On Lexicographic Max-Min Node Lifetime for Wireless Sensor Networks

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Abstract— **In this paper, we study the network lifetime problem by considering not only maximizing the time until the first node fails, but also maximizing the lifetime for all the nodes in the network, which we define as the *Lexicographic Max-Min (LMM) node lifetime* problem. The main contributions of this paper are two-fold. First, we develop a polynomial-time algorithm to derive the LMM-optimal node lifetime vector, which effectively circumvents the computational complexity problem associated with an existing state-of-the-art approach, which is exponential. Second, we present a simple (also polynomial-time) algorithm to calculate the flow routing schedule such that the LMM-optimal node lifetime vector can be achieved. Our results in this paper advance the state-of-the-art algorithmic design to network-wide node lifetime problems.**

Index Terms— **Energy constraint, node lifetime, lexicographic max-min, flow routing, power control, wireless sensor networks.**

I. Introduction

Wireless sensor networks promise to have a significant impact on society that could quite possibly dwarf previous milestones in the information revolution. Although there have been significant improvements in processor design and computing, advances in battery technology still lag behind, making energy resource the fundamental challenge in wireless sensor networking. As a consequence of the energy constraint, a new performance metric, namely, the *network lifetime,* has become a vitally important benchmark for wireless sensor networks. There have been active research efforts recently at the networking layer on devising flow routing algorithms to maximize network lifetime, e.g. [3], [4], [6], [18]. However, the network lifetime objective in most of these efforts has been centered around maximizing the time until the first node fails. Although the time until the first node fails is an important measure from the complete network coverage point of view, this performance metric alone cannot measure the lifetime performance behavior for all nodes in the network. For wireless sensor networks that are primarily designed for environmental monitoring or surveillance, the loss of a single node will only affect the coverage of one particular area and will not affect the monitoring or surveillance capabilities of the remaining nodes in the network. Consequently, it is important to investigate how to maximize the lifetime for, not only the first node, but also *all* the nodes in the network. We call this the *Lexicographic Max-Min (LMM) node lifetime* problem, which will be formally defined in Section II-C.

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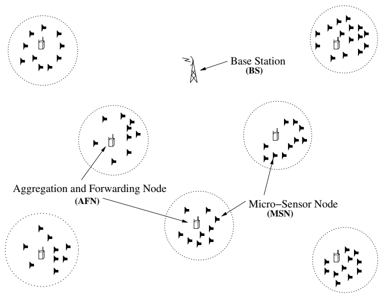
Recently, Brown *et al.* [5] studied this problem under the so-called "maximum node lifetime curve" problem, which is equivalent to the Lexicographic Max-Min (LMM) node lifetime problem. Informally, the maximum node life curve attempts to *maximize* the time until a set of nodes drain their energy (*drop point*) while *minimizing* the number of nodes that drain their energy at each drop point. The main contribution by Brown *et al.* [5] is the development of a procedure to solve the maximum node lifetime curve problem. Although this approach can solve the LMM node lifetime problem, its computational complexity is shown to be exponential, which could be a potential problem for large-scale networks.

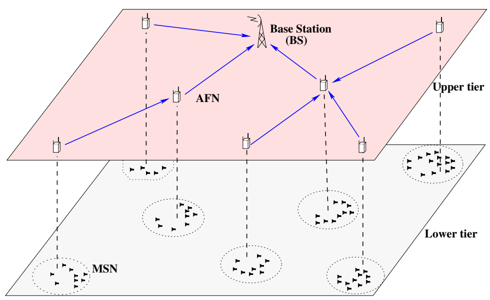
Inspired by the important work in [5] on the LMM node lifetime, we develop a polynomial-time algorithm to derive the LMM-optimal node lifetime vector. We demonstrate that, for any given network configuration and initial condition, our approach is computationally more efficient than the slack variable based (SV-based) approach in [5]. The computational effectiveness of our approach accrues from two important techniques. First, we employ a *link-based* formulation, which significantly reduces the problem size in comparison with a flow-based formulation used in [5]. Second, which is also the most significant contribution in this paper, we exploit the so-called *parametric analysis* technique to determine the minimum energy-drained node set at each drop point. When the problem is non-degenerate, we show that this technique is a powerful tool in determining the minimum node set for each drop point. It has a quadric time complexity in contrast with the SV-based approach proposed in [5], which requires solving multiple additional LPs at each drop point (with much higher order complexity). Even for the rare case, when the problem is degenerate, using the parametric analysis technique still is more efficient than the SV-based approach as it decreases the number of additional LPs that need to be solved at each drop point.

In addition to providing an efficient polynomial-time algorithm for the LMM-optimal node lifetime vector computation, we also develop a simple polynomial-time algorithm that provides a corresponding flow routing schedule at each stage such that the LMM-optimal node lifetime vector can indeed be achieved.

The remainder of this paper is organized as follows. Sec-



(a) Physical topology.



(b) A hierarchical view.

Fig. 1. Reference architecture for a two-tiered wireless sensor network.

tion II describes the system model and problem statement for this research. We also describe a naive approach to address this problem and discuss why it usually gives an incorrect solution. Section III presents the link-based formulation and our efficient serial LP algorithm based on parametric analysis, which we call SLP-PA. In Section IV, we present a simple algorithm to calculate the flow routing schedule at each stage such that the LMM-optimal node lifetime vector can indeed be achieved. Section V compares the complexity of our algorithm with that in [5]. Numerical results using the SLP-PA approach and the corresponding flow routing schedule are given in Section VI. Section VII reviews related work and Section VIII concludes this paper.

II. System Modeling and Problem Formulation

A. Reference Network Architecture

We focus on a two-tiered architecture for wireless sensor networks. The two-tiered network architecture is motivated by recent advances in *distributed source coding* (DSC) [7], [12]. Figures 1 (a) and (b) show the *physical* and *hierarchical* network topology for such a network, respectively. Here, we have three types of nodes in the network: *micro-sensor nodes* (MSNs), *aggregation and forwarding nodes* (AFNs), and a *base-station* (BS). The MSNs can be application-specific sensor nodes (*e.g.,* temperature sensor nodes (TSNs), pressure sensor nodes (PSNs), and video sensor nodes (VSNs)) and they constitute the lower tier of the network. They are deployed in groups (or clusters) at strategic locations for surveillance or monitoring applications. The objective of an MSN is very simple: Once triggered by an event, (*e.g.,* the detection of motion or biological/chemical agents), it starts to capture live information (*e.g.,* video), which it sends directly to the local AFN in one hop.

For each cluster of MSNs, there is one AFN, which is different from an MSN in terms of both its physical properties and functions. The primary functions of an AFN are: (1) *data aggregation* (or "fusion") for data flows from the local cluster of MSNs, and (2) *forwarding* (or relaying) the aggregated data to the next hop AFN toward the base-station. Although an AFN is expected to be provisioned with much more energy than an MSN, it also consumes energy at a substantially higher rate (due to wireless communication over large distances). Consequently, an AFN has limited lifetime. Upon the depletion of energy at an AFN, we expect that the *coverage* for the particular area under surveillance will be lost, despite the fact that some of the MSNs within the cluster may still have remaining energy.[1] Therefore, it is essential to maximize the lifetime of each AFN, which is the main focus of this paper.

The third component in the two-tiered architecture is the base-station. The base-station is, essentially, the *sink* node for data streams from all the AFNs in the network. We assume that a base-station has sufficient power resource and does not have energy constraint as the MSNs and AFNs. In summary, the main functions of the lower tier MSNs are data acquisition and compression while the upper-tier AFNs are used for data fusion and relaying the information to the base-station.

B. Power Control and Consumption Model

An effective technique to control network routing topology is to adjust the power level of a node's transmitter [9], [14], [16], [17]. This in turn will control the distance coverage of an AFN and form different network routing topologies.

For an AFN, the radio-related power consumption (*i.e.,* in transmitter and receiver) is the dominant factor [1]. The power consumption at a transmitter can be modeled as:

$$p_t(i,\ k) = c_{ik} \cdot f_{ik}\ , \tag{1}$$

where $p_t(i,\ k)$ is the power dissipated at omni-directional antenna when AFN i is transmitting to k, f_{ik} is the rate of the data stream sent by AFN i to k, c_{ik} is the power consumption cost of link (i,k) and

$$c_{ik} = \alpha + \beta \cdot d_{ik}^m\ , \tag{2}$$

where α is a *distance-independent* term, β is a coefficient associated with the *distance-dependent* term, d_{ik} is the distance between these two nodes, m is the path loss index and $2 \leq m \leq 4$ [15]. Typical values for these parameters are $\alpha = 50$ nJ/b and $\beta = 0.0013$ pJ/b/m^4 for $m = 4$ [10]. In this paper, we use $m = 4$ in all of our numerical results.

[1] We assume that each MSN can only forward data to its local AFN for processing.

The power dissipation at the receiver of AFN j is [15]:

$$p_r(j) = \rho \cdot \sum_{k \neq j} f_{kj} \ , \tag{3}$$

where f_{kj} (in b/s) is the incoming rate of received data stream from AFN k. Typical value of ρ is 50 nJ/b [10].

C. The Lexicographic Max-Min Node Lifetime Problem

It is important to maximize the time until any AFN runs out of energy (also known as the network lifetime), but it is even more important to concurrently maximize the time that all AFNs run of energy. That is, it is important to find a flow routing schedule among the AFNs such that the lifetimes of all AFNs in the network can achieve the optimal *Lexicographic Max-Min* (LMM) vector. A formal definition for the LMM-optimal node lifetime vector is given as follows.

Definition 1: A sorted network node lifetime vector $[\tau_1, \tau_2, \cdots, \tau_N]$ with $\tau_1 \leq \tau_2 \leq \cdots \leq \tau_N$ is LMM-optimal if and only if for any other sorted node lifetime vector $[\hat{\tau}_1, \hat{\tau}_2, \cdots, \hat{\tau}_N]$ with $\hat{\tau}_1 \leq \hat{\tau}_2 \leq \cdots \leq \hat{\tau}_N$, there exists a $k \in [1, N]$, such that $\tau_i = \hat{\tau}_i$ $(i = 1, 2, \cdots, k-1)$ and $\tau_k > \hat{\tau}_k$.

A naive approach to the LMM node lifetime problem would be to apply a max-min-like iterative procedure. Under this approach, an iterative LP for alive nodes could be employed to find the maximum time until the next node fails. By calculating the remaining energy at each node at the end of the iteration, one would attempt to move on to the next iteration, until all the nodes drain their energy. Although this approach seems appealing and intuitive, we now show that it usually gives an incorrect solution. This is because the LMM node lifetime problem implicitly embeds (or couples) a flow routing problem, and due to this coupling, *any iterative LMM node lifetime algorithm requiring energy reservation among the nodes during each iteration is incorrect.* Starting from the first iteration, there usually exist *non-unique* flow routing solutions corresponding to the same drop point. Consequently, each of these flow routing schedules, once chosen, will yield *different* remaining energy at the AFNs for future iterations and so forth, leading to a different node lifetime vector, which may not be the LMM-optimal node lifetime vector (see Section VI for numerical results).

Recently, Brown *et al.* [5] studied the LMM node lifetime problem under the notion of a "node lifetime curve". They first identified the uniqueness of the LMM-optimal node lifetime vector. Based on this property, they developed an iterative procedure to solve the LMM node lifetime problem. A key step in their procedure is to use multiple independent LPs to determine the minimum node set at each drop point. During each iteration, only the drop point and the corresponding minimum set of nodes are determined, and there is no resource reservation among the nodes at each stage. Although their proposed approach solves the LMM node lifetime problem and constitutes a major advance in the basic understanding of node lifetime problem, this approach is shown to be of *exponential* computational complexity, which could become problematic when the scale of the network becomes large.

III. An Efficient Serial LP Algorithm Based on Parametric Analysis

In this section, we present a polynomial-time algorithm for the LMM node lifetime problem. Moreover, for *any* given network configuration and initial condition, our approach is much simpler than the slack variable based (SV-based) approach in [5]. The computational effectiveness of our approach hinges upon two important techniques. First, we employ a link-based problem formulation that significantly reduces the problem size in comparison with a flow-based formulation adopted in [5]. Second, we invoke a *parametric analysis* procedure at each stage to determine the minimum node set at each drop point. For non-degenerate case, this parametric analysis results in only a quadric time computational complexity, while the SV-based approach in [5] requires solving multiple independent LPs to determine the minimum set of nodes at each drop point. Even for the rare case, when the problem is degenerate, using our parametric analysis technique still is more efficient than the SV-based approach because it decreases the number of additional LPs that need to be solved at each drop point. In the remainder of this section, we elaborate on the details of our serial LP algorithm based on parametric analysis (SLP-PA).

A. Link-based Formulation

Suppose that $[\tau_1, \tau_2, \cdots, \tau_N]$ with $\tau_1 \leq \tau_2 \leq \cdots \leq \tau_N$ is LMM-optimal. To keep a track of *distinct* node lifetimes, we remove all repetitive elements in the vector and rewrite it as $[a_1, a_2, \cdots, a_n]$ such that $a_1 < a_2 < \cdots < a_n$, where $a_1 = \tau_1$, $a_n = \tau_N$, and $n \leq N$. Denote S_i as the set of nodes that drain their energy at a_i $(1 \leq i \leq n)$. Clearly, $|S_1| + |S_2| + \cdots + |S_n| = |S| = N$ where S denotes the set of all N AFNs in the network. The problem is to find the LMM-optimal values of $a_1, a_2, \cdots, a_n$ and the corresponding sets $S_1, S_2, \cdots, S_n$.

To formulate this problem into an iterative form, we define $a_0 = 0$ and $S_0 = \phi$. Furthermore, denote $\delta_l = a_l - a_{l-1}$. Then, the iterative optimization problem (starting with $l = 1$) for the LMM node lifetime problem becomes,

LP-LMM: Max δ_l

s.t.

$$V_{iB} + \sum_{k \neq i} V_{ik} - \sum_{m \neq i} V_{mi} - \delta_l g_i = a_{l-1} g_i \quad (i \in S - \textstyle\bigcup_{j=0}^{l-1} S_j) \tag{4}$$

$$V_{iB} + \sum_{k \neq i} V_{ik} - \sum_{m \neq i} V_{mi} = a_h g_i \quad (i \in S_h, h < l) \tag{5}$$

$$\sum_{m \neq i} \rho V_{mi} + \sum_{k \neq i} c_{ik} V_{ik} + c_{iB} V_{iB} \leq e_i \quad (i \in S - \textstyle\bigcup_{j=0}^{l-1} S_j) \tag{6}$$

$$\sum_{m \neq i} \rho V_{mi} + \sum_{k \neq i} c_{ik} V_{ik} + c_{iB} V_{iB} = e_i \quad (i \in S_h, h < l) \tag{7}$$

$$V_{ik}, V_{iB}, \delta_l \geq 0 \quad (1 \leq i \neq k \leq N).$$

The set of constraints in (4) and (5) state that the total incoming and local data bit volumes are equal to the total outgoing data bit volumes for each node with and without remaining energy at time a_{l-1}, respectively. The set of constrains in (6) and (7) state that the total energy consumed for receiving and transmitting data bit volumes is no more than or equal to the initial energy for each node with and without remaining energy at time a_{l-1}, respectively.

The above LP formulation can be rewritten in the form: **Max** cx, **s.t.** $Ax = b$ and $x \geq 0$, the dual problem for which is given by: **Min** wb, **s.t.** $wA \geq c$ and w unrestricted [2]. Both can be solved simultaneously by standard LP techniques (e.g., [2]), in polynomial-time. Although solving LP-LMM gives the optimal value for δ_l, we need yet to determine the *minimum* set of nodes corresponding to this δ_l, which is the main task in this investigation. In the remainder of this section, we effectively exploit post-LP parametric analysis techniques [2] to determine the minimum node set for each drop point.

B. Minimum Node Set Determination with Parametric Analysis

Denote $\hat{S}_l \neq \phi$ to be the set of nodes that achieve *equality* in (6). Some of the nodes in $\hat{S}_l$ may still be further "stretched" to live longer under alternative flow routing schedules. Therefore, we need to determine the *minimum* set of S_l ($S_l \subseteq \hat{S}_l$) that achieves the LMM-optimal solution.

The so-called *parametric analysis* (PA) technique [2] is most effective in addressing this type of problems. Considering a small increase in the right-hand-side (RHS) of (4), *i.e.,* changing b_i to $b_i + \epsilon_i$, where $\epsilon_i > 0$, node i belongs to S_l *if and only if* $\frac{\partial^+ \delta_l}{\partial \epsilon_i}(0) < 0$, *i.e.,* a small increase in node i's lifetime (in terms of total bit volume generated at node i) leads to a decrease in the next drop point.

To compare $\frac{\partial^+ \delta_l}{\partial \epsilon_i}(0)$ with 0, we resort to an important duality relationship in LP theory. If x and w are the respective optimal solutions to the primal and dual problems, then based on the parametric duality property [2], we have

$$\frac{\partial^+ \delta_l}{\partial \epsilon_i}(0) = \frac{\partial^+ (cx)}{\partial b_i}(b_i) \leq w_i \ . \tag{8}$$

Recall that these w_i can be easily obtained at the same time when we solve the primal LP problem. Note that by the nature of the problem, we have $w_i \leq 0$ for an optimal dual solution. Therefore, if $w_i < 0$, then we can determine immediately that $i \in S_l$. On the other hand, if we find that $w_i = 0$, it is not clear whether $\frac{\partial^+ \delta_l}{\partial \epsilon_i}(0)$ is strictly negative or 0 and further analysis is thus needed.

For each node i with $w_i = 0$, we must perform a complete PA to see whether $i \in S_l$ or not. Assume that the optimal solution is $(x_{\mathcal{B}}, x_{\mathcal{Z}})$, where $x_{\mathcal{B}}$ and $x_{\mathcal{Z}}$ denote the set of basic and non-basic variables; $\mathcal{B}$ and $\mathcal{Z}$ denote the columns corresponding to the basic and non-basic variables. Denote $c_{\mathcal{B}}$ and $c_{\mathcal{Z}}$ the objective function coefficient vectors for the basic and non-basic variables and q the objective value. Then the corresponding canonical equations yield

$$\begin{aligned} q + (c^t_{\mathcal{B}}\mathcal{B}^{-1}\mathcal{Z} - c^t_{\mathcal{Z}})x_{\mathcal{Z}} &= c^t_{\mathcal{B}}\mathcal{B}^{-1}b \ , \\ x_{\mathcal{B}} + \mathcal{B}^{-1}\mathcal{Z}x_{\mathcal{Z}} &= \mathcal{B}^{-1}b \ . \end{aligned}$$

If b is replaced by $b + \epsilon_i I_i$, where the column vector I_i has a single 1 corresponding to node i in (4) and has 0 elements otherwise, then the only change in the constraints due to this perturbation is that $\mathcal{B}^{-1}b$ will be replaced by $\mathcal{B}^{-1}(b + \epsilon_i I_i)$. Consequently, the objective value for the current basis becomes $c^t_{\mathcal{B}}\mathcal{B}^{-1}(b + \epsilon_i I_i)$. Furthermore, as long as $\mathcal{B}^{-1}(b + \epsilon_i I_i)$ is nonnegative, the current basis remains optimal. Denote $\bar{b} = \mathcal{B}^{-1}b$ and $\mathcal{B}^{-1}_i = \mathcal{B}^{-1}I_i$ and let $\hat{\epsilon}_i$ be an upper bound for ϵ_i such that the current basis remains optimal, we have

$$\hat{\epsilon}_i = \min_j \{ \frac{\bar{b}_j}{-\mathcal{B}^{-1}_{ij}} : \mathcal{B}^{-1}_{ij} < 0 \} \ . \tag{9}$$

If $\hat{\epsilon}_i > 0$, the optimal objective value varies according to $c^t_{\mathcal{B}}\mathcal{B}^{-1}(b + \epsilon_i I_i)$ for $0 < \epsilon_i \leq \hat{\epsilon}_i$. Since $w = c^t_{\mathcal{B}}\mathcal{B}^{-1}$ and $w_i = 0$, we have $c^t_{\mathcal{B}}\mathcal{B}^{-1}I_i = w_i = 0$. Thus, the objective value will *not* change for $\epsilon_i \in (0, \hat{\epsilon}_i]$, and consequently, node i can live longer beyond current drop point a_l. That is, node i does not belong to the minimum node set S_l.

For most problems in practice, this process can determine whether $i \in S_l$ or $i \notin S_l$ for all $i \in \hat{S}_l$. But in the rare event where $\hat{\epsilon}_i = 0$, the problem is degenerate. To develop a polynomial-time algorithm, denote W_l as the set of all nodes with $w_i < 0$ and U_l the set of all nodes with $w_i = 0$ and $\hat{\epsilon}_i = 0$. Then we solve the following LP to maximize the slack variables (SV) for nodes in U_l.

MSV: Max $\sum_{i \in U_l} \epsilon_i$

s.t.

$$\begin{aligned}
V_{iB} + \sum_{k \neq i} V_{ik} - \sum_{m \neq i} V_{mi} - \epsilon_i g_i &= a_l g_i, && (i \in U_l) \\
V_{iB} + \sum_{k \neq i} V_{ik} - \sum_{m \neq i} V_{mi} &= a_h g_i, && (i \in \textstyle\bigcup_{h=1}^{l-1} S_h) \\
V_{iB} + \sum_{k \neq i} V_{ik} - \sum_{m \neq i} V_{mi} &= a_l g_i, && (i \notin U_l \textstyle\bigcup_{h=1}^{l-1} S_h) \\
\sum_{m \neq i} \rho V_{mi} + \sum_{k \neq i} c_{ik} V_{ik} + c_{iB} V_{iB} &= e_i, && (i \in U_l \textstyle\bigcup W_l \bigcup_{h=1}^{l-1} S_h) \\
\sum_{m \neq i} \rho V_{mi} + \sum_{k \neq i} c_{ik} V_{ik} + c_{iB} V_{iB} &\leq e_i, && (i \notin U_l \textstyle\bigcup W_l \bigcup_{h=1}^{l-1} S_h) \\
V_{ik}, V_{iB}, \epsilon_i &\geq 0, && (1 \leq i \neq k \leq N).
\end{aligned}$$

If the optimal objective value is 0, then no node in U_l can have a positive ϵ_i. That is, these nodes should all belong to S_l and we have $S_l = W_l + U_l$. On the other hand, if the optimal objective value is positive, then some nodes in U_l must have positive ϵ_i, *i.e.,* these nodes should not belong to S_l. Consequently, we remove these nodes from U_l and if $U_l \neq \phi$, we solve another MSV. This procedure will terminate when the optimal objective value is 0 or $U_l = \phi$.

To ensure that MSV determinate the minimum node set correctly, we need the following lemma, the proof of which is omitted due to space limitation.

Lemma 1: The minimum node set at each drop point under the LMM-optimal solution is unique.

In a nutshell, the complete PA procedure for the determination of whether or not a node i ($\in \hat{S}_l$) belongs to the minimum node set S_l can be summarized as follows.

Algorithm 1: **(Minimum Node Set Determination)**

1) *Initialize* $W_l = \phi$ *and* $U_l = \phi$.

2) *For each node $i \in \hat{S}_l$,*
   a) *if $w_i < 0$, add node i to W_l;*
   b) *otherwise, using $\mathcal{B}^{-1}$ (which is readily available after solving an LP-LMM), compute $\bar{b} = \mathcal{B}^{-1}b$, $\mathcal{B}_i^{-1} = \mathcal{B}^{-1}I_i$, and $\hat{\epsilon}_i$ according to (9). If $\hat{\epsilon}_i = 0$, add i to U_l.*
3) *If $U_l = \phi$, let $S_l = W_l$ and stop, else build and solve an MSV.*
4) *If the optimal objective value is 0, let $S_l = W_l + U_l$ and stop. Otherwise, remove all nodes with $\epsilon_i > 0$ from U_l and go to Step 3.*

IV. LMM-OPTIMAL FLOW ROUTING SCHEDULE

In this section, we present a simple polynomial-time algorithm that provides an LMM-optimal flow routing schedule. The main task of this algorithm is to define flows based on the bit volumes (V_{ik} and V_{iB} values), which are obtained upon the completion of the LP-LMM in our SLP-PA approach. The main result here is that for all the remaining alive nodes at each stage, if we let the total amount of out-going flow at a node be distributed *proportionally to the bit volumes* on each out-going link, then we can achieve the drop points $a_1, a_2, \cdots, a_n$ as well as the corresponding minimum node sets $S_1, S_2, \cdots, S_n$. The algorithm is formally described as follows.

Algorithm 2: **(An Optimal Flow Routing Schedule)** *Upon the completion of the SLP-PA algorithm for the LMM node lifetime vector, we have the drop points (in strictly increasing order) $a_1, a_2, \cdots, a_n$, the corresponding minimum physical node sets $S_1, S_2, \cdots, S_n$, and the total amount of bit volume on each radio link (*i.e., *V_{ik} and V_{iB}). The following iterations give an LMM-optimal flow routing schedule for the corresponding time interval $(a_{l-1}, a_l]$, where $a_0 = 0$ and $l = 1, 2, \cdots, n$.*

1) *Denote $U_l = S - \bigcup_{j=0}^{l-1} S_j$, with $S_0 = \phi$. Initialize all flows to zero,* i.e., *$f_{ik}^{(l)} = 0$, $f_{iB}^{(l)} = 0$ for $1 \le i \ne k \le N$.*
2) *If $U_l = \phi$, then stop, else choose a node $i \in U_l$ such that*[2]
   - *either node i does not receive data from any other node, or*
   - *all nodes from which node i receives data are not in U_l.*
3) *The flow routing at node i during $(a_{l-1}, a_l]$ is then defined as*

$$f_{ik}^{(l)} = \frac{V_{ik}}{V_{iB} + \sum_{k \ne i} V_{ik}} (\sum_{m \ne i} f_{mi}^{(l)} + g_i) \ , \quad (\forall k \ne i)$$

$$f_{iB}^{(l)} = \frac{V_{iB}}{V_{iB} + \sum_{k \ne i} V_{ik}} (\sum_{m \ne i} f_{mi}^{(l)} + g_i) \ ,$$

   where the $f_{mi}^{(l)}$ values, if not zero, have all been defined before calculating the flow routing for node i.
4) *Let $U_l = U_l - \{i\}$ and go to Step 2.*

[2]It can be shown that an LMM-optimal solution is cycle free in terms of flow routing. Consequently, the node i under consideration must exist when $U_l \ne \phi$.

The proof that Algorithm 2 will indeed give the LMM-optimal node lifetime vector is omitted due to space limit. As shown in this algorithm, for each time interval $(a_{l-1}, a_l]$, $l = 1, 2, \cdots, n$, we initialize U_l as the set of remaining alive nodes at this stage, which is represented by $U_l = S - \bigcup_{j=0}^{l-1} S_j$. For these nodes, we compute a flow routing by starting with the "boundary" nodes and then move to the "interior" nodes. More precisely, we calculate the flow routing for a node i if and only if we have calculated the flow routing for each node m that has traffic coming into node i. The out-going flow at node i is calculated by distributing the aggregated in-coming flow *proportionally* according to the overall bit volume along its out-going radio links.

V. COMPUTATIONAL COMPLEXITY ANALYSIS

It is clear that SLP-PA is strictly polynomial due to the polynomial complexity of LP. Here, we compare the complexity of our approach with the SLP-SV approach in [5]. First of all, SLP-SV needs to keep track of each *sub-flow* along its route from the source node toward the base-station. Such a flow-based (or more precisely, sub-flow based) approach makes the size of the LP coefficient matrix exponential, which leads to an exponential-time algorithm even with the most efficient LP technique (*e.g.,* [2]). Second, even if a link-based LP formulation such as ours is adopted in [5], the computational efficiency of slack variable based (SV-based) approach would be still worse than SLP-PA. This is because that at each stage, the SV-based approach in [5] solves several *additional* LPs (up to $|\hat{S}_l - S_l|$) to determine S_l, in contrast with the simpler parametric analysis for the SLP-PA approach, which only involves $O(N^2)$ effort for the non-degenerate case. Even for the degenerate case, the number of additional LPs are up to $|U_l^{(0)} - S_l|$ ($\le |\hat{S}_l - S_l|$). Consequently, for any problem, our approach is computational more efficient than the SLP-SV approach in [5].

Finally, we discuss a hybrid link-flow approach mentioned in [5]. This approach requires a sub-flow accounting on each link and results in an order of magnitude more constraints than the link-based approach proposed in this paper. Although this approach can solve the LMM node lifetime problem in polynomial-time (*e.g.,* using interior point methods [2]), the overall complexity is still orders of magnitude higher than that for our proposed SLP-PA approach. Furthermore, there remains the additional burden associated with the SLP-SV approach for solving the additional LPs even using the hybrid link-flow based approach.

VI. NUMERICAL INVESTIGATION

In this section, we use numerical results to illustrate the solution to the LMM node lifetime problem and compare our SLP-PA to some other approaches. In particular, we will compare SLP-PA with the naive approach (see Section II-C) that uses a serial LP "blindly" to solve the LMM node lifetime problem. We call this naive approach *Serial LP* (SLP).

TABLE I

LOCATIONS (IN METERS) FOR EACH AFN IN A 10-NODE NETWORK.

AFN	Location	AFN	Location
1	(400, -320)	6	(-500, 100)
2	(300, 440)	7	(-400, 0)
3	(-300, -420)	8	(420, 120)
4	(320, -100)	9	(200, 140)
5	(340, -120)	10	(220, -340)

TABLE II

LIFETIME (IN DAYS) FOR THE 10-AFN NETWORK.

Sorted	SLP-PA		SLP		MPR	
Index	AFN	τ_i	AFN	τ_i	AFN	τ_i
1	3	45.71	1	45.71	7	28.91
2	6	45.71	2	45.71	3	46.09
3	7	45.71	3	45.71	6	61.63
4	1	146.08	5	45.71	9	87.75
5	2	146.08	6	45.71	4	92.77
6	4	146.08	7	45.71	5	118.79
7	5	146.08	10	45.71	8	142.96
8	8	146.08	4	303.70	2	150.29
9	9	146.08	8	303.70	10	157.62
10	10	146.08	9	303.70	1	182.55

We also compare our SLP-PA approach with the *Minimum-Power Routing* (MPR) approach that has been considered in the literature (see, e.g. [8], [9], [11], [13]) and is used to achieve energy efficiency. Under the MPR approach, an AFN always chooses the path that consumes the minimum amount of power toward the base-station.[3]

A. Network Configuration and Parameters

We consider a network consisting of 10 AFNs. The base-station B is assumed to locate at the origin $(0,\ 0)$ (in meters). The locations for the 10 AFNs are generated at random and are shown in Tables I.

B. Results

We assume that the initial energy at each AFN is 50 kJ and local data generated by each AFN is 0.2 kb/s. The power dissipation behaviors for transmission and reception are defined in (1) and (3), respectively.

Table II gives each AFN's lifetime under each approach. The "sorted index" column represents the node index, in which AFNs are sorted in node lifetimes nondecreasing order. Clearly, the node lifetime vector under SLP-PA is much more superior than that under the SLP and MPR approaches with respect to the LMM-optimal node lifetime vector definition (see Definition 1). For example, comparing the node lifetime vector under SLP-PA and SLP, we find that $\tau_1^{SLP-PA} = \tau_1^{SLP}$, $\tau_2^{SLP-PA} = \tau_2^{SLP}$, $\tau_3^{SLP-PA} = \tau_3^{SLP}$, and $\tau_4^{SLP-PA} > \tau_4^{SLP}$. Similarly, comparing the node lifetime vector under SLP-PA and MPR, we have $\tau_1^{SLP-PA} > \tau_1^{MPR}$. In general, τ_1^{MPR} (28.91 days) is the smallest among the three approaches (45.71 days under both SLP-PA and SLP) since minimum power routing does not guarantee a good performance with respect to node lifetime performance. Although SLP and SLP-PA have the same node lifetime (45.71 days) at the first stage, SLP-PA gives a smaller AFN set ($|S_1^{SLP-PA}| = 3$) at this drop point than SLP ($|S_1^{SLP}| = 7$), which shows that the naive SLP approach cannot offer the correct solution to the LMM node lifetime problem.

[3]The results for the SLP-SV approach is not shown since they are the same as those under SLP-PA. The difference is in the computational complexity.

We now show how to use Algorithm 2 to calculate a flow routing schedule that achieves the LMM-optimal node lifetime vector for the 10-AFN network. Under the SLP-PA approach, we have $a_1 = 45.71$ days with $S_1 = \{3, 6, 7\}$ and $a_2 = 146.08$ days with $S_2 = \{1, 2, 4, 5, 8, 9, 10\}$. Also, we obtain the following bit volumes (all in 10^4 kb) among the nodes from the last LP-LMM solution:

$V_{1,5} = 320.0419$, $V_{1,B} = 46.7550$;
$V_{2,9} = 233.8006$, $V_{2,B} = 18.6306$;
$V_{3,7} = 48.6548$, $V_{3,B} = 30.3317$;
$V_{4,B} = 303.3560$;
$V_{5,4} = 50.9249$, $V_{5,8} = 390.6881$, $V_{5,B} = 130.8601$;
$V_{6,7} = 22.2673$, $V_{6,B} = 56.7191$;
$V_{7,B} = 149.9086$;
$V_{8,9} = 576.2578$, $V_{8,B} = 66.8615$;
$V_{9,B} = 1062.4895$;
$V_{10,1} = 114.3658$, $V_{10,B} = 138.0654$.

We now find the flow routing schedule for each interval, *i.e.*, $[0,\ a_1]$ and $(a_1,\ a_2]$, respectively. For time interval $[0, a_1]$, we obtain the following.

- Nodes 2, 3, 6, and 10 do not receive any data. Using Algorithm 2, node 2 sends 0.185 kb/s to node 9 and 0.015 kb/s to the base-station B. Similarly, we can calculate the flows for node 3, 6, and 10.
- Now, since the in-coming flow to nodes 1 and 7 are defined, we can calculate their out-going flow rates. For example, node 1 sends 0.254 kb/s to node 5 and 0.037 kb/s to the base-station B.
- Next, we consider node 5. After calculation, we find that node 5 should send 0.040 kb/s to node 4, 0.310 kb/s to node 8, and 0.104 kb/s to the base-station B.
- Following this, we consider nodes 4 and 8. We find that node 4 sends 0.240 kb/s to the base-station B; node 8 sends 0.457 kb/s to node 9 and 0.053 kb/s to the base-station B.
- Finally, we consider node 9. Using Algorithm 2, we find that node 9 sends 0.842 kb/s to the base-station B.

In summary, during $[0,\ a_1] = [0, 45.71]$, we have the following flow rates (all in kb/s):

$f_{1,5} = 0.254$, $f_{1,B} = 0.037$;
$f_{2,9} = 0.185$, $f_{2,B} = 0.015$;
$f_{3,7} = 0.123$, $f_{3,B} = 0.077$;
$f_{4,B} = 0.240$;
$f_{5,4} = 0.040$, $f_{5,8} = 0.310$, $f_{5,B} = 0.104$;
$f_{6,7} = 0.057$, $f_{6,B} = 0.143$;
$f_{7,B} = 0.380$;

$f_{8,9} = 0.457, \quad f_{8,B} = 0.053;$
$f_{9,B} = 0.842;$
$f_{10,1} = 0.091, \quad f_{10,B} = 0.109.$

The application of Algorithm 2 for the interval $(a_1,\ a_2] = (45.71, 146.08]$ is similar and is thus omitted here due to space limit. It is easy to verify that the flow routing schedule will indeed achieve the LMM-optimal node lifetime vector.

VII. Related Work

The closest work related to ours is that in [5], which has been discussed in detail in the paper. In this section, we briefly review relevant work that contributed to the background for our investigation.

There have been many recent efforts in the area of *power-aware routing* (see *e.g.,* [8], [9], [11], [13]). Most schemes under power-aware routing use a shortest path algorithm with a power-based metric, rather than a hop-count based metric. However, as we have shown in the numerical results section, power-aware routing (*e.g.,* minimum-power path) cannot ensure good performance in maximizing network lifetime. For example, using the most energy-efficient route may still result in a premature depletion of energy at certain nodes, which is not optimal from the network lifetime perspective.

The notion of network lifetime for wireless sensor networks has been studied in [3], [4], [6], [18]. The notion of network lifetime discussed in these work focuses on the time until the first node fails without further consideration of the remaining nodes in the network. As wireless sensor networks will typically remain useful even if some nodes run out of energy, it is essential to further investigate how to maximize the lifetime for all the remaining nodes in the network, which is the focus of this paper.

VIII. Conclusions

In this paper, we have considered the problem of how to maximize the lifetime for all the nodes in a wireless sensor network. We formally defined this optimization problem as the Lexicographic Max-Min (LMM) node lifetime problem and investigated approaches to solve it. The main contributions in this paper are two-fold. First, we developed a polynomial-time algorithm to obtain the LMM-optimal node lifetime vector, which theoretically improves upon the exponential computational complexity associated with a state-of-the-art approach. Second, we presented a simple (also polynomial-time) algorithm to calculate the flow routing schedule among the AFNs such that the LMM-optimal node lifetime vector can be achieved. The results in this paper lay the essential groundwork on studying network lifetime problems in energy-constrained wireless sensor networks.

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