Flow Routing for Variable Bit Rate Source Nodes in Energy-constrained Wireless Sensor Networks

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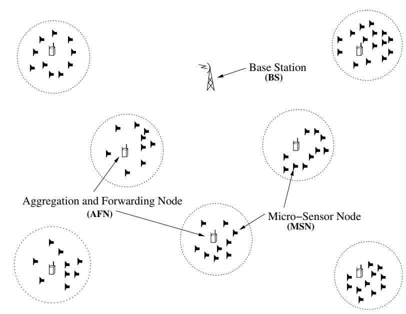
***Abstract*— We consider a two-tier wireless sensor network and focus on the flow routing problem for the upper tier aggregation and forwarding nodes (AFNs). Assuming each AFN is equipped with directional antennas for transmission, we are interested in how to perform flow routing at each node such that the network lifetime is maximized. We present a flow routing algorithm that is provably to have the following property: (1) When the average source rate of each AFN is known a priori, the flow routing algorithm is optimal and gives maximum network lifetime performance; (2) When the average source rate of each AFN is unknown but is within a fraction of ϵ of an estimated rate value, then the network lifetime given by the proposed flow routing algorithm is no more than $\frac{2\epsilon}{1-\epsilon}$ from optimal. As a result, the proposed flow routing algorithm can provide predictable lifetime performance even when the source bit rate can be time-varying.**

***Index Terms*— Network lifetime, energy constraint, directional antenna, power control, flow routing, variable bit rate, wireless sensor networks.**

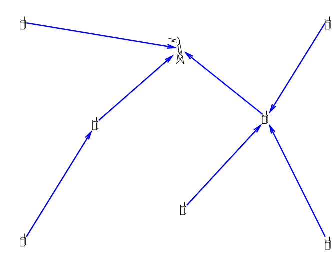
I. Introduction

Wireless sensor network is a special form of wireless networks dedicated to surveillance and monitoring applications and is characterized by severe constraint on battery resource. We consider a two-tiered wireless sensor network that can be deployed for various sensing applications (see Fig. 1). This network consists of a number of sensor clusters and a base-station. Each cluster is deployed around a strategic location and consists of a number of wireless *micro-sensor nodes* (MSNs) and one *aggregation and forwarding node* (AFN). Each MSN is used to capture and transmit data stream to an AFN that performs in-network processing by aggregating all correlated information within the cluster (which is also known as "fusion"). The AFN then sends the composite data stream to the base-station via single or multi-hop transmission.

A key performance measure for a wireless sensor network is *network lifetime*. For the two-tiered wireless sensor networks considered in this paper, whenever an AFN runs out of energy, the sensing capability for that local area is lost. Therefore, the most stringent definition for network lifetime would be the time until any AFN fails. To conserve energy consumption, we assume each AFN is equipped with directional antennas [12], which is capable of forming multiple beams for flow routing in the network. Further, we assume each beam's distance coverage can be controlled by the transmission power of the



(a) Physical network topology.



(b) Flow routing among AFNs.

Fig. 1. Reference architecture for a two-tier wireless sensor network.

beam. To conserve energy, we assume the beam-width for each directional beam is set to the minimum and remained fixed.

In this network setting, we investigate the flow routing problem for the upper-tier AFNs such that the network lifetime is maximized. The case where the bit rate generated by each AFN is a constant can be solved by the LP approach [3]. In this paper, we explore the more difficult problem where the bit rate from each AFN can be time-varying. The main result of this paper is a flow routing algorithm that has the following guaranteed network lifetime performance: (1) When the average rate of each AFN's bit rate is known a priori, the flow routing algorithm is shown to be optimal and provides maximum network lifetime performance; (2) When the average rate of each AFN's bit rate is unknown but can be estimated within an error margin of ϵ, then the network lifetime given by the proposed flow routing algorithm is no more than $\frac{2\epsilon}{1-\epsilon}$ from optimal.

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The remainder of this paper is organized as follows. In Section II, we present a flow routing algorithm when the average data rate at each AFN is known. We also prove that this algorithm is optimal in terms of network lifetime performance. In Section III, we consider the case when the average bit rate at each AFN is unknown. We show that as long as the estimated bit rate is within ϵ from the unknown average rate, the network lifetime performance by the same flow routing algorithm is no more than $\frac{2\epsilon}{1-\epsilon}$ from the optimal. Section IV reviews related work and Section V concludes this paper.

II. Optimal Flow Routing with Given Average Bit Rate

A. Network Model and Energy Consumption

We focus on the two-tiered architecture for wireless sensor networks. Figure 1(a) and (b) show the physical network topology and a flow routing among AFNs for such a network, respectively. As shown in these figures, we have three types of nodes in the network, namely, *micro sensor nodes* (MSNs), *aggregation and forwarding nodes* (AFNs), and a *base-station* (BS). The MSNs constitute the lower-tier of the network. They are deployed in groups (or clusters) at strategic locations for various sensing applications. The objective of an MSN is very simple: once triggered by an event, it starts to capture live information (video, audio, or other scalar measurements) and sends it directly to the local AFN.

Within each cluster of MSNs, there is one AFN, which is different from an MSN in terms of physical structure and functions. The primary functions of an AFN are: 1) *data aggregation* (or "fusion") for flows coming from the local cluster of MSNs, and 2) *forwarding* (or relaying) the aggregated data to the next hop AFN toward the base-station. Due to energy consumption, each AFN has a limited lifetime. Upon the depletion of energy at an AFN, the *coverage* for the particular area is lost. In this paper, we define network lifetime as the time instance until any AFN runs out of energy.

The last component in the two-tier architecture is the base-station, which is the *sink* node for data streams generated at all AFNs in the network. We assume that it has sufficient power resource and is not subject to energy constraint.

In summary, the function of the lower-tier MSNs is data acquisition, while the upper-tier AFNs are used for data fusion and forwarding the aggregated data toward the base-station. Our focus in this paper is on the flow routing problem for the upper tier AFNs.

For an AFN, the radio-related power consumption (*i.e.,* in transmitter and receiver) is the dominant factor. The transmission power can be characterized by

$$p_t(i,\ k) = c_{ik} \cdot f_{ik} \ , \qquad (1)$$

where $p_t(i,k)$ is the power dissipated when AFN i is transmitting to k, f_{ik} is the rate of the data stream from AFN i to k. c_{ik} is the power consumption cost for directional antennas over link (i,k) and

$$c_{ik} = \alpha + \frac{\theta}{2\pi}\beta \cdot d_{ik}^n \ , \qquad (2)$$

where α is a distance-independent term, β is a coefficient associated with the distance-dependent term, θ is the beam-width of the directional antenna and is a constant in this paper, d_{ik} is the distance between nodes i and k, n is the path loss index and $2 \leq n \leq 4$ [8]. Typical values for these parameters are $\alpha = 50$ nJ/b and $\beta = 0.0013$ pJ/b/m^4 for $n = 4$ [4]. In this paper, we use $n = 4$ and $\theta = \frac{\pi}{6}$ in all of our numerical results.

The power dissipation at the receiver of AFN j can be modeled as

$$p_r(j) = \sum_{k \neq j} \rho f_{kj} \ , \qquad (3)$$

where f_{kj} (in b/s) is the incoming rate of received data stream from AFN k. Typical value of ρ is 50 nJ/b [4].

Our main focus in this paper is the flow routing problem when the bit rate from each AFN is time-varying. Denote the rate from AFN i as $g_i(t)$ and the initial energy at AFN i as e_i. Let $\bar{g}_i$ be the average of $g_i(t)$, *i.e.*

$$\bar{g}_i = E[g_i(t)] \ . \qquad (4)$$

Denote P as the flow routing problem under variable bit rate source $g_i(t)$ and $\bar{P}$ as the flow routing problem for $\bar{g}_i$ (i.e., constant bit rate source), $1 \leq i \leq N$, with the same network configuration and the same initial energy on each AFN.

In this section, we study the optimal flow routing problem when $g_i(t)$, $1 \leq i \leq N$, are time-varying but with known average $\bar{g}_i$. We show that the optimal flow routing solution for problem P (i.e., time-varying source bit rate) can be obtained by studying an optimal flow routing solution for an auxiliary network with constant source rate $\bar{g}_i$, $1 \leq i \leq N$.

B. Preliminary

For the constant source rate $\bar{g}_i$, $1 \leq i \leq N$, we first formulate the optimal flow routing problem $\bar{P}$ as a linear programming (LP) problem. Denote $\bar{T}$ the network lifetime, i.e., time until any AFN runs out of energy. Then we have the following flow balance equations and energy constraints for each AFN i, $1 \leq i \leq N$.

$$\bar{g}_i + \sum_{m \neq i} \bar{f}_{mi} = \sum_{k \neq i} \bar{f}_{ik} + \bar{f}_{iB} \ , \qquad (5)$$

$$\left(\sum_{m \neq i} \rho \bar{f}_{mi} + \sum_{k \neq i} c_{ik} \bar{f}_{ik} + c_{iB} \bar{f}_{iB} \right) \bar{T} \leq e_i \ . \qquad (6)$$

where $\bar{f}_{ik}$ and $\bar{f}_{iB}$ denote the flow rate from AFN i to AFN k and base-station B respectively. The first set of N equations in (5) state that, at each AFN i, the data stream $\bar{g}_i$ generated by i, plus the amount of total received data streams from other

AFNs sent to i, are equal to the total bit rate transmitted from i. The second set of N inequalities in (6) state that the energy required to receive and transmit all these data streams at each AFN i, at the end of network lifetime $\bar{T}$, cannot exceed its energy constraint. Our objective is to maximize the network lifetime $\bar{T}$ while both (5) and (6) are satisfied.

Denoting $\bar{H} = 1/\bar{T}$, we formulate the flow routing problem into the following LP.

Min $\quad \bar{H}$

s.t.
$$\sum_{k\neq i} \bar{f}_{ik} + \bar{f}_{iB} - \sum_{m\neq i} \bar{f}_{mi} = \bar{g}_i \qquad (1 \le i \le N)\ , \quad (7)$$

$$\sum_{m\neq i} \rho \bar{f}_{mi} + \sum_{k\neq i} c_{ik} \bar{f}_{ik} + c_{iB} \bar{f}_{iB} - e_i \bar{H} \le 0 \qquad (1 \le i \le N)\ , \quad (8)$$

$$\bar{f}_{ik}, \bar{f}_{iB}, \bar{H} \ge 0\ , \quad (1 \le i, k \le N, i \neq k)$$

where (7) follow from the balance equations (5) and (8) follow from the energy constraints (6). Note that $\bar{H}$, $\bar{f}_{mi}$, $\bar{f}_{ik}$, and $\bar{f}_{iB}$ are variables and $\bar{g}_i$, ρ, c_{ik}, c_{iB}, and e_i are all constants. This LP problem is similar to that in [3] and can be solved by standard techniques in polynomial time.

C. Optimal Flow Routing for Problem P

In this subsection, we present a flow routing algorithm for problem P with a network lifetime $T^* = \bar{T}$, where $\bar{T}$ is the maximum network lifetime for $\bar{P}$ obtained in the last subsection. Further, we show that T^* is the maximum network lifetime for problem P.

Denote $\bar{\pi}$ an optimal flow routing solution (obtained via LP) to the constant bit rate problem $\bar{P}$ with maximum network lifetime $\bar{T}$. The following algorithm defines a flow routing solution ψ for the variable bit source problem P, whose network lifetime T^* will be shown to be the same as $\bar{T}$, i.e., $T^* = \bar{T}$. The main idea in constructing flow routing ψ is as follows. We first find that, for each source AFN j, how many percentage of data generated by AFN j is transmitted by flow $\bar{f}_{ik}$ and $\bar{f}_{iB}$ in the flow routing solution $\bar{\pi}$, which we denote as w^j_{ik} and w^j_{iB}, respectively. Then for problem P, we keep the same w^j_{ik} and w^j_{iB} when constructing the flow routing solution ψ.

Algorithm 1: **(Flow Routing ψ for Problem P)** *Denote $f_{ik}(t)$ and $f_{iB}(t)$ the flow rates from AFN i to AFN k and to the base-station B at time t under π, respectively. We now define w^j_{ik} and w^j_{iB} for each source node AFN j as follows.*

1) *Identify an AFN s such that w^j_{sk} and w^j_{sB} have not been defined so far.*
   If no such AFN exists, we already have defined all w^j_{ik} and w^j_{iB}, $1 \le i, k \le N$ and $i \neq k$, and we can continue to (12) and (13).
   Otherwise, we have
   - *either $s = j$, or*
   - *w^j_{ms} are already defined (for all flows with $\bar{f}_{ms} > 0$) during previous iterations.*
2) *Denote w^j_s the percentage of data generated by AFN j are transmitted by AFN s under $\bar{\pi}$. We have*
$$w^j_s = \begin{cases} \sum_{m\neq s} w^j_{ms} & s \neq j\ , \\ 1 & s = j\ . \end{cases} \quad (9)$$
   Now we define w^j_{sk} and w^j_{sB} as
$$w^j_{sk} = \frac{\bar{f}_{sk}}{\bar{f}_{sB} + \sum_{r\neq s} \bar{f}_{sr}} w^j_s\ . \quad (10)$$
$$w^j_{sB} = \frac{\bar{f}_{sB}}{\bar{f}_{sB} + \sum_{r\neq s} \bar{f}_{sr}} w^j_s\ . \quad (11)$$
3) *Go to Step 1.*

Given these weight definitions w^j_{ik} and w^j_{iB} for all $1 \le i, k \le N$ and $i \neq k$, we now define the flow routing on each wireless link ψ as
$$f_{sk}(t) = \sum_{j\neq k} f^j_{sk}(t) = \sum_{j\neq k} w^j_{sk} \cdot g_j(t)\ , \quad (12)$$
$$f_{sB}(t) = \sum_{j=1}^{N} f^j_{sB}(t) = \sum_{j=1}^{N} w^j_{sB} \cdot g_j(t)\ , \quad (13)$$
where $f^j_{sk}(t)$ and $f^j_{sB}(t)$ are the data generated by AFN j and transmitted by AFN s to AFN k and to base-station B, respectively.

Denote $\bar{f}^j_{sr}$ the percentage of data generated by AFN j are transmitted by AFN s to r under $\bar{\pi}$, we have
$$\bar{f}_{sk} = \sum_{j\neq k} \bar{f}^j_{sk} = \sum_{j\neq k} w^j_{sk} \bar{g}_j\ , \quad (14)$$
$$\bar{f}_{sB} = \sum_{j=1}^{N} \bar{f}^j_{sB} = \sum_{j=1}^{N} w^j_{sB} \bar{g}_j\ . \quad (15)$$

For Algorithm 1, we have the following lemma.

Lemma 1: The flow routing solution ψ produced by Algorithm 1 is feasible and gives a network lifetime $T^* = \bar{T}$.

Proof: The feasibility part can be proved by showing that at each node, the flow balance property is maintained throughout $\bar{T}$ and the energy constraint is also met by time $\bar{T}$. To show that the network lifetime T^* under ψ is identical to $\bar{T}$, it is sufficient to show that the remaining energy on each node i under both ψ and $\bar{\pi}$ at time $\bar{T}$ is identical.

We first consider the part of incoming and outgoing data flows at AFN s that originates from source node j.

If $s = j$, we have
$$\begin{aligned} g_s(t) &= \left(\frac{\bar{f}_{sB}}{\bar{f}_{sB} + \sum_{r\neq s} \bar{f}_{sr}} + \sum_{k\neq s} \frac{\bar{f}_{sk}}{\bar{f}_{sB} + \sum_{r\neq s} \bar{f}_{sr}} \right) w^s_s g_s(t) \\ &= (w^s_{sB} + \sum_{k\neq s} w^s_{sk}) g_s(t) = f^s_{sB}(t) + \sum_{k\neq s} f^s_{sk}(t)\ . \quad (16) \end{aligned}$$

The first equality holds by $w^s_s = 1$. The second equality holds by the definitions of w^j_{sk} and w^j_{sB} in (10) and (11). The third equality holds by the definitions of $f^j_{sk}(t)$ and $f^j_{sB}(t)$ in (12) and (13).

If $s \neq j$, we have

$$\sum_{m\neq s} f^j_{ms}(t) = \sum_{m\neq s} w^j_{ms} g_j(t) = w^j_s g_j(t)$$
$$= \left(\frac{\bar{f}_{sB}}{\bar{f}_{sB} + \sum_{r\neq s} \bar{f}_{sr}} + \sum_{k\neq s} \frac{\bar{f}_{sk}}{\bar{f}_{sB} + \sum_{r\neq s} \bar{f}_{sr}} \right) w^j_s g_j(t)$$
$$= (w^j_{sB} + \sum_{k\neq s} w^j_{sk}) g_j(t) = f^j_{sB}(t) + \sum_{k\neq s} f^j_{sk}(t) \ . \quad (17)$$

The first equality holds by the definitions of $f^j_{sk}(t)$ and $f^j_{sB}(t)$ in (12) and (13). The second equality holds by the definition of w^j_s in (9). The fourth equality holds by the definitions of w^j_{sk} and w^j_{sB} in (10) and (11). The fifth equality holds by the definitions of $f^j_{sk}(t)$ and $f^j_{sB}(t)$ in (12) and (13).

For flow balance, we have

$$g_s(t) + \sum_{m\neq s} f_{ms}(t) = g_s(t) + \sum_{j\neq s}\sum_{m\neq s} f^j_{ms}(t)$$
$$= f^s_{sB}(t) + \sum_{k\neq s} f^s_{sk}(t) + \sum_{j\neq s}[f^j_{sB}(t) + \sum_{k\neq s} f^j_{sk}(t)]$$
$$= \sum_{j=1}^{N}[f^j_{sB}(t) + \sum_{k\neq s} f^j_{sk}(t)] = f_{sB}(t) + \sum_{k\neq s} f_{sk}(t) \ .$$

The first equality holds by the definitions of $f_{sk}(t)$ and $f_{sB}(t)$ in (12) and (13). The second equality holds by our results in (16) and (17). The fourth equality holds by the definitions of $f_{sk}(t)$ and $f_{sB}(t)$ in (12) and (13). Therefore, flow balance on AFN s holds for any time $t < \bar{T}$.

For energy constraint, we first consider a flow $f_{sk}(t)$,

$$\int_{t=0}^{\bar{T}} f_{sk}(t)dt = \int_{t=0}^{\bar{T}} \sum_{j\neq k} w^j_{sk} g_j(t)dt$$
$$= \sum_{j\neq k} w^j_{sk} \int_{t=0}^{\bar{T}} g_j(t)dt = \sum_{j\neq k} w^j_{sk} \bar{g}_j \bar{T} = \bar{f}_{sk}\bar{T} \ .$$

The first equality holds by the definitions of $f_{sk}(t)$ and $f_{sB}(t)$ in (12) and (13). The third equality holds since the estimated average rate $\bar{g}_j$ exactly matches the actual average bit rate over time interval $\bar{T}$. The fourth equality by (14) and (15). Similarly, we have $\int_{t=0}^{\bar{T}} f_{sB}(t)dt = \bar{f}_{sB}\bar{T}$. Thus, we have

$$\int_0^{\bar{T}} \left[\sum_{m\neq s} \rho f_{ms}(t) + \sum_{k\neq s} c_{sk} f_{sk}(t) + c_{sB} f_{sB}(t) \right] dt$$
$$= \sum_{m\neq s} \rho \bar{f}_{ms}\bar{T} + \sum_{k\neq s} c_{sk}\bar{f}_{sk}\bar{T} + c_{sB}\bar{f}_{sB}\bar{T} \leq e_s \ .$$

Therefore, at time $\bar{T}$ the energy consumption at each AFN i under ψ for problem P is exactly the same as that under $\bar{\pi}$, for problem $\bar{P}$. Since at time $\bar{T}$, there is at least one node with zero remaining energy under $\bar{P}$, $\bar{T}$ is thus also the network lifetime for problem P. The proof is now complete. ■

The following theorem shows that T^* is the *maximum* network lifetime for problem P. That is, there does not exist a flow routing solution that can produce a network lifetime greater than T^* for problem P

Theorem 1: The flow routing solution ψ obtained by Algorithm 1 is optimal for problem P in terms of maximizing network lifetime.

We omit the details of the proof for Theorem 1 due to space limitation. Instead, we give a sketch of the proof for completeness. The theorem can be proved by showing that problems P and $\bar{P}$ have the same maximum network lifetime. Since Algorithm 1 shows that a flow routing solution ψ for problem P has a network lifetime $T^* = \bar{T}$, where $\bar{T}$ is the maximum network lifetime for problem $\bar{P}$, then the maximum network lifetime for problem P is no less than that of problem $\bar{P}$. To show that the maximum network lifetime for problem $\bar{P}$ is also no less than that of problem P, we show that for a given network flow routing solution ψ under P with the maximum network lifetime T, we can find a flow routing solution $\bar{\pi}$ to problem $\bar{P}$ with the same network lifetime T. Using the same notation in Algorithm 1 for flow rates, such a flow routing solution $\bar{\pi}$ can be constructed by

$$\bar{f}_{sk} = \frac{\int_0^T f_{sk}(t)dt}{T} \quad \text{and} \quad \bar{f}_{sB} = \frac{\int_0^T f_{sB}(t)dt}{T} \ .$$

Therefore, problems P and $\bar{P}$ have the same network lifetime and the theorem is proved.

In a nutshell, the procedure to obtain an optimal flow routing solution ψ to problem P has the following two steps: (1) First, we solve an LP problem and find an optimal flow routing solution $\bar{\pi}$ for problem $\bar{P}$. (2) Second, we apply Algorithm 1 to get an optimal flow routing solution ψ for problem P.

III. Near Optimal Flow Routing with Estimated Average Rate

Our investigation in the last section assumes that we know the average rate $\bar{g}_i$ for the time-varying source bit rate $g_i(t)$, $1 \leq i \leq N$. In practice, $\bar{g}_i$ may not be readily available and the best thing we can do is to make an estimate for the average rate of $g_i(t)$, denoted as $\hat{g}_i$. In this section, we show that as long as the actual average is within a small constant fraction ϵ of the estimated average, i.e.,

$$|\bar{g}_i - \hat{g}_i| \leq \epsilon \hat{g}_i \ , \quad (18)$$

for each node i, $1 \leq i \leq N$, then the flow routing solution given by Algorithm 1 is near optimal. More precise, we will show that the produced network lifetime is bounded within a margin of $\frac{2\epsilon}{1-\epsilon}$ to the maximum (unknown) network lifetime.

Again, we can define problems P and $\bar{P}$. However, in this case, since average rate $\bar{g}_i$ for each AFN i is not available, flow routing $\bar{\pi}$ is thus not obtainable. Consequently, we cannot obtain an optimal flow routing solution for P using the procedure discussed in the last section.

For the estimated average rate $\hat{g}_i$ (constant), denote $\hat{P}$ as the corresponding flow routing problem with the same network topology and initial energy as those under problem P. In this case, we can find $\hat{\pi}$ following the LP similar to that for $\bar{\pi}$ discussed in the last section. Consequently, we can find a flow routing solution π from $\hat{\pi}$ using Algorithm 1.

Denote T the network lifetime to problem P under flow routing π, $f_{ik}(t)$ and $f_{iB}(t)$ are rates of flow from AFN i to AFN k and base-station B, respectively. Similarly, we define $\hat{T}$, $\hat{f}_{ik}$, and $\hat{f}_{iB}$ for $\hat{\pi}$. Despite that the average bit rate is unknown, we denote $\bar{T}$, $\bar{f}_{ik}$, $\bar{f}_{iB}$, and $\bar{\pi}$ for reference purpose. Note that the optimal network lifetime for problem P is $\bar{T}$ via Theorem 1 (if $\bar{T}$ was known). The following theorem is the main result of this section.

Theorem 2: If $|\bar{g}_i - \hat{g}_i| \le \epsilon \hat{g}_i$ for some small constant $\epsilon > 0$, then the network lifetime T under our flow routing solution π does not deviated from the maximum network lifetime $\bar{T}$ (unknown) by a fraction of

$$\frac{\bar{T} - T}{\bar{T}} \le \frac{2\epsilon}{1-\epsilon} \ .$$

Proof: Since $\frac{\bar{T}-T}{\bar{T}} \le \frac{|\bar{T}-\hat{T}|}{\bar{T}} + \frac{|\hat{T}-T|}{\bar{T}}$, we will analyze $\frac{|\bar{T}-\hat{T}|}{\bar{T}}$ and $\frac{|\hat{T}-T|}{\bar{T}}$ separately.

1) In this part, we will show that $\frac{|\bar{T}-\hat{T}|}{\bar{T}} \le \epsilon$.

To obtain $\hat{H}$ $(= 1/\hat{T})$, we solve an LP problem for problem $\hat{P}$, which can be rewritten in the form **Max** cx, **s.t.** $Ax = \hat{b}$ and $x \ge 0$, where $-H = cx$. The dual problem is **Min** $v\hat{b}$, **s.t.** $vA \ge c$ with v being unrestricted in sign [1]. Suppose that the optimal primal and dual solution are x^* and v^*, respectively. The LP problem to obtain $\bar{H}$ $(= 1/\bar{T})$ is in the form **Max** cx, **s.t.** $Ax = \bar{b}$ and $x \ge 0$, where $-H = cx$. For $\hat{b}$ and $\bar{b}$, we have

$$\hat{b}_i = \hat{g}_i \ \text{ and } \ \bar{b}_i = \bar{g}_i \quad (1 \le i \le N) \ , \tag{19}$$

$$\hat{b}_i = 0 \ \text{ and } \ \bar{b}_i = 0 \quad (N+1 \le i \le 2N) \ . \tag{20}$$

The following dual property exists for x^* and v^*,

$$-\hat{H} = cx^* = v^*\hat{b} \ . \tag{21}$$

Based on the nature of problem $\hat{P}$, we have $v^* \le 0$.

Note that $-\bar{H}$ can be obtained through $-\hat{H}$ by changing $\hat{b}_i$ to $\bar{b}_i$, $1 \le i \le N$. We now analyze the effect of changing $\hat{b}_i$ to $\bar{b}_i$.

If the LP problem for problem $\hat{P}$ is non-degenerate, we have

$$\frac{\partial(cx)}{\partial b_i}(\hat{b}_i) = v_i^* \ ,$$

where $\hat{b}_i$ and v_i^* are i-th element in $\hat{b}$ and v^*. Moreover, $\frac{\partial(cx)}{\partial b_i}(b_i)$ remains v_i^* in certain area $[l_i, u_i]$, where $l_i \le \hat{b}_i \le u_i$. Therefore, changing $\hat{b}_i$ to $\bar{b}_i$, the objective value will change by $v_i^* \cdot (\bar{b}_i - \hat{b}_i) \le \epsilon(-v_i^*\bar{b}_i)$.

If the LP problem for problem $\hat{P}$ is degenerate, we have

$$\frac{\partial^+(cx)}{\partial b_i}(\hat{b}_i) \le v_i^* \ \text{ and } \ \frac{\partial^-(cx)}{\partial b_i}(\hat{b}_i) \ge v_i^* \ .$$

Again, $\frac{\partial(cx)}{\partial b_i}(b_i) \le v_i^*$ holds in certain area $[\hat{b}_i, u_i]$ and $\frac{\partial(cx)}{\partial b_i}(b_i) \ge v_i^*$ holds in certain area $[l_i, \hat{b}_i]$. Therefore, when $\bar{b}_i > \hat{b}_i$, increasing $\hat{b}_i$ to $\bar{b}_i$, the objective value will increase by at most $\frac{\partial^+(cx)}{\partial b_i}(\hat{b}_i) \cdot (\bar{b}_i - \hat{b}_i)$; when $\bar{b}_i < \hat{b}_i$, and $\hat{b}_i$ is decreased to $\bar{b}_i$, the objective value will decrease by at most $\frac{\partial^-(cx)}{\partial b_i}(\hat{b}_i) \cdot (\hat{b}_i - \bar{b}_i)$. Thus, changing $\hat{b}_i$ to $\bar{b}_i$, the objective value will change by at most $\left|v_i^* \cdot (\bar{b}_i - \hat{b}_i)\right| \le \epsilon(-v_i^*\bar{b}_i)$.

Therefore, regardless of problem $\hat{P}$ is non-degenerate or degenerate, we have

$$\begin{aligned} \left|\frac{1}{\bar{T}} - \frac{1}{\hat{T}}\right| &= |\bar{H} - \hat{H}| \le \sum_{i=1}^{N} \epsilon(-v_i^*\hat{b}_i) \\ &= \epsilon(-v^*\hat{b}) = \epsilon\hat{H} = \frac{\epsilon}{\hat{T}} \ , \end{aligned} \tag{22}$$

where the third equality holds by (20), and the fourth equality holds by (21). Therefore, we have

$$\frac{|\bar{T} - \hat{T}|}{\bar{T}} \le \epsilon \ .$$

2) In this part, we show that $\frac{|\hat{T}-T|}{\bar{T}} \le \frac{\epsilon(1+\epsilon)}{1-\epsilon}$.

Assume node s first runs out of energy under π. We have

$$\begin{aligned} &\int_0^T \left[\sum_{m\ne s} \rho f_{ms}(t) + \sum_{k\ne s} c_{sk} f_{sk}(t) + c_{sB} f_{sB}(t)\right] dt \\ = &\sum_{m\ne s} \rho \hat{f}_{ms}\hat{T} + \sum_{k\ne s} c_{sk}\hat{f}_{sk}\hat{T} + c_{sB}\hat{f}_{sB}\hat{T} = e_s \end{aligned}$$

Consider $\hat{f}_{ms}\hat{T}$ and $\int_{t=0}^{T} f_{ms}(t)dt$, we have

$$\begin{aligned} \hat{f}_{ms}\hat{T} &= \sum_{j\ne s} w_{ms}^j \hat{g}_j \hat{T} \ , \\ \int_{t=0}^{T} f_{ms}(t)dt &= \int_{t=0}^{T} \sum_{j\ne s} w_{ms}^j g_j(t)dt \\ &= \sum_{j\ne s} w_{ms}^j \int_{t=0}^{T} g_j(t)dt \\ &= \sum_{j\ne s} w_{ms}^j \bar{g}_j T \\ &\le (1+\epsilon)\sum_{j\ne s} w_{ms}^j \hat{g}_j T \ . \end{aligned}$$

The first equality holds by (14) and (15). The second equality holds by the definitions of $f_{sk}(t)$ and $f_{sB}(t)$ in (12) and (13). The fourth equality holds since the actual average rate is $\bar{g}_j$. The fifth inequality holds by (18). Thus,

$$\begin{aligned} &\left(\sum_{m\ne s}\rho\sum_{j\ne s} w_{ms}^j\hat{g}_j + \sum_{k\ne s} c_{sk}\sum_{j\ne k} w_{sk}^j\hat{g}_j + c_{sB}\sum_{j=1}^{N} w_{sB}^j\hat{g}_j\right)\hat{T} \\ = &\sum_{m\ne s}\rho\hat{f}_{ms}\hat{T} + \sum_{k\ne s} c_{sk}\hat{f}_{sk}\hat{T} + c_{sB}\hat{f}_{sB}\hat{T} \\ = &\int_0^T \left[\sum_{m\ne s}\rho f_{ms}(t) + \sum_{k\ne s} c_{sk} f_{sk}(t) + c_{sB} f_{sB}(t)\right] dt \end{aligned}$$

$$\leq \quad (1+\epsilon)\left(\sum_{m\neq s}\rho\sum_{j\neq s}w_{ms}^{j}\hat{g}_j+\sum_{k\neq s}c_{sk}\sum_{j\neq k}w_{sk}^{j}\hat{g}_j + c_{sB}\sum_{j=1}^{N}w_{sB}^{j}\hat{g}_j\right)T$$

That is, $\hat{T} \leq (1+\epsilon)T$. Similarly, we have $\hat{T} \geq (1-\epsilon)T$. Therefore,

$$\frac{|\hat{T}-T|}{\bar{T}} \leq \frac{\epsilon T}{\hat{T}} \cdot \frac{\hat{T}}{\bar{T}} \leq \frac{\epsilon}{1-\epsilon} \cdot \frac{\bar{H}}{\hat{H}} \leq \frac{\epsilon(1+\epsilon)}{1-\epsilon} .$$

The first and second inequalities hold by the above result on T and $\hat{T}$. The third inequality holds by (22).

Combining the results in both parts, we have

$$\frac{\bar{T}-T}{\bar{T}} \leq \frac{|\bar{T}-\hat{T}|}{\bar{T}} + \frac{|\hat{T}-T|}{\bar{T}} \leq \epsilon + \frac{\epsilon(1+\epsilon)}{1-\epsilon} = \frac{2\epsilon}{1-\epsilon}$$

and the theorem is proved. ■

IV. Related Work

There has been active research on addressing issues associated with energy constraints in wireless sensor networks. In this section, we briefly summarize related research efforts on power control, power-aware routing, and network lifetime maximization.

Power control capability has been under intensive research at different layers in recent years. At the *network* layer, most work on the power control problem can be classified into two areas. The first area is comprised of strategies to find an optimal transmitter power to control the *connectivity* properties of the network (see, e.g., [6], [7], [9], [11]). A common theme in these strategies is to formulate power control as a network layer problem and then to adjust each node's transmission power so that different network connectivity topologies can be achieved for different objectives. The second area could be called *power-aware routing.* Most schemes use a shortest path algorithm with a power-based metric, rather than a hop-count based metric (see e.g., [5], [10]). However, energy-aware (e.g., minimum energy path) routing cannot ensure good performance in maximum network lifetime.

The notion of network lifetime has been a focus in sensor networking research in recent years [2]. The most relevant work on network lifetime related to our research have been described in [3]. As discussed, the LP formulation in [3] can only address the simple constant source bit rate case. On the other hand, we explored the more difficult problem of time-varying source bit rate in this paper.

V. Conclusions

In this paper, we studied flow routing problem for a energy-constrained wireless sensor network where the source bit rate could be time-varying. Our objective is to find flow routing algorithm such that the network lifetime can be maximized. We presented a flow routing algorithm that has the following performance guarantees: (1) When the average source rate of each AFN is known a priori, the flow routing algorithm is optimal and gives maximum network lifetime performance; (2) When the average source rate of each AFN is unknown but is within a fraction of ϵ of an estimated rate value, then the network lifetime given by the proposed flow routing algorithm is no more than $\frac{2\epsilon}{1-\epsilon}$ from optimal. The result in this paper constitutes an important step in algorithmic research for flow routing problems in energy-constrained sensor networks.

Acknowledgements

This research has been supported in part by the National Science Foundation (NSF) under Grants ANI-0312655, and CNS-0347390, and by the Office of Naval Research (ONR) under Grants N00014-03-1-0521 and N00014-05-1-0179.

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