AFN 1 sends its outgoing flow to AFN 4; during $[220.33, 302.88]$ days, AFN 1 sends its flow to AFN 5. Following Algorithm 1, we proceed to transform AFN 3 as follows: during $[0,\ 155.56)$ days, AFN 3 sends all its flow to base-station B; during $[155.56,\ 302.88]$ days, AFN 3 sends all its flow to AFN 4.

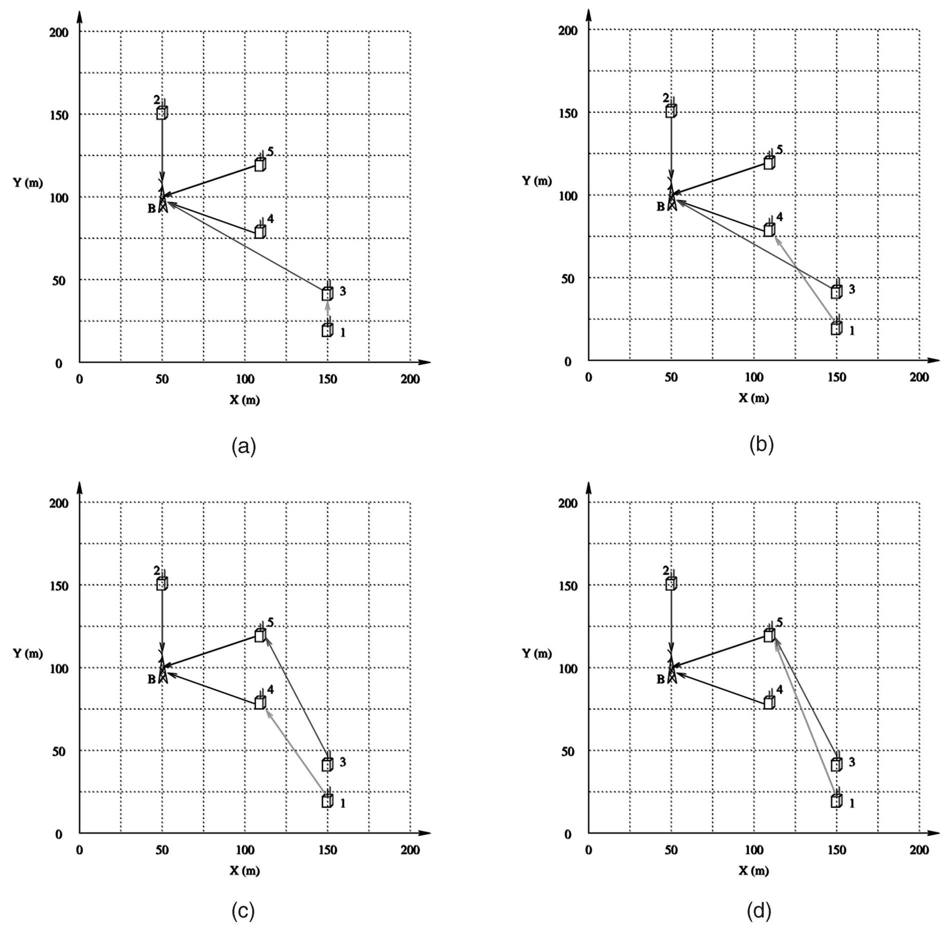

Fig. 3. An equivalent single-session flow routing schedule during $[0,\ 302.88]$ days for Example 1. (a) $[0,\ 37.79)$ days. (b) $[37.79,\ 155.56)$ days. (c) $[155.56,\ 220.33)$ days. (d) $[220.33,\ 302.88]$.

Fig. 3 shows the entire single-session flow routing schedule during network lifetime of 302.88 days. It is easy to verify that the flow balance equation at each AFN is satisfied throughout $[0,\ 302.88]$ days, and that at the end of 302.88 days, the energy consumption at each AFN is the same as that under the multisession flow routing solution.

3.4 Discussions

It is important to note that the single-session flow routing solution developed in this paper is fundamentally different from a TDM-based scheme. First and foremost, under a TDM-based scheme, there is a *regular* time-frame that each sender shall follow to send information in a specific time-slot within the frame *periodically*. Under single-session flow routing, an AFN can send flows to one node only within a specific time duration, and will no longer send to this node again at any other time. Second, the time scale of a TDM-based scheme is typically small with deterministic patterns. Under single-session flow routing, the time scale to change next hop node is much larger (see Example 1 in Section 3.3). Third, our single-session flow routing solution meets the stringent requirement of satisfying flow balance and more important, the energy constraint at AFNs, which may not be the focus under a TDM-based scheme. Finally, our single-session flow routing solution offers a perfect match when directional antennas are employed by AFNs, since a directional antenna has even greater potential to achieve energy saving by focusing transmission beam only toward its next hop node. However, the direction of antenna should be only adjusted in a large timescale. On the other hand, a TDM-based scheme is typically limited to omni-directional antennas and may not be suitable for working with directional antennas.

4 Extension to Variable Bit-Rate

The results in the last section show that an optimal multisession flow routing solution can be achieved by an equivalent single-session flow routing solution with power control on a large time scale instead of per-packet level. This provides an important methodology on further research for energy-constrained flow routing in wireless sensor networks. In this section, we relax the constant bit-rate constraint for g_i at each AFN i. We show that as long as the *average* bit-rate (denoted as $\bar{g}_i$) for $g_i(t)$ can be estimated, the optimal single-session flow routing solution is also obtainable. As an example, if the bit rate from an AFN follows an on/off process with known average bit-rate, we show how to obtain an optimal single-session flow routing solution to maximize network lifetime. In addition, we show that, as long as the estimated bit-rate $\bar{g}_i$ does not deviate too much from the actual value, the network lifetime obtained through single-session flow routing is near-optimal.

4.1 Perfect Knowledge of Average Bit-Rate

We begin with the ideal case that we have perfect knowledge of the average bit-rate of the flow generated by AFN i, denoted as $\bar{g}_i$. In this section, we show that an optimal single-session flow routing solution for a sensor network of variable bit-rate AFNs can be obtained by studying the optimal single-session flow routing solution for an auxiliary network of constant bit-rate AFNs, with the optimal solution for the auxiliary network being obtained with the procedures described in the previous section.

Denote P as the problem of variable bit-rate AFNs. The initial energy at AFN i is e_i and each AFN generates a flow at rate $g_i(t)$. Denote $\bar{P}$ as the problem of constant bit-rate AFNs with the same network configuration and initial energy at each AFN. Under $\bar{P}$, each AFN is assumed to generate a constant bit-rate composite flow with rate $\bar{g}_i$, which is the estimated average of $g_i(t)$, i.e.,

$$\bar{g}_i = E[g_i(t)]. \tag{10}$$

The following theorem shows that, for a flow solution for $\bar{P}$ with maximum network lifetime T, there exists an equivalent feasible solution for P with the same network lifetime T:

Theorem 2. *For a constant bit-rate problem $\bar{P}$ with maximum network lifetime T and the corresponding optimal flow routing solution $\bar{\pi}$, there exists an equivalent single-session flow routing solution π for the equivalent variable bit-rate problem P with the same network lifetime T.*

Theorem 2 can be proved by constructing a single-session flow routing solution for P with the same network lifetime as that obtained for $\bar{P}$. In the following algorithm, we show that an optimal flow routing solution (with constant bit-rate $\bar{g}_i$) with maximum network lifetime T can be transformed into a single-session flow routing solution for P with the same network lifetime T. Not surprisingly, this algorithm follows closely to Algorithm 1, with the difference being that g_i is now replaced by $g_i(t)$. Again, we need to first perform the cycle detection and removal procedure to ensure that the multisession flow routing solution $\bar{\pi}$ for $\bar{P}$ is cycle-free before the transformation.

Algorithm 2. Given a flow routing solution $\bar{\pi}$ for constant bit-rate problem $\bar{P}$ with maximum network lifetime T, the following iterative algorithm obtains an equivalent single session flow routing solution π for variable bit-rate problem P with the same network lifetime T.

Denote $\bar{f}_{ik}$ and $\bar{f}_{iB}$ as the flow rates from AFN i to AFN k and to base-station B under $\bar{\pi}$, and $f_{ik}(t)$ and $f_{iB}(t)$ as the flow rates from AFN i to AFN k and to base-station B at time t under π, respectively.

1. Under $\bar{\pi}$ for $\bar{P}$, identify a multisession AFN s such that
   a) either s is not receiving flows from any other AFN (i.e., a nonrelay AFN) or
   b) the incoming flows for AFN s in P are already defined.

   If no such AFN exists, we already have an equivalent single-session flow routing solution π for P; otherwise, define the following outgoing flows for s in P.

2) For AFN s, denote $R_s = r_1, r_2, \cdots, r_{|R_S|}$ as the set of relay nodes of s in $\bar{P}$ (the base-station is also included if s sends flow to B under $\bar{\pi}$). Here, $|R_s|$ denotes the number of AFNs in R_s. Define $|R_s|$ durations, $T_{s,r_1} = [0, t_1)$, $T_{s,r_2} = [t_1, t_2)$, $\cdots$, $T_{s,r_k} = [t_{k-1}, t_k)$, $\cdots$, $T_{s,r_{|R_s|}} = [t_{|R_s|-1}, t_{|R_s|}]$, with $t_{|R_s|} = T$. Again, it can be shown that $t_{|R_s|} = T$. T_{s,r_k} $(k = 1, 2, \cdots, |R_s|)$ are defined as follows:

$$\int_{T_{s,r_k}} \left[g_s(t) + \sum_{m \neq s} f_{ms}(t) \right] dt = \bar{f}_{s,r_k} T. \tag{11}$$

During T_{s,r_k}, AFN s will only transmit to AFN r_k. Then, the single-session flow routing schedule at AFN s for P is

$$f_{s,r_k}(t) = \begin{cases} g_s(t) + \sum_{m \neq s} f_{ms}(t) & t \in T_{s,r_k}, \\ 0 & \text{otherwise.} \end{cases} \tag{12}$$

3) Go to Step 1.

The correctness proof for Algorithm 2 follows the same token as the correctness proof for Algorithm 1 and is, thus, omitted here to conserve paper length. There is one detail that we should pay special attention to. In the correctness proof for Algorithm 2, we assume that

$$\bar{g}_s = \frac{1}{T} \int_0^T g_s(t) dt,$$

which means that the estimated bit-rate $\bar{g}_s$ is the actual average bit-rate over time interval T. In practice, $\bar{g}_s$ may deviate slightly from $\frac{1}{T}\int_0^T g_s(t)dt$, which we will discuss in Section 4.2.

Theorem 2 and Algorithm 2 show that, for problem P, we can obtain a single session flow routing solution π with the same network lifetime T, where T is the maximum network lifetime that is achievable for problem $\bar{P}$ with multisession flow routing solution $\bar{\pi}$. The next theorem shows that this network lifetime T is also the maximum achievable network lifetime for P. Consequently, the single-session flow routing solution π obtained by Algorithm 2 is also optimal.

Theorem 3 (π is Optimal). *The single-session flow routing solution π obtained by Algorithm 2 is optimal in terms of maximizing network lifetime for problem P.*

Proof. It is sufficient to show that the maximum network lifetime for problem P is the same as the maximum network lifetime for problem $\bar{P}$. First, since Theorem 2 shows that there is a solution for problem P with lifetime T, where T is the maximum network lifetime for problem $\bar{P}$, then the maximum network lifetime for problem P should be greater than or equal to T.

We now show that the maximum network lifetime for problem $\bar{P}$ is also greater than or equal to the maximum network lifetime for problem P. With these two findings, consequently, we can conclude that the maximum network lifetime for problem P is the same as the maximum network lifetime for problem $\bar{P}$.

To show that the maximum network lifetime for problem $\bar{P}$ is indeed greater than or equal to the maximum network lifetime for problem P, it is sufficient to prove that, for any given network flow routing solution π under P with network lifetime τ, we can find an equivalent flow routing solution $\bar{\pi}$ under $\bar{P}$ with the same network lifetime τ.

Since π is a network flow routing solution for P, for each AFN i, we have the following flow balance:

$$f_{iB}(t) + \sum_{k \neq i} f_{ik}(t) = g_i(t) + \sum_{m \neq i} f_{mi}(t). \quad (13)$$

We also have the following energy constraint inequality:

$$\sum_{m \neq i} \rho \int_0^{\tau} f_{mi}(t)dt + \sum_{k \neq i} c_{ik} \int_0^{\tau} f_{ik}(t)dt + c_{iB} \int_0^{\tau} f_{iB}(t)dt \leq e_i. \quad (14)$$

We now construct a flow routing solution $\bar{\pi}$ for $\bar{P}$ that has the same network lifetime τ. For $\bar{\pi}$, we define

$$\bar{f}_{ik} = \frac{\int_0^{\tau} f_{ik}(t)dt}{\tau}, \quad (15)$$

$$\bar{f}_{iB} = \frac{\int_0^{\tau} f_{iB}(t)dt}{\tau}. \quad (16)$$

We show that through such a construction, both the flow balance equation and energy constraint are satisfied for $\bar{P}$. Consequently, $\bar{\pi}$ is a feasible flow routing solution for $\bar{P}$. For flow balance, we have

$$\bar{g}_i + \sum_{m \neq i} \bar{f}_{mi} = \frac{1}{\tau} \left[\int_0^{\tau} g_i(t)dt + \sum_{m \neq i} \int_0^{\tau} f_{mi}(t)dt \right]$$
$$= \frac{1}{\tau} \left[\int_0^{\tau} f_{iB}(t)dt + \sum_{k \neq i} \int_0^{\tau} f_{ik}(t)dt \right] = \bar{f}_{iB} + \sum_{k \neq i} \bar{f}_{ik}.$$

The first equality holds by our assumption that $g_s = \frac{1}{\tau}\int_0^{\tau} g_s(t)dt$ and by (15). The second equality holds due to the flow balance (13). The third equality holds due to (15) and (16).

Similarly, for the energy constraint, we have

$$\sum_{k \neq i} \rho \bar{f}_{ki} \tau + \sum_{k \neq i} c_{ik} \bar{f}_{ik} \tau + c_{iB} \bar{f}_{iB} \tau$$
$$= \sum_{k \neq i} \rho \int_0^{\tau} f_{ki}(t)dt + \sum_{k \neq i} c_{ik} \int_0^{\tau} f_{ik}(t)dt + c_{iB} \int_0^{\tau} f_{iB}(t)dt$$
$$\leq e_i.$$

The first equality holds due to (15) and (16) and the inequality holds due to (14). Thus, at time τ, the energy consumption at each AFN i under $\bar{\pi}$ for problem $\bar{P}$ is the same as that under π for problem P, i.e., the network lifetime under $\bar{\pi}$ is also τ for $\bar{P}$. Therefore, for any achievable network lifetime τ under P, we can also find a flow routing solution under $\bar{P}$ that has the same network lifetime. This completes the proof. □

The significance of Theorem 2 and Theorem 3 is that they enable us to obtain an optimal single-session flow routing solution for a general sensor network of variable bit-rate AFNs (e.g., following an on/off process), as long as the estimated average bit-rate of each AFN is the same as its actual value. In a nutshell, this approach takes the following two steps:

- First, we find an optimal multisession flow routing solution $\bar{\pi}$ for problem $\bar{P}$ (from the LP problem described in Section 3.1).
- Second, we apply Algorithm 2 to get an optimal single-session flow routing solution for problem P.

TABLE 3
Traffic "On" Periods (in Days) and Bit Rate (kb/s) during "On" Periods for Each AFN (k Is Nonnegative Integer)

AFN i	Source "on" period	Rate during "on" period
1	$[k,\ k+0.4] \bigcup [k+0.8,\ k+1]$	15
2	$[k,\ k+0.3] \bigcup [k+0.6,\ k+1]$	10
3	$[k+0.4,\ k+0.9]$	10
4	$[k+0.2,\ k+0.4]$	5
5	$[k,\ k+0.3]$	7.5

4.2 Imperfect Estimate of Average Bit-Rate

Our investigation in Section 4.1 assumes that the estimated average bit-rate $\bar{g}_i$ matches perfectly with the actual value, i.e., $\bar{g}_i = \frac{1}{T}\int_0^T g_i(t)dt$. In practice, the estimated average bit-rate for $g_i(t)$, i.e., $\bar{g}_i$, could deviate from the actual value for $\bar{g}_i(t)$ over network lifetime T.

We now show that as long as this discrepancy is not substantial, the procedure developed in Section 4.1 can still yield a near-optimal single-session flow routing solution. Furthermore, the deviation between the actual network lifetime and the expected maximum network lifetime is negligible, as long as the estimated average bit-rate $\bar{g}_i$ is not far away from the actual value $\frac{1}{T}\int_0^T g_i(t)dt$, where T is the actual network lifetime. We use the following example to illustrate this result, which has the dual purpose of illustrating the procedures to obtain a single-session flow routing solution in Section 4.1:

Example 2. We use the sample network configuration in Fig. 2, where there are 5 AFNs and a base-station (B). Each AFN's coordinates and initial energy are the same as those in listed Table 1. The base-station is also located at the same location (i.e., $(50, 100)$ m). The local flow bit-rate g_i listed in Table 1 now represents the estimated average bit-rate $\bar{g}_i$ for AFN i, i.e., $\bar{g}_1 = 9$ kb/s for AFN 1, $\bar{g}_2 = 7$ kb/s for AFN 2, $\bar{g}_3 = 4$ kb/s for AFN 3, $\bar{g}_4 = 1$ kb/s for AFN 4, and $\bar{g}_5 = 3$ kb/s for AFN 5. Assume that $g_i(t)$ (in kb/s) follows a periodic on/off process (see Table 3).

Clearly, depending on the actual network lifetime T, the average rate for each AFN i over time T (i.e., $\frac{1}{T}\int_0^T g_i(t)dt$) could be slightly different from its estimated average $\bar{g}_i$. We will show such slight discrepancy results in negligible difference between the actual network lifetime T and the estimated maximum network lifetime (denoted as $\bar{T}$).

Denote the flow routing problem for the network of variable bit-rate AFNs as P and the flow routing problem for the network of constant bit-rate AFNs as $\bar{P}$. Under $\bar{P}$, we assume that each AFN i generates a constant bit-rate flow $\bar{g}_i$, which is the estimated average bit-rate for AFN i. We can build an LP problem (see Section 3.1) to get an

TABLE 4
Single-Session Flow Routing Schedule (in Days) for Example 2

AFN i	Time Duration	Next-Hop Node	Flow Bit-Rate
1	$[0,\ 37.87)$	3	$g_1(t)$
	$[37.87,\ 220.20]$	4	$g_1(t)$
	$[220.20,\ 302.38]$	5	$g_1(t)$
2	$[0,\ 302.38]$	B	$g_2(t)$
3	$[0,\ 37.87)$	B	$g_1(t) + g_3(t)$
	$[37.87,\ 155.68)$	B	$g_3(t)$
	$[155.68,\ 302.38]$	5	$g_3(t)$
4	$[0,\ 37.87)$	B	$g_4(t)$
	$[37.87,\ 220.20)$	B	$g_1(t) + g_4(t)$
	$[220.20,\ 302.38]$	B	$g_4(t)$
5	$[0,\ 155.68)$	B	$g_5(t)$
	$[155.68,\ 220.20)$	B	$g_3(t) + g_5(t)$
	$[220.20,\ 302.38]$	B	$g_1(t)+g_3(t)+g_5(t)$

optimal multisession flow routing solution for $\bar{P}$ (see Fig. 2) with exactly the same $\bar{f}_{ik}$ and $\bar{f}_{iB}$ as listed in Table 2. Again, the maximum network lifetime for $\bar{P}$ of the sample sensor network is $\bar{T} = 302.88$ days.

Now, we move on to obtain a single-session flow routing solution for P. According to Algorithm 2, since AFNs 2, 4, and 5 are already in single-session mode, there is no need to perform transformation on these AFNs. For AFN 1, since it sends flows to AFNs 3, 4, and 5 under $\bar{P}$, we calculate T_{13}, T_{14}, and T_{15} using (11) in Algorithm 2. That is, since $\int_{T_{13}} g_1(t)dt = f_{13}T$ and only T_{13} is unknown, we obtain $T_{13} = [0,\ 37.87)$ (in days). Therefore, during $[0,\ 37.87)$ days, AFN 1 sends its flows to AFN 3. Similarly, we obtain that $T_{14} = [37.87,\ 220.20)$ and $T_{15} = [220.20,\ 302.93]$ (in days) with $\int_{T_{14}} g_1(t)dt = f_{14}T$ and $\int_{T_{15}} g_1(t)dt = f_{15}T$, respectively. That is, AFN 1 sends its flows to AFN 4 during $[37.87,\ 220.20)$ days and sends its flows to AFN 5 during $[220.20,\ 302.93]$ days. Note that the actual lifetime for AFN 1 (302.93 days) is slightly different from the expected maximum network lifetime (302.88 days), due to the imperfect average bit-rate estimation for $g_i(t)$ with $\bar{g}_i$.

For AFN 3, since it sends flows to AFN 5 and base-station B under $\bar{P}$, we calculate T_{35} and T_{3B} under P. Since $\int_{T_{3B}} [g_3(t) + f_{13}(t)]dt = f_{3B}T$, we obtain $T_{3B} = [0,\ 155.68)$ (in days). Similarly, since $\int_{T_{35}} [g_3(t) + f_{13}(t)]dt = f_{35}T$, we obtain $T_{35} = [155.68,\ 302.84]$ (in days). Therefore, AFN 3 sends all its flows to base-station B during time $[0,\ 155.68)$ and sends all its flows to AFN 5 during time $[155.68,\ 302.84]$. Again, we note that the actual lifetime for AFN 3 (302.84 days) is slightly different from the expected maximum network lifetime (302.88 days), due to the same average bit-rate estimation error.

We can easily compute the node lifetimes of AFNs 2, 4, and 5, and find that AFN 4 has the smallest life 302.38. Since AFN 4 has the smallest lifetime among all the AFNs, it is also the network lifetime. Note that this is very close to the maximum network lifetime under $\bar{P}$ (302.88 days). We now have completed a single-session flow routing solution for P, which is summarized in Table 4. It is easy to verify that the incoming/outgoing flow balance holds for each AFN at any time during $[0,\ 302.38]$, with the bit-rate of composite flows generated by each AFN, $g_i(t)$, defined in Table 3.

We can also verify that there is indeed a tiny deviation here between the estimated average bit-rate $\bar{g}_i$ and the actual average bit-rate for each AFN i during $[0,\ 302.38]$ days. For example, for AFN 1, the actual average bit rate over $[0,\ 302.38]$ is

$$\frac{1}{302.38}\int_0^{302.38} g_1(t)dt = 9.0075,$$

which is very close the the estimated average bit-rate for $g_1(t)$, 9. Similarly, the actual average bit-rates for AFNs 2, 3, 4, and 5 over time interval $[0,\ 302.38]$ days are 7.0011, 4.9937, 1.0017, and 3.0062 (all in kb/s), which are very close to the estimated averages bit-rates 7, 5, 1, and 3, respectively.

5 Related Work

There has been active research on addressing energy conservation issues in wireless sensor networks. Several review papers (e.g., [25], [30]) have examined various issues when designing an energy-aware sensor network. In this section, we briefly summarize related research efforts on power control, power-aware routing, and network lifetime maximization.

Power control capability has been studied at different layers in recent years. At the *network* layer, most work on the power control problem can be classified into two categories. The first category is comprised of strategies to find an optimal transmitter power to control the *connectivity* properties of the network (see, e.g., [12], [18], [23], [28], [31], [34], [40]). A common theme in these strategies is to formulate power control as a network layer problem and, then, to adjust each node's transmission power, so that a different network connectivity topology can be formed for different objectives. For example, in [12], [18], the authors propose to use power control to improve network throughput, whereas, in [31], Ramanathan and Rosales-Hain's objective is to keep the number of one-hop neighbors bounded. Algorithmic issues for minimizing total power have been explored in [23]. In [28], Narayanaswamy et al. present one of the first implementations of a power control protocol that uses a common power level. In [34], [40], the authors aim to design a distributed algorithm to achieve minimum power routing topology while still maintaining the desired network connectivity properties.

The second category is usually referred to as *power-aware routing*. Most schemes use a shortest path algorithm with a power-based metric, rather than a hop-count based metric (see, e.g., [11], [15], [21], [24], [26], [29], [36], [38]). In [36], Singh et al. make some suggestions on developing a metric for power-aware routing, including energy consumed per-packet, time to network partition, variance in battery life of nodes, cost per packet, and node cost. In [38], Stojmenovic and Lin propose a localized routing algorithm based on the node's lifetime and distance-based power-metrics, with the aim of extending a node's worst-case battery lifetime. Energy-aware routing algorithms have also been explored in the context of broadcasting and multicasting (see, e.g., [39], [41], [42], and references therein).

However, energy-aware (e.g., minimum energy path) routing may not ensure good performance in maximum network lifetime [35]. For example, using the most energy-efficient routes may still result in premature depletion of energy at certain nodes, which is not optimal in some performance measures such as network lifetime.

The notion of network lifetime for wireless sensor networks has been discussed in [3], [5]. The network lifetime definition in this paper is consistent with that proposed in [3], [5]. The most relevant work on network lifetime related to our research have been described in [7], [8]. Here, we describe some additional relevant work on maximizing network lifetime. In [4], Bhardwaj and Chandrakasan attempt to develop a bound for maximum network lifetime through the notion of *role assignment,* which corresponds to the single-session solution discussed in this paper. But since the transformation from multisession solution to single-session solution is not explored, their approach results in prohibitively complex problem formulation, and polynomial solutions only exist for very simple scenarios. In [19], Kalpakis et al. propose a so-called *GETTREE* algorithm, which can be extended to give a single-session solution. The algorithm is obtained by applying results from graph theory, without exploring some unique properties of these networks (e.g., bit-volume conservation between equivalent solutions). Consequently, such an approach results in rather complex solutions.

6 Conclusions

In this paper, we explored the flow routing problem for two-tier wireless sensor networks with the objective of maximizing network lifetime of upper-tier aggregation and forwarding nodes (AFNs). Existing flow routing solutions for maximizing network lifetime require data generated at a node to be split into multiple subflows and to be transported along different paths. For a node with single transceiver, this would require a packet-level power control to conserve energy, which calls for considerable overhead in synchronization among the AFNs. In this paper, we show that the packet-level power control is not necessary. Instead, it is possible to achieve the same maximum network lifetime by employing power control in a much larger timescale with the so-called single-session flow routing solutions. As a result, the synchronization requirement is extremely low and its overhead is negligible when compared to packet-level multisession flow routing solution. In addition, we show how to perform optimal single-session flow routing when the bit-rate generated by AFNs is time-varying, as long as the average bit-rate can be estimated. These results contribute new understanding on lifetime-centric flow routing for energy-constrained wireless sensor networks.

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Appendix

Correctness Proof for Algorithm 1

Since the flow balance for each AFN s under $\hat{\psi}$ is satisfied by the definition for the single-session flow routing solutions in (9), we only need to show that the energy consumption at each AFN under $\hat{\psi}$ is the same as that under ψ.

We first prove (by induction) that the following two equations hold:

$$t_{|R_s|} = T, \tag{17}$$

$$\int_0^T \hat{f}_{sk}(t)dt = f_{sk}T. \tag{18}$$

1. Base case: If s is a nonrelay AFN, then

$$\begin{aligned} g_s t_{|R_s|} &= \sum_{k=1}^{|R_s|} \int_{T_{s,r_k}} g_s dt + \int_{T_{sB}} g_s dt \\ &= \sum_{k=1}^{|R_s|} f_{s,r_k} T + f_{sB} T \\ &= \left[\sum_{k=1}^{|R_s|} f_{s,r_k} + f_{sB} \right] T = g_s T. \end{aligned}$$

   The second equality above holds by the definition for T_{s,r_k} in (8). The last equality holds by the flow balance equation at AFN s under multisession flow routing solution ψ. Therefore, we have $t_{|R_s|} = T$. To show that (18) holds, we have

$$\int_0^T \hat{f}_{s,r_k}(t)dt = \int_{T_{s,r_k}} \hat{f}_{s,r_k}(t)dt = \int_{T_{s,r_k}} g_s dt = f_{s,r_k} T.$$

   The first equality holds since we only need to consider the time (in integration) when $\hat{f}_{s,r_k}(t)$ is nonzero. The second equality holds since s is a nonrelay AFN and $\hat{f}_{ms}(t) = 0$ in (9). The last equality holds by applying the definition for T_{s,r_k} in (8).
2. Induction step: If s is a relay AFN, suppose that all AFNs m from which AFN s receives flows under ψ are already transferred in single-session mode and that

$$\int_0^T \hat{f}_{ms}(t)dt = f_{ms}T,$$

   i.e., (18) holds for those AFNs m. We now show that, for AFN s, (17) and (18) also hold. To show this, we have

$$\begin{aligned} &\int_0^{t_{|R_s|}} \left[g_s + \sum_{m \neq s} \hat{f}_{ms}(t) \right] dt \\ &= \sum_{k=0}^{|R_s|} \int_{T_{s,r_k}} \left[g_s + \sum_{m \neq s} \hat{f}_{ms}(t) \right] dt \\ &= \sum_{k=0}^{|R_s|} f_{s,r_k} T = \left[\sum_{k=0}^{|R_s|} f_{s,r_k} \right] T \\ &= \left[g_s + \sum_{m \neq s} f_{ms} \right] T = \int_0^T \left[g_s + \sum_{m \neq s} \hat{f}_{ms}(t) \right] dt. \end{aligned}$$

The second equality holds by applying the definition for T_{s,r_k} in (8). The fourth equality holds by the flow balance under ψ. The last equality holds by the induction assumption. Therefore, we have $t_{|R_s|} = T$. To show that (18) holds for AFN s, we have

$$\int_0^T \hat{f}_{s,r_k}(t)dt = \int_{T_{s,r_k}} \hat{f}_{s,r_k}(t)dt = \int_{T_{s,r_k}} \left[g_s + \sum_{m \neq s} \hat{f}_{ms}(t) \right] dt = f_{s,r_k} T.$$

The first equality holds since we only need to consider the time (in integration) when $\hat{f}_{s,r_k}(t)$ is nonzero. The second equality holds by applying the definition for $\hat{f}_{s,r_k}(t)$ in (9). The third equality holds by applying the definition for T_{s,r_k} in (8).

By 1 and 2, we have proved that (17) and (18) holds for all AFNs under single-session solution $\hat{\psi}$. We are now ready to show that the consumed energy at each AFN at T under single-session solution $\hat{\psi}$ is the same as that under multisession solution ψ. For AFN s, we have

$$\sum_{m \neq s} \rho \int_0^T \hat{f}_{ms}(t)dt + \sum_{k \neq s} c_{sk} \int_0^T \hat{f}_{sk}(t)dt + c_{sB} \int_0^T \hat{f}_{sB}(t)dt = \rho \sum_{m \neq s} f_{ms} T + \sum_{k \neq s} c_{sk} f_{sk} T + c_{sB} f_{sB} T \leq e_i.$$

The equality holds by (18). The correctness proof is now complete.

References

[1] I.F. Akyildiz, W. Su, Y. Sankarasubramaniam, and E. Cayirci, "Wireless Sensor Networks: A Survey," *Computer Networks*, vol. 38, no. 4, pp. 393-422, 2002.

[2] J. Albowicz, A. Chen, and L. Zhang, "Recursive Position Estimation in Sensor Networks," *Proc. IEEE Int'l Conf. Network Protocols*, pp. 35-41, Nov. 2001.

[3] M. Bhardwaj, T. Garnett, and A.P. Chandrakasan, "Upper Bounds on the Lifetime of Sensor Networks," *Proc. IEEE Int'l Conf. Comm.*, pp. 785-790, June 2001.

[4] M. Bhardwaj and A.P. Chandrakasan, "Bounding the Lifetime of Sensor Networks via Optimal Role Assignments," *Proc. IEEE Infocom*, pp. 1587-1596, June 2002.

[5] D. Blough and S. Paolo, "Investigating Upper Bounds on Network Lifetime Extension for Cell-Based Energy Conservation Techniques in Stationary Ad Hoc Networks," *Proc. ACM MobiCom*, pp. 183-192, Sept. 2002.

[6] N. Bulusu, J. Heidemann, and D. Estrin, "GPS-Less Low-Cost Outdoor Localization for Very Small Devices," *IEEE Personal Comm.*, vol. 7, no. 5, pp. 28-34, Oct. 2000.

[7] J.-H. Chang and L. Tassiulas, "Routing for Maximum System Lifetime in Wireless Ad-Hoc Networks," *Proc. 37th Ann. Allerton Conf. Comm., Control, and Computing*, pp. 22-31, Sept. 1999.

[8] J.-H. Chang and L. Tassiulas, "Energy Conserving Routing in Wireless Ad-Hoc Networks," *Proc. IEEE Infocom*, pp. 22-31, Mar. 2000.

[9] J. Chou, D. Petrovis, and K. Ramchandran, "A Distributed and Adaptive Signal Processing Approach to Reducing Energy Consumption in Sensor Networks," *Proc. IEEE Infocom*, pp. 1054-1062, Mar-Apr. 2003.

[10] T.H. Cormem, C.E. Leiserson, and R.L. Rivest, *Introduction to Algorithms*, chapter 23. New York: McGraw-Hill Book Company, 1989.

[11] S. Doshi, S. Bhandare, and T.X. Brown, "An On-Demand Minimum Energy Routing Protocol for a Wireless Ad Hoc Network," *ACM Mobile Computing and Comm. Rev.*, vol. 6, no. 3, pp. 50-66, July 2002.

[12] T.A. ElBatt, S.V. Krishnamurthy, D. Connors, and S. Dao, "Power Management for Throughput Enhancement in Wireless Ad-Hoc Networks," *Proc. IEEE Int'l Conf. Comm.*, pp. 1506-1513, June 2000.

[13] J. Elson and K. Romer, "Wireless Sensor Networks: A New Regime for Time Synchronization," *Proc. First Workshop Hot Topics in Networks (HotNets-I)*, Oct. 2002.

[14] S. Ganeriwal, R. Kumar, S. Adlakha, and M. Srivastava, "Timing-Sync Protocol for Sensor Networks," *Proc. ACM Sensys*, pp. 138-149, Nov. 2003.

[15] J. Gomez, A.T. Campbell, M. Naghshineh, and C. Bisdikian, "Conserving Transmission Power in Wireless Ad Hoc Networks," *Proc. IEEE Int'l Conf. Network Protocols*, pp. 24-34, Nov. 2001.

[16] T. He, C. Huang, B.M. Blum, J.A. Stankovic, and T. Abdelzaher, "Range-Free Localization Schemes for Large Scale Sensor Networks," *Proc. ACM MobiCom*, pp. 81-95, Sept. 2003.

[17] W. Heinzelman, *Application-Specific Protocol Architectures for Wireless Networks*, PhD thesis, Dept. of Electrical Eng. and Computer Science, Massachusetts Inst. of Technology, June 2000.

[18] L. Hu, "Topology Control for Multihop Packet Radio Networks," *IEEE Trans. Comm.*, vol. 41, no 10, pp. 1474-1481, Oct. 1993.

[19] K. Kalpakis, K. Dasgupta, and P. Namjoshi, "Maximum Lifetime Data Gathering and Aggregation in Wireless Sensor Networks," *Proc. IEEE Int'l Conf. Networking (ICN '02)*, pp. 685-696, Aug. 2002.

[20] K. Langendoen and N. Reijers, "Distributed Localization in Wireless Sensor Networks: A Quantitative Comparison," *Computer Networks*, vol. 43, no. 4, pp. 499-518, Nov. 2003.

[21] Q. Li, J. Aslam, and D. Rus, "Online Power-Aware Routing in Wireless Ad-Hoc Networks," *Proc. ACM MobiCom*, pp. 97-107, July 2001.

[22] Q. Li and D. Rus, "Global Clock Synchronization in Sensor Networks," *Proc. IEEE Infocom*, pp. 564-574, Mar. 2004.

[23] E.L. Lloyd, R. Liu, and M.V. Marathe, "Algorithmic Aspects of Topology Control Problems for Ad Hoc Networks," *Proc. ACM MobiHoc*, pp. 123-134, June 2002.

[24] A. Michail and A. Ephremides, "Energy Efficient Routing for Connection-Oriented Traffic in Ad-Hoc Wireless Networks," *Proc. IEEE Int'l Symp. Personal Indoor and Mobile Radio Comm.*, pp. 762-766, Sept. 2000.

[25] R. Min, M. Bhardwaj, S.-H. Cho, N. Ickes, E. Shih, A Sinha, A. Wang, and A. Chandrakasan, "Energy-Centric Enabling Technologies for Wireless Sensor Networks," *IEEE Wireless Comm. Magazine*, vol. 9, no. 4, pp. 28-39, Aug. 2002.

[26] A. Misra and S. Banerjee, "MRPC: Maximizing Network Lifetime for Reliable Routing in Wireless Environments," *Proc. IEEE Wireless Comm. and Networking Conf.*, pp. 800-806, Mar. 2002.

[27] R. Nagpal, H. Shrobe, and J. Bachrach, "Organizing a Global Coordinate System from Local Information on an Ad Hoc Sensor Network," *Proc. Second Int'l Workshop Information Processing in Sensor Networks*, pp. 333-348, Apr. 2003.

[28] S. Narayanaswamy, V. Kawadia, R.S. Sreenivas, and P.R. Kumar, "Power Control in Ad-Hoc Networks: Theory, Architecture, Algorithm and Implementation of the COMPOW Protocol," *Proc. European Wireless Conf.—Next Generation Wireless Networks: Technologies, Protocols, Services and Applications*, pp. 156-162, Feb. 2002.

[29] M.B. Pursley, H.B. Russell, and J.S. Wysocarski, "Energy-Efficient Routing in Frequency-Hop Radio Networks with Partial-Band Interference," *Proc. IEEE Wireless Comm. and Networking Conf.*, pp. 79-83, Sept. 2000.

[30] V. Raghunathan, C. Schurgers, S. Park, and M.B. Srivastava, "Energy-Aware Wireless Microsensor Networks," *IEEE Signal Processing Magazine*, vol. 19, no. 2, pp. 40-50, Mar. 2002.

[31] R. Ramanathan and R. Rosales-Hain, "Topology Control of Multihop Wireless Networks Using Transmit Power Adjustment," *Proc. IEEE Infocom*, pp. 404-413, Mar. 2000.

[32] K. Ramchandran, "Distributed Sensor Networks: Opportunities and Challenges in Signal Processing and Comm.," *US Nat'l Science Foundation Workshop Distributed Comm. and Signal Processing for Sensor Networks*, http://www.ece.northwestern.edu/~pappas/nsf_workshop/presentations/ramchandran_workshop_DDSP.ppt, Dec. 2002.

[33] T.S. Rappaport, *Wireless Communications: Principles and Practice.* New Jersey: Prentice Hall, 1996.

[34] V. Rodoplu and T.H. Meng, "Minimum Energy Mobile Wireless Networks," *IEEE J. Selected Areas in Comm.*, vol. 17, no. 8, pp. 1333-1344, Aug. 1999.

[35] R.C. Shah and J.M. Rabaey, "Energy Aware Routing for Low Energy Ad Hoc Sensor Networks," *Proc. IEEE Wireless Comm. and Networking Conf.,* pp. 350-355, Mar. 2002.
[36] S. Singh, M. Woo, and C.S. Raghavendra, "Power-Aware Routing in Mobile Ad Hoc Networks," *Proc. ACM MobiCom,* pp. 181-190, Oct. 1998.
[37] F. Sivrikaya and B. Yener, "Time Synchronization in Sensor Networks: A Survey," *IEEE Network,* pp. 45-50, July/Aug. 2004.
[38] I. Stojmenovic and X. Lin, "Power-Aware Localized Routing in Wireless Networks," *IEEE Trans. Parallel and Distributed Systems,* vol. 12, no. 11, pp. 1122-1133, Nov. 2001.
[39] P.-J. Wan, G. Galinescu, X.-Y. Li, and O. Frieder, "Minimum-Energy Broadcast Routing in Static Ad Hoc Wireless Networks," *Proc. IEEE Infocom,* pp. 1162-1171, Apr. 2001.
[40] R. Wattenhofer, L. Li, P. Bahl, and Y.-M. Wang, "Distributed Topology Control for Power Efficient Operation in Multihop Wireless Ad Hoc Networks," *Proc. IEEE Infocom,* pp. 1388-1397, Apr. 2001.
[41] J.E. Wieselthier, G.D. Nguyen, and A. Ephremides, "Resource Management in Energy-Limited, Bandwidth Limited, Transceiver-Limited Wireless Networks for Session-Based Multicasting," *Computer Networks,* vol. 39, no. 2, pp. 113-131, June 2002.
[42] J.E. Wieselthier, G.D. Nguyen, and A. Ephremides, "Energy-Limited Wireless Networking with Directional Antennas: The Case of Session-Based Multicasting," *Proc. IEEE Infocom,* pp. 190-199, June 2002.

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